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MODELS OF JOINT DYNAMICS OF OPINIONS AND ACTIONS IN ONLINE SOCIAL NETWORKS. PART II: Linear Models

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Abstract. Based on *VKontakte* data, we study the influence of various factors on the dynamics of opinions and actions both at the macro level ("public opinion") and at the micro level (the opinions and actions of individual agents). A model of collective decision-making is briefly considered; in this model, interconnected parameters reflect both the mental and behavioral components of the agents' activity. Identification results are presented for two special modifications of the model, namely, linear macro- and micro models of the dynamics of opinions and actions in a social network. We estimate the influence of various factors on the opinions and actions of agents: aggregated social influence (public opinion), the agent's individual opinions and actions, the opinions and actions of the social environment, and the mechanisms of the agent's trust in information sources and information content.

Keywords: social network, agent, opinion, action, social influence, cognitive dissonance, trust in information.

INTRODUCTION

In part I of this study [1], primary analysis results were presented for the joint dynamics of opinions and actions[1] of agents on the example of their attitude toward wearing medical masks in the *VKontakte* online social network during the first year of the COVID-19 pandemic (the period from March 2020 to February 2021 inclusive). A sample containing over 60 thousand related posts, over 2 million comments to the posts, and over 7 million likes to the posts was used to examine social network behavior: how much support individual users and the online community as a whole have for wearing masks, how much public opinion changes over time, etc. The following questions were formulated:

1) How consistent are the opinions and actions of agents with each other?

2) Do agents change their opinions and actions over time?

3) Who are these (opinion- and action-changing) agents? Do they differ from others in their socio-demographic characteristics?

4) Which models better describe the dynamics of the opinions and actions of agents (linear, threshold, etc.)?

5) Are the influence of actions on opinions (*cognitive dissonance*) and the converse effect significant?

6) Under which factors do the opinions and actions of agents change? Among such factors, we considered:

- the agent's previous opinions or (and) actions;
- social influence:

  – *public opinion* (the averaged shares of certain opinions and actions of the entire social network, i.e., the so-called *macro model*, where the network is conventionally treated as one agent);

[1] *An opinion* was conventionally interpreted as the "tone" of an agent's comment, as assessed by an automatic classifier; *an action* was conventionally interpreted as the tone of a comment with an agent's like.

– the opinions or (and) actions of the agent's environment (the agents with the friendship relation to a given agent), i.e., the averaged and (or) individual ones (the so-called *micro model*);

- some unobservable (*latent*) characteristics of the agent.

7) Does an agent's change in the opinion (action) depend on his trust in the source of information? Does it depend on the content of that information?

In part I, Questions nos. 1–3 were answered.

This paper is organized as follows. Section 1 briefly describes models of the joint dynamics of opinions and actions from [2]. In Section 2, key factors for the analysis and modeling of network interactions are considered. In Section 3, we present identification results for linear macro- and micro models of the joint dynamics of opinions and actions, including their discussion. This section provides answers to Questions nos. 5–7. Some intermediate outcomes are outlined in the Conclusions.

1. MENTAL AND BEHAVIORAL COMPONENTS OF ACTIVITY: KNOWN MODELS OF JOINT DYNAMICS

In this paper, the basic model is the mathematical model of the joint dynamics of opinions and actions of the agents proposed in [2]. Following [2], consider the process of joint decision-making by interacting subjects and original models of the joint dynamics of mental and behavioral components in the process of their individual and collective activity. The set $N = \{1, 2, ..., n\}$ of interacting subjects, called *agents*, is introduced. The agent number is indicated by the subscript whereas the time instant (period) by the superscript.

The parameter $r_i \in R_i$, called the (internal) *state* of agent i, $i \in N$, is defined to reflect all his essential individual characteristics, including the personality structure parameters [3]. The agent's state in applications can be interpreted as his opinion, belief, or attitude (e.g., an assessment of some object or subject), the efficiency of his activity, his learning rate, or the most desirable result of the activity for him, etc.

Agent $i \in N$ can choose *actions* y_i from the set A_i of admissible actions. All agents choose their actions, and *the results* of their activity are then realized; they are denoted by $z_i \in A_{0i}$, where A_{0i} is the set of admissible results of agent i. A possible discrepancy between the agent's action and result can be caused by the influence of his *environment* (with the state $\theta \in \Omega$) or by the actions of other agents.

According to [2], the agent's action and result may have a complex relationship. For simplicity, let the result z_i of agent i be a known deterministic real function $w_i(y_i, y_{-i}, \theta)$ depending on his action y_i, the vector $y_{-i} = (y_1, ..., y_{i-1}, y_{i+1}, ..., y_n)$ containing the actions of all other agents (the so-called *opponents' action profile* for agent i), and the state of the environment θ.

Each agent knows his state, and his action is completely observable to him and all other agents. When choosing an action, the agent is guided by his preferences on the set of results A_{0i} and by the potential effect of the chosen action on his result. Given his state, the state of the environment, and the actions of all other agents, agent i will choose the action y_i^* maximizing his utility:

$$y_i^*\left(y_{-i}^*, r_i, \theta\right) = \arg\max_{y_i \in A_i} f_i\left(w_i\left(y_i, y_{-i}^*, \theta\right), r_i\right), \; i \in N, \quad (1)$$

where f_i: $A_{0i} \times R_i \to \Re^1$ is the utility function of agent i.

Table 1 shows the factors influencing the agent's decision-making (columns) and the scientific disciplines dealing with decision-making models (rows). All these factors were considered in [2]. (The number of pluses conventionally reflects the degree of consideration of the factor within the corresponding discipline.) The columns of Table 1 with yellow filling are the factors studied in this paper.

The author [2] introduced different assumptions about the behavior of agents with practical interpretations and considered the corresponding mathematical models of the joint dynamics of agents' states and actions (in particular, the models of informational influence and control). The assumptions are as follows.

A.1. $A_i = A_{0i} = R_i = U_i = [0, 1]$, $i \in N$.

A.2. $w_i(y_i, y_{-i}, \theta) = w(y_i, y_{-i})$, $i \in N$.

A.3. Given a fixed state r_i of agent i, his utility function f_i: $[0, 1]^2 \to \Re$ is *single-peaked* with the *peak point* r_i, $i \in N$.

A.4. The function $w(\cdot)$ is continuous, strictly monotonically increasing in all variables, and satisfies *the unanimity condition*

$$\forall a \in [0, 1] \; w(a, ..., a) = a.$$

Table 1

Decision-making factors and scientific disciplines [4]

Disciplines	Factors						
	Utility	Action	Actions of others	Social environment	Internal state	History	Control
Individual decision-making [5, 6]	+++	++		++	+	+	
Game theory [7], collective behavior theory [8–10], behavioral economics [11]	++	+++	+++	+	+	+	+
Social psychology [12–14], Personality psychology [15–17] Mathematical psychology [18–21]	+	++	+	++	+++	+	+
Multi-agent systems [22, 23]		+++	+	++	++	+	+
Control of social and organizational systems [24, 25]	++	++	++	+++	+	+	+++

Assumption A.1 is "technical." Assumption A.2 plays a more significant role: first, the result (*collective decision*) $z = w(y_i, y_{-i})$ is the same for all agents; second, there is no uncertainty about the state of the environment. According to Assumption A.3, the agent's utility function defined on the set of results has a unique maximum, which is reached when the result coincides with the agent's state. The agent's state can be interpreted as his *assessment*, *opinion*, or *attitude* toward certain results. Assumption A.4 is meaningfully transparent: if the goals of all agents coincide, then the corresponding result of their joint activity is achievable.

The expression (1) describes one-time decision-making by agents (the one-time choice of their actions). Additional assumptions have to be introduced to consider repeated situations of decision-making. In [2], the dynamics of decision-making were studied under the following assumption.

A.5. *The dynamics* of agents' *actions* obey the procedure of *indicator behavior*

$$y_i^t = \left(1-\gamma_i^t\right) y_i^{t-1} + \gamma_i^t y_i^*\left(y_{-i}^{t-1}, r_i^t\right), \quad t=1,2,\ldots, \tag{2}$$

with given *initial values* $\left(y_i^0, r_i^0\right)$, $i \in N$, where $\gamma_i^t \in (0,1]$ are known constants. At each time instant, the agent takes a "step" proportionally to γ_i^t from his previous state to his best response y_i^* to the environment at the previous time instant.

A.6. The dynamics of agents' states obey the procedure

$$\begin{aligned} r_i^t = & \Big[1 - b_i B_i\left(r_i^{t-1}, u_i^t\right) - c_i C_i\left(r_i^{t-1}, y_i^{t-1}\right) \\ & - d_i D_i\left(r_i^{t-1}, z^{t-1}\right) - e_i\Big] r_i^{t-1} + b_i B_i\left(r_i^{t-1}, u_i^t\right) u_i^t \\ & + c_i C_i\left(r_i^{t-1}, y_i^{t-1}\right) y_i^{t-1} + d_i D_i\left(r_i^{t-1}, z^{t-1}\right) z^{t-1} \\ & + e_i E_i\left(r_i^{t-1}, y_{-i}^{t-1}\right), \ t = 1, 2, \ldots, \ i \in N, \end{aligned} \tag{3}$$

where $u_i^t \in U_i$ is the external influence (control) applied to agent $i \in N$ at time instant t.

The expressions (3) will be used below to construct and identify linear models of the joint dynamics of agents' opinions and actions.

A.7. The nonnegative *true constants* (b_i, c_i, d_i, e_i) satisfy the constraints

$$b_i + c_i + d_i + e_i \le 1, \ i \in N.$$

A.8. *The trust functions* $B_i(\cdot)$, $C_i(\cdot)$, $D_i(\cdot)$, and $E_i(\cdot)$, $i \in N$, take values from the interval [0, 1]; $\forall\, a \in [0, 1]$ $E_i(a, \ldots, a) = a$, $i \in N$.

A.9. The nonnegative trust constants (b_i, c_i, d_i, e_i) and the trust functions $B_i(\cdot)$, $C_i(\cdot)$, and $D_i(\cdot)$, $i \in N$, satisfy the condition

$$\forall x_1, x_2, x_3, x_4 \in [0, 1]$$
$$b_i B_i(x_1, x_2) + c_i C_i(x_1, x_3) + d_i D_i(x_1, x_4) + e_i \le 1,$$
$$i \in N.$$

Assumptions A.7–A.9 ensure that the state of the dynamic system (2), (3) will not leave the set of admissible values.

In a possible interpretation, the constant "weights" (b_i, c_i, d_i, e_i) reflect the attitude (*trust*) of agent i to the corresponding *information source* whereas the trust functions $B_i(\cdot)$, $C_i(\cdot)$, $D_i(\cdot)$, and $E_i(\cdot)$ reflect the trust of agent i in *the information content*. The coefficient $\left[1 - b_i B_i\left(r_i^{t-1}, u_i^t\right) - c_i C_i\left(r_i^{t-1}, y_i^{t-1}\right) - d_i D_i\left(r_i^{t-1}, z^{t-1}\right) - e_i\right]$ at the first term on the right-hand side of (3) conditionally describes *the strength of the* agent's *own beliefs.*

The five terms on the right-hand side of the expression (3) can be interpreted as follows. According to (3), the state r_i^t of agent i at time instant t is generally defined as a linear combination of:

I) his state r_i^{t-1} at the previous time instant ($t - 1$);

II) his action y_i^{t-1} at the previous time instant ($t - 1$);

III) the actions y_{-i}^{t-1} and (in general) results z_{-i}^{t-1} of the other agents at the previous time instant ($t - 1$); this influence can be indirect, i.e., "through" the result of this agent;

IV) the result z^{t-1} at the previous time instant ($t - 1$);

V) a purposeful external influence (*control*) or generalized social influence u_i^t on him at time instant t.

The paper [2] examined special cases of the general model (2), (3) with only one influence factor for the agent's state (I, II, III, IV, or V). The results of equilibria analysis in such models were presented (the active expertise model, the informational control model, the consensus model, the conformity behavior model, and social influence models, in particular, cognitive dissonance, hindsight, and the mutual influence of agents).

Thus, the model of the interrelation of agents' states, actions, and results has been briefly described. Note that the class of models under consideration requires experimental verification, including analysis of the relationship between states and actions, as well as other factors affecting the dynamics of agents' states and actions. The results of such research are demonstrated below for the agents of an online social network. In contrast to the paper [2], the expression (2) below is replaced by the linear dependences (similar to (3)) of the agent's actions on his previous states and actions as well as on the states and actions of other agents and other factors.

2. AN OUTLINE OF KEY FACTORS TO MODEL AND ANALYZE NETWORK INTERACTIONS

Recall the formalization of the factors necessary to analyze and identify the models of the joint dynamics of opinions and actions; for details, see part I of this study [1]. Let the network participants be *agents* from a set $N = \{1, 2, ..., n\}$. They commit some *acts* from a fixed set $K = \{1, 2, ..., k\}$ at certain time instants t of an interval T. Our considerations are restricted to the following types of acts ($K = \{1, 2\}$):

– publishing a comment on a post or another comment,

– liking a comment.

We denote by Δ the set of acts. Each act $a \in \Delta$ is described by three parameters: the agent who committed it, the type of the act, and the time instant when it was committed. We introduce the following functions to characterize acts:

- f_a: $\Delta \to N$, associating with each act $a \in \Delta$ the agent $i \in N$ who committed it;
- f_t: $\Delta \to T$, associating with each act $a \in \Delta$ the time instant $t \in T$ when it was committed;
- f_k: $\Delta \to K$, associating with each act $a \in \Delta$ its type $j \in K$.

