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ROBUST CONTROLLER DESIGN ENSURING THE DESIRED APERIODIC STABILITY DEGREE OF A CONTROL SYSTEM WITH AFFINE UNCERTAINTY

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Abstract. This paper considers a system whose characteristic polynomial coefficients are linear combinations of the interval parameters of a plant forming a parametric polytope. A linear robust controller is parametrically designed to place a dominant pole of the system within the desired interval of the negative real semi-axis and ensure an aperiodic transient in the system. The parametric design procedure involves a low-order controller with dependent and free parameters: the former serve to place the dominant pole within the desired interval on the complex plane whereas the latter to shift the other poles to some localization regions beyond a given bound (to the left of the dominant pole to satisfy the pole dominance principle). To evaluate the dependent parameters of the controller, the originals of the interval bounds of the dominant pole are determined for the plant's parametric polytope based on a corresponding theorem (see below). The free parameters of the controller are chosen using the robust vertex or edge *D*-partition method, depending on the boundary edge branches of the localization regions of the free poles. A numerical example of the parametric design procedure is provided: a PID controller is built to ensure an acceptable aperiodic transient time in a load-lifting mechanism with interval values of cable length and load weight.

Keywords: robust control, affine uncertainty, modal control, aperiodic transient.

INTRODUCTION

As is known, ensuring a given performance of an automatic control system (ACS) is possible based on the desired placement of its poles, implemented by the modal controller design using the characteristic polynomial of the system. In such cases, the most frequently solved problem is to ensure an aperiodic transient of a given duration in the system. With aperiodic transients in the system, one decreases the amount of energy to bring the plant to the desired state as well as reduces the wear of the actuator. Let us consider known methods for solving this problem for systems with deterministic parametric uncertainty [1–29]. It is logical to classify the design approaches described therein by the type of controller and the order of the plant. In particular, linear controllers with constant parameters [1–10], adaptive controllers [12–14], controllers based on fuzzy logic [15–17], and neural network controllers [18–20] were applied. In turn, linear plants of given orders or nonlinear plants with the linear part of a given order were considered in [4–10]; no constraints on the order of the plant or its linear part were imposed in the other works. A criterion for the aperiodicity of systems with interval parameters was given in [11].

According to the analysis of the publications cited above, an aperiodic transient is most often ensured using linear controllers of different structures whose order coincides with that of the plant or its linear part: common controllers in the classical or modified form and polynomial controllers. In such cases, it is possible to derive symbolic expressions for calculating the controller parameters [2, 4, 5, 8, 10]. The criterion presented in [11] verifies whether all poles of a system with interval parameters lie on the real axis. The desired placement of all poles requires controllers whose order depends on the number of system poles. The implementation of full-order controllers is often complicated due to the impossibility of directly measuring the derivatives of the output, which are necessary to form the control signal.

In addition, there are known methods for designing linear low-order controllers, which have no constraints on the order of the plant and are based on the pole dominance principle [21–23]. The disadvantage of these methods is the increased conservatism of the designed system due to the interval uncertainty of the characteristic polynomial coefficients.

Controllers based on fuzzy logic and neural network controllers were also designed for plants of arbitrary order [15–20]. However, such controllers are more difficult to implement than common linear controllers of both low and full order.

Thus, it is topical to design common linear controllers of low order that ensure an aperiodic transient of a given duration in systems with deterministic parametric uncertainty without restricting the order of the plant. To reduce the conservatism of the designed system, it is topical to consider the methods of placing the poles of systems with affine uncertainty of the coefficients of the interval characteristic polynomial.

If aperiodic transients of a given duration are required in an ACS, the pole dominance principle should be applied in the controller design: the real pole corresponding to the transient time is selected as the dominant one, and the other (free) poles are shifted on the left of it beyond some bound.

If the plant of the ACS has uncertain parameters whose values change during the system operation within specified intervals according to a priori unknown laws, there arises the problem of preserving aperiodic transients for any possible values of the interval parameters. When solving this problem, one should keep in mind that the system poles migrate inside their localization regions. Therefore, with the designed controller, the real dominant pole should be localized within some interval on the real negative semi-axis. At the same time, the localization regions of the free poles should be removed to a sufficient distance from this interval. Such localization of the poles ensures the completion of the ACS transients in an admissible time for any values of the interval parameters of the plant.

