TECHNICAL CONDITION MONITORING METHODS TO MANAGE THE REDUNDANCY OF SYSTEMS. PART III: Nonclassical Models in Fault Diagnosis

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Abstract. Redundancy management in a technical system involves a monitoring procedure to reconfigure the system as needed. The four-part survey presents modern diagnosis methods for dynamic systems as an integral function of monitoring. Part III is devoted to diagnosis methods employing neural networks, fuzzy models, structural models, set-based models, and a statistical approach. The fundamentals of creating and training neural networks to perform diagnostic functions are considered. The approach with fuzzy models is described, including general modeling rules and the features of their use in diagnosis tasks. The approach with structural models is demonstrated, including its features in failure detection. The fundamentals of set-theory methods, particularly the formalism of zonotopes, are presented. Finally, the approach based on statistical pattern recognition is briefly discussed.

Keywords: artificial neuron, neural network, fuzzy models, membership function, fuzzy clustering, structural models, Dulmage–Mendelsohn decomposition, diagnosability, zonotopes, gradations in discrete feature space, empirical likelihood ratio estimate.

INTRODUCTION

The main limitation of approaches with analytical models is the need for precise knowledge of such models (both their structure and parameters), which is often impossible in practical applications. In the case of mathematical models with partial uncertainty, there exist well-developed procedures for their parametric or structural identification [1–3]. However, here the focus is on the "initial" elimination of this drawback (i.e., the one inherent in the approach itself). Such approaches include the use of neural networks, fuzzy models, structural models, the apparatus of set theory, and statistical estimates.

Proponents of such methods, deliberately or due to insufficient information, refrain from a detailed description of the processes occurring in a system diagnosed. In one way or another, this makes the models used rough (inaccurate, approximate) but provides greater flexibility in fault diagnosis, reducing or completely bypassing negative effects. That is why such approaches are applied for the effective monitoring of dynamic systems [4].

1. NEURAL NETWORK-BASED DIAGNOSIS METHODS

Neural networks (NNs) are widely used in fault diagnosis [5–9]. One modification of such networks [10, 11] uses *Nonlinear AutoRegressive with eXogenous inputs* (NARX) models in combination with learning algorithms.

1.1. Formation of an ARX Neural Network

One approach involves developing and training a set of NN-based estimators to reproduce the behavior of a system under consideration. The structure of the *i*th individual neuron [12] uses a *Multiple Input Single Output* (MISO) system in which the output signal y_i is calculated as a function of the weighted sum of all inputs u_i of the neuron, $u_{i.1},\ldots,\ u_{i.n_i}$, with corresponding weights $w_{i.1},\ldots,\ w_{i.n_i}$ (Fig. 1). The function f is called the activation function.

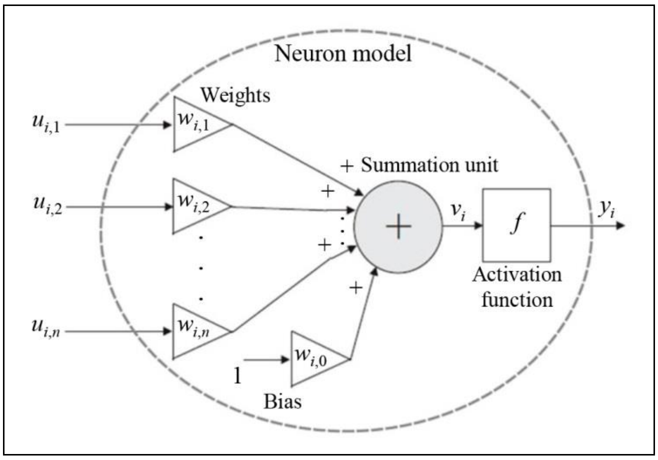

Fig. 1. One example of an artificial neuron.

NNs are classified according to the ways of connecting their elements [13, 14]. In a feedforward NN, the elements are grouped into unidirectional layers. The first (input) layer receives information directly from the network inputs; then, each sequential hidden layer receives input data from the neurons of the previous layer and transmits the output data to the neurons of the next layer, up to the last output layer, where the final network output data are generated. Therefore, neurons are connected layer-to-layer but not within the same layer. The only limitation is the number of neurons in the output layer, which must coincide with the number of actual output channels of the network.