On the set of acts, we define a binary partial-order relation of the form "a causes b": $a \to b$. If $a \to b$, $a \neq b$, and there does not exist $c \in \Delta$ such that $a \to c$ and $c \to b$, then a is *the direct cause of* b: $a \downarrow b$. The binary relation $a \to b$ is supposed to hold in the following cases:

- a is a comment and b is a like to it.
- a is a comment and b is a comment on it.
- a and b coincide.

For each agent $i \in N$, we define the set of all his acts $\delta_i = \{a \in \Delta \mid f_a(a) = i\}$ and the set of his friends $N_i \subseteq N$. (The formal "friendship" relation in an online social network implies that an agent can receive information about the comments posted by his friends, the likes they give, etc.).

Opinions and actions. When modeling the joint dynamics of opinions and actions, we conventionally interpret the agent's *opinion* as his attitude to wearing medical masks, expressed in a comment.

The agent's opinion in a comment $b \in \Delta$ ($f_k(b)$ = 1) is formally defined in three ways as follows:

- $r' \in \{0, 1, 2\}$, where the classification results 0, 1, and 2 correspond to "against masks" (or "–"), "for masks" (or "+"), and "neutral/irrelevant" (or "="). This result is determined using the stochastic vector $(p_-, p_+, p_=)$ calculated by the classifier. In machine learning, such a vector consists of the probabilities with which the object belongs to appropriate classes.

- $r'' = \dfrac{p_+}{p_+ + p_-} \in [0, 1]$, the confidence that the comment reflects the "for masks" opinion. Note that r' = 0 or r' = 1 for this comment.

- $r = \dfrac{p_+ - p_-}{p_+ + p_-} \in [-1, 1]$, where r = 1 and r = –1 indicate strong confidence in expressing the "for masks" and "against masks" opinions, respectively. Note that either r' = 0 or r' = 1 for this comment.

Let a like to some comment be *an action* as well; its assessment coincides with that of the corresponding comment liked: $y' \in \{0, 1, 2\}$, $y'' \in [0,1]$, and $y \in [-1,1]$. For example, for a like $a \in \Delta$, $y'(a) = r'(b)$, where b is the corresponding comment liked (i.e., $b \downarrow a$). To simplify further notations, we adopt the conventions $r'(a) = y'(a)$, $r''(a) = y''(a)$, and $r(a) = y(a)$. Assume that the instant of liking coincides with the instant of publishing the corresponding comment liked.

3. LINEAR MICRO- AND MACRO MODELS OF JOINT DYNAMICS OF AGENTS' OPINIONS AND ACTIONS

In part I of this study [1], it was shown that the "average" opinions and actions of agents in a social network are interrelated. Now we identify the linear models of the dynamics of opinions and actions in an online social network and answer Questions nos. 5–7 of the Introduction.

3.1 Macro models

Consider equal consecutive time intervals τ_1, τ_2, ... (It can be an hour, a day, etc.; in this section, a weekly interval is used.) Each act in the network is committed at a certain time instant. Therefore, it is possible to define the set of acts committed during an interval τ_m:

$$\Delta(\tau_m) = \Delta^m = \{a \in \Delta \mid f_t(a) \in \tau_m\}.$$

We introduce the notations necessary to model the joint dynamics of opinions and actions at the macro level (at the level of the entire network):

- $r_+^m = \dfrac{\left|\{a \in \Delta^m | f_k(a) = 1, r'(a) = 1\}\right|}{\left|\{a \in \Delta^m | f_k(a) = 1\}\right|}$, the share of "for" opinions;

- $r_-^m = \dfrac{\left|\{a \in \Delta^m | f_k(a) = 1, r'(a) = 0\}\right|}{\left|\{a \in \Delta^m | f_k(a) = 1\}\right|}$, the share of "against" opinions;

- $y_+^m = \dfrac{\left|\{a \in \Delta^m | f_k(a) = 2, r'(a) = 1\}\right|}{\left|\{a \in \Delta^m | f_k(a) = 2\}\right|}$, the share of "for" actions;

- $y_-^m = \dfrac{\left|\{a \in \Delta^m | f_k(a) = 2, r'(a) = 0\}\right|}{\left|\{a \in \Delta^m | f_k(a) = 2\}\right|}$, the share of "against" actions; here, $m \in \mathbb{Z}_+$ is the current time instant.

Consider the macro models of joint dynamics where the opinions and actions of agents in the network (i.e., their shares) at the next time instant (m + 1) depend on those at the current time instant m = 1, 2, ... First of all, Fig. 1 shows the real dynamics of these variables for the annual period.

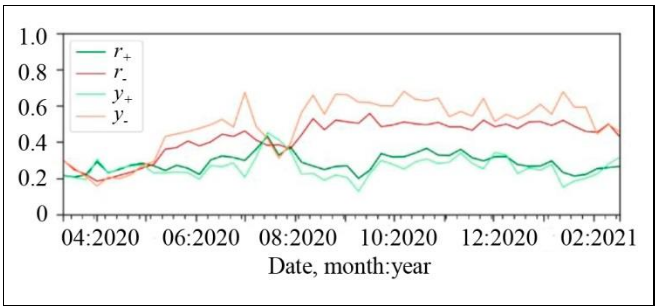

Fig. 1. The dynamics of opinions and actions.

At the beginning of the period under consideration (Fig. 1), the share of opinions and actions increases, both "for" and "against." At some point, the share of "against" opinions and actions (r_-, y_-) diverges from the share of "for" ones; by the end of summer, the shares of "for" and "against" become equal; subsequently, the share of "against" shows another appreciable divergence. The values of Pearson's correlation of the variables for this annual period are presented in Fig. 2.

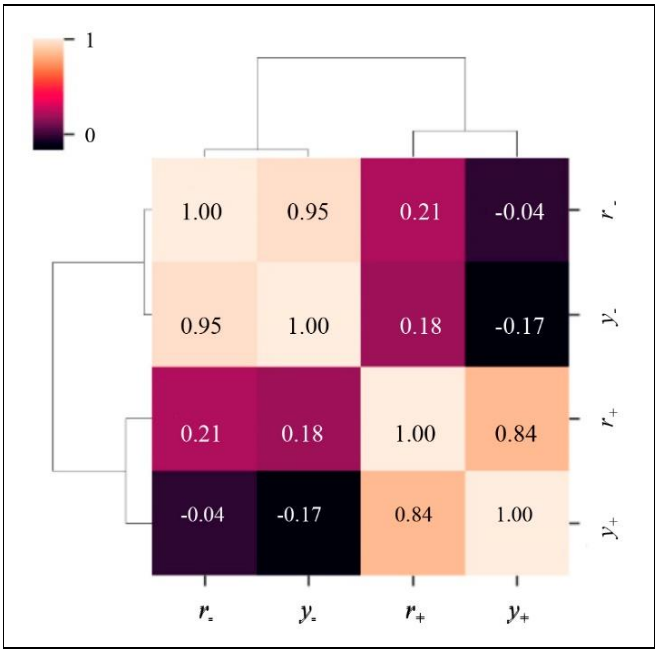

Fig. 2. Pearson's correlations of opinions and actions.

The shares of actions and opinions of the same orientation correlate well with each other: the best results are for "against" opinions and actions; the "for" opinions correlate to some extent with the "against" opinions and actions. However, there are two points of changing dynamics in Fig. 1: early July and early September 2020. Will the correlations of the variables change for the corresponding time periods? See the tables in Fig. 3.

The data in Fig. 3 confirm the strong positive correlation between the opinions and actions of the same orientation. Note that the correlation between the variables r_- and y_- weakens over time. Also, there is a "monotonic" growth of "against" opinions (with the exception of a monthly drop in summer) and wave fluctuations of "for" opinions. The conclusions for different periods are as follows:

T_1) a positive correlation between the variables r_+ and r_-, which can be explained by the gradual polarization of society (due to the reduction of neutral opinions), and a weak correlation between the variables y_+ and y_-;

T_2) a strong negative correlation between the variables r_+ and r_- as well as between the variables y_- and y_+ (probably, the growth limits reached due to neutral opinions);

T_3) no correlation between the variables r_+ and r_- and a decreased negative correlation between the variables y_- and y_+, possibly due to the disappearance of sharp fluctuations in the network.

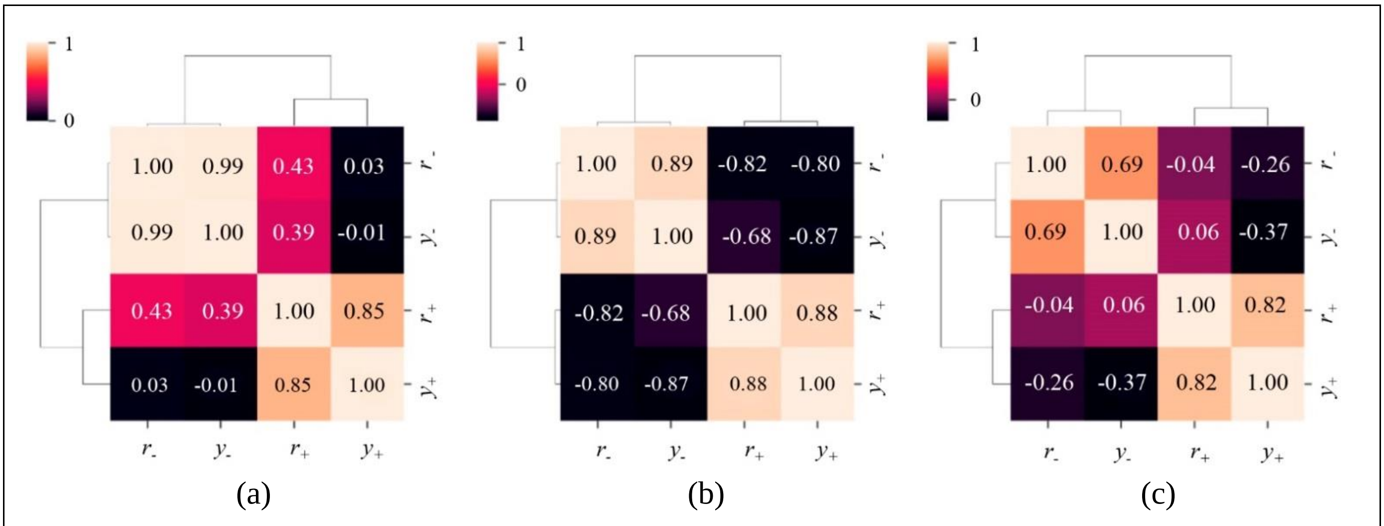

Fig. 3. The correlations of opinions and actions: (a) March–June 2020 (T_1), (b) July–August 2020 (T_2), and (c) September 2020–February 2021 (T_3).

The conclusions are confirmed by the dynamics of opinions in Fig. 4 (a correlation of 0.21) and the dynamics of actions in Fig. 5 (a correlation of –0.17).

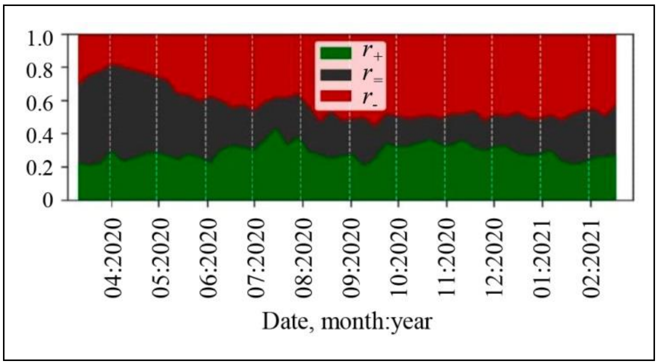

Fig. 4. The dynamics of opinions.

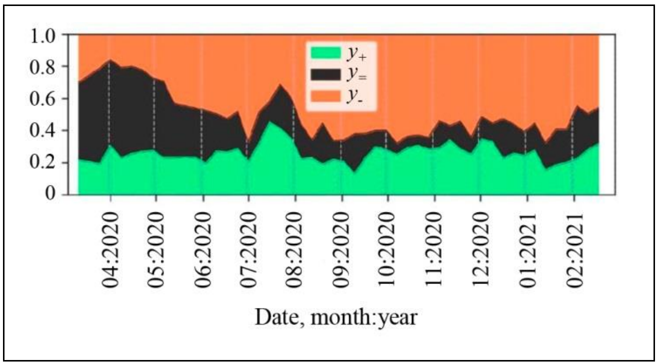

Fig. 5. The dynamics of actions.

Returning to formal models of the dynamics of opinions and actions, we consider Markov models[2], i.e., only with the influence of the previous time instant on the current one. Possible dependences of the variables are presented in Fig. 6.

For such dependences, we examine all possible linear models of the dynamics of opinions and actions (also, see the expression (3) in Section 2). In particular,

$$r_+^{m+1} = (1-\beta_{+1}-\beta_{+2}-\beta_{+3})r_+^m + \beta_{+1}y_+^m + \beta_{+2}r_-^m + \beta_{+3}y_-^m, \tag{4}$$

where the coefficients are $\beta_{+1}, \beta_{+2}, \beta_{+3} \ge 0$,

$$\beta_{+1}+\beta_{+2}+\beta_{+3} \le 1. \tag{5}$$

[2] The corresponding models with memory seem to be a promising line for future research.

