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ON A DECOMPOSITION METHOD FOR DESIGNING COMMUNICATION NETWORKS[1]

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Abstract. This paper presents a communication network design algorithm for finding a guaranteed transportation plan of a given volume under uncertain factors. The volumes of production and the capacities of communication lines are expressed as linear functions of invested resources. The well-known Dantzig–Wolfe decomposition algorithm is applied to solve the dual problem due to its stepped block structure. In view of their specifics, the linear problems arising in iterations are solved using effective network and graph theory methods: the maximum flow, the minimum cutset in the network, the connectivity components, and the minimum spanning trees of the graphs are found. The existing algorithms for these problems have the complexity estimates $O(mn^2)$, $O(n^2m)$, and $O(n + m)$, where n is the number of graph vertices and m is the number of edges.

Keywords: supply and demand problem, communication networks, linear design, decomposition methods, maximum flow, minimum cutset, minimum spanning tree.

INTRODUCTION

Network structure problems arise in the design of transport or other real networks as well as many other spheres of human activities. The general statement of such problems was given, e.g., in the books [1, 2].

This paper considers the linear design problem for the Gale supply and demand model [3] under finitely many uncertain factors, a linear programming problem of high dimension. The pronounced peculiarity of this problem—the stepped block structure of the constraint matrix—suggests finding special algorithms to solve it, different from those commonly used in linear programming problems. As is known, special algorithms with higher efficiency and the capability to increase the dimension were developed for many analysis problems of communication networks as linear programming problems (transportation, the shortest path, the maximum flow, the minimum cost flow, and others). One example is the method of potentials for the standard transport problem and its various modifications. This method concretizes the simplex method for a special type of linear programming problems [4]. The modified method of potentials for the problem with capacity constraints can be also found in the literature [5]. Some researchers described an algorithm of the method of potentials for the multi-index transport problem [6, 7] and the transportation problem (the problem with an arbitrary-structure network [8]). All these methods refer to the analysis problems of communication networks. The first coauthor developed an algorithm of generalized potentials for the linear design problems of communication networks [9] and a modification of this method for the linear design problem of communication networks under uncertain factors [10]. The latter problem is studied below.

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cation lines are expressed as linear functions of invested resources. The well-known Dantzig–Wolfe decomposition algorithm is applied to solve the dual problem due to its stepped block structure. In view of their specifics, the linear problems arising in iterations are solved using effective network and graph theory methods: the maximum flow, the minimum cutset in the network, the connectivity components, and the minimum spanning trees of the graphs are found. The existing algorithms for these problems have the complexity estimates $O\,(mn^2)$, $O\,(n^2m)$, and $O\,(n+m)$, where n is the number of graph vertices and m is the number of edges. Some examples of such algorithms were presented in [11–20]. This result significantly excels the exponential complexity of general linear programming methods and the polynomial complexity [15] of special methods, e.g., $O(mn^4+m^2n^3+m^3n^2)$, where m is the number of problem constraints and n is the number of variables [14]. Therefore, the proposed method is more efficient.

1. PROBLEM STATEMENT

We formulate the problem in mathematical terms. Consider the following components of the problem:

– a direct graph (G, Γ) with a set of arcs Γ and a set of vertices G;

– a unique product that can be manufactured in the vertices of a set A and then consumed in the vertices of a set C;

– a set B of intermediate vertices in which the product is neither manufactured nor consumed;

– a homogeneous separable resource, which is allocated over the sets A (vertices) and Γ (arcs).

Assume that the volumes of production in the vertices of the set A and the capacities of the arcs of the set Γ have a known dependence on the invested resources. The problem is to allocate the limited resource among the production points and communication lines of the network so that: (a) for any value of the uncertain factors, the network allows a flow satisfying the demand and (b) the cost of this resource allocation achieves minimum. Note that the resource is homogeneous and separable, i.e., arbitrarily divisible.

