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FUZZY VOLATILITY MODELS WITH APPLICATION TO THE RUSSIAN STOCK MARKET

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Abstract. Volatility modeling and forecasting is a topical problem both in scientific circles and in the practice. This paper develops an approach combining the GARCH model and fuzzy logic. The Takagi–Sugeno fuzzy inference scheme is adopted to fuzzify an original autoregression model (the conditional heteroskedasticity model). As a result, several different local GARCH models can be used in different input data domains with soft switching between them. This approach allows considering such phenomena as volatility clustering and asymmetric volatility (the properties of real financial markets). The proposed algorithm is applied to the historical values of the RTS Index and compared with the classical GARCH model. As demonstrated below, in several cases, fuzzy models have advantages over traditional ones, namely, higher forecasting accuracy. Thus, the proposed method should be considered among others when modeling the volatility of the Russian financial market instruments: it demonstrates qualities superior to the conventional counterparts.

Keywords: fuzzy systems, forecasting, time series, volatility.

INTRODUCTION

The translation of several well-known econometric models from the probabilistic to fuzzy formulation has recently attracted increasing attention; for example, see the papers [1–9]. This also applies to volatility modeling, an important problem from both scientific and applied points of view. Many models imply a single dependence of output values on input data, but the factual dependence may vary in different domains of the input variables. Introducing fuzziness into the system eliminates this drawback by using several local models and aggregating them through fuzzy membership functions.

Volatility modeling somewhat differs from other problems of financial econometrics: unlike, e.g., the prices of financial instruments or interest rates, volatility is unobservable. Moreover, there is no consensus on what to call volatility. However, the capability to calculate the current volatility of an instrument or portfolio and forecast its future value correctly is crucial for financial institutions: it underlies market risk assessment. In turn, a sufficiently accurate assessment of market risks improves the stability of a financial institution and allows avoiding fatal losses in turbulent markets.

One of the most well-known and widespread methods for measuring volatility is the implied volatility within the Black–Scholes model [10, 11]. It is calculated from the market prices of European options (financial derivatives giving the right to buy or sell an underlying asset, e.g., a stock, at a definite price on a definite date). This approach has several disadvantages. First of all, a liquid options market is needed; otherwise, the option price with a high probability can be unfair, and the entire procedure will make no sense. In addition, the Black–Scholes model contains several strong assumptions, which are often not satisfied in reality. For example, we mention the assumption of a time-constant risk-free interest rate to borrow and lend money, or the assumption of a time-constant volatility of the underlying asset price. Due to the latter assumption, implied volatility may vary for different option exercise prices. For stocks, this effect is known as the volatility smile or volatility smirk. Thus, the method is somewhat contradictory and, in addition, it requires rather strict preconditions for application.

Another well-known approach is econometric modeling based on historical volatility values. As a rule, volatility here is understood as the rate of return of an asset. In this direction, we note GARCH (*generalized autoregressive conditional heteroskedasticity*) [12, 13], a generally recognized model pioneered in [14] and based on the ARCH model. However, the classical GARCH model neglects an effect demonstrated by modern financial markets. It consists in the skewed distribution of asset returns in the markets: negative external shocks cause a sharper fall and higher volatility, whereas positive external shocks cause less sharp growth and less high volatility. Different researchers proposed quite a wide range of modifications for the classical GARCH model to take asymmetric volatility into account: NAGARCH (nonlinear asymmetric GARCH) [15], EGARCH (exponential GARCH) [13], QGARCH (quadratic GARCH) [16], GJR-GARCH (Glosten, Jagannathan, and Runkle GARCH) [17], TGARCH (threshold GARCH) [18], VSGARCH (volatility-switching GARCH) [19], and others. However, these approaches disregard, e.g., the presence of four volatility clusters.