1. PROBLEM STATEMENT

For a time-invariant ACS, the pole placement principle (see above) can be implemented by a low-order controller based on the methodology proposed in [21]. According to it, the controller parameters are divided into dependent and free: the former set the dominant poles, whereas the latter shift all other poles to a certain region of the complex plane by the *D*-partition method.

Based on this approach, a methodology for designing a controller that ensures an aperiodic transient in a time-varying system with an interval characteristic polynomial was developed in [22, 23]. The coefficients of this polynomial are defined by limits found from the known intervals of the plant parameters and interval arithmetic rules. The coefficients form a parametric polytope, and the robust vertex *D*-partition method is applied at its vertices to select the free parameter of the controller.

However, the approach proposed in [21–23] allows an independent variation of the polynomial coefficients inside their polytope, making the resulting robust controller conservative. To reduce the conservatism of this approach, it is desirable to pass from the interval uncertainty of the characteristic polynomial to the affine one. Such a possibility exists if the polynomial coefficients are a linear combination of the interval parameters. In this case, the polytope of the interval parameters of the plant is considered instead of the polytope of the interval coefficients when designing the controller. It is mapped into the interval of the real dominant pole of the ACS and the localization regions of its free poles.

This paper aims to design a robust controller ensuring the aperiodic stability degree of an ACS with affine uncertainty based on determining, for the polytope of the plant parameters, the originals of the interval bounds of the system's dominant pole and the interval bounds of its free poles for their placement by the robust *D*-partition method [24].

2. MAPPING THE EDGES OF THE POLYTOPE OF THE INTERVAL PLANT PARAMETERS INTO THE ROOT PLANE

We write the characteristic equation of a system with interval parameters of the plant in the form

$$D(s)=\sum_{i=1}^{m}[T_i]A_i(s)+B(s)=0, \qquad (1)$$

where $[T_i]$ are the interval parameters, $\underline{T_i}=\min(T_i)$, and $\overline{T_i}=\max(T_i)$. Let $A_i(s)$ be polynomials of s, which corresponds to the affine uncertainty of the polynomial (1). We denote by $B(s)$ the sum of the characteristic polynomial terms not containing the interval parameters.

Since the m interval parameters T_i are given by bounds, they will vary arbitrarily within a parametric polytope representing the rectangular hyperparallelepiped $P_T=\left\{T_i \mid \underline{T_i}\le T_i\le \overline{T_i},\ i=\overline{1,m}\right\}$ with 2^m vertices. The coordinates of any point P_T relative to a vertex V_q, $q=\overline{1,2^m}$, are given by

$$T_i = T_i^q + \Delta T_i,\ i=\overline{1,m}, \qquad (2)$$

where ΔT_i is the increment of the ith parameter; T_i^q is the value of the ith parameter at the vertex V_q. Substituting the sum (2) into the expression (1) yields the equation

$$D^q(s) + \Delta T_1 A_1(s) + \Delta T_2 A_2(s) + ... + \Delta T_m A_m(s) = 0, \qquad (3)$$

where $D^q(s) = \sum_{i=1}^{m} T_i^q A_i(s) + B(s)$ is the polynomial at the vertex V^q. Based on formula (3), we write the mapping equation of an edge P_T outgoing from the vertex V^q when changing the value of the parameter T_i:

$$D^q(s) + \Delta T_i A_i(s) = 0. \qquad (4)$$

Using equation (4) and the root locus theory [4], we form the transfer function to construct the edge branch by the parameter T_i:

$$W_i^q(s) = \frac{\Delta T_i A_i(s)}{D^q(s)}. \qquad (5)$$

Due to the expression (5), the roots of the equation $\Delta T_i A_i(s) = 0$ are the zeros of the edge transfer function $W_i^q(s)$, and the roots of the vertex polynomial $D^q(s)$ are its poles.