The problem with n production points and limited product supply can be reduced to the problem with one production point and unlimited product supply [2]. For this purpose, one vertex (number 0) is added to the graph and is connected to each vertex i of the set A by an arc with a capacity $\varphi_i\,(x_i,\,k)$. The extended graph will also be denoted by (G, Γ). Assume that the capacity functions are linear: $\varphi_i(x_i,k) = b_i^k + a_i^k x_i$, where b_i^k and a_i^k indicate the coefficients of the corresponding linear dependences.

We introduce the following notations: d_i is the product demand at point i; y_j^k is the flow along arc j under uncertain factor k; x_j is the resource invested in arc j; $D(i)$ is the numbers of the incoming arcs for vertex i; finally, $C(i)$ is the numbers of the outgoing arcs for vertex i.

The mathematical problem statement is as follows:

$$
\begin{gathered}
\min_{x,y}\Big(\sum_{j\in\Gamma} x_j\Big),\\
\sum_{j\in C(i)} y_j^k - \sum_{j\in D(i)} y_j^k = 0,\ i\in A\cup B,\\
\sum_{j\in D(i)} y_j^k - \sum_{j\in C(i)} y_j^k \ge d_i,\ i\in C, \qquad (1)\\
y_j^k - a_j^k x_j \le b_j^k,\ j\in\Gamma,\\
x_j\ge 0,\ y_j^k\ge 0,\quad j\in\Gamma,\ k=1,\ldots,l.
\end{gathered}
$$

According to the well-known rules of the duality theory of linear programming, the dual problem for (1) has the form

$$
\begin{gathered}
\max_{\lambda,\mu}\left(\sum_{i\in C}\Big(\sum_{k=1}^{l}\lambda_i^k\Big)d_i - \sum_{j\in\Gamma}\sum_{k=1}^{l}\mu_j^k b_j^k\right),\\
1-\sum_{k=1}^{l}\mu_j^k a_j^k \ge 0,\ j\in\Gamma\setminus C(0),\\
-\lambda_{n_2(j)}^k + \mu_j^k \ge 0,\ j\in C(0), \qquad (2)\\
\lambda_{n_1(j)}^k - \lambda_{n_2(j)}^k + \mu_j^k \ge 0,\ j\in\Gamma,\\
\mu_j^k\ge 0,\ j\in\Gamma,\ \lambda_i^k\ge 0,\ i\in C,\ k=1,\ldots,l,
\end{gathered}
$$

where the vectors λ and μ are the variables of the dual problem and $n_1(j)$ and $n_2(j)$ are the start and end vertices of arc j, respectively.

Consider the additional sets of variables $z, z^1,\ldots, z^l$ to transform the problem constraints into equivalent equalities. Figure 1 shows the structure of the coefficient matrix for the constraints of problem (2), where E denotes an identity matrix, IN is the incidence matrix of the network graph, and

$$
A^i = \begin{pmatrix} a_1^i & \ldots & 0 & 0\\ 0 & \ldots & 0 & 0\\ \ldots & \ldots & \ldots & \ldots\\ 0 & \ldots & 0 & a_n^i \end{pmatrix}.
$$

λ^1	μ^1	z^1	λ^2	μ^2	z^2	...	z		b
0	A^1	0	0	A^2	0	...	E	x	1 ... 1
IN^{T}	E	$-E$	0	0	0	...	0	y^1	0 ... 0
...	...	...	...	...	...	...	...	...	...
0	0	0	0	IN^{T}	E	$-E$	0	y^l	0 ... 0

Fig. 1. The block structure of the coefficient matrix.

According to Fig. 1, the coefficient matrix of problem (2) has a stepped block structure with some set of connecting sinks. There exist effective decomposition methods for such problems, e.g., the Dantzig–Wolfe decomposition method [12]. Consider its modification with an unbounded set given by block constraints. The idea of this method is to decompose a high-dimensional problem into several subproblems of lower dimensions and solve them sequentially at each step of an iterative algorithm. The following optimization problem arises at each step of the decomposition method:

$$\max_{\lambda,\mu}\left(\sum_{i\in C} d_i\lambda_i^k-\sum_{j\in\Gamma}\mu_j^k\tilde{b}_j^k\right),$$

$$-\lambda_{n_2(j)}^k+\mu_j^k\ge 0,\ j\in C(0), \tag{3}$$

$$\lambda_{n_1(j)}^k-\lambda_{n_2(j)}^k+\mu_j^k\ge 0,\ j\in\Gamma\setminus C(0),$$

$$\mu_j^k\ge 0,\ j\in\Gamma,\ \lambda_i^k\ge 0,\ i\in C,$$

where $\tilde{b}_j^k$ are variable coefficients. According to the Dantzig–Wolfe method, it is necessary to find the optimal basis solution (λ, μ) (if the problem is solvable) or the basis ray (λ,μ) on which the objective function is unbounded (otherwise).