Due to some shortcomings of conventional methods, we propose a model incorporating elements of fuzzy logic. The fuzzy system used in this paper, the Takagi–Sugeno fuzzy inference scheme, stems from [20]. The authors [21] introduced the fuzzy model parameters estimation procedure via the least squares method. These two papers conditioned the wide application of such models in various fields and the fuzzy formulation added to classical econometric models. Fuzzy systems were described, e.g., in [22, 23].

Fuzzy models are adopted to forecast stock index volatility as well. According to the works on this subject, there is a wide range of algorithms and input data. For example, the study [1] presented an asymmetric fuzzy GARCH model with a fuzzy inference scheme to determine the switching threshold. The paper demonstrated the effectiveness of the proposed method on the returns of several stock indices: NASDAQ (the USA), Nikkei 225 (Japan), the Taiwan Weighted Index, and the Hang Seng Index (Hong Kong). The original idea of [1] was further developed in [2]. The same asymmetric fuzzy GARCH model was used (as before, to determine the switching threshold), but fuzziness was also introduced into the characteristic function. (In the modified model, it can take any value in the interval [0, 1].) Three variants were proposed to fuzzify the characteristic function. The authors compared the effectiveness of the presented methods with the GJR-GARCH model and the model [1]. The MOEX Russia Index (formerly the MICEX Index) and the RTS Index were studied as time series.

The authors [3] proposed an *adaptive fuzzy inference* system (AdaFIS), which dynamically determines the required number of fuzzy rules and their parameters. The model was applied to the Bovespa Index (Brazil's main stock index), the BRL/USD exchange rate, and the Petrobras preferred stock prices. The paper [4] described *evolving participatory learning* (ePL), a dynamic estimation method for model parameters. The researchers tested their method on historical values of the S&P 500 and Bovespa stock indices. Note that this method is an extension of the *evolving Takagi–Sugeno* (eTS) model [24]. The authors [5] continued studying the evolving fuzzy GARCH model in the paper [4], but they applied another fuzzy clustering method (the eClustering algorithm). The values of the S&P 500 and Bovespa indices were taken as the real series as well. The fuzzy GARCH model was also presented in [6]. In the cited work, asymmetry was considered by using known returns as explanatory variables: since the return (not its square) is employed in fuzzy clustering, the sign is taken into account. The authors applied the model to the Dow Jones Industrial Average Index.

In this paper, we propose a model combining the standard GARCH model and fuzzy logic. The model can be briefly described as follows. The input data are divided into several fuzzy clusters, and a different local GARCH model is applied within each cluster. The outputs of each local model are then aggregated into one via a pre-selected membership function. We conducted the empirical part of this research on historical values of the RTS Index, one of the main stock indices of the Russian market. Two models (benchmarks) are used to compare the forecasting properties of the proposed model: GARCH without recalculation and GARCH with recalculation. (The latter model will be defined below.) According to the calculation results, there exist fuzzy GARCH models with a higher forecasting accuracy than their classical counterparts.

This paper is organized as follows. In Section 1, we describe theoretically the proposed model and the approach to input data clustering. Section 2 presents the initial data, the problem statement, and the results of the empirical study. The Conclusions section summarizes the outcomes and outlines possible lines for further research.

1. METHODOLOGY

1.1. The Fuzzy GARCH Model

The classical GARCH model was described, e.g., in [13]. The proposed fuzzy model is based on the GARCH model but also includes soft switching between fuzzy rules. Each rule corresponds to a fuzzy cluster in the input data space. With C clusters, the fuzzy GARCH model can be represented as a set of C fuzzy IF-THEN rules:

$$\text{IF } x_t \in F_k \text{ THEN}$$
$$h_t^{(k)} = \alpha_{k0} + \sum_{i=1}^{q} \alpha_{ki} y_{t-i}^2 + \sum_{j=1}^{p} \beta_{kj} h_{t-j},$$
$$\alpha_{k0} > 0, \qquad (1)$$
$$\alpha_{ki} > 0 \forall k, i,$$
$$\beta_{kj} \ge 0 \forall k, j.$$