3. THE INTERVALS OF REAL POLES OF THE SYSTEM WITH AFFINE UNCERTAINTY: SOME PROPERTIES

According to the root locus theory [25], the branches of a root locus are in definite parts of the real axis depending on the number of real zeros and poles of the system. For the interval extension of this property, it is necessary to determine, for the polytope of the interval parameters of the system, the vertices mapped into the interval bounds of the real poles. For this purpose, we establish the following result.

Proposition. *The right bound of the interval* $[s_j^L, s_j^R]$ *of the real pole* s_j *of the system with the interval parameters* T_i *is the image of the vertex* V_q *with the coordinates* $T_i^q = \underline{T_i}$ *provided that the total number of the intervals of the other system poles located to the right of* s_j^R *is even and the zeros of the edge transfer function* (5) *are constant. If the total number of the right intervals and zeros is odd, then* $T_i^q = \overline{T_i}$. *In this case, the coordinates of the vertex original of the left bound* s_j^L *have the opposite limits of the interval parameters.*

The proof of this proposition is provided in the Appendix.

Corollary 1. *The real pole* s_1 *determining the robust aperiodic stability degree of the system with the interval parameters* T_i *has the vertex original* V_q *with the coordinates* $T_i^q = \underline{T_i}$ *provided that the number of the real zeros of the edge transfer function* (5) *located to the right of* s_1 *is even. If the number of the real zeros is odd, then* $T_i^q = \overline{T_i}$.

Corollary 2. *If there are no real zeros of the edge transfer function* (5) *between the two intervals of the real poles of the system, then the right and left bounds of these intervals are associated with the vertices with the opposite limits of the interval parameters.*

A numerical example below illustrates the application of the proposition and its corollaries.

Example. Consider the characteristic polynomial of an ACS with affine uncertainty in which the polynomial coefficients are a linear combination of three interval parameters:

$$[T_1]s^4 + (6[T_1]+[T_2])s^3 + (11[T_1]+5[T_2])s^2 + (6[T_1]+6[T_2]+[T_3])s + 3[T_3]+1 = 0, \qquad (6)$$

where $[T_1]=[5, 10]$, $[T_2]=[30, 70]$, and $[T_3]=[10, 20]$. We transform the polynomial (6) to (1):

$$[T_1]A_1(s)+[T_2]A_2(s)+[T_3]A_3(s)+1=0, \qquad (7)$$

where $A_1(s)=s^4+6s^3+11s^2+6s$, $A_2(s)=s^3+5s^2+6s$, and $A_3(s)=s+3$.

For this fourth-degree polynomial, the multiparametric interval root locus is represented by four intervals on the negative real semi-axis: $s_1=[-0.38, -0.07]$, $s_2=[-1.92, -1.44]$, $s_3=[-3.03, -3.01]$, and $s_4=[-15, -4.1]$. These intervals are shown in Fig. 1 (for clarity, without precise scale), together with the real roots of the polynomials $A_i(s)$ (7), namely:

– the roots of $A_1(s): s_1=0$, $s_2=-1$, $s_3=-2$, and $s_4=-3$;

– the roots of $A_2(s)$: $s_1=0$, $s_2=-2$, and $s_3=-3$;

– the root of $A_3(s): s_1=-3$.

Based on the mutual arrangement of the intervals and these roots, by the proposition, we obtain the coordinates of the vertices mapped into the bounds of the root intervals. Obviously, these vertices coincide with those determined when constructing the intervals of the real roots of the interval characteristic polynomial (Fig. 1).