2. THE SOLUTION ALGORITHM

The analysis begins with the constraint matrix of problem (3), namely, its rank. The following result is true.

Proposition 1. *Let A be the coefficient matrix of problem* (3). *Then,* rank $A = n + m - 1$*, where n is the number of vertices and m is the number of arcs in the graph* (G, Γ).

P r o o f.

Let (G, Γ_0) be some connecting subtree of the graph (G, Γ), which obviously exists due to the connectivity of this graph. Then, as is known, $|\Gamma_0| = n-1$. Consider a system of $(n + m - 1)$ inequality constraints:

$$-\lambda_{n_2(j)}^k+\mu_j^k\ge 0,\ j\in C(0),$$

$$\lambda_{n_1(j)}^k-\lambda_{n_2(j)}^k+\mu_j^k\ge 0,\ j\in\Gamma_0\setminus C(0),$$

$$\mu_j^k\ge 0,\quad j\in\Gamma .$$

The coefficient matrix of this system has the block form

$$\tilde{A}=\begin{pmatrix} IN & -\tilde{E}\\ 0 & E\end{pmatrix},$$

where the matrices IN and E are the same as above and $\tilde{E}$ is some partial-identity matrix.

The matrix $\tilde{A}$ has dimension $(m + n - 1)$, with $(n - 1)$ rows in the first group and m rows in the second one. By the well-known theorems of graph theory [1], the rows of the incidence matrix of a direct graph are linearly independent if and only if the corresponding arcs do not form a circuit. The connecting subtree contains no circuits; therefore, the first group of rows in the matrix $\tilde{A}$ is linearly independent. Due to the zero block in the second group of rows, the whole system of rows in the matrix $\tilde{A}$ is linearly independent and rank $A \ge n + m - 1$. Since the matrix A contains $(m + n - 1)$ columns, rank $A \le n + m - 1$. Considered jointly, these inequalities imply rank $A = n + m - 1$. The proof of Proposition 1 is complete. ♦

Consider two alternatives for the coefficients $\tilde{b}_{j_0}^k$ of the objective function as follows.

In the first alternative, these coefficients contain a negative one, i.e., $\exists\ j_0 : \tilde{b}_{j_0}^k < 0$. Then problem (3) has no solution due to the unboundedness of the objective function. To prove this result, we construct a vector (λ, μ) of the form

$$\lambda_i^k=0,\ i\in G\cup C(0),\ \mu_i^k=0,\ j\ne j_0,\ \mu_{j_0}^k=1.$$

Obviously, the vectors $p(\lambda, \mu)$, $p > 0$, are admissible in problem (3) and the value of the objective function tends to $+\infty$ as $p\to+\infty$. Within the decomposition method, it is then necessary to design a basis ray on which the objective function tends to $+\infty$. According to the linear programming theory, a ray is a basis one if and only if it turns into equalities $(n + m - 2)$ linearly independent constraints of problem (3).