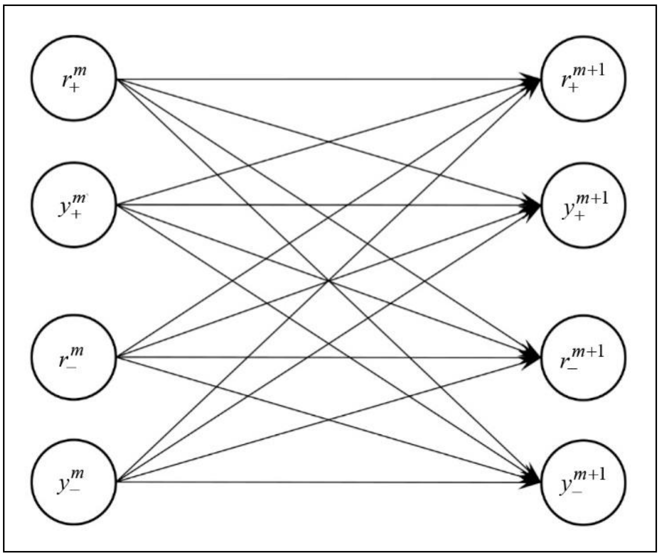

Fig. 6. Possible dependences of the variables.

Let the discretization step be 7 days. For each explanatory variable, we build a tree of possible models (Figs. 7–10): from the models with one explanatory variable to the model including all explanatory variables. For comparison, two models are added: *the average model* (with the annual average forecast) and *the trend model* (with the forecast depending linearly on the time step).

The *error* is calculated as

$$100\left(1-R_0^2\right), \tag{6}$$

where $R_0^2 = 1 - \frac{\sum_i (x_i-\hat{x}_i)^2}{\sum_i x_i^2}$, x_i is the actual value of the explained variable, and $\hat{x}_i$ is the forecasted value.

Each vertex of the *quality graphs of the models* in Figs. 7–10 shows the explanatory variables, the coefficients corresponding to these variables in model (4), and the error (bottom line in the vertex). The vertices lying on the “critical path” (the maximum error reduction) are highlighted in green. These vertices form *the optimal sequence of increasing the number of explanatory variables*; see Question no. 6 in the Introduction. In the bar graphs of Figs. 7–10, the error of the inertial model is considered to be a reference (100%) when assessing the quality of other models (the error variations). They include the models without stochastic constraints (5) as well as the models with additional explanatory variables: the constant, the time step, and the number of posts per step (standardized with respect to the entire annual period).

The variable r_+ (the share of "for" opinions): the variable r_+^{m+1} is worse explained by the average and trend models compared to the one-variable model r_+^m but is even better explained by the dynamics model $0.96r_+^m + 0.04r_-^m$ and the model $0.91r_+^m + 0.04r_-^m + 0.04y_+^m$ (see Fig. 7).

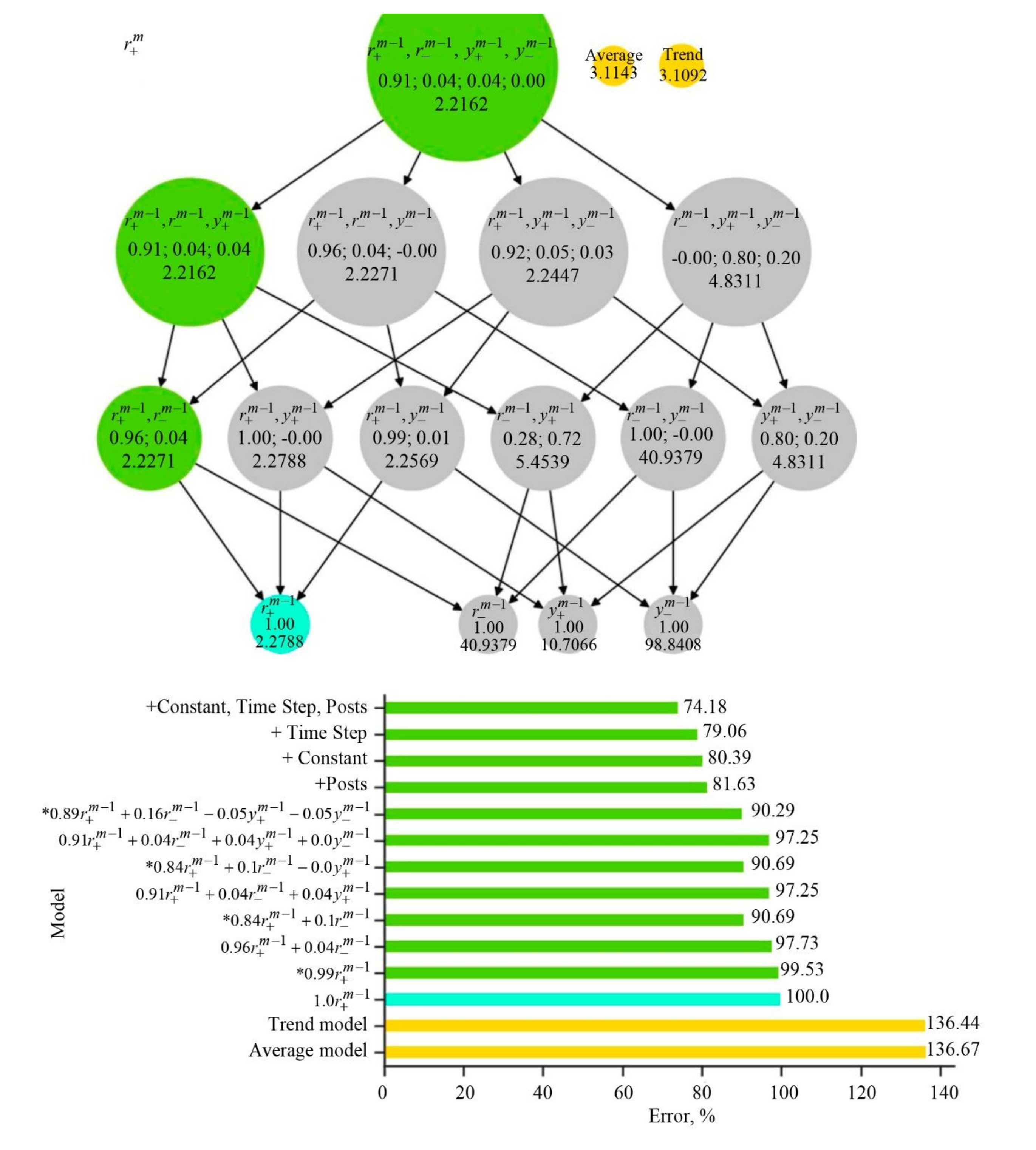

Fig. 7. The quality of models for the variable r_+ . (Prefix "*"—no stochastic constraints imposed on the model coefficients; prefix "+"—additional variables included in the most complex model (4).)

The variable r_- (the share of "against" opinions). Among the models with one explanatory variable, the best results are demonstrated by the inertial model (the average and trend models explain the dynamics much worse). Among the models with two explanatory variables, the highest quality is observed for the model taking into account both the opinion and action of those "against wearing masks": $0.95r_-^m + 0.05y_-^m$. The three-variable model $0.1r_+^m + 0.75r_-^m + 0.15y_-^m$ ensures a balance between quality and complexity (see Fig. 8).

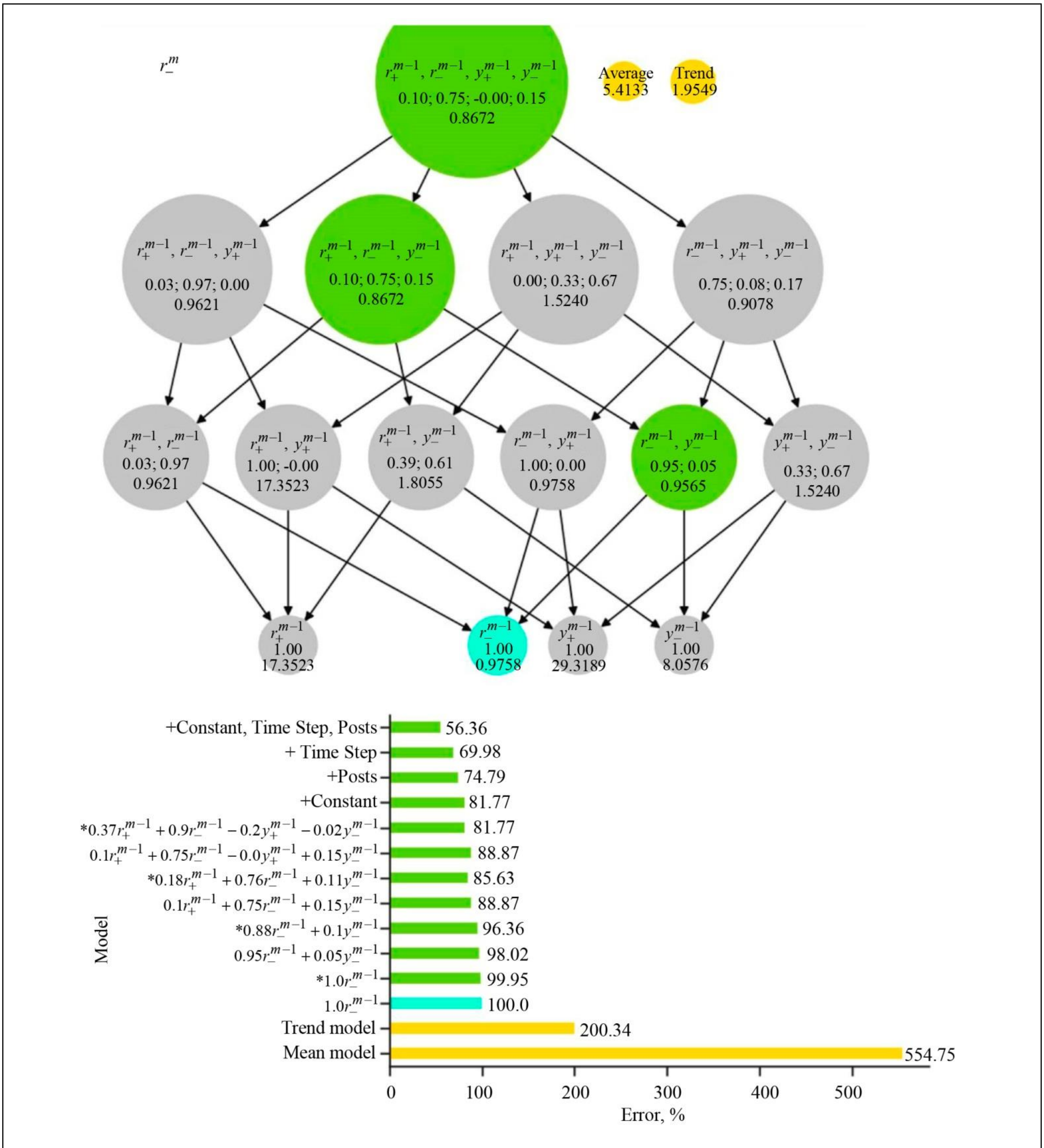

Fig. 8. The quality of models for the variable r_-.

The variable y_+ (the share of "for" actions): the variable y_+^{m+1} is more accurately explained by the average and trend models compared to the model with one variable r_-^m. In the class of models with stochastic constraints, the two-variable model $0.51r_+^m + 0.49y_+^m$ shows the best results (see Fig. 9).

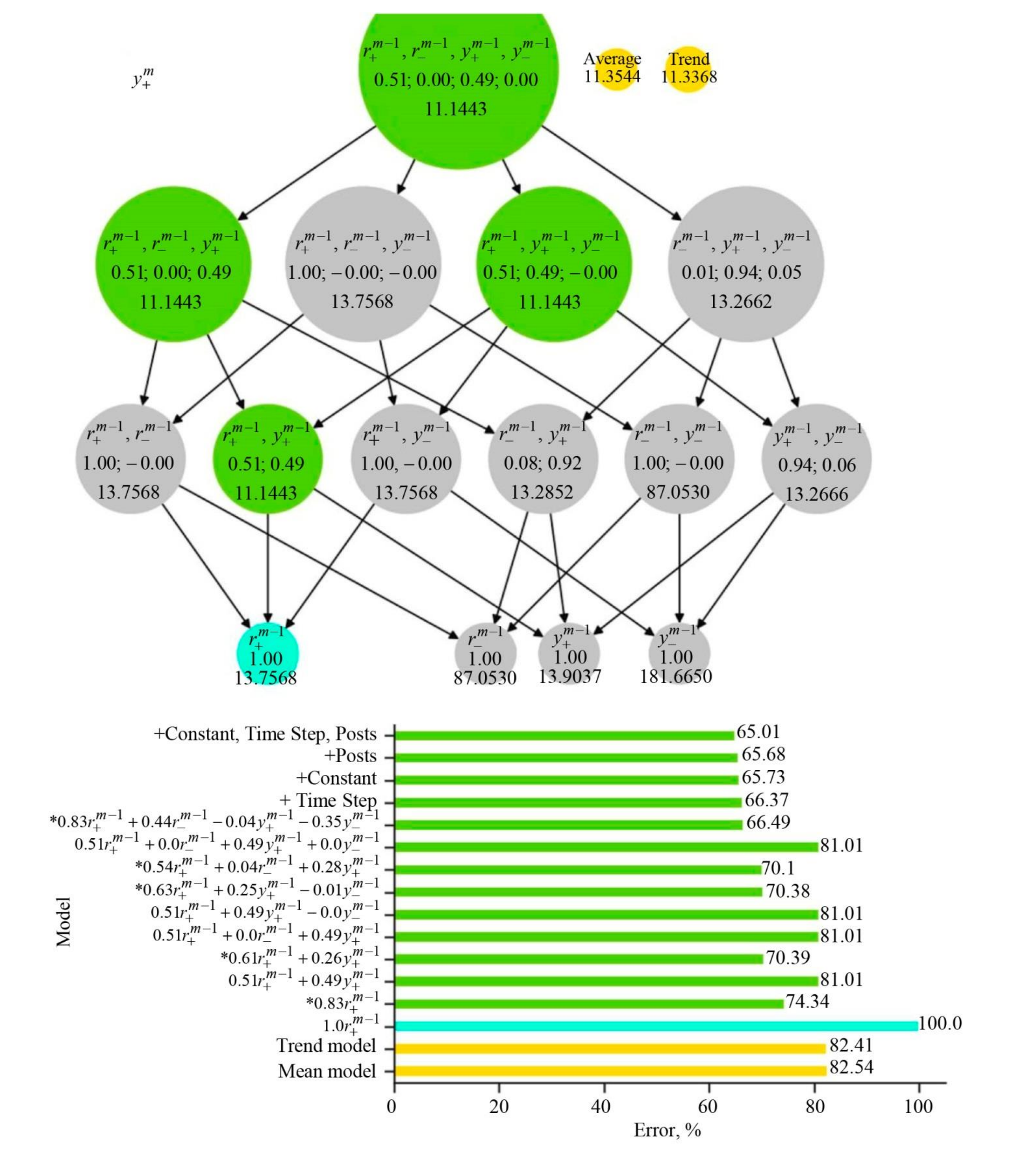

Fig. 9. The quality models for the variable y_+.