The notations are as follows: k is the cluster number ($k = 1,..., C$); F_k denotes the kth fuzzy cluster; $x_t = (x_t^1,..., x_t^n)'$ is the variable vector to determine the membership function at a time instant t. In addition, y_t gives the time series under consideration; $h_t^{(k)}$ is the conditional variance corresponding to the kth fuzzy rule at a time instant t, and h_t is the conditional variance at a time instant t (see below); α_{k0}, α_{ki}, and β_{kj} are the estimated model parameters. From this point onwards, the symbol ' means transposition. Generally speaking, the vector x_t may have an arbitrary dimension n depending on t. The parameters α_{ki} and β_{kj} will be called the consequent parameters.

The expression $x_t \in F_k$ is understood in a fuzzy sense (the degree of membership of the vector x_t to a cluster F_k, a real number from the interval [0, 1]). The degree of membership to a certain cluster may have different functional forms. In this paper, we use the Gaussian membership function, which analytically coincides with the density of the multivariate Gaussian distribution:

$$\mu_k(x_t) = \frac{1}{(2\pi)^{n/2} (\det(\Sigma_k))^{1/2}} e^{-\frac{1}{2}(x_t - c_k)' \sum_k^{-1} (x_t - c_k)}.$$

Here, $\mu_k : \mathbb{R}^n \to \mathbb{R}$ is the membership function of a vector to the kth cluster, whereas $c_k \in \mathbb{R}^n$ and $\Sigma_k^* \in \mathbb{R}^n$ are the center and covariance matrix of the kth cluster. This paper considers diagonal positive definite covariance matrices. The matrix Σ_k and the vector c_k, the parameters of the kth cluster, completely define it. For all k together, they will be called the antecedent parameters.

The degrees of membership are normalized so that at each point, their sum over all clusters is 1:

$$\mu_k^*(x_t) = \frac{\mu_k(x_t)}{\sum_{k=1}^{C} \mu_k(x_t)}.$$

All variances $h_t^{(k)}, k = 1,..., C$, are aggregated into a single value h_t using the membership functions:

$$h_t = \sum_{k=1}^{C} \mu_k^*(x_t) h_t^{(k)},$$

where $h_t^{(k)}$ are given by (1).

The value y_t^2 is forecasted. In this paper, the series y_t consists of the returns on some financial instrument. The forecast has the form

$$\hat{y}_t^2 = h_t.$$

Let T be the number of elements in an aggregate sample (including training and test samples). The consequent parameters are estimated using the least squares method: we choose the parameters α_{ki} and β_{kj} by minimizing the sum

$$\sum_{t=1}^{T} (y_t^2 - \hat{y}_t^2)^2,$$

where y_t are known values from the sample.

Hereinafter, y_t describes the logarithmic return on some asset or index, expressed as a percentage. For a given series z_t of some values (the prices of a financial instrument or index values),

$$y_t = \ln \frac{z_t}{z_{t-1}} \cdot 100. \qquad (2)$$

In all calculations below, $C = 4$.

1.2. Clustering and Antecedent Parameter Estimation

In this study, we cluster the entire initial series of known returns y_t at time instants t, i.e., $x_t = (y_1,..., y_t)'$ $\forall t$. Within the proposed approach, the dimension n of the vector x_t is a function of time: $n = n(t) = t$. Thus, we build a family of fuzzy systems: at each time instant, when a new value of the return becomes known, the entire series is clustered again with this new value, generating a new fuzzy system.

The input data are divided into two parts: training and test samples.

There exist different approaches to data clustering, which essentially means estimating the antecedent parameters. In this paper, the grid search method is used. A detailed description of the grid is provided in subsection 2.1.

Let T_{train} and T_{test} denote the sizes of the training and test samples, respectively.