There are only two cases concerning the structure of the network graph. In the first case, the graph (G, Γ) preserves connectivity after removing arc j_0. The algorithm to check graph connectivity has the complexity estimate $O(n + m)$, where n is the number of graph vertices and m denotes the number of edges [17]. In this case, it is possible to construct a minimum spanning subtree (G, Γ_0) of the graph (G, Γ) such that $j_0 \notin \Gamma_0$. The ray (λ, μ) designed above is a basis one since it turns into equality $(n + m - 2)$ linearly independent constraints:

$$\begin{aligned} -\lambda^k_{n_2(j)} + \mu^k_j = 0,\ j \in \Gamma_0 \cap C(0), \\ \lambda^k_{n_1(j)} - \lambda^k_{n_2(j)} + \mu^k_j = 0,\ j \in \Gamma_0 \setminus C(0), \\ \mu^k_j = 0,\ j \in \Gamma,\ j \neq j_0. \end{aligned} \tag{4}$$

In the second case, the graph loses the connectivity property after removing arc j_0. Obviously, the only possible outcome here is the formation of two connected components (G_1, Γ_1) and (G_2, Γ_2). Consider the following ray (λ, μ):

$$\lambda^k_i = 0,\ i \in G_1,\ \lambda^k_i = 1,\ i \in G_2, \quad \mu^k_i = 0,\ j \neq j_0,\ \mu^k_{j_0} = 1.$$

As is easily checked, all the vectors $p(\lambda, \mu)$, $p > 0$, satisfy the problem constraints and the value of the objective function is unbounded (from above) when increasing the parameter p. The constructed ray is a basis one. To show this fact, consider a connecting subtree (G, Γ_0) of the graph (G, Γ). In this case, system (4) is the linearly independent group of equality constraints corresponding to this ray. Thus, the case of a negative component $j_0 : \tilde{b}^k_{j_0} < 0$ has been fully investigated.

In the second alternative, all coefficients of the arcs are nonnegative, i.e., $\forall\ j \in \Gamma\ \ \tilde{b}^k_{j_0} \geq 0$. We introduce an additional vertex of the graph (G, Γ) (vertex η) and a group of arcs between this vertex and those of the set C with the capacities d_i. The coefficients $\tilde{b}^k_j$ will be interpreted as the capacities of the corresponding arcs. The resulting network is a communication network with source 0, sink η, and known capacities of all its communication lines (Fig. 2).

According to the rules of the duality theory, the dual problem for (3) has the form

$$\begin{aligned} &\min_{x,y} 0, \\ &\sum_{j \in C(i)} y^k_j - \sum_{j \in D(i)} y^k_j = 0,\ i \in A \cup B, \\ &\sum_{j \in D(i)} y^k_j - \sum_{j \in C(i)} y^k_j \geq d_i,\ i \in C, \\ &y^k_j \leq \tilde{b}^k_j,\ j \in \Gamma,\ y^k_j \geq 0, j \in \Gamma. \end{aligned} \tag{5}$$

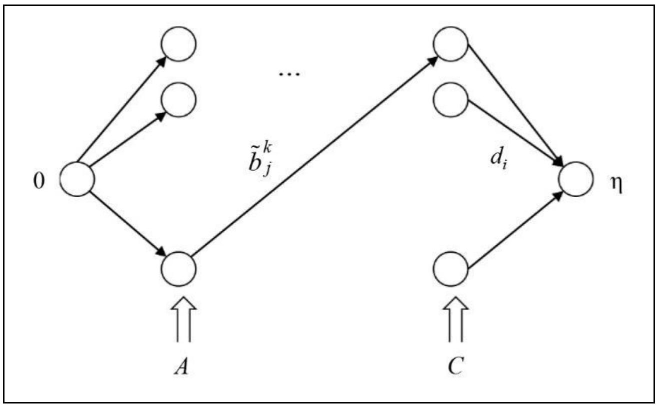

Fig. 2. The general structure of the formed graph.

As is well known, problem (3) is solvable if and only if problem (5) has a non-empty set of admissible solutions. Therefore, it is of interest to establish a non-emptiness criterion for this set. We formulate it as follows.

Proposition 2. *The set of admissible solutions in problem (5) is non-empty if and only if the two-pole network designed above has the maximum flow*

$$\sum_{i \in C} d_i.$$

P r o o f.

Sufficiency. Assume that the maximum flow in this network is given by

$$\sum_{i \in C} d_i.$$

The additional arcs introduced above form a cutset of the network. Hence, the maximum flow is achieved when the values of flows on each of them equal their capacities. Therefore, the balance equations

$$\sum_{j \in D(i)} y^k_j - \sum_{j \in C(i)} y^k_j = d_i$$

hold for all $i \in C$.

