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PEAK-MINIMIZING DESIGN FOR LINEAR CONTROL SYSTEMS WITH EXOGENOUS DISTURBANCES AND STRUCTURED MATRIX UNCERTAINTIES[1]

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Abstract. A major characteristic of transients in linear dynamic systems with non-zero initial conditions is the maximum deviation of the trajectory from zero, which has a direct engineering meaning. If the maximum deviation is large, the so-called peak effect occurs. This paper completes a series of research works devoted to the peak effect in linear control systems. We consider a linear control system with non-random bounded exogenous disturbances and system uncertainties. A regular approach is proposed to design a stabilizing static state-feedback control law that minimizes the peak effect. The approach is based on the technique of linear matrix inequalities and reduces the original problem to a parameterized semidefinite programming one, which can be easily solved numerically. The proposed approach can be extended to new classes of problems, in particular, to the case of output feedback using an observer or a dynamic controller.

Keywords: linear control system, peak effect, structured matrix uncertainty, bounded exogenous disturbances, linear matrix inequalities, semidefinite programming.

INTRODUCTION

April 2023 was remarkable for the 75th anniversary of Feldbaum's famous paper [1], which pioneered studies of transients in linear systems with non-zero initial conditions. Among many characteristics of transients, a major one is the maximum deviation of the trajectory from zero, which has a direct engineering meaning. If this deviation is large, the so-called peak effect occurs.

To date, various approaches to estimating the deviations of the trajectories of dynamic systems have been proposed. In this context, let us mention Russian researchers A.P. Krishchenko and A.N. Kanatnikov [2–4], A.V. Ushakov and N.A. Vunder (Polinova) [5–7], I.B. Furtat [8], and P.S. Shcherbakov [9].

Although the peak effect is associated with linear systems, it also plays an important role in the nonlinear systems theory. Really, if the trajectories of a system linearized in a neighborhood of some point leave this neighborhood by undergoing a peak, it becomes difficult to give any guaranteed estimates for the behavior of the original nonlinear system.

This paper completes a series of research works on the peak effect in linear control systems. The series began with the publication [10], where a linear matrix inequalities-based approach was proposed to minimize the deviations of linear dynamic systems, and their upper bounds were derived. Further, in [11, 12], these results were extended to continuous- and discrete-time linear systems with structured matrix uncertainty; the paper [13] considered continuous-time systems subjected to non-random bounded exogenous disturbances. Within the developed approach, the technique of linear matrix inequalities (LMIs) [14–16] proved to be a very effective tool to design a peak-minimizing feedback control law. As was demonstrated by numer-

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ous examples, the degree of conservatism of the upper bounds is rather small.

Below, we study linear control systems in continuous (Section 1) and discrete (Section 2) time that are subjected to system uncertainties and non-random bounded exogenous disturbances. For each class of systems, we estimate the maximum deviation of trajectories and propose a regular approach to design a state-feedback control law minimizing this deviation. A numerical example in Section 3 illustrates the effectiveness of the developed approach.

The following notations are used throughout this paper: $|\bullet|$ stands for the Euclidean norm of a vector; $\|\bullet\|$ is the spectral norm of a matrix; the symbol $^{\mathrm{T}}$ indicates the transpose operation; I is an identity matrix of appropriate dimensions. All matrix inequalities are understood in the sense of positive or negative (semi)definiteness of the corresponding matrices. (In particular, the expression $A \preceq 0$, where $A \in \mathbb{R}^{n\times n}$, means that $x^{\mathrm{T}} A x \le 0$ for all $x \in \mathbb{R}^n$.)