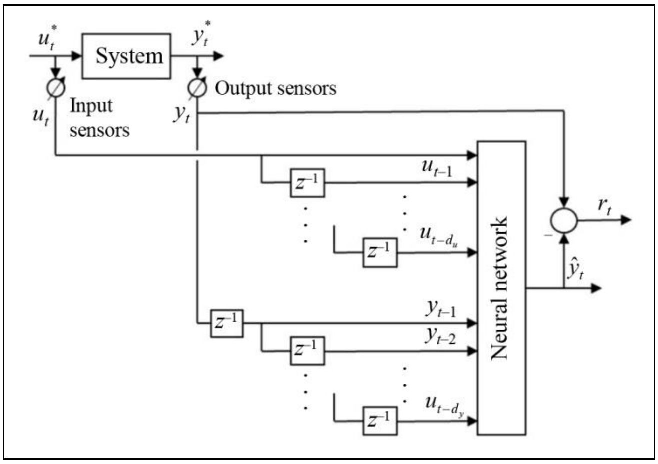

Fig. 2. An open-loop dynamic NN.

On the other hand, recurrent NNs [13] are multilayer networks in which the output data of some neurons is fed back to neurons of previous layers. Thus, information is transmitted both forward and backward and provides dynamic memory within the network.

An intermediate solution is an NN with a connected delay line. Networks of this type are suitable for modeling or predicting the evolution of a dynamic system. In particular, a properly trained open-loop NARX network can estimate current (or future) results based on past measurements of the system's input and output data.

Generally speaking, for *Multiple Input Multiple Output* (MIMO) systems, open-loop NARX networks follow the law

$$\hat{y}_t = f_{\text{net}}(u_t,\ldots,u_{t-d_u},\ y_{t-1},\ldots,\ y_{t-d_y}), \tag{1}$$

where $\hat{y}_t$ is the estimate of the system output at step t; u and y are the measured system inputs and outputs, respectively; d_u and d_y are the number of input and output delays, respectively; finally, f_{net} is a function implemented by the network that depends on the architecture of the layers, the number of neurons, their weights, and activation functions. Figure 2 shows the structure of an open-loop NARX network used as an estimator.

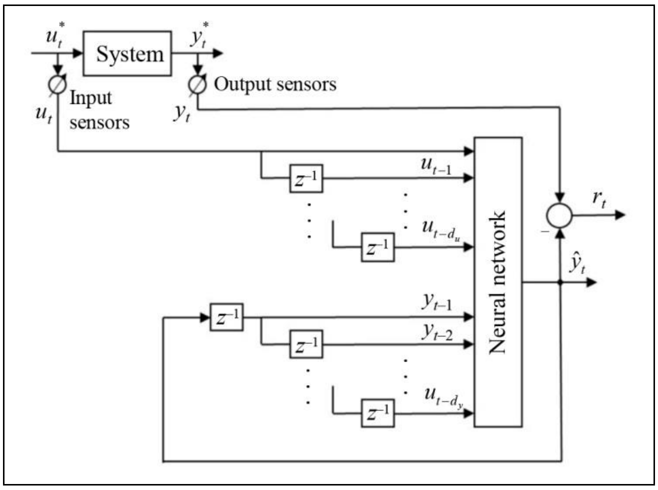

Fig. 3. A closed-loop dynamic NN.

The difference between the measured output y_t and its estimate (1) is taken as the diagnosis residual r_t.

When only input measurements are available, the NARX network can become a dynamic NN by closing the feedback loop of the network outputs to the inputs (Fig. 3).

The developer of a fault diagnosis system can vary the number of neurons and the connections between them, while the weights $w_{i.j}$ within each neuron are assigned by training the network.

Since NN components are often chosen subjectively, they may be incompletely adequate for the system or process modeled, which affects NN implementations.

1.2. Neural Network Training

An NN becomes a diagnosis tool after its training (assignment of the weights w) on pre-selected examples of the functioning of the system diagnosed, both in operable and inoperable states (in particular, due to the occurrence of a single fault or a combination of faults). Besides the basic network with fixed weights after training, there are modifications with refinement (retraining) of network parameters during its operation. In addition, an NN should be retrained each time, even for homogeneous objects operating in different conditions and with different historical operation data.