The flow constraints for the remaining vertices $i \in A \cup B$ are also implemented in the form of strict equations.

Necessity. Assume that the set of admissible solutions in problem (5) is non-empty. In this case, we can form a flow as an admissible solution of problem (5) that satisfies the system of equalities

$$\sum_{j \in D(i)} y^k_j - \sum_{j \in C(i)} y^k_j = d_i,\ i \in C. \tag{6}$$

Now, suppose that there exists a vertex i for which the flow balance constraint is a strict inequality:

$$\sum_{j \in D(i)} y^k_j - \sum_{j \in C(i)} y^k_j = d_i + p, \quad p > 0.$$

In this case, there exists a path between the network vertices 0 and i without zero-flow arcs. Let Δy denote the minimum flow along this path; this value is positive. Next, we introduce the value $Y = \min(\Delta y,\ p)$ and subtract it from the flows along the given path. Obviously, the arc flows changed in this way will remain nonnegative and will not violate the constraints of problem (5), including the balance equations of the intermediate vertices:

$$\sum_{j\in C(i)} y_j^k - \sum_{j\in D(i)} y_j^k = 0,\ i\in A\cup B.$$

If there still exist some network vertices violating any of equations (6), the described procedure should be repeated. An admissible flow of problem (5) will be generated in a finite number of iterations. This flow is given by

$$\sum_{i\in C} d_i$$

and satisfies all the relations (6).

By the well-known Ford–Fulkerson theorem [18], the admissible flows of problem (5) have the upper bound

$$\sum_{i\in C} d_i.$$

Therefore, the designed flow is maximum, and the proof of Proposition 2 is complete. ♦

The algorithm for finding the maximum flow in a two-pole network has the complexity estimate $O(mn^2)$ [16]. Due to Proposition 2, if the maximum flow is

$$\sum_{i\in C} d_i,$$

then the set of admissible solutions in problem (5) is non-empty and the minimum value of the objective function in problem (5) is 0. According to the duality theory of linear programming, the maximum value of the objective function in problem (3) is also 0, and the optimal basis solution of the problem is given by the vector (λ,μ) such that

$$\lambda_i^k = 0,\ i\in G,\quad \mu_i^k = 0,\ j\in\Gamma.$$

As has been noted, the maximum flow does not exceed the value

$$\sum_{i\in C} d_i.$$

Now, assume that the maximum flow is strictly smaller than

$$\sum_{i\in C} d_i.$$

We construct the minimum cutset of this flow. By the definition, a cutset is a partition of the set of vertices into two subsets $G_1 : 0\in G_1$ and $G_2 : \eta\in G_2$. Let B denote the arc base of the cutset:

$$B = \left\{ j\in\Gamma : n_1(j)\in G_1,\ n_2(j)\in G_2 \right\}.$$

The algorithm for finding the minimum cutset in a two-pole network has the complexity estimate $O(nm^2)$ [16]. We form the graph $(G_2\setminus\eta,\ \Gamma_2\setminus D(\eta))$. Possibly, it has several connectivity components, further denoted by $(G_2^1,\Gamma_2^1),\ldots,(G_2^p,\Gamma_2^p)$. The algorithms for finding connected components in a graph have the complexity estimates $O(n^2)$ or $O(m+n)$ [18]. Also, we form the set $C_0 = G_2\cap C$. Note that $C_0\neq\varnothing$: otherwise, the capacity of the cutest would be

$$\sum_{i\in C} d_i.$$

Consider the partition of the set C_0 into components $C_1,\ldots,C_p$, where $C_i = C_0\cap G_2^i$, and the sets $B_q = \left\{ j\in B : n_2(j)\in C_q \right\}$ (Fig. 3). The following result is true.