1. THE CONTINUOUS CASE

Consider a continuous-time linear control system described by

$$\dot{x} = \big(A + F\Delta(t)H\big)x + Bu + Dw, \quad x(0) = x_0, \quad (1)$$

where $A \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times p}$, $H \in \mathbb{R}^{q\times n}$, $B \in \mathbb{R}^{n\times r}$, and $D \in \mathbb{R}^{n\times m}$ are given matrices, with the state vector $x(t) \in \mathbb{R}^n$, the control action $u(t) \in \mathbb{R}^r$, a bounded exogenous disturbance $w(t) \in \mathbb{R}^m$,

$$|w(t)| \le \gamma \quad \text{for all } t \ge 0, \quad (2)$$

and a matrix uncertainty

$$\|\Delta(t)\| \le \delta \quad \text{for all } t \ge 0. \quad (3)$$

The so-called "framing" matrices F and H define the uncertainty structure in the system matrix, whereas the values γ and δ determine the ranges of exogenous disturbances and system uncertainties.

The problem is to find a stabilizing static linear state-feedback control law

$$u = Kx \quad (4)$$

that minimizes the peak value

$$\xi = \max_{t\ge 0} \max_{|x_0|=1} \max_{\|\Delta(t)\|\le\delta} \max_{|w|\le\gamma} |x(t)|$$

in the closed loop system (1) under all admissible uncertainties $\Delta(t)$ and all admissible exogenous disturbances $w(t)$.

Recall the following result [16], presented here in a slightly modified statement.

Lemma 1. *For the solutions of a dynamic system*

$$\dot{x} = Ax + Dw, \ |w| \le 1,$$

with a stable (Hurwitz) matrix A, *the peak value satisfies the upper bound*

$$\xi = \max_{t\ge 0} \max_{|x_0|=1} \max_{|w|\le 1} |x(t)| \le \sqrt{\|P_*\|},$$

where P_* *is the solution to the parametric semidefinite programming problem*

$$\|P\| \to \min,$$
$$AP + PA^{\mathrm{T}} + \alpha P + \frac{1}{\alpha} DD^{\mathrm{T}} \preceq 0,$$
$$P \succeq I$$

with respect to the matrix variable $P = P^{\mathrm{T}} \in \mathbb{R}^{n\times n}$ *and the scalar parameter* $\alpha > 0$.

Applying the state-feedback control law (4) to the system (1), we obtain the closed loop system

$$\dot{x} = \big(A + BK + F\Delta(t)H\big)x + Dw. \quad (5)$$

Using Lemma 1, we arrive at the minimization problem for $\|P\|$ subject to the constraints

$$\big(A + BK + F\Delta(t)H\big)P + P\big(A + BK + F\Delta(t)H\big)^{\mathrm{T}}$$
$$+\alpha P + \frac{\gamma^2}{\alpha} DD^{\mathrm{T}} \preceq 0 \ \text{ and } \ P \succeq I.$$

The first constraint is a matrix inequality nonlinear jointly in the variables P and K.

The nonlinearity can be rid of by introducing an auxiliary matrix variable $Y = KP \in \mathbb{R}^{r\times n}$ and eliminating the variable K. In this case, the variable K is reconstructed unambiguously: $K = YP^{-1}$. As a result, we have the inequality

$$AP + PA^{\mathrm{T}} + BY + Y^{\mathrm{T}}B^{\mathrm{T}} + \alpha P + \frac{\gamma^2}{\alpha} DD^{\mathrm{T}}$$
$$+F\Delta(t)HP + PH^{\mathrm{T}}\Delta^{\mathrm{T}}(t)F^{\mathrm{T}} \preceq 0 \quad (6)$$

for all values of the matrix uncertainty $\Delta(t): \|\Delta(t)\| \le \delta$.

The definiteness of the resulting matrix family can be verified by solving one matrix inequality with respect to an additional scalar variable. To this end, we adopt the so-called Petersen lemma [17], presented here in the following statement.