An NN can be trained only to the "diagnostic events" (normal, operable, pre-fault, and inoperable states) that are initially known and used in the training process. If the set of admissible states (including faulty ones) is not specified for an NN, it will not distinguish between them or find an unforeseen state.

The use of NNs allows performing self-diagnosis in practice, since a properly trained NN is robust to anomalies in data and recognizes a diagnostic event itself, even under inaccuracies in the source data.[1] This will not happen when using "hard" diagnosis algorithms.

The formal objective of training is to minimize the loss function (residual) E [14], which depends on the vector of weights w, and it can be implemented in two different modes:

– the incremental mode (each input-target pair independently generates an update of the network weights);

– the batch mode (all input data and the loss function are applied to the network simultaneously).

Although the second training mode requires more memory than the first, it is characterized by faster convergence and results with smaller errors.

Given a set of S patterns corresponding to both the absence and presence of various faults in the system, for each of $P = C_P^2 = S!/2(S-2)!$ possible input-output pairs, the error vector can be written as

$$e_p = y_p - \hat{y}_p = \begin{bmatrix} e_{p.1} & \cdots & e_{p.M} \end{bmatrix}^{\mathrm{T}},\ p = \overline{1, P}, \quad (2)$$

where M is the number of outputs (i.e., the residuals of the system diagnosed). The total loss function, which depends on the choice of all weights $w = (w_1, ..., w_N)$ (see formula (1) and Fig. 1), takes the form:

$$E(w) = \frac{1}{2}\sum_{p=1}^{P}\sum_{m=1}^{M}(y_{p.m} - \hat{y}_{p.m}(w))^2 = \frac{1}{2}\sum_{p=1}^{P}\sum_{m=1}^{M}e_{p.m}^2(w).$$

Any standard numerical optimization algorithm [14–16] can be applied to update the values of the parameters w_i in order to minimize the residual $E(w)$. Iterative algorithms are most widespread; they use such characteristics as:

– the gradient of the loss function,

$$\mathrm{grad}E = \frac{\partial E}{\partial w} = \begin{bmatrix} \frac{\partial E}{\partial w_1} & \cdots & \frac{\partial E}{\partial w_N} \end{bmatrix}^{\mathrm{T}};$$

– the Hessian of the loss function,

$$H(E) = \begin{bmatrix} \partial^2 E / \partial w_1^2 & \cdots & \partial^2 E / \partial w_1 \partial w_N \\ \vdots & \ddots & \vdots \\ \partial^2 E / \partial w_N \partial w_1 & \cdots & \partial^2 E / \partial w_N^2 \end{bmatrix};$$

– the Jacobian of the estimation errors,

$$J(e) = \begin{bmatrix} \partial e_{1.1} / \partial w_1 & \cdots & \partial e_{1.1} / \partial w_N \\ \partial e_{1.2} / \partial w_1 & \cdots & \partial e_{1.2} / \partial w_N \\ \vdots & \ddots & \vdots \\ \partial e_{P.M} / \partial w_1 & \cdots & \partial e_{P.M} / \partial w_N \end{bmatrix}.$$

Sequential iterations of these algorithms consist of updating the parameter values and calculating the new value of the loss function until the stopping condition is satisfied.

The achievable sensitivity of the diagnosis procedure to individual faults is a complex issue that is unlikely to have a generalized assessment.

Note finally that NNs approximate any nonlinear and dynamic function under a suitable structure of weights. Moreover, online training makes it easy to modify the diagnosis system when changes are made to the physical process or control system. An NN can generalize when available input data are not present in the training data and make reasonable decisions in cases of noisy or corrupted data. NNs are also easily applicable to multiparameter systems and have a high level of structural parallelism. An NN can work with both qualitative and quantitative data simultaneously. In addition, an NN can be very useful in the absence of any mathematical model of the system (i.e., when analytical models cannot be applied for some reason). As expected, all these factors together may provide a higher degree of fault tolerance.