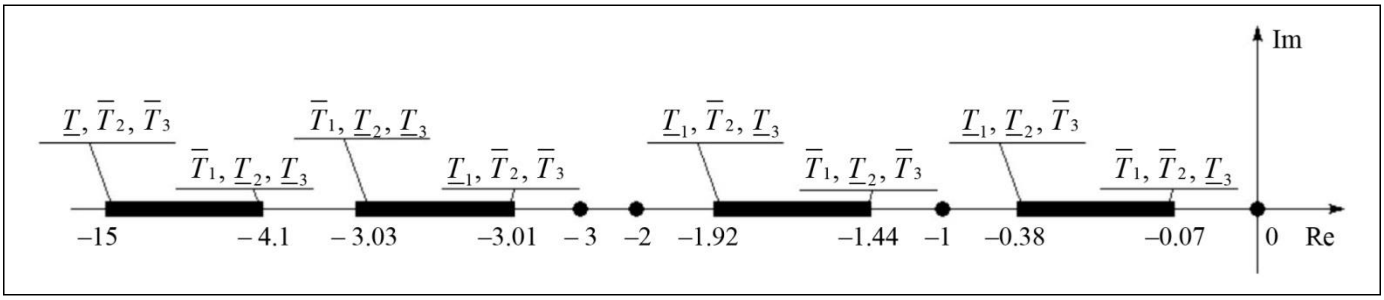

Fig. 1. Interval bounds for the real roots of the characteristic polynomial and the coordinates of their vertex originals.

4. FREE POLES LOCALIZATION BY THE ROBUST *D*-PARTITION METHOD

When changing one of the plant's interval parameters ΔT_i in the corresponding interval, the characteristic polynomial roots form a one-parameter interval root locus according to the root locus theory. The branches of this locus are called edge branches (RS_i^q), and their beginnings and ends are called root nodes (U_q). In addition, the edges and vertices of the parametric polytope are mapped into the branches and root nodes of the root locus, respectively: $\varphi(R_i^q) = RS_i^q$ and $\varphi(V_q) = U_q$.

By the edge theorem [26, 27], the localization regions of the roots of a characteristic polynomial with the affine interval uncertainty of its coefficients are bounded by the edge branches RS_i^q, i.e., the images of definite edges of the plant's parametric polytope. Following [23], we consider an edge branch RS_i^q formed by the motion of a complex conjugate root. If one of its ends is nearest to the imaginary axis, then the branch belongs to the first type; if one of its inner roots is nearest to the imaginary axis, and the original of this root is a priori unknown, then the branch under consideration belongs to the second type. To determine the type of an edge branch, we should check the following condition [28]: if the polynomial $A_i(s)$ under the interval uncertain parameter T_i is a polynomial of degree one, or of only an even degree, or of only an odd degree (of the variable s), or the product of such polynomials, then the edge branch RS_i^q belongs to the first type.

In the problem under study, the type of the edge branches bounding the localization regions of free poles is important for the robust *D*-partition [24] by the free parameter of the controller with a selected boundary of these regions. For instance, if all branches belong to the first type, then it suffices to perform *D*-partition at all vertices of the plant's parametric polytope and choose the value of the free parameter from the intersection of the regions obtained for each vertex.

If the above condition fails for some branch, then this branch belongs to the second type. In this case, it is necessary to apply *D*-partition by two parameters: the free parameter of the controller and the parameter of the corresponding branch of the polytope edge. (The coefficients of the characteristic polynomial must be a linear combination of these parameters.) By drawing the known bounds of the interval parameter in the parametric region, we obtain the admissible region of the controller's free parameter ensuring the desired placement of the free poles. After obtaining such regions for each branch of the second type, it is necessary to find their intersection and choose the value of the free parameter from it.

5. A NUMERICAL EXAMPLE: PID CONTROLLER DESIGN FOR A CABLE TENSION STABILIZATION SYSTEM OF A LOAD-LIFTING MECHANISM

Using the proposition above, we design a robust controller placing the dominant pole of a system within the desired interval and the other poles in a given region.

For this purpose, we consider the automatic cable tension stabilization system of a load-lifting mechanism [29]; see the block diagram in Fig. 2.

The notations are as follows: ΔF_{in} and ΔF_{out} are the increments of the cable tension force at the system input and output, respectively; DCM is a direct current motor; PA is a power amplifier.