Lemma 2. *Let* $G = G^{\mathrm{T}} \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{n\times p}$, *and* $N \in \mathbb{R}^{q\times n}$ *be nonzero matrices. The matrix inequality*

$$G + M\Delta N + N^{\mathrm{T}}\Delta^{\mathrm{T}}M^{\mathrm{T}} \preceq 0$$

holds for all $\Delta: \|\Delta\| \le \delta$ *if and only if there exists a number* $\varepsilon > 0$ *such that*

$$G + \varepsilon\delta^2 MM^{\mathrm{T}} + \frac{1}{\varepsilon} N^{\mathrm{T}}N \preceq 0.$$

Applying Lemma 2 to the matrix inequality (6) with

$$G = AP + PA^{\mathrm T} + BY + Y^{\mathrm T}B^{\mathrm T} + \alpha P + \frac{\gamma^2}{\alpha}DD^{\mathrm T},$$

$$M = F, \quad N = HP$$

yields an equivalent condition for the existence of a positive number ε such that

$$AP + PA^{\mathrm T} + BY + Y^{\mathrm T}B^{\mathrm T} + \alpha P + \frac{\gamma^2}{\alpha}DD^{\mathrm T}$$

$$+\varepsilon\delta^2 FF^{\mathrm T} + \frac{1}{\varepsilon}PH^{\mathrm T}HP \preceq 0.$$

Using the Schur complement lemma, we finally write this condition as an equivalent LMI in the variables P, Y, and ε:

$$\begin{pmatrix} AP + PA^{\mathrm T} + BY + Y^{\mathrm T}B^{\mathrm T} + \alpha P + \frac{\gamma^2}{\alpha}DD^{\mathrm T} + \varepsilon\delta^2 FF^{\mathrm T} & PH^{\mathrm T} \\ HP & -\varepsilon I \end{pmatrix} \preceq 0.$$

Thus, the following result has been established.

Theorem 1. *Let* P_*, Y_* *be the solution of the constrained optimization problem*

$$\|P\| \to \min,$$

$$\begin{pmatrix} AP + PA^{\mathrm T} + BY + Y^{\mathrm T}B^{\mathrm T} + \alpha P + \frac{\gamma^2}{\alpha}DD^{\mathrm T} + \varepsilon\delta^2 FF^{\mathrm T} & PH^{\mathrm T} \\ HP & -\varepsilon I \end{pmatrix} \preceq 0,$$

$$P \succeq I$$

with respect to the matrix variables $P = P^{\mathrm T} \in \mathbb{R}^{n\times n}$ *and* $Y \in \mathbb{R}^{r\times n}$, *the scalar variable* ε, *and the scalar positive parameter* α.

Then for the solutions of the closed loop system (1), (4) *with the controller*

$$K_* = Y_* P_*^{-1},$$

the peak value satisfies the upper bound

$$\xi \le \sqrt{\|P_*\|}$$

under all admissible exogenous disturbances (2) *and system uncertainties* (3).

Remark 1. According to [16], the quadratic form

$$V(x) = x^{\mathrm T}P^{-1}x, \quad P \succ 0,$$

is a quadratic Lyapunov function for the closed loop system (1). Therefore, the ellipsoid

$$E = \left\{x \in \mathbb{R}^n:\ x^{\mathrm T}P^{-1}x \le 1\right\}$$

is *invariant*: the trajectory of the closed loop system evolving from an arbitrary point of this ellipsoid will remain inside it at all subsequent time instants under all admissible exogenous disturbances and system uncertainties.

The first constraint in the problem of Theorem 1 is nonlinear jointly in the variables and represents a parameterized LMI with the scalar parameter α. For a fixed value α, the optimization problem turns into a semidefinite programming one. Its solution is easily found numerically on a one-dimensional grid for the parameter α.

2. THE DISCRETE CASE

Passing to the discrete-time problem, we consider a linear control system of the form

$$x_{k+1} = \left(A + F\Delta_k H\right)x_k + Bu_k + Dw_k, \qquad (7)$$

where $A \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times p}$, $H \in \mathbb{R}^{q\times n}$, $B \in \mathbb{R}^{n\times r}$, and $D \in \mathbb{R}^{n\times m}$ are given matrices, with an initial condition x_0, the state vector $x_k \in \mathbb{R}^n$, the control action $u_k \in \mathbb{R}^r$, a bounded exogenous disturbance $w_k \in \mathbb{R}^m$,