On the other hand, a fundamental feature of NNs is that they operate as a "black box" without qualitative and quantitative information about the model they represent. This circumstance currently restrains develop-

[1] This statement belongs to one of the paper's reviewers. We have accepted it with gratitude.

ers and customers from applying such solutions, especially for high-responsibility tasks (e.g., in civil aviation, for safety-critical tasks). It is generally recognized that there exists an ambiguity regarding the operation of NNs in unforeseen situations [17].

The approach described above has a structural problem. It is practically impossible to preselect a combination of measurable parameters and network structure that would guarantee reliable discrimination of faults from a given list. This is usually achieved through trial and error. In addition, the set of training examples must be representative, i.e., contain a sufficient number of combinations of conditions for the task tackled.

2. DIAGNOSIS WITH FUZZY MODELS

The fuzzy model approach also helps in dealing with the precise knowledge problem for the model of a system diagnosed. This approach can be applied at two levels: first, fuzzy descriptions are employed to generate residuals, and then faults are detected using fuzzy logic again [18–24].

Fuzzy logic systems offer a linguistic model of system dynamics that can be easily understood using definite rules. They are also able to handle inaccurate or noisy data.

2.1. Fuzzy Model Formation

An effective approach to designing a fuzzy logic system begins with partitioning the available data into subsets (clusters) [19] characterized by simpler (linear or affine) behavior. A cluster can be defined as a group of data that are more similar to each other than to data from another cluster. Data similarity is usually expressed in terms of their distance to a given element within a cluster (used as the latter's prototype). Fuzzy clustering is an effective data partition tool with smooth, rather than abrupt, transitions between subsets, determined by the so-called membership functions.

In general, nonlinear MISO systems can be approximated using fuzzy inference [25, 26]. However, according to the approach proposed in [27], implications become crisp functions of the input data, and each fuzzy rule takes the form

$$R_i : \text{IF } x \in X_i \text{ THEN } \hat{y}_i = a_i^{\mathrm{T}} x + b_i, \; i = \overline{1, n_{\mathrm{c}}}, \quad (3)$$

where R_i is the ith rule; X_i is the ith cluster; n_{c} is the total number of clusters; a_i is the parameter vector; b_i is a scalar bias; $\hat{y}_i$ is the inference of the ith rule; finally, y_i is the output of the system diagnosed. Figure 4 illustrates the rule (3) for a one-dimensional variable x with $n_{\mathrm{c}} = 5$.

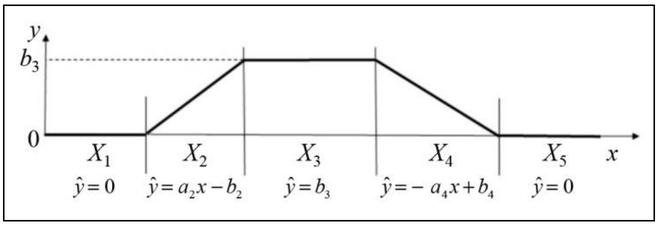

Fig. 4. The graphical illustration of the rule (3).

With the membership function $\lambda_i(x)$ considering the logic of cluster selection (IF … THEN …), the resulting formula for the entire set of rules (3) can be written as the weighted sum

$$\hat{y}_t = \frac{\sum_{i=1}^{n_{\mathrm{c}}} \lambda_i(x_t) y_t^i}{\sum_{i=1}^{n_{\mathrm{c}}} \lambda_i(x_t)} \quad (4)$$

with the following notation: x_t is a suitable combination of the input and output signals; y_t^i is the output signal of the local linear (or affine) ith model, defined as

$$y_t^i = \sum_{k=1}^{n} a_{i.k} y_{t-k} + \sum_{k=0}^{n-1} b_{i.k} u_{t-k} + c_i, \quad (5)$$

where $a_{i.k}$, $b_{i.k}$, and c_i are the parameters of the ith model in the subspace (cluster) X_i.

Formula (5) contains both the latest obtained u_t, y_{t-1} and the previous u_{t-1}, $y_{t-2}, \ldots$ samples of the input and output data, which reflects the dynamic behavior of the system. Therefore, the observation is treated as the linear autoregressive model with exogenous inputs (ARX) [9] of order s in which the regressor vector takes the form

$$x_t = \Big[\underbrace{y_{t-1} \;\cdots\; y_{t-s}}_{\text{output sample}} \underbrace{u_