On the same training sample used for the fuzzy model, we construct the classical GARCH(p, q) model with the same values of the parameters p and q as in the fuzzy model. The forecasting accuracy of the classical model is estimated using two approaches as follows. The first approach is producing the usual model forecast for T_{test} days ahead. The second approach consists in reestimating the GARCH model parameters daily for T_{test} days and obtaining the forecast for the next day only. (For convenience, the latter approach will be called GARCH with recalculation). Note that the recalculation procedure allows reducing the forecasting error; see the empirical results below (Tables 1 and 2 in Section 2). For each approach mentioned, we find the mean square errors: $MSE_{n/r}$ for GARCH without recalculation and $MSE_{w/r}$ for GARCH with recalculation. The two resulting errors are used as benchmarks for the fuzzy model. For comparing the fuzzy model with the classical one, we introduce $ratio_{w/r} = \frac{MSE_{w/r}}{MSE_{fuzzy}}$ and $ratio_{n/r} = \frac{MSE_{n/r}}{MSE_{fuzzy}}$. If $ratio_i > 1$, the error of the fuzzy method is smaller than that of the classical ith method. (Here, i = "w/r" or i = "n/r"). In this case, the higher value $ratio_t$ takes, the "better" the fuzzy model will be compared to the ith classical model.

To define the fuzzy model, we use the set $\Theta = (\Theta, ..., \Theta_C) \in \mathbb{R}^{n \times n \times C}$ of possible values of the antecedent parameters. Each $\Theta_k = \{c_k, \Sigma_k \mid c_k \in \mathbb{R}^n, \Sigma_k \in \mathbb{R}^n\}$ represents all possible combinations of the values c_k and Σ_k. The procedure of estimating the antecedent parameters begins with constructing a particular fuzzy model for each element Θ and calculating the value MSE_{fuzzy}. To find the best antecedent parameters c_k and Σ_k, we maximize $ratio_{w/r} = ratio_{w/r}(c_1, ..., c_C, \Sigma_1, ..., \Sigma_C)$.

Thus, these steps yield the best set of antecedent parameters, i.e., the best fuzzy model compared to the classical GARCH model.

2. EMPIRICAL STUDY

2.1. The Set of Antecedent Parameters

Let p = 1 and q = 1. Assume that the number of clusters is fixed: C = 4. The center of each cluster $c_k \in \mathbb{R}^n$ is a vector with the same components c_k^*:

$$c_k = \begin{pmatrix} c_k^* \\ \dots \\ c_k^* \end{pmatrix}.$$

The covariance matrix is a diagonal real-valued matrix of dimensions $n \times n$, with the same element $\Sigma_k^* > 0$ on the main diagonal:

$$\Sigma_k = \begin{pmatrix} \Sigma_k^* & 0 & \dots & 0 \\ 0 & \Sigma_k^* & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \Sigma_k^* \end{pmatrix}.$$

This element can be treated as the variance of the kth cluster.

Let $c^* = (c_1^*, c_2^*, c_3^*, c_4^*)$ and $\Sigma^*(\Sigma_1^*, \Sigma_2^*, \Sigma_3^*, \Sigma_4^*)$. These four-dimensional vectors completely define all four clusters. We employ them to parameterize the admissible vector space instead of the vector Θ.

In this paper, the set of centers is not varied but set expertly: in all calculations, c^* = (–7.5; –1.5; 1.5; 4). Intuitively, the centers c^* can be interpreted as follows: center 1.5 corresponds to small positive returns whereas center 4 to large returns; centers –1.5 and –7.5 correspond to small and large negative returns, respectively (in absolute terms). The fact that |–7.5| > |4| reflects a characteristic feature of capital markets: the growth under positive external shocks is smoother than the drop under external negative shocks.

The variances are estimated by the grid search method. The grid was constructed with the following ranges: for Σ_1^*, from 4 to 12; for Σ_2^* and Σ_3^*, from 1 to 6; for Σ_4^*, from 2 to 10. The grid step was set equal to 1 for all Σ_k^*. Thus, 2916 grid nodes were considered in total.