$$|w_k| \le \gamma \quad \text{for all } k = 0, 1, 2, \ldots, \qquad (8)$$

and a matrix uncertainty

$$\|\Delta_k\| \le \delta \quad \text{for all } k = 0, 1, 2, \ldots \qquad (9)$$

It is required to find a stabilizing static linear state-feedback control law

$$u_k = Kx_k, \qquad (10)$$

that minimizes the peak value

$$\xi = \max_{k=0,1,2,\ldots}\ \max_{|x_0|=1}\ \max_{\|\Delta_k\|\le\delta}\ \max_{|w_k|\le\gamma} |x_k|$$

in the closed loop system (7) under all admissible uncertainties Δ_k and all admissible exogenous disturbances w_k.

According to [18], Lemma 1 has a discrete analog as follows.

Lemma 3. *For the solutions of the dynamic system* $x_{k+1} = Ax_k + Dw_k$, $|w_k| \le 1$, *with a stable (Schur) matrix* A, *the peak value satisfies the upper bound*

$$\xi = \max_{k=0,1,2,\ldots}\ \max_{|x_0|=1}\ \max_{|w_k|\le 1} |x_k| \le \sqrt{\|P_*\|},$$

where P_* *is the solution to the parametric semidefinite programming problem*

$$\|P\| \to \min,$$

$$\frac{1}{\alpha}APA^{\mathrm T} - P + \frac{1}{1-\alpha}DD^{\mathrm T} \preceq 0,$$

$$P \succeq I$$

with respect to the matrix variable $P = P^{\mathrm T} \in \mathbb{R}^{n\times n}$ *and the scalar parameter* $0 < \alpha < 1$.

The closed loop system (7), (10) has the form

$$x_{k+1} = \left(A + BK + F\Delta_k H\right)x_k + Dw_k.$$

Using Lemma 3, we obtain the minimization problem for $\|P\|$ subject to the constraints

$$\frac{1}{\alpha}\left(A+BK+F\Delta(t)H\right) P\left(A+BK+F\Delta(t)H\right)^{\mathrm{T}}$$

$$-P+\frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}} \preceq 0 \text{ and } P\succeq I.$$

The first constraint can be equivalently transformed to

$$\begin{pmatrix} -\alpha P & P\left(A+BK+F\Delta(t)H\right)^{\mathrm{T}} \\ \left(A+BK+F\Delta(t)H\right)P & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P \end{pmatrix} \preceq 0,$$

representing a matrix inequality nonlinear jointly in the variables P and K. As in the continuous case, eliminating the variable K by introducing the auxiliary matrix variable $Y=KP\in\mathbb{R}^{r\times n}$ gives

$$\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}}+PH^{\mathrm{T}}\Delta^{\mathrm{T}}(t)F^{\mathrm{T}} \\ AP+BY+F\Delta(t)HP & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P \end{pmatrix} \preceq 0. \quad (11)$$

This inequality must hold for all matrix uncertainties $\Delta_k: \|\Delta_k\|\le\delta.$

Writing the relation (11) as

$$\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}} \\ AP+BY & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P \end{pmatrix} + \begin{pmatrix} 0 \\ F \end{pmatrix}\Delta(t)\begin{pmatrix} HP & 0 \end{pmatrix}$$

$$+\begin{pmatrix} PH^{\mathrm{T}} \\ 0 \end{pmatrix}\Delta^{\mathrm{T}}(t)\begin{pmatrix} 0 & F^{\mathrm{T}} \end{pmatrix} \preceq 0$$

and applying the Petersen lemma with

$$G=\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}} \\ AP+BY & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P \end{pmatrix},$$

$$M=\begin{pmatrix} 0 \\ F \end{pmatrix}, \quad N=\begin{pmatrix} HP & 0 \end{pmatrix},$$

we arrive at an equivalent condition for the existence of a positive number ε such that

$$\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}} \\ AP+BY & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P \end{pmatrix} + \varepsilon\delta^2\begin{pmatrix} 0 \\ F \end{pmatrix}\begin{pmatrix} 0 & F^{\mathrm{T}} \end{pmatrix}$$

$$+\frac{1}{\varepsilon}\begin{pmatrix} PH^{\mathrm{T}} \\ 0 \end{pmatrix}\begin{pmatrix} HP & 0 \end{pmatrix} \preceq 0.$$

Using the Schur complement lemma again, we finally obtain

$$\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}} & PH^{\mathrm{T}} \\ AP+BY & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P+\varepsilon\delta^2FF^{\mathrm{T}} & 0 \\ HP & 0 & -\varepsilon I \end{pmatrix} \preceq 0.$$

Thus, the following result has been established.