The characteristic polynomial of the system has the form

$$a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0,$$

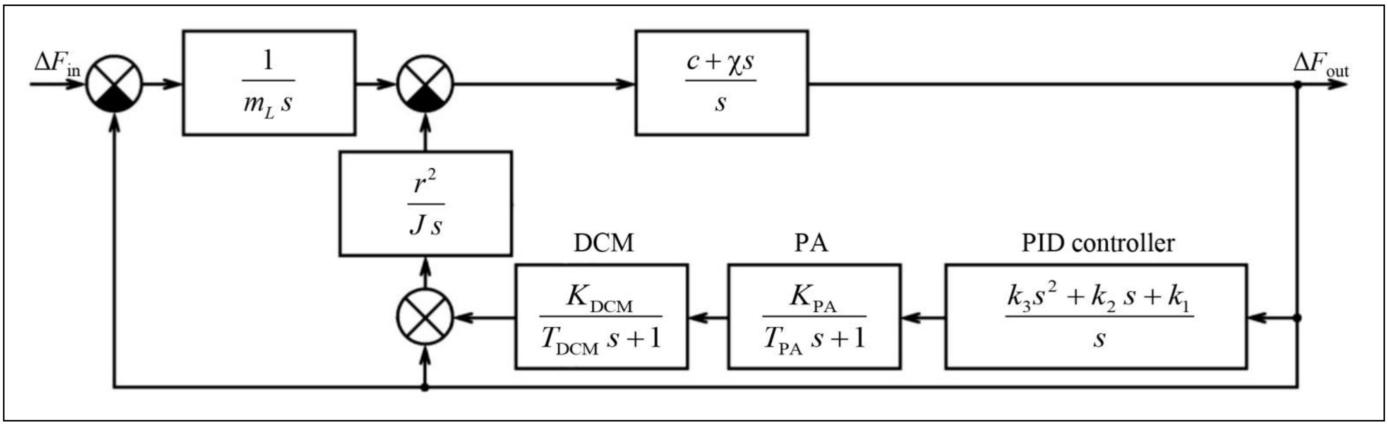

Fig. 2. The block diagram of the cable tension stabilization system of the load-lifting mechanism.

where $a_5 = J\, T_{\text{DCM}} T_{\text{PA}}\, l\, m_L$, $a_4 = T_{\text{DCM}} T_{\text{PA}}\, \chi\, m_L r^2 + J\, T_{\text{DCM}} T_{\text{PA}}\, \chi + J T_{\text{DCM}}\, l m_L + J T_{\text{PA}}\, l\, m_L$, $a_3 = J l m_L + J T_{\text{DCM}}\chi + J T_{\text{PA}}\chi + J T_{\text{DCM}} T_{\text{PA}} c + T_{\text{DCM}}\chi\, m_{\text{L}} r^2 + T_{\text{PA}}\,\chi\, m_L r^2 + T_{\text{DCM}} T_{\text{PA}} c\, m_L r^2 + k_3 K_{\text{DCM}} K_{\text{PA}} \chi\, m_L r^2$, $a_2 = J\,\chi + \chi\, m_L\, r^2 + J\, T_{\text{DCM}}\, c + J\, T_{\text{PA}}\, c + T_{\text{PA}}\, c\, m_L\, r^2 + T_{\text{DCM}}\, c\, m_L\, r^2 + k_3 K_{\text{DCM}} K_{\text{PA}} c\, m_L r^2 + k_2\, K_{\text{DCM}}\, K_{\text{PA}}\, \chi\, m_L\, r^2$, and $a_1 = J\, c + c\, m_L\, r^2 + k_1 K_{\text{DCM}} K_{\text{PA}}\, \chi\, m_L r^2 + k_2\, K_{\text{DCM}}\, K_{\text{PA}}\, c\, m_L\, r^2$, $a_0 = k_1\, K_{\text{DCM}}\, K_{\text{PA}}\, c\, m_L\, r^2$; m_L is the load mass, in kg; l is the cable length, in m; J is the moment of inertia of the electric drive, in $\text{kg}\cdot\text{m}^2$; χ is the specific damping coefficient of the cable, in $\text{N}\cdot\text{s}$; c is the specific stiffness of the cable, in N; r is the radius of the drive pulley of the electric drive, in m; K_{DCM} is the gain of the DC motor, in $\text{rad}\cdot\text{s}^{-1}\cdot\text{V}^{-1}$; T_{DCM} is the time constant of the DC motor, in s; K_{PA} is the gain of the PA; finally, T_{PA} is the time constant of the PA, in s. The constant parameters of the plant have the following values: $J = 0.5\ \text{kg}\cdot\text{m}^2$, $\chi = 10^4\ \text{N}\cdot\text{s}$, $c = 2\cdot 10^4\ \text{N}$, $r = 0.1\ \text{m}$, $K_{\text{DCM}} = 5\ \text{rad}\cdot\text{s}^{-1}\cdot\text{V}^{-1}$, $T_{\text{DCM}} = 0.01\ \text{s}$, $K_{\text{PA}} = 10$, and $T_{\text{PA}} = 0.001\ \text{s}$. The interval parameters of the plant are $m_{\text{L}} = [50,\ 500]\ \text{kg}$ and $l = [50,\ 100]\ \text{m}$. Thus, the parametric polytope of the system has four vertices.