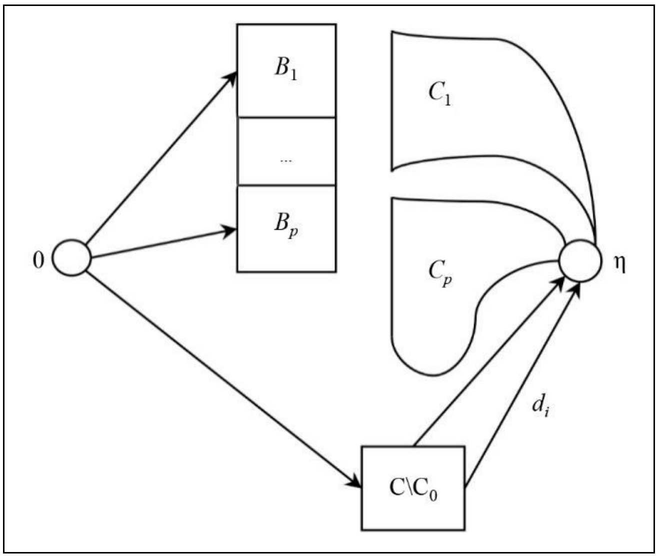

Fig. 3. The structure of the connectivity components of the graph.

Proposition 3. *There exists at least one partition component p_0 such that*

$$\sum_{i\in C_{p0}} d_i > \sum_{i\in B_{p0}} \tilde{b}_j^k.$$

P r o o f.

Assume on the contrary that

$$\sum_{i\in C_m} d_i \le \sum_{i\in B_m} \tilde{b}_j^k \quad \forall m = 1,\ldots,p.$$

Summing these inequalities over all m gives the integral inequality

$$\sum_{m=1}^{p}\sum_{i\in C_m} d_i \le \sum_{m=1}^{p}\sum_{i\in B_m} \tilde{b}_j^k.$$

Adding the same value

$$\sum_{i\in C\setminus C_0} d_i$$

to both parts of this inequality, we obtain the new relation

$$\sum_{i\in C} d_i \le \sum_{i\in B} \tilde{b}_j^k.$$

But it contradicts the previous considerations: B is the arc base of the minimum cutset, and the maximum flow in the network is strictly less than

$$\sum_{i\in C} d_i.$$

Hence, the original assumption is false, and the conclusion follows. ♦

Now, we can investigate the case where the maximum flow is strictly smaller than the value

$$\sum_{i\in C} d_i.$$

We form the vector (λ, μ) by the following rules:

$$\lambda_i^k = 1,\ i \in G_2^{p_0},\ \lambda_i^k = 0,\ i \in G \setminus G_2^{p_0},$$
$$\mu_i^k = 1,\ j \in B_{p_0},\ \mu_i^k = 0,\ j \in \Gamma \setminus B_{p_0}.$$

As is easily checked, all the vectors $p(\lambda, \mu)$, $p > 0$, satisfy the constraints of problem (3). Let us show that the objective function of the problem is unbounded from above when increasing the parameter p. At the points of the vector $p(\lambda, \mu)$, the objective function of problem (3) has the form

$$p\left(\sum_{i\in C_{p_0}} d_i - \sum_{j: n_2(j)\in G_{p_0}^2} \tilde{b}_j^k\right).$$

By Proposition 3, the bracketed expression is strictly positive; hence, the objective function is unbounded when increasing the parameter p.

Next, it is necessary to establish that the ray (λ, μ) is a basis one. We introduce the following notations for the dimensions of the sets under consideration: $|\Gamma_0| = r_0$, $|G_{p_0}^2| = n_0$, $|\Gamma_{p_0}^2| = m_0$. Let us form arbitrary spanning subtrees: $\left(G_{p_0}^2, \Gamma_{p_0}^*\right)$ in the graph $\left(G_{p_0}^2, \Gamma_{p_0}^2\right)$ and $(\tilde{G}, \tilde{\Gamma})$ in the graph $\left(G \setminus G_{p_0}^2, \Gamma \setminus \left(\Gamma_{p_0}^2 \cup B_{p_0}^2\right)\right)$. As has been mentioned above, the algorithms have the complexity estimates $O(mn^2)$ and $O(n^2 m)$, where n is the number of vertices and m is the number of graph edges. The following relations are obvious:

$$\begin{gathered}
-\lambda_{n_2(j)}^k + \mu_j^k = 0,\ j \in \tilde{\Gamma} \cap C(0),\\
\lambda_{n_1(j)}^k - \lambda_{n_2(j)}^k + \mu_j^k = 0,\ j \in \tilde{\Gamma} \setminus C(0),\\
\lambda_{n_1(j)}^k - \lambda_{n_2(j)}^k + \mu_j^k = 0,\ j \in \Gamma_{p_0}^*,\\
\lambda_{n_1(j)}^k - \lambda_{n_2(j)}^k + \mu_j^k = 0,\ j \in B_{p_0} \setminus C(0),\\
-\lambda_{n_2(j)}^k + \mu_j^k = 0,\ j \in B_{p_0} \cap C(0),\\
\mu_j^k = 0,\ j \in \Gamma \setminus B_{p_0}.
\end{gathered} \tag{7}$$

According to the well-known theorems of graph theory, $|\tilde{\Gamma}| = n - n_0 - 1$, $|\Gamma_{p_0}^*| = n_0 - 1$, and $|\Gamma_0| = r_0$; therefore, the total number of equalities in system (7) is $(n - n_0 - 1) + (n_0 - 1) + r_0 + (m - r_0) = n + m - 2$. This fact justifies the basis property of the ray (λ, μ).

The method can be described by the flowchart in Fig. 4.

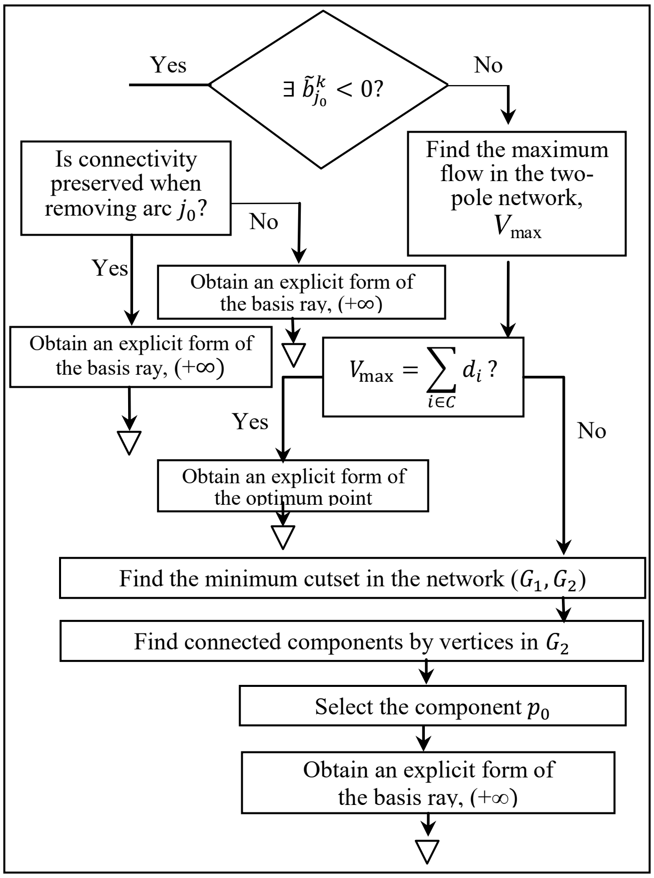

Fig. 4. The flowchart of the method.

CONCLUSIONS

This paper has proposed a communication network design algorithm for finding a guaranteed transportation plan of a given volume under uncertain factors.

The volumes of production and the capacities of communication lines are expressed as linear functions of invested resources.

This method involves algorithms for finding the maximum flow and minimum cutest and algorithms for selecting connectivity components and designing the minimum spanning tree; see the Introduction. Efficient computational schemes to implement such algorithms are well-known [13, 14]. Complexity estimates have been also presented for some of their implementations. Note that the algorithm solves the dual problem (5) instead of the original one (3). However, the transition to the primal problem (3) is carried out using the general duality theory of linear programming and, apparently, does not require separate consideration. The complexity estimates of the proposed algorithm substantially excel those of common linear programming methods (exponential complexity) and special methods (polynomial complexity). Therefore, it will be useful for solving high-dimensional problems of this class.

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