2.2. Results

The daily logarithmic returns of the RTS Index were used as the initial series. Figure 1 shows the closing index data for a long historical period.

The beginning of the training sample coincides with the first trading day of 2014 (January 6, 2014). Figure 2 presents the values of the RTS Index for the historical period used in the calculations (the series z_t), about 3 years from the beginning of the training sample. The corresponding logarithmic returns (the series y_t (2)) are demonstrated in Fig. 3.

Fig. 1. Daily values of the RTS Index since 2001.

Fig. 2. Daily values of the RTS Index: January 2014–December 2016.

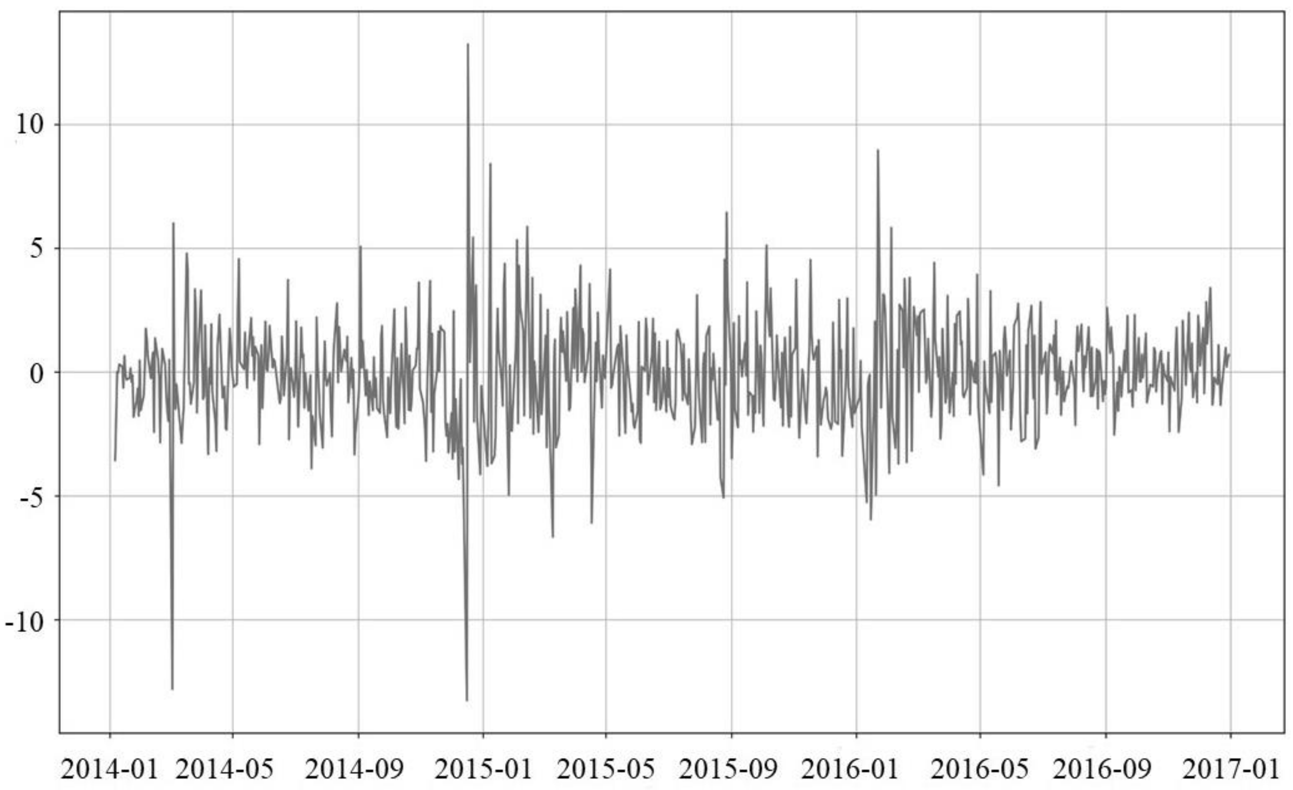

Fig. 3. Daily logarithmic returns of the RTS Index: January 2014–December 2016.