The system uses a PID controller with the transfer function

$$W_{\text{PID}}(s) = \frac{k_3\, s^2 + k_2\, s + k_1}{s},$$

where k_1, k_2, and k_3 are the integral, proportional, and differential gains of the PID controller, respectively. We divide the controller parameters into dependent and free: the dependent parameters k_1 and k_3 determine the position of the dominant pole bounds whereas the free one k_2 the position of the other poles to the left of a given bound. Let it be required to localize the dominant pole within the interval $[-0.7,\ -0.5]$ of the real axis and to place the other poles to the left of the vertical line passing through the point $(-1,\ j0)$ parallel to the imaginary axis.

We substitute the constant parameters of the system into the characteristic polynomial, writing it as

$$l\, A_1(s) + \frac{1}{m_L}\, A_2(s) + A_3(s) = 0,$$

where $A_1(s) = s^3\ (5\cdot 10^{-10}\, s^2 + 5.5\cdot 10^{-7}\, s + 5\cdot 10^{-5})$, $A_2(s) = s(5\cdot 10^{-6}\, s^3 + 5.51\cdot 10^{-3}\, s^2 + 0.511 s + 1)$, $A_3(s) = 10^{-7}\, s^4 + (0.5 k_3 + 1.102\cdot 10^{-4})\, s^3 + (0.5 k_2 + k_3 + 0.0102)\, s^2 + (0.5\, k_1 + k_2 + 0.02)\, s + k_1$.

Let us determine the coordinates of the vertex originals for the bounds of the real dominant pole of the system. The polynomial $A_1(s)$ has five roots: $s_1 = s_2 = s_3 = 0$, $s_4 = -100$, and $s_5 = -1000$. The polynomial $A_2(s)$ has four roots: $s_1 = 0$, $s_2 = -2$, $s_3 = -100$, and $s_4 = -1000$. Therefore, by the proposition above, the right bound $s^R = -0.5$ of the dominant pole is the projection of the parametric polytope vertex with the coordinates $(\bar{l};\ \underline{m_{\text{L}}})$; the left bound $s^L = -0.7$ of the dominant pole is the projection of the vertex $(\underline{l};\ \overline{m_{\text{L}}})$. Substituting the vertex coordinates and the interval bounds into the characteristic polynomial of the system yields two algebraic equations for the PID controller gains. By solving these equations, we obtain the following expressions for the dependent parameters of the controller:

$$\begin{cases} k_1(k_2) = 0.292\ k_2 + 0.025 \\ k_3(k_2) = 0.833\ k_2 - 0.017. \end{cases}$$

Note that $k_2 \geq 0.0206$ for a physically implementable controller.

Let us evaluate the free parameter k_2 of the controller using the *D*-partition method. Due to the form of the polynomials, the edge branches by the interval parameters of the system belong to the first type. Therefore, the ends of the edges are nearer to the imaginary axis than any of its interior points. Consequently, it suffices to perform *D*-partition at the vertices of the parametric polytope to locate the free poles of the system. To obtain the equations of the *D*-partition curves, we substitute the values $k_1(k_2)$ and $k_3(k_2)$ as well as the bound $s = -1 + j\omega$ of the free poles into the characteristic polynomial of the system. The *D*-partition curves are shown in Fig. 3.