The variable y_- (the share of "against" actions): the variable y_-^{m+1} is explained significantly worse by the average and trend models compared to the model with one variable y_-^m (the inertial model). The "balanced" quality is shown by the model with two explanatory variables $0.22r_-^m + 0.78y_-^m$ (see Fig. 10).

Now we forecast the joint dynamics of opinions and actions using a macro model without stochastic constraints on the coefficients and with the intercept term (a constant); see Fig. 11.

As it turns out, the system reaches equilibrium rather quickly:

- $r_+^* = 0.28$, $r_-^* = 0.47$,
- $y_+^* = 0.24$, $y_-^* = 0.57$.

Generally speaking, the influence of external factors on the network should be considered when investigating complex patterns of opinion and action dynamics; nonlinear models seem promising when solving the forecasting problem.

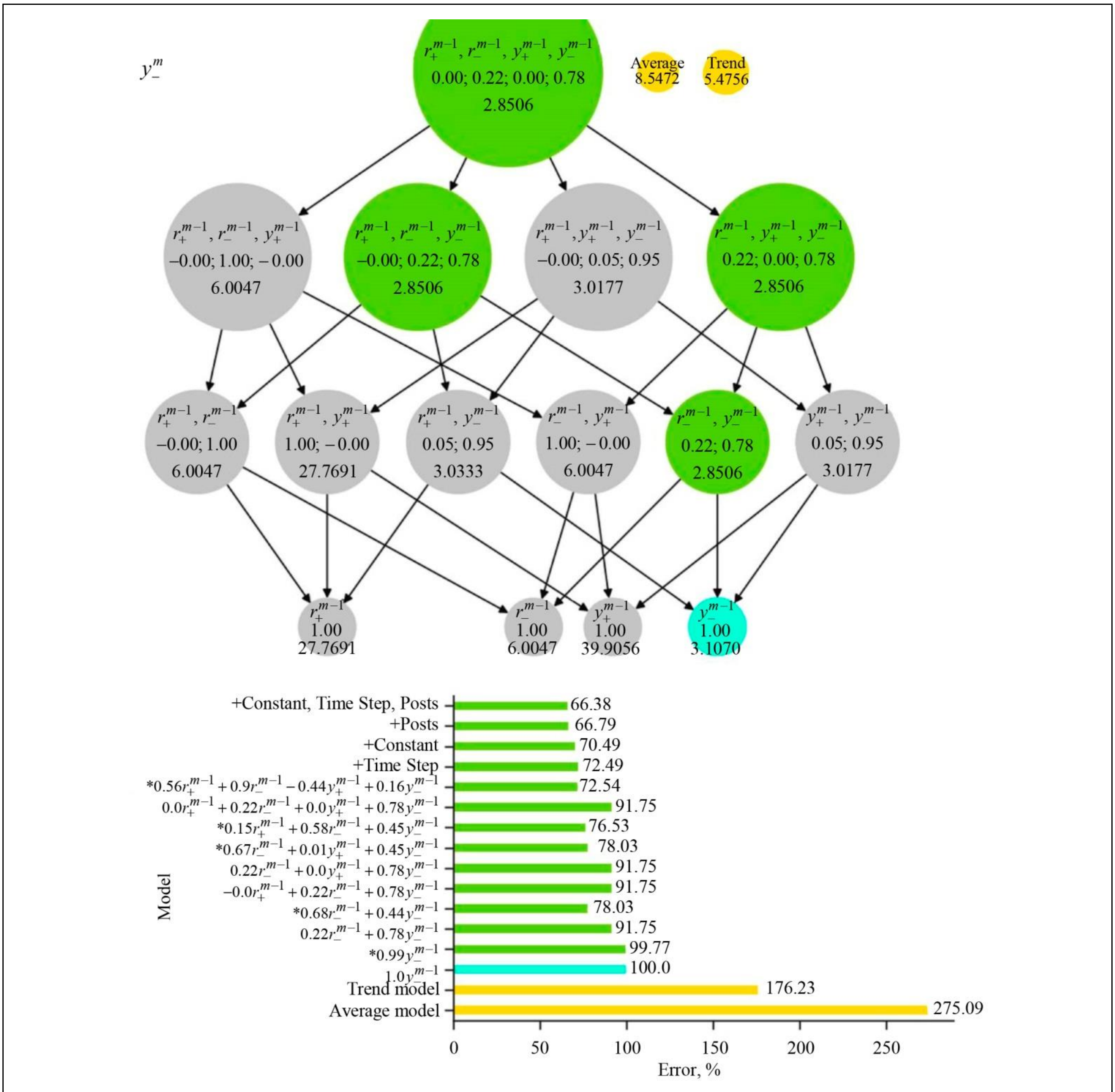

Fig. 10. The quality of models for the variable y_-.

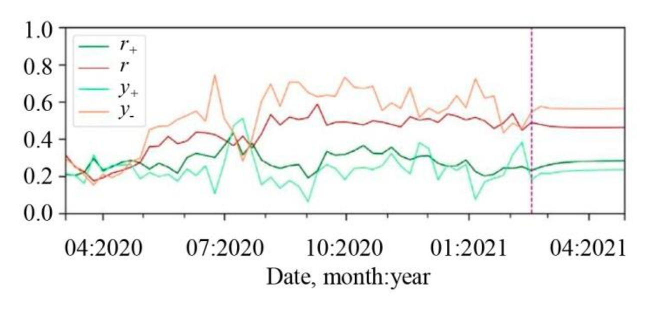

Fig. 11. The forecasted dynamics of opinions and actions. (The vertical line separates the actual and forecasted values.)

Let us discuss informally the results of this section for *the macro model* (the "causal" relationship of the "averaged" characteristics of agents).

First, even the "inertial" model with one explanatory variable better reflects the dynamics of the corresponding variable than the average and trend models (except for the inertial dynamic model of y_+, the share of "for" actions). Of course, including additional explanatory variables (opinions and actions) in the model improves its quality. The dynamic model of r_+ is almost completely inertial, although being partially determined by the variables r_- and y_+. The value of the variable y_+ at the current time instant is "equally" influenced by the values of the variables r_+ and y_+ at the previous time instant. The variable r_- "depends" on r_- by 75%, on y_- by 15%, and on r_+ by 10%. (The influence of this variable probably reflects the reactance effect.) The variable y_- is determined by the variables y_- (78%) and r_- (22%).

Second, there is an explicit increase in quality when eliminating the stochastic constraints (5) on the model coefficients: less for the models of opinion dynamics (by ~7%) and more for the models of action dynamics (by ~15%). Probably, such a relaxation of the constraints for the latter models allows better considering bidirectional influences: opinions (the excitatory effect) and opposite actions (the inhibitory effect).

Third, adding the constant, the time step (possibly reflecting the change of attitudes in society with the pandemic evolution), and the volume of initial posts (possibly explaining an external informational "control") further improves the quality of the models: by 18–31% for opinion dynamics and by 2–8% for action dynamics. These factors seem to have an opinion-mediated effect on the actions of agents.

Thus, the following conclusions can be drawn. On the one hand, there is a two-way relationship between the actions and opinions of agents in the network; on the other, models should incorporate the factors external to the network as well.

3.2 Micro models

Let us model and analyze the dynamics of opinions and actions *at the micro level*. Such models describe the dynamics of opinions and actions of a separate agent. Consider agent $i \in N$ committing "for" and "against" acts during a time interval τ:

• $\delta_i^{\pm}(\tau) = \{a \in \delta_i \mid f_t(a) \in \tau,\ r'(a) \in \{0, 1\}\}$ is the set of his acts,

• $\delta_{i,1}^{\pm}(\tau) = \{a \in \delta_i^{\pm}(\tau) \mid f_k(a) = 1\}$ is the set of his comments, and

• $\delta_{i,2}^{\pm}(\tau) = \{a \in \delta_i^{\pm}(\tau) \mid f_k(a) = 2\}$ is the set of his likes.

Agent i is subjected to the following factors:

• *The influence of the entire network*, given by

○ $$\bar{r}_{-i}(\tau) = \frac{\sum_{a \in \Delta(\tau) \mid f_k(a)=1, r'(a) \in \{0,1\}} r(a)}{\left|\{a \in \Delta(\tau) \mid f_k(a) = 1, r'(a) \in \{0,1\}\}\right|} \in [-1, 1],$$

○ $$\bar{y}_{-i}(\tau) = \frac{\sum_{a \in \Delta(\tau) \mid f_k(a)=2, r'(a) \in \{0,1\}} r(a)}{\left|\{a \in \Delta(\tau) \mid f_k(a) = 2, r'(a) \in \{0, 1\}\}\right|} \in [-1, 1].$$

The network influence is mass or background for the agent: all opinions and actions of the network are considered regardless of the agent's knowledge of them.

• *The influence of the agent's own actions and opinions on himself*, given by

○ $$\bar{r}_i(\tau) = \frac{\sum_{a \in \delta_{i,1}^{\pm}(\tau)} r(a)}{\left|\delta_{i,1}^{\pm}(\tau)\right|},$$

○ $$\bar{y}_i(\tau) = \frac{\sum_{a \in \delta_{i,2}^{\pm}(\tau)} r(a)}{\left|\delta_{i,2}^{\pm}(\tau)\right|}.$$

• The indirect influence of friends on the agent's opinion/action $h \in [-1,1]$, given by

○ $$\bar{r}_{N_i}(\tau) = \sum_{j \in N_i,\ \delta_{j,1}^{\pm}(\tau) \neq \varnothing} e_{ij} \frac{\sum_{a \in \delta_{j,1}^{\pm}(\tau)} E_i(h, r(a)) r(a)}{\left|\delta_{j,1}^{\pm}(\tau)\right|},$$

○ $$\bar{y}_{N_i}(\tau) = \sum_{j \in N_i,\ \delta_{j,2}^{\pm}(\tau) \neq \varnothing} e_{ij} \frac{\sum_{a \in \delta_{j,2}^{\pm}(\tau)} E_i(h, r(a)) r(a)}{\left|\delta_{j,2}^{\pm}(\tau)\right|},$$

where $e_{ij} \in [0, 1]$ is the trust of agent i in his friend j, $\sum_{j \in N_i} e_{ij} = 1$, and E_i denotes the information trust function of agent i (his trust in the information content with the range [0, 1]). Here, the influence on the agent's opinion (or action) is estimated at the beginning of the interval τ.

As a result, the change in the opinion of agent $i \in N$ between successive time instants (m – 1) and m (on the time interval $\tau = [t_{m-1}, t_m)$) is determined by the influence of the entire network ($\bar{r}_{-i}^{m-1} = \bar{r}_{-i}(\tau)$, $\bar{y}_{-i}^{m-1} = \bar{y}_{-i}(\tau)$), the influence of the agent's own actions ($\bar{y}_i^{m-1} = \bar{y}_i(\tau)$), and the influence of his friends ($\bar{r}_{N_i}^{m-1} = \bar{r}_{N_i}(\tau)$, $\bar{y}_{N_i}^{m-1} = \bar{y}_{N_i}(\tau)$). In turn, the change in the action of agent $i \in N$ between successive time instants l – 1 and l (on the time interval $\tau = [t_{l-1}, t_l)$) is determined by the influence of the entire network $(\bar{r}_{-i}^{l-1} = \bar{r}_{-i}(\tau)$, $\bar{y}_{-i}^{l-1} = \bar{y}_{-i}(\tau))$, the influence of the agent's own opinions ($\bar{r}_i^{l-1} = \bar{r}_i(\tau)$), and the influence of his friends $(\bar{r}_{N_i}^{l-1} = \bar{r}_{N_i}(\tau)$, $\bar{y}_{N_i}^{l-1} = \bar{y}_{N_i}(\tau))$.