Theorem 2. *Let* P_*, Y_* *be the solution of the optimization problem*

$$\|P\|\to\min,$$

$$\begin{pmatrix} -\alpha P & PA^{\mathrm{T}}+Y^{\mathrm{T}}B^{\mathrm{T}} & PH^{\mathrm{T}} \\ AP+BY & \frac{\gamma^2}{1-\alpha}DD^{\mathrm{T}}-P+\varepsilon\delta^2FF^{\mathrm{T}} & 0 \\ HP & 0 & -\varepsilon I \end{pmatrix} \preceq 0,$$

$$P\succeq I,$$

with respect to the matrix variables $P=P^{\mathrm{T}}\in\mathbb{R}^{n\times n}$ *and* $Y\in\mathbb{R}^{r\times n}$, *the scalar variable* ε, *and the scalar parameter* $0<\alpha<1$.

Then for the solutions of the closed loop system (7), (10) *with the controller*

$$K_*=Y_*P_*^{-1},$$

the peak value satisfies the upper bound

$$\xi\le\sqrt{\|P_*\|}$$

under all admissible exogenous disturbances (8) *and system uncertainties* (9).

Remark 1 remains valid in the discrete case as well.

3. AN EXAMPLE

We illustrate the proposed approach by an example of a two-mass system. It consists of two solid bodies with masses m_1 and m_2 connected by a spring with an elastic coefficient k. The bodies slide without friction along a fixed horizontal rod. A control action is applied to the left body to compensate an exogenous disturbance w, $|w|\le 0.25$, affecting the right body (Fig. 1).

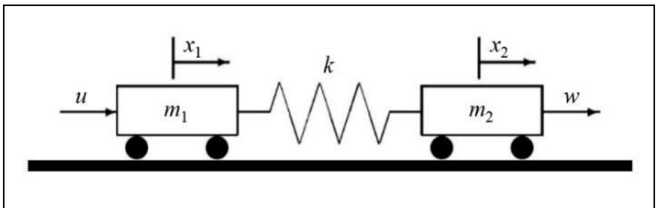

Fig. 1. The two-mass system.

Let x_1 and x_2 be the coordinates of the left and right bodies, respectively, and let v_1 and v_2 be their velocities. Assume that the bodies have unit masses and the uncertainty is concentrated in the elastic coefficient of the spring:

$$k = 1 + \Delta, |\Delta| \le \delta = 0.1.$$

The dynamics of this system are described by equation (1) with

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}, \; B = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \; D = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$F = \begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}, \; H = \begin{pmatrix} -1 & 1 & 0 & 0 \end{pmatrix},$$

where the state vector has the form

$$x = \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix}.$$

Using Theorem 1, we find the matrix

$$P_* = \begin{pmatrix} 2.4483 & 1.0584 & -0.5874 & -1.1188 \\ 1.0584 & 3.7785 & 0.7298 & -0.4198 \\ -0.5874 & 0.7298 & 4.3554 & -0.8388 \\ -1.1188 & -0.4198 & -0.8388 & 2.8905 \end{pmatrix},$$

and the corresponding controller

$$K_* = \begin{pmatrix} -2.0410 & 0.2025 & -1.4978 & -0.8333 \end{pmatrix}$$

ensures the upper bound

$$\xi \le \sqrt{\|P_*\|} = 2.3131 \qquad (12)$$

for the peak value under all admissible exogenous disturbances and system uncertainties.