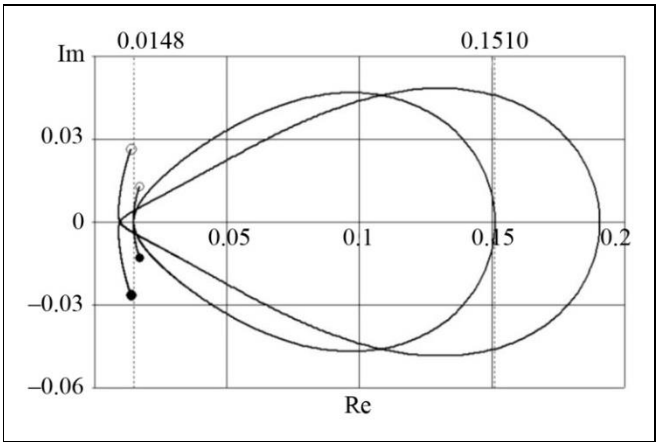

Fig. 3. The vertex *D*-partition curves by the free parameter of the controller.

Due to the *D*-partition, the desired pole placement is achieved for $k_2 \in [0.0148,\ 0.1510]$. Let us choose $k_2 = 0.1$. In this case, $k_1 = 0.054$ and $k_3 = 0.066$. The pole placement of the system with the designed PID controller is shown in Fig. 4.

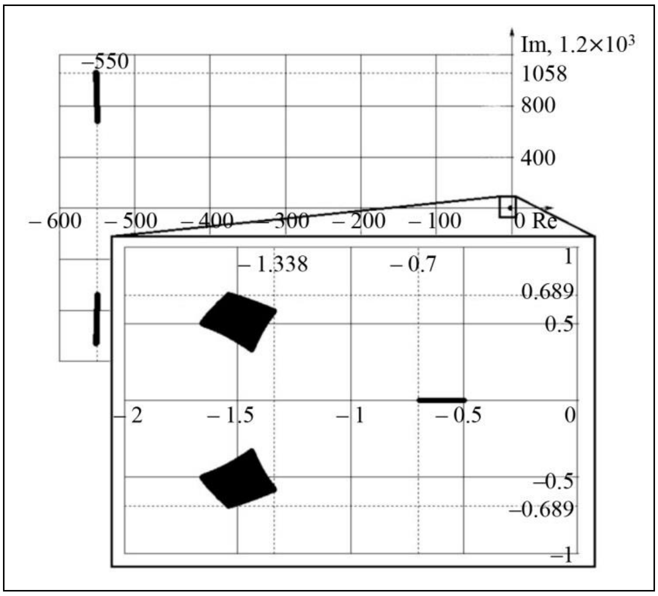

Fig. 4. The localization regions of the poles of the designed system.

With this placement, the design problem has been solved successfully: the real dominant pole is located within the desired interval, and the free poles are located to the left of the given bound. Next, Fig. 5 presents the family of transient characteristics of the designed system for different value combinations of the interval parameters.

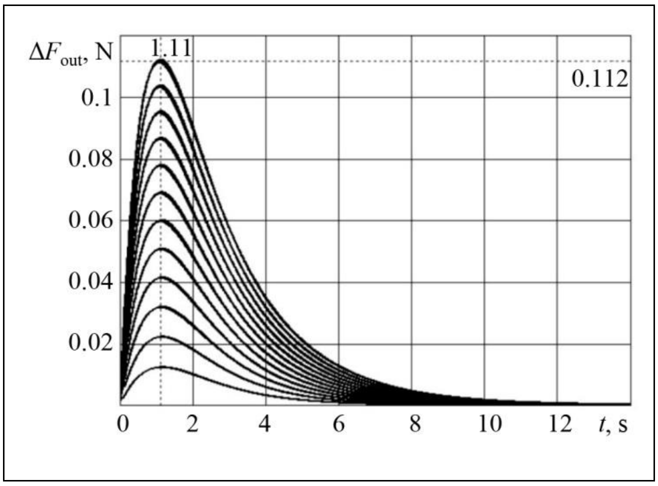

Fig. 5. The transient characteristics of the designed system.

According to Fig. 5, the designed controller ensures aperiodic transients in the system. The tension force occurring in the cable of length up to 100 m when suspending a load with mass up to 500 kg and weight up to 4900 N is compensated with a maximum deviation of 0.112 N or $2.2 \cdot 10^{-3}\%$. Also, considering the transient characteristics in Fig. 5, the pole dominance principle holds despite a small distance between the free poles and the dominant one.