We construct the corresponding micro models of the joint dynamics of opinions and actions. For example, a possible model of opinion dynamics has the form

$$r_i^m = (1 - \beta_{i1} - \beta_{i2} - \beta_{i3}) r_i^{m-1} + \beta_{i1}\bar{r}_{-i}^{m-1} + \beta_{i2}\bar{y}_i^{m-1} + \beta_{i3}\bar{y}_{-i}^{m-1}, \quad (7)$$

where $m = 1, 2, \ldots$, $\beta_{i1}, \beta_{i2}, \beta_{i3} \ge 0$, $\beta_{i1} + \beta_{i2} + \beta_{i3} \le 1$.

Possible modifications of the models of opinion dynamics (7) are combined in Table 2. It is necessary to estimate the corresponding dependences between their variables. A similar table can be compiled for possible modifications of the models of action dynamics.

The micro models of the dynamics of opinions and actions presented below were built for *significant agents*.[3] Additional information was collected about the friends of significant agents to assess their influence: the posts published by friends (for the period specified earlier) as well as the comments and likes for them. The collected data were used to identify the opinions and actions of friends.

Linear models. In linear micro models, the agent "averages" his opinion with the opinions of agents he interacts with and trusts (agrees his opinion with them). We consider several linear micro models, special cases of model (2)–(3), in ascending order of their complexity:

(I) micro models where changes in opinions and actions are due to the influence of the network;

[3] The agents who showed the minimum network activity required for modeling. The criteria for selecting significant agents were described in part I of the study [1].

Table 2

Micro models of opinion dynamics: some modifications

Dependences between variables		$\bar{r}_{-i}^{m-1}$	$\bar{y}_i^{m-1}$	$\bar{y}_{-i}^{m-1}$
I	$r_i^m(r_i^{m-1})$			
II	$r_i^m(r_i^{m-1}, \bar{r}_{-i}^{m-1})$	+		
III	$r_i^m(r_i^{m-1}, \bar{y}_i^{m-1})$		+	
IV	$r_i^m(r_i^{m-1}, \bar{y}_{-i}^{m-1})$			+
V	$r_i^m(r_i^{m-1}, \bar{r}_{-i}^{m-1}, \bar{y}_i^{m-1})$	+	+	
VI	$r_i^m(r_i^{m-1}, \bar{r}_{-i}^{m-1}, \bar{y}_{-i}^{m-1})$	+		+
VII	$r_i^m(r_i^{m-1}, \bar{y}_i^{m-1}, \bar{y}_{-i}^{m-1})$		+	+
VIII	$r_i^m(r_i^{m-1}, \bar{r}_{-i}^{m-1}, \bar{y}_i^{m-1}, \bar{y}_{-i}^{m-1})$	+	+	+

(II) unified micro models, which include the influence of the network and friends;

(III) personalized micro models (besides the influence of the network and friends, the agents' trust functions reflect their individual characteristics).

(I) Linear micro models with macro variables

In these models, we estimate the change in the agent's opinions/actions at the current time instant depending on his opinions and actions at the previous time instant and the background influence of the entire network:

$$r_i^m = \beta_{i10} + \beta_{i11} r_i^{m-1} + \beta_{i12}\bar{r}_{-i}^{m-1} + \beta_{i13}\bar{y}_i^{m-1} + \beta_{i14}\bar{y}_{-i}^{m-1}, \quad (8)$$

where $m = 1, 2, \ldots$[4] and $\beta_{i10}, \beta_{i11}, \beta_{i12}, \beta_{i13}, \beta_{i14} \in \mathbb{R}$;

$$y_i^l = \beta_{i20} + \beta_{i21}\bar{r}_i^{l-1} + \beta_{i22}\bar{r}_{-i}^{l-1} + \beta_{i23} y_i^{l-1} + \beta_{i24}\bar{y}_{-i}^{l-1}, \quad (9)$$

where $l = 1, 2, \ldots$, and $\beta_{i20}, \beta_{i21}, \beta_{i22}, \beta_{i23}, \beta_{i24} \in \mathbb{R}$.

For the purposes of this paper, it is of little importance to analyze the coefficients of the micro models (hence, their statistical significance). We are interested in the contributions of certain factors assessed by comparing different micro models of the dynamics of opinions and actions.

Figure 12 shows *the quality graph* of the family of models (8), (9); each vertex corresponds to the models with the explanatory variables indicated. Here, the constant is zero, and the other coefficients are "stochastic" (nonnegative values making up 1 in sum). For each vertex, the average error of its models is given.[5]

[4] For each agent, of course, these time instants differ.

[5] Recall that the error is given by (6).

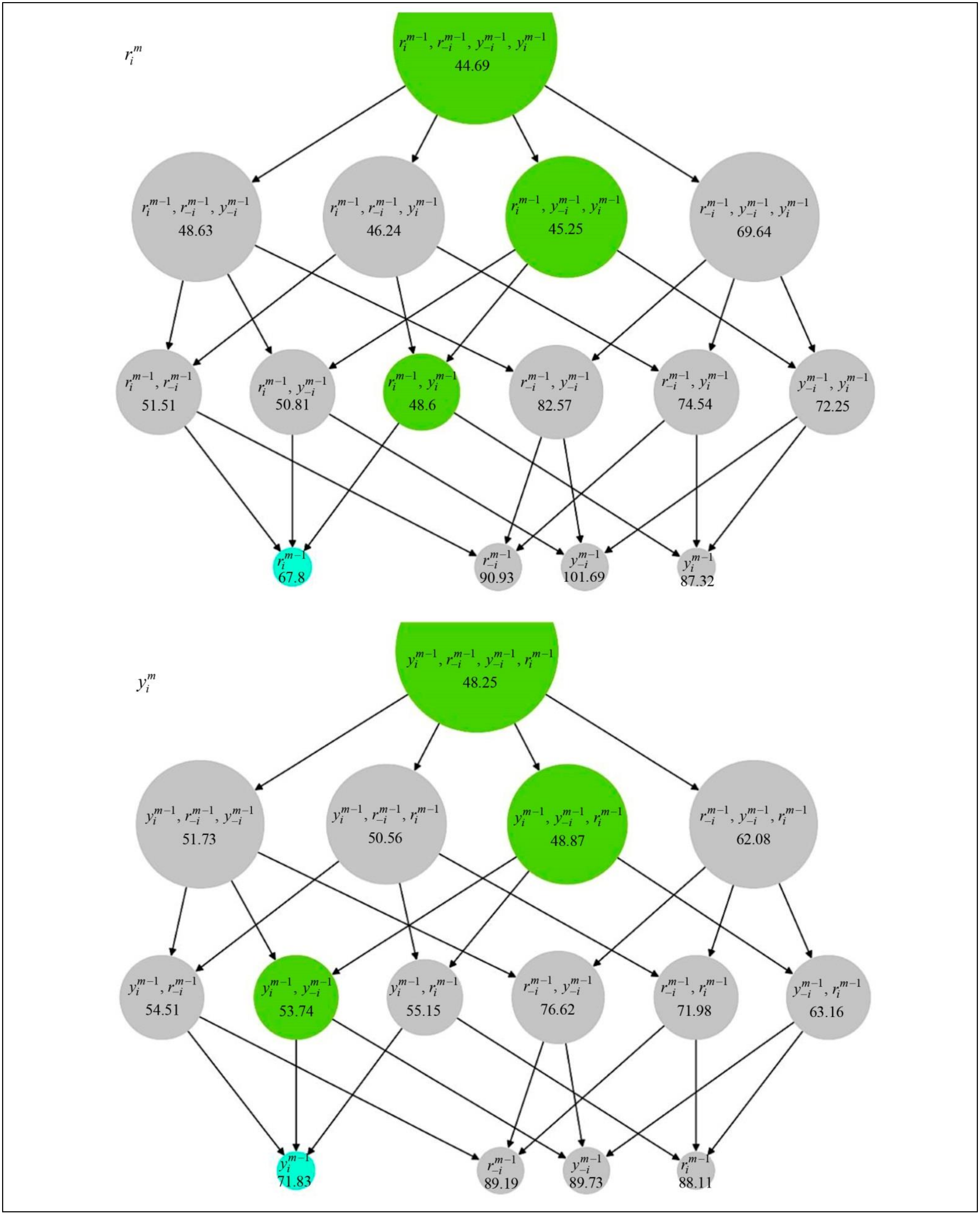

Fig. 12. The average quality of models (8), (9).

Figure 13 presents the quality graph of models (8), (9) without stochastic constraints on the coefficients.

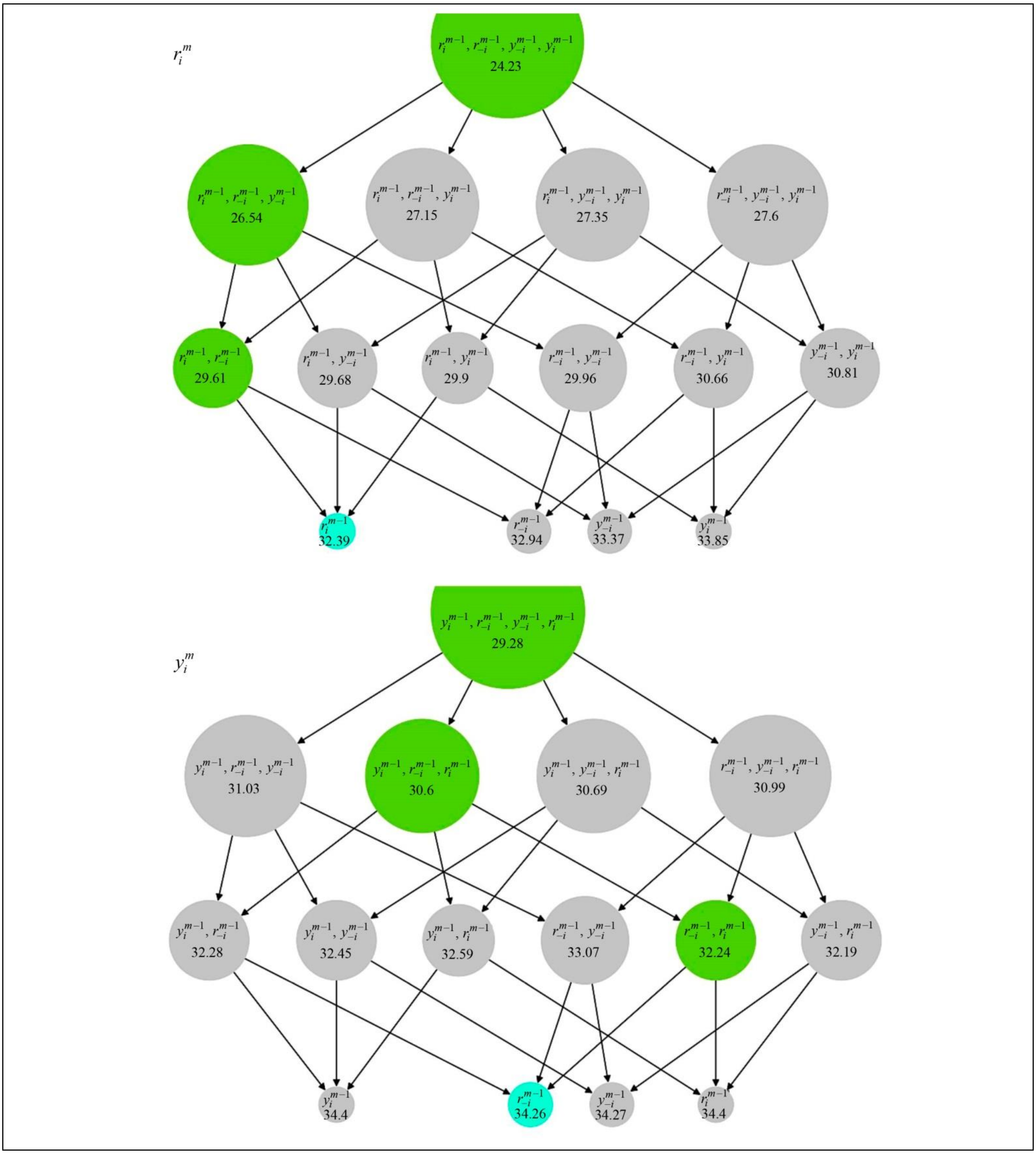

Fig. 13. The average quality of models (8), (9) (without stochastic constraints and with the constant).

We compare models (8), (9) with the *inertial model*[6] by quality. Figure 14 shows the distribution of these models by the error reduction.

[6] Recall that in the inertial model, "tomorrow" coincides with "today."

Considering the influence of the network and the agent's self-influence significantly improves the quality of the micro models of opinions and actions: for half of the agents, the error value is reduced by at least 60% (opinion dynamics) and by 57% (action dynamics) compared to the inertial model; for one-fifth of the agents, by at least 70% and 68% respectively. Figure 15 demonstrates an example of the dynamics for one agent.

According to this example, the influence of the network (the opinions and actions of the other agents) changes the agent's opinion.