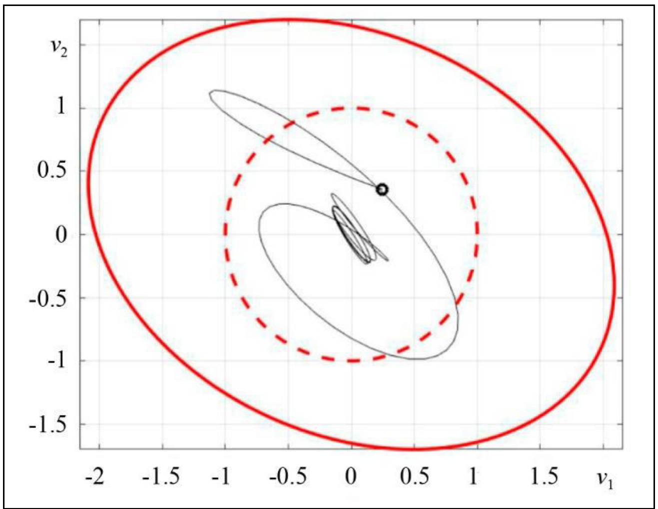

Fig. 2. The projection of the closed loop system trajectory on the plane (v_1, v_2).

Figure 2 shows the projection of the trajectory $x_*(t)$ of the closed loop system with the controller K_* on the plane (v_1, v_2) under the following conditions: the initial state

$$x_0 = \begin{pmatrix} 0.6493 \\ -0.5893 \\ -0.1444 \\ 0.4585 \end{pmatrix}, \; |x_0| = 1, \qquad (13)$$

the admissible exogenous disturbance $w(t) = 0.25\sin(2t)$, and the system uncertainty realization $\Delta = 0.1$. Also, see the projections of the invariant ellipsoid with the matrix P_* and the unit ball of initial conditions in this figure.

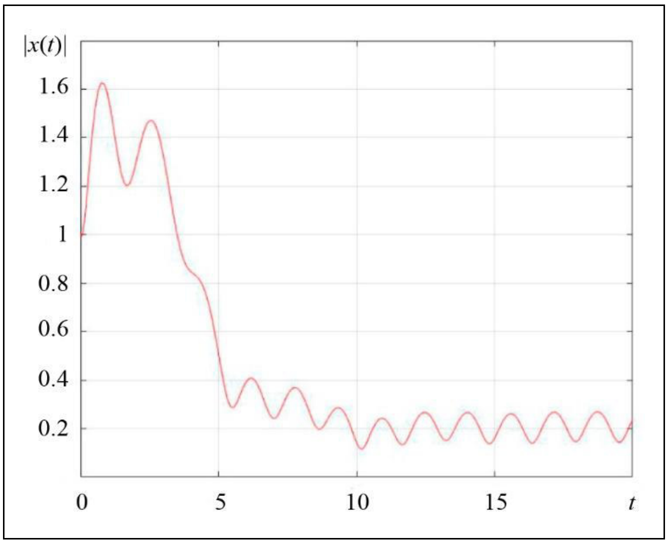

Fig. 3. Dynamics of $|x(t)|$.

Figure 3 presents the dynamics of the norm of the trajectory $x_*(t)$; the peak in the system reaches a value of 1.6281.

The calculations were carried out in Matlab R2019b v 9.7 using CVX [19], a free software package for convex programming.

What is the "worst-case" initial condition causing the largest peak value? This question has a rather complicated nature. In some particular cases, however, it can be answered meaningfully; for example, see the papers [1, 9, 10]. Of course, the worst-case initial condition can be found numerically, but its analytical calculation in a relatively general statement is still an open problem.

CONCLUSIONS

We have proposed an approach to peak minimization in linear control systems subjected to arbitrary bounded exogenous disturbances and system uncertainties. This approach is based on the technique of linear matrix inequalities and reduces the original problem to a parametric semidefinite programming one.

The approach has a high potential for generalizations: it can be extended to new classes of problems, in particular, to the case of output feedback using an observer or a dynamic controller.

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