The antecedent parameters (i.e., the variances under fixed centers) were estimated using 100 elements in the training sample and 10 elements in the test sample. The best set of the variances was (7, 6, 3, 5). The model with these parameters was then applied to the samples of other sizes; see the corresponding results in Table 2. The sample sizes were selected from the following considerations: 252 is the approximate number of trading days in one year (504 in two years, 21 in one month, 42 in two months, and 63 in three months).

The results of the empirical study are tabulated below. Table 1 combines the characteristics of the systems with the best results compared to the classical model. Note that $ratio_{w/r} > 1$ in all these models. The first row of this table contains information about the best fuzzy model. Next, Table 2 shows the results of applying the best system from Table 1 to the samples of other sizes. According to the calculations, these systems have a higher error than the classical model ($ratio_{w/r} < 1$).

Good results were achieved for 100 elements in the training sample and 10 elements in the test sample. The quality of the fuzzy model gradually declines when increasing the size of either sample. This phenomenon can be explained as follows: the antecedent parameters need to be reestimated on other sample sizes. (This was not done due to high computational costs.) Moreover, adding variability to the cluster centers may contribute to finding a model with higher accuracy.

Thus, for sufficiently short time series, there exist fuzzy GARCH systems superior to the classical GARCH model in their forecasting properties. With increasing the time series length, the classical GARCH model becomes a “stronger” competitor. Most probably, the reason lies in the advantages of the maximum likelihood method, which show up with increasing the time series length. However, it seems that fuzzy GARCH systems superior to the classical GARCH model can be obtained even for longer time series using broader classes of membership functions.

Table 1

The best models on the training and testing samples of 100 and 10 elements, respectively

T_{train}	T_{test}	Σ^*	MSE_{fuzzy}	$MSE_{n/r}$	$MSE_{w/r}$	$ratio_{n/r}$	$ratio_{w/r}$
100	10	[7, 6, 3, 5]	9.66	17.89	12.75	1.85	1.32
100	10	[6, 4, 4, 6]	9.72	17.89	12.75	1.84	1.31
100	10	[8, 6, 4, 6]	10.18	17.89	12.75	1.76	1.25
100	10	[8, 1, 4, 6]	11.31	17.89	12.75	1.58	1.13

Table 2

The best model among the ones applied to the samples of other sizes

T_{train}	T_{test}	Σ^*	MSE_{fuzzy}	$MSE_{n/r}$	$MSE_{w/r}$	$ratio_{n/r}$	$ratio_{w/r}$
252	21	[7, 6, 3, 5]	353.14	273.58	189.09	0.77	0.51
252	42	[7, 6, 3, 5]	251.84	201.34	137.60	0.80	0.55
252	63	[7, 6, 3, 5]	190.30	141.74	99.21	0.74	0.52
504	21	[7, 6, 3, 5]	542.75	382.64	300.49	0.70	0.55
504	42	[7, 6, 3, 5]	290.84	203.56	160.35	0.70	0.55
504	63	[7, 6, 3, 5]	202.47	143.12	112.76	0.70	0.56

CONCLUSIONS

In this paper, a fuzzy GARCH system has been applied to model volatility. The classical GARCH models with and without recalculation were considered as alternatives to the proposed model (benchmarks). The effectiveness of the fuzzy model has been tested on the main index of the Russian stock market (the RTS Index). As shown above, there exist fuzzy systems producing more accurate forecasts compared to traditional models.

Nevertheless, the forecasting properties of the fuzzy system obviously tend to deteriorate (compared to the benchmark models) with increasing the sample size. This can be due to the parameter estimation methods used in the fuzzy model and in the classical GARCH model.

A possible line for further research is practical methods for finding such fuzzy systems. Of particular interest are other forms of membership functions as well as more universal methods for estimating the antecedent parameters.

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