(II) Unified linear micro models

In such models, we estimate the change in the agent's opinions and actions depending on his opinions and actions at the previous time instant, the influence of the entire network, and the influence of his friends. Each agent does not "distinguish" between friends (treating friends as one meta-agent) and trusts the information source regardless of its content:

$$\begin{aligned} r_i^m &= \beta_{i10} + \beta_{i11} r_i^{m-1} + \beta_{i12} \overline{r}_{-i}^{m-1} + \beta_{i13} \overline{y}_i^{m-1} \\ &+ \beta_{i14} \overline{y}_{-i}^{m-1} + \beta_{i15} \overline{r}_{N_i}^{m-1} + \beta_{i16} \overline{y}_{N_i}^{m-1}, \end{aligned} \tag{10}$$

where $m = 1, 2, \ldots, \beta_{i10}, \beta_{i11}, \beta_{i12}, \beta_{i13}, \beta_{i14}, \beta_{i15}, \beta_{i16} \in \mathbb{R}$;

$$\begin{aligned} y_i^l &= \beta_{i20} + \beta_{i21} \overline{r}_i^{l-1} + \beta_{i22} \overline{r}_{-i}^{l-1} + \beta_{i23} y_i^{l-1} \\ &+ \beta_{i24} \overline{y}_{-i}^{l-1} + \beta_{i25} \overline{r}_{N_i}^{l-1} + \beta_{i26} \overline{y}_{N_i}^{l-1}, \end{aligned} \tag{11}$$

where $l = 1, 2, \ldots,$ β_{i20}, β_{i21}, β_{i22}, β_{i23}, β_{i24}, β_{i25}, $\beta_{i26} \in \mathbb{R}$.

In addition to the coefficients, the types of trust functions are selected in personalized models when calculating the influence of friends (the variables $\overline{r}_{N_i}^{m-1}$ and $\overline{y}_{N_i}^{m-1}$). The unified models use the same trust function for all agents when calculating the variables.

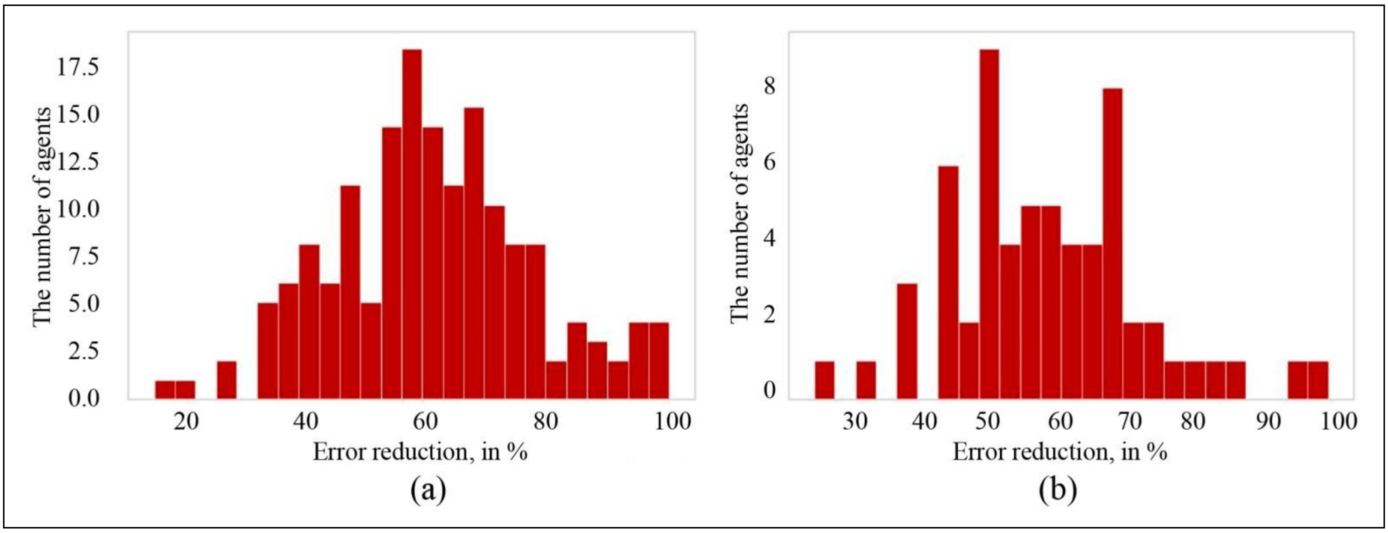

Fig. 14. The distribution of models by error reduction compared to the inertial model: (a) opinion dynamics and (b) action dynamics.

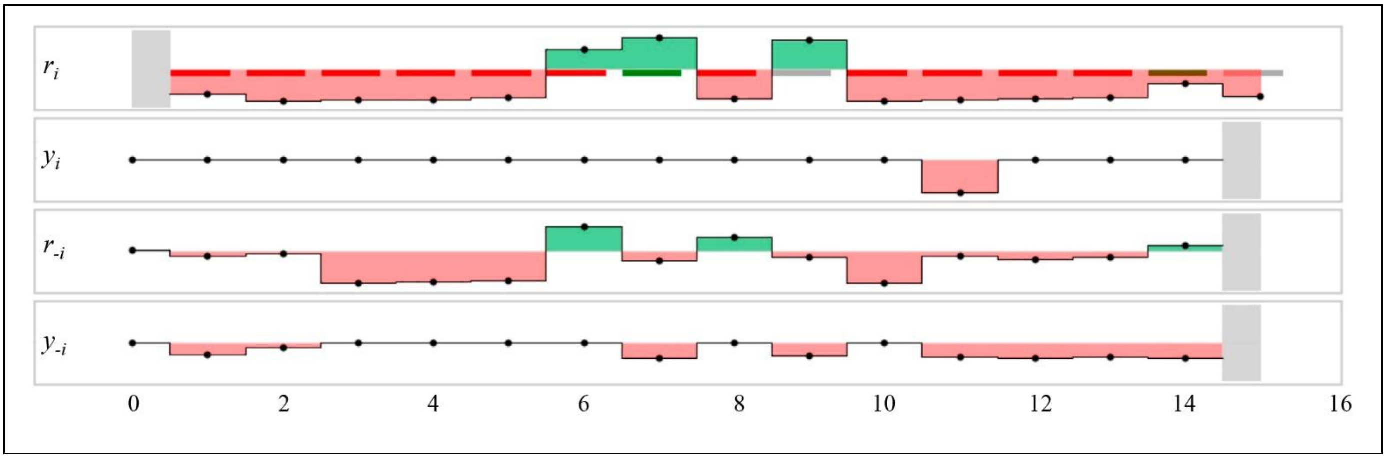

Fig. 15. The dynamics of variables for one agent: an example.

We compare models (10), (11) with the *inertial model* by the error reduction (Fig. 16).

Considering the opinions and actions of agent's friends improves the quality of the micro models of opinions and actions: for half of the agents, the error value is reduced by at least 67% (opinion dynamics) and 61% (action dynamics) compared to the inertial model; for one-fifth of the agents, by at least 85% and 75%, respectively (on average, by 68% and 62%).

If there is no need or possibility to include all variables in the model, then it suffices to choose some subset of variables with the greatest error reduction. To assess this approach, we determine the best-on-average models depending on the number of explanatory variables. The average error of the models depending on the number of variables is shown in Fig. 17. (The horizontal axis corresponds to the variables included in the model in the optimal sequence: the decreasing curves are convex.)

Note that the inertial model is the best-on-average micro model of opinion dynamics with one explanatory variable; the model where the action depends on the opinions of the entire network is the best-on-average model of action dynamics with one explanatory variable.

Next, let us estimate the significance of each individual variable. Fixing one of the independent variables, we consider the best-on-average models without it. How much will the quality of each class of models (with one, two, three variables, etc.) be improved by adding the fixed variable? The gains in the quality of models are shown in Fig. 18 (opinion dynamics) and Fig. 19 (action dynamics). The model error is given by (6).

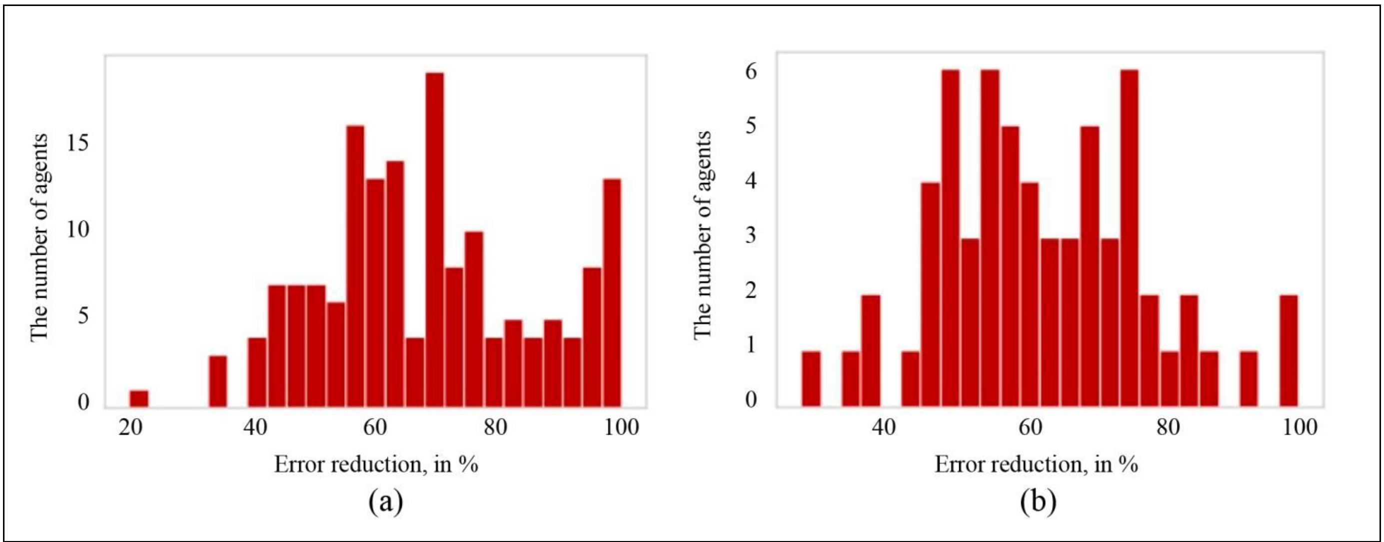

Fig. 16. The distribution of models by error reduction compared to the inertial model: (a) opinion dynamics and (b) action dynamics.

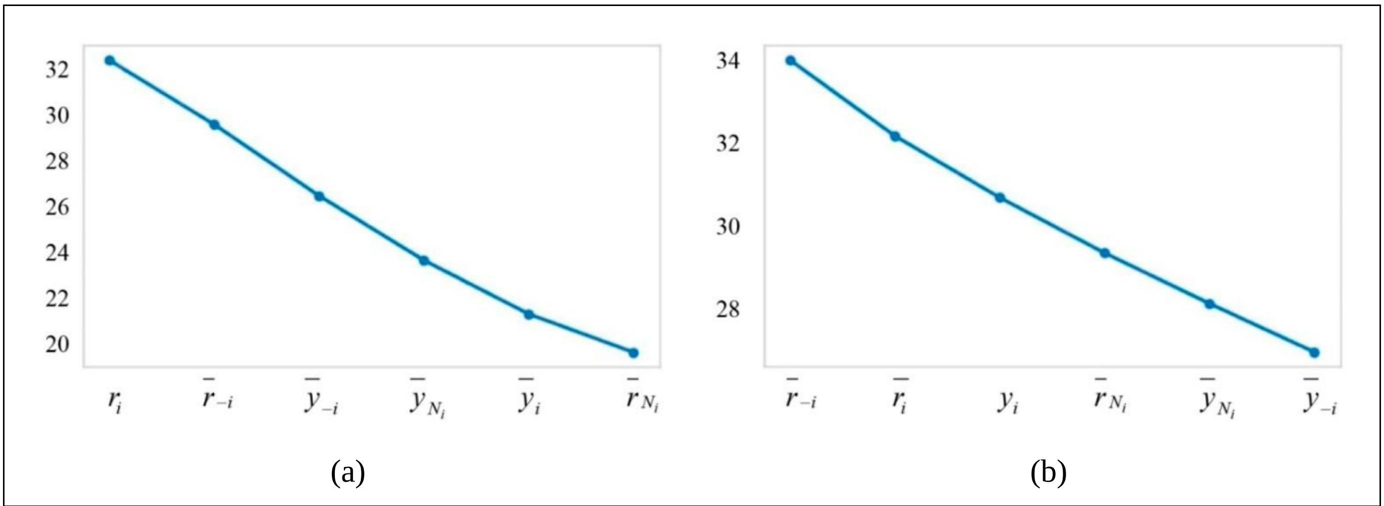

Fig. 17. The average quality of the family of models (10), (11): (a) r_i^m and (b) y_i^m .