CONCLUSIONS

For technological reasons, the required aperiodic transient in an ACS often should be close to monotonic and contain, on the initial time interval, as little as possible oscillations from the free complex conjugate poles of the system. Therefore, to reduce the impact of the free poles on the transient, it is desirable to shift them from the interval of the real dominant pole to a sufficient distance defined by a given bound. If the shift cannot be performed using the *D*-partition method by one free parameter, then this method should be applied by two free parameters to expand the possibility of the desired localization of the free poles. For this purpose, if a typical PID controller with three parameters is used as a low-order controller without any constraints on the minimum transient time, one of the two dependent parameters of the controller should be made a free parameter. In this case, the remaining dependent parameter is used to ensure the given right bound of the dominant pole interval, which is enough to provide the maximum acceptable transient time in the ACS.

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APPENDIX

P r o o f the Proposition. The phase equation of the root locus has the form [25]

$$\sum_{i=1}^{m}\Theta_i^0-\sum_{i=1}^{n}\Theta_i=\pm(2n+1)\pi,$$

where Θ_i^0 is the angle of exit of the edge branch from the ith zero of the edge transfer function; Θ_i is the angle of exit of the edge branch from the ith pole of the edge transfer function; m is the number of zeros; n is the number of poles. Based on this equation, we write the equation of the angle Θ_1 of exit of the edge branch of the root locus from the right bound of the real pole s_j^R:

$$\Theta_1=r\pi-\sum_{i=1}^{m}\Theta_i^0+\sum_{i=2}^{n}\Theta_i,$$

where r is a parameter equal to 0 or 1 depending on the direction of motion along the edge branch of the root locus.

The angles in the equation depend on the location of the zeros and poles of the edge transfer function relative to the root locus point under study. Suppose that k of n poles and p of m zeros of the edge transfer function are located to the right of s_j^R. Let Θ_{Ri}^0 and Θ_{Ri} denote the angles corresponding to the right zeros and poles, and let Θ_{Li}^0 and Θ_{Li} denote the angles corresponding to the left zeros and poles. With these notations, the last equation takes the form

$$\Theta_1=r\pi-\left(\sum_{i=1}^{m-p}\Theta_{Li}^0+\sum_{i=1}^{p}\Theta_{Ri}^0\right)+\left(\sum_{i=1}^{n-k-1}\Theta_{Li}+\sum_{i=1}^{k}\Theta_{Ri}\right).$$

The zeros and poles of the edge transfer function can be either complex conjugate or real. Obviously, the angles corresponding to complex conjugate zeros or poles are equal in magnitude and opposite in sign. Consequently, when calculating the exit angle of an edge branch, they are mutually reduced and do not affect the final result. For the angles corresponding to the real zeros and poles, we have $\Theta_{Ri}^0=\Theta_{Ri}=\pi$ and $\Theta_{Li}^0=\Theta_{Li}=0$. Hence, the expression for Θ_1 can be written as

$$\Theta_1=r\pi-(m-p)\cdot 0-p\pi+(n-k-1)\cdot 0+k\pi=\pi(r+k-p).$$

The problem is to find the original V_q of the right bound of the real pole s_j^R. Consequently, it is necessary that $\Theta_{1i}=\pi$. To fulfill this condition, for even $(k-p)$ one should choose $r=1$ and, accordingly, $\underline{T_i}$ in the coordinates V_q. For odd $(k-p)$, one should choose $r=0$ and, accordingly, $\overline{T_i}$ in the coordinates V_q. Obviously, if the difference between the nonnegative integers k and p is even, then their sum is also even, and vice versa. Therefore, for the sake of convenience, when analyzing the placement of zeros and poles, we consider the sum of k and p, i.e., the total number of the zeros and poles of the edge transfer function located to the right of the point under study.

To find the original of the left bound s_j^L, it is necessary to set the exit angles of the edge branches as $\Theta_{1i}=0$. Hence, the values of the parameter r and the bounds of the interval parameters T_i in the coordinates V_q are set in opposite ways, and the originals of the right and left bounds of the real pole have opposite coordinates.

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