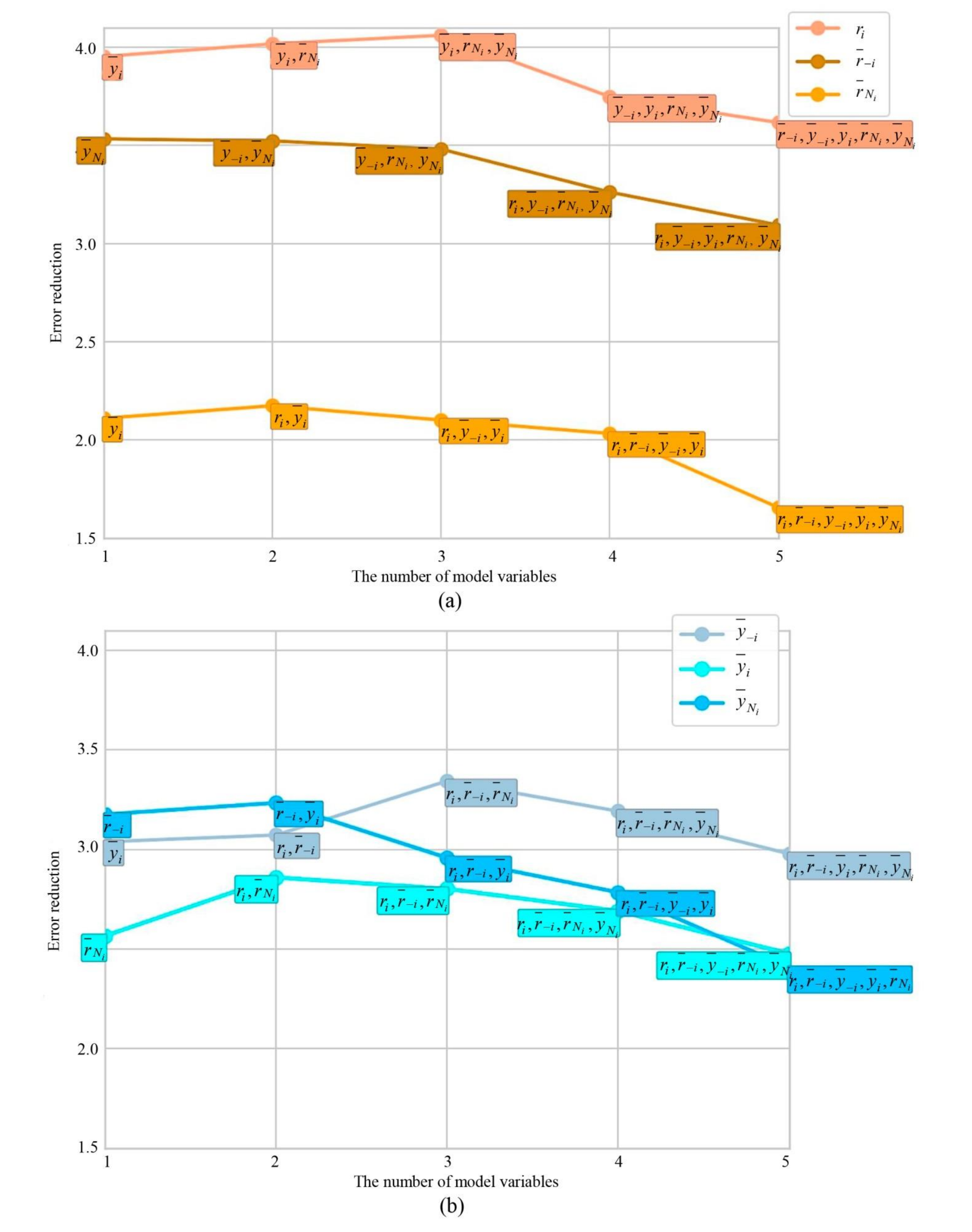

Fig. 18. The models of opinion dynamics: quality improvement using given variables: (a) fixed opinion variables and (b) fixed action variables. (The horizontal axis corresponds to the variables included in the model whereas the vertical axis to the error reduction.)

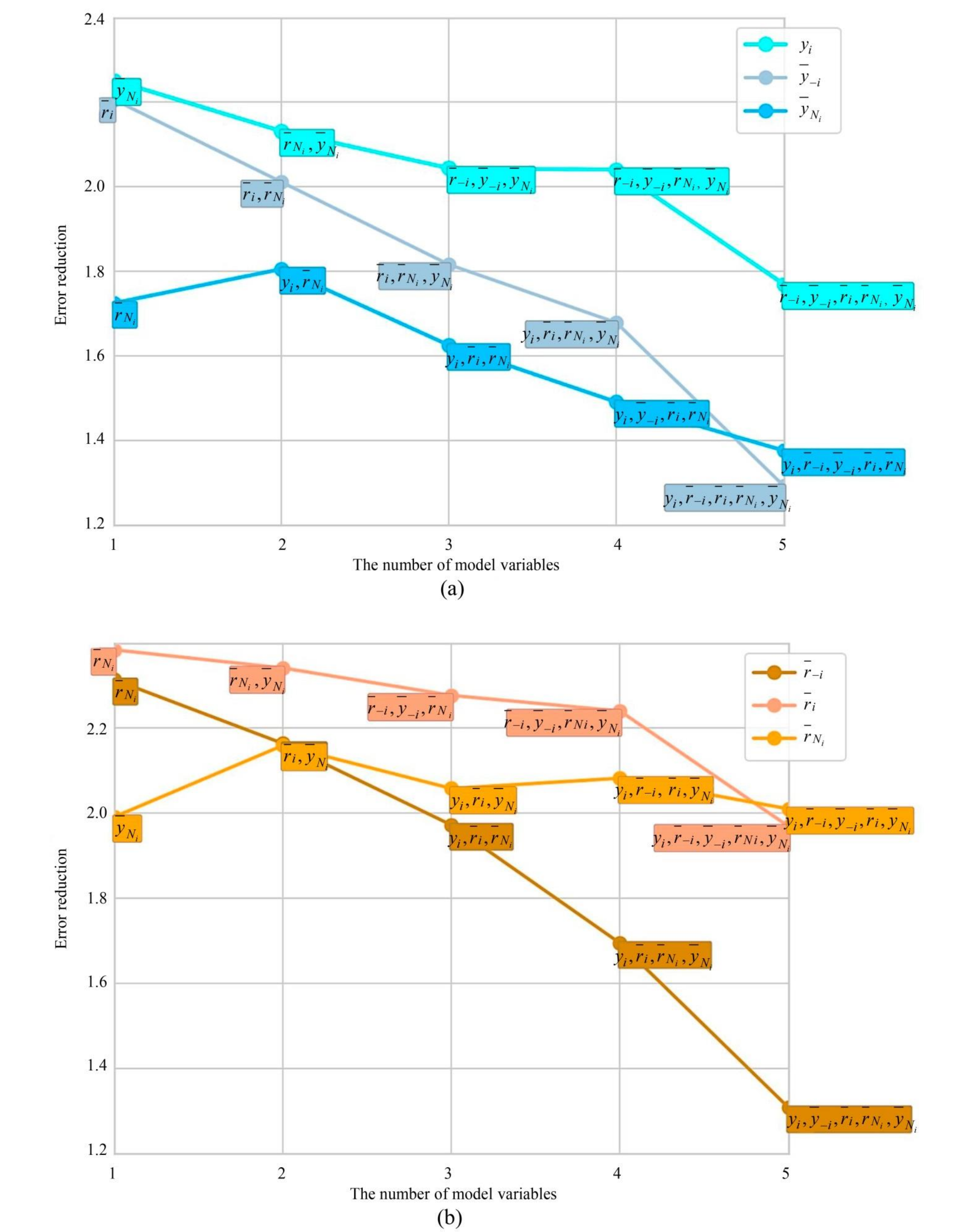

Fig. 19. The models of action dynamics: quality improvement using given variables: (a) fixed opinion variables and (b) fixed action variables. (The horizontal axis corresponds to the variables included in the model whereas the vertical axis to the error reduction.)

Obviously, *for the micro models of opinion dynamics, the greatest effect is given by the variable* r_i^{m-1} (*the opinion at the previous time instant*); *the smallest one, by the variable* $\bar{r}_{N_i}$ (*the opinions of friends*).

Generally speaking, including individual variables in the models of action dynamics yields a lower gain than including individual variables in the models of opinion dynamics. For the models of action dynamics, the largest error reduction is given by the variable $\bar{r}_i^{m-1}$ (the agent's opinions); the smallest one, by the variable $\bar{y}_{N_i}$ (the actions of friends). Note that the effect from adding variables in the model decreases but non-monotonically.

(III) Personalized linear micro models

In personalized models, the best configuration of hyperparameters is chosen for each agent: the types of his trust functions for information and friends.

Consider the following common types of information trust functions[7]:

- $E_C(h,g)=1$ (*simpletons*);
- $E_\varepsilon^+(h,g)=\begin{cases}1-(h-g)^2, & h,g\in(0,1),\\ \varepsilon>0, & (h-g)^2=1,\end{cases}$

where ε > 0 is an arbitrarily small and strictly positive constant (*conservatives*);

- $E_\varepsilon^-(h,g)=\min\{\varepsilon+(h-g)^2,1\}$, where ε > 0 (*innovators*).

As a common type of the friend *trust* function *e*(·), we consider a "non-differentiating" function (the agent treats his friends as one meta-agent) and functions where the trust in a friend is proportional to[8]:

– unity;
– the number of friends of this friend;
– the number of friends shared with this friend;
– unity if this friend is active (i.e., has an opinion or commits acts);
– the number of friends of this friend if he is active;
– the number of friends shared with this friend if he is active;
– the friend's activity by comments;
– the friend's activity by likes or his popularity.

Models of opinion dynamics. Due to the individually selected trust functions, the error value of the inertial model was reduced by 77% on average (by at least 78% for half of the agents and by 97% for one-fifth of the agents; see Fig. 20).

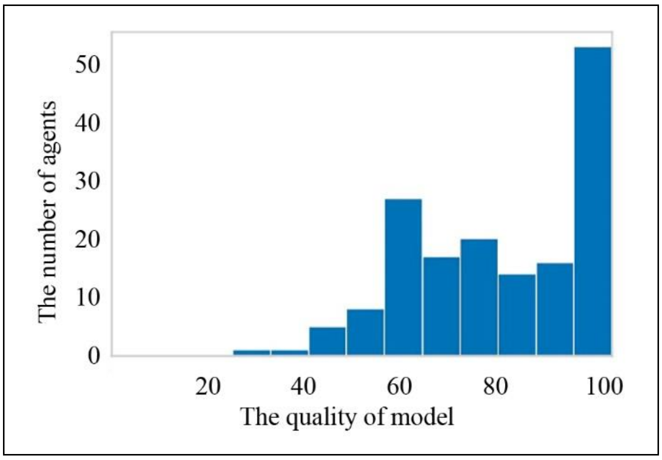

Fig. 20. The distribution of personalized models of opinion dynamics by error reduction (in % compared to the inertial model).

In general, the average error of the linear micro models of opinion dynamics decreases as they become more complex (Fig. 21): the largest error is produced by the inertial model; a moderate error, by the model with macro variables and the unified model; the smallest error, by the personalized model. The transition from the unified model to the personalized one reduces the error from 20 to 14. (Recall that the error (6) takes values between 0 and 100.)

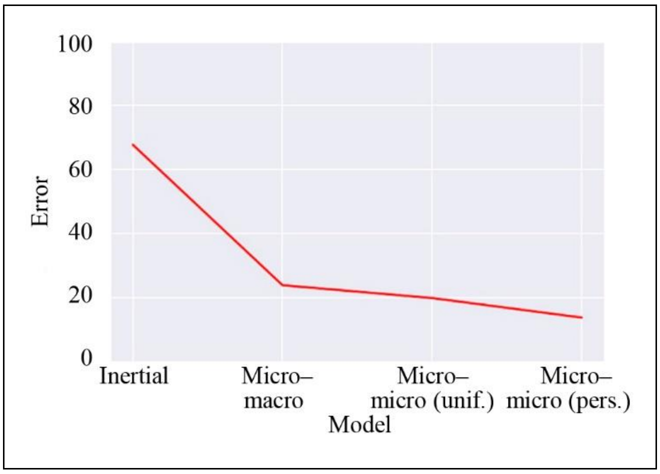

Fig. 21: Quality improvement for linear opinion models.

Models of action dynamics. Due to the individually selected trust functions, the error value of the inertial model was reduced by 74% on average (by at least 74% for half of the agents and by 92% for one-fifth of the agents; see Fig. 22). In this case, the transition from the unified model to the personalized one reduces the error from 26 to 17.

[7] These trust functions reflect the main hypotheses about the behavior of individuals in social networks that are accepted in modern studies.

[8] It is needed to ensure the "stochastic" constraint.

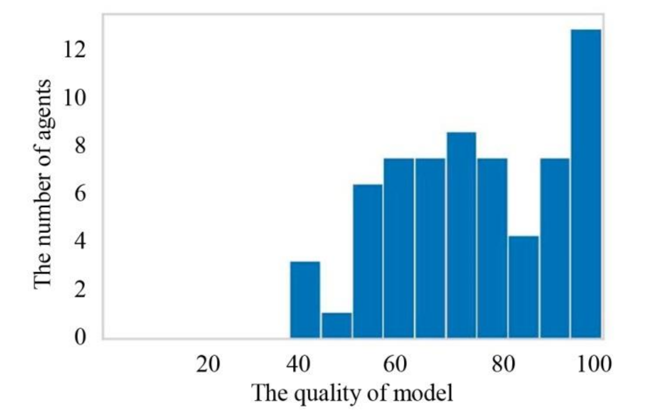

Fig. 22. The distribution of personalized models of action dynamics by error reduction (in % compared to the inertial model).

The conclusions for the micro models of action dynamics are similar to those for the micro models of opinion dynamics (Fig. 23): the largest average error corresponds to the inertial model; a moderate error, to the model with macro variables and the unified model; the smallest error, to the personalized model.

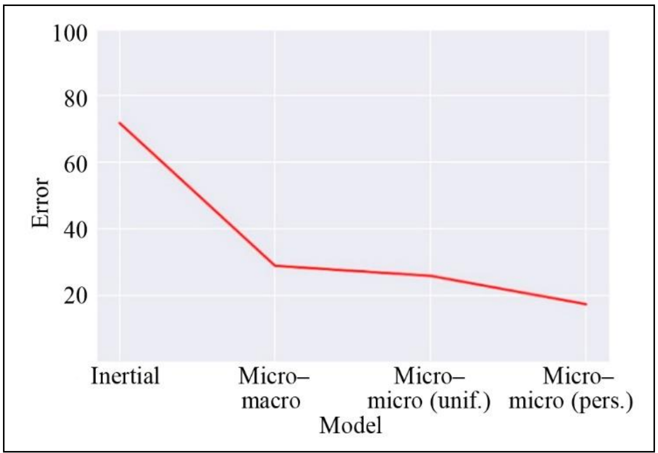

Fig. 23: Quality improvement for linear action models.

These results provide a partial answer to Question no. 6 from the Introduction. Let us proceed to Question no. 7.

Significance analysis of trust functions, the classes of agents. We assess the preferability of different types of trust functions and separate the classes of agents based on their trust in friends and the incoming information. Note that the properties of the agent's trust in his friends are one of the most important issues in the contemporary modeling of opinion dynamics in social networks.

A. Personalized models of opinion dynamics

The preferable type of the information trust function. As it turns out, for 30% of agents, the choice of the trust function does not affect the quality of the best model for the agent. For one-third of the agents (36%), the best choice is the trust function E_{ε}^{-} (the more the estimates differ, the greater the trust will be). For one-quarter of the agents (25%), the best choice is the function E_{ε}^{+} (the less the estimates differ, the greater the trust will be). Figure 24 shows the Euler–Venn diagram for three sets of agents with the most preferable trust functions E_{ε}^{+}, E_{ε}^{-}, and E_C, respectively.

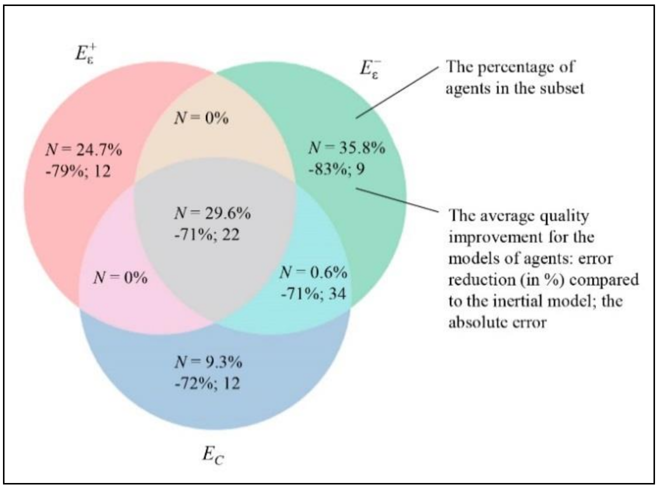

Fig. 24. Information trust functions in the models of opinion dynamics: the Euler–Venn diagram of preferability.

Considering the models for female (69) and male (93) agents separately shows the following. The share of those for whom the type of the information trust function does not matter is higher[9] among males (30% vs. 25%); the share of conservatives is higher in this group as well (29% vs. 20%). On the other hand, the share of innovators is higher among females (45% vs. 30%). About 10% of the agents in each group are simpletons.

The preferable type of the friend trust function. The preferability of three types of the trust function $e_{ij}(\cdot)$ are estimated as follows:

- e_M (the agent perceives the friends as a whole; his trust in such a meta-agent is 1);
- e_U (all friends are equally significant for the agent ($e_{ij} = 1/|N_i|,\ j \in N_i$);
- e_D (the friends with more friends have higher significance: $e_{ij} = \dfrac{|N_j|}{\sum_{k \in N_i} |N_k|}$, $j \in N_i$).

For 28% of agents, the type of the friend trust function is not important. A larger group of agents

[9] In the sense of improving the quality of the best model.

(35%) does not "distinguish" between friends. About 16% of agents better trust friends with more friends, and 15% of agents trust friends equally (Fig. 25).

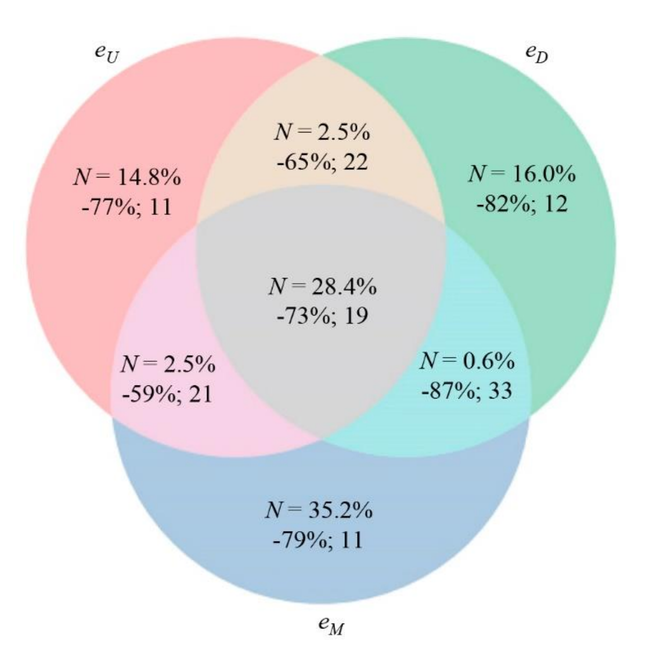

Fig. 25. Friend trust functions in the models of opinion dynamics: the Euler–Venn diagram of preferability.

Considering the models for female and male agents gives the following results. The share of those for whom the type of the friend trust function does not matter is higher[10] among males (31% vs. 22%); the share of those who do not distinguish between friends is higher in this group as well (40% vs. 30%). On the other hand, the share of those who equally trust friends is higher among females (23% vs. 9%). About 16% of agents in each group better trust friends with more friends.

B. Personalized models of action dynamics

The preferable type of the information trust function. For 15% of the agents, the type of the information trust function is not important. The majority of the agents (61%) are "pure" innovators. One-fifth of the agents (19%) are "pure" conservatives. Figure 26 shows the Euler–Venn diagram for the three sets of agents (conservatives, innovators, and simpletons).

Considering the models for female (30) and male (32) agents separately shows the following. The share of those for whom the type of the information trust function does not matter is higher among males (16% vs. 10%); the share of simpletons is higher in this group as well (10% vs. 0%). On the other hand, the share of innovators is higher among females (67% vs. 56%). About 20% of the agents in each group are conservatives.

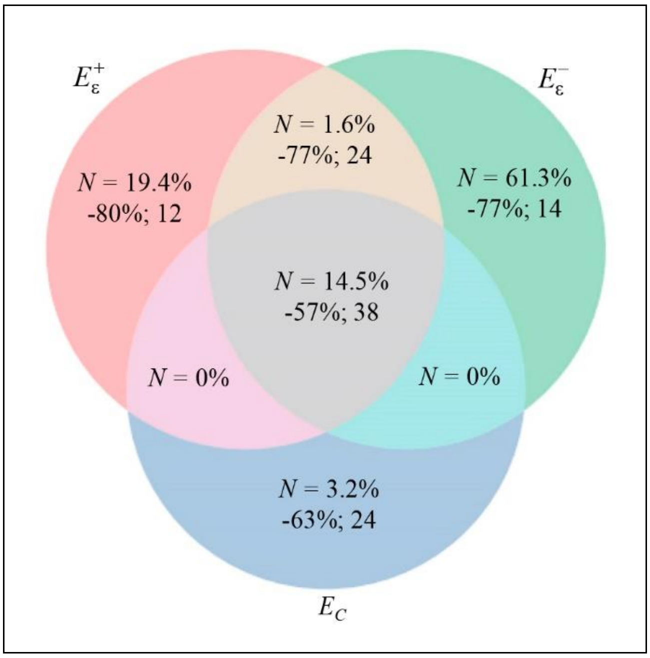

Fig. 26. Information trust functions in the models of action dynamics: the Euler–Venn diagram of preferability.

The preferable type of the friend trust function.

The preferability of three types of the trust function $e_{ij}(\cdot)$ is estimated as follows:

- e_M (the agent perceives the friends as a whole);
- e_U (all friends are equally significant for the agent);
- e_D (the friends with more friends have higher significance).

For 13% of the agents, the type of the friend trust function is not important. The largest group of agents (40%) does not "distinguish" between friends. About one-quarter of the agents (26%) better trust friends with more friends (Fig. 27).

Considering the models for female and male agents gives the following results. The share of those for whom the type of the friend trust function does not matter is higher among males (16% vs. 10%); the share of those who equally trust friends is higher in this group as well (19% vs. 13%). On the other hand, the share of those who do not distinguish between friends is higher among females (47% vs. 38%). The shares of agents better trusting their friends with more friends are almost the same: 27% (females) and 25% (males).

[10] In the sense of improving the quality of the best model.

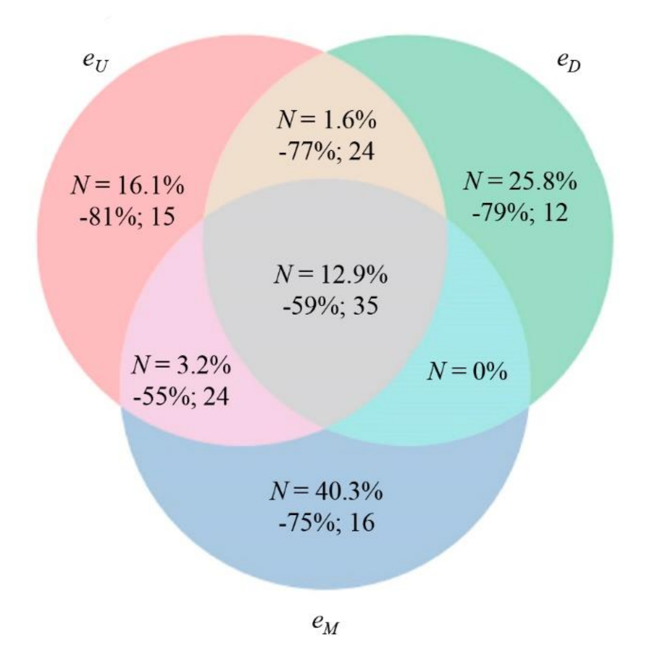

Fig. 27. Friend trust functions in the models of action dynamics: the Euler–Venn diagram of preferability.

General conclusions on personalized linear models

For the majority of males, the type of the trust function does not matter. The share of such agents increases when considering opinion dynamics and decreases when considering action dynamics.

The share of those who trust the information diverging from their opinion or action prevails; for females, this share is higher. Simpletons form the minority. (Recall that they trust the information regardless of its content.)

The share of those who do not distinguish between friends prevails as well (35–40%). When considering the dynamics of actions, the share of those who better trust friends with more friends increases (from 16% to 26% compared to the dynamics of opinions); this share is the same for both males and females. The shares of those who equally trust their friends are nearly the same (15% and 16%) in the models of opinion and action dynamics.

Finally, we illustrate the quality of the linear micro models of agents' opinions and actions in Fig. 28:

– the inertial model ("Inertial"),

– the models without the variables reflecting the influence of friends (with one, two, three, and four explanatory variables);

– the unified model with the influence of friends ("Unif.");

– the personalized model with the influence of friends ("Pers.").

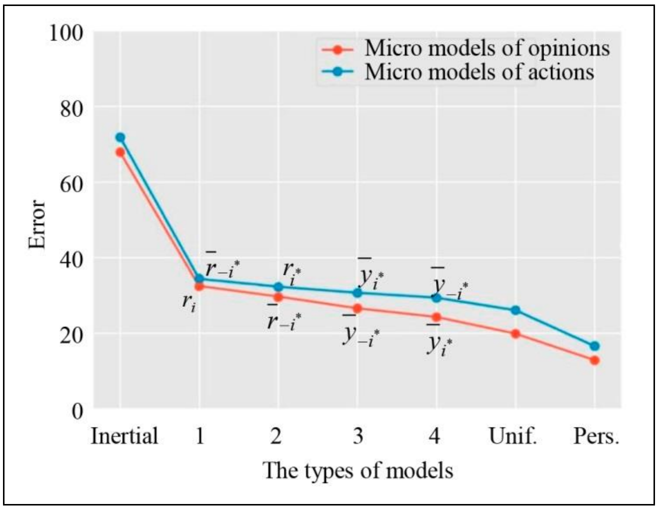

Fig. 28. The quality of linear micro models of agents' opinions and actions.

CONCLUSIONS

This paper has investigated linear models of the joint dynamics of opinions and actions of agents on the example of their attitude toward wearing medical masks in the *VKontakte* online social network during the first year of the COVID-19 pandemic. The following results have been established.

The formal models of opinion and action dynamics have been verified (see Question no. 6 in the Introduction). First, different modifications of macro models have been considered, where "public" opinion and action in the network (i.e., the share of "for" or "against" opinions and actions) at the current time instant depends on the opinions and actions at the previous time instant (subsection 3.1). Second, different modifications of linear micro models have been considered, where the influence of the entire network, the influence of agent's friends and individual characteristics are reflected (subsection 3.2). According to the identification results for the macro models, first, there is a relationship between the actions and opinions in the network; second, external factors should be included in the network model.

Micro models are considered only for significant agents. The quality of such micro models is acceptable and becomes even better when increasing their complexity. In the class of linear micro models, the largest error is given by the inertial model; a moderate error, by the model with macro variables and the unified model; the smallest error, by the personalized model (in which each agent can have its own trust function).

In the case of personalized models, the prevailing share of agents corresponds to those who better trust the information diverging from their opinion or action. Note that the share of such agents is higher among females. Fewer agents trust information regardless of its content. Concerning trust in friends, agents predominantly do not distinguish the actions of their friends; many agents better trust friends with more friends; a few agents equally trust their friends (Question no. 7).

Part III of this study, the final one, will be devoted to the identification of binary micro models and the comparison of linear and threshold models (Questions nos. 4, 6, and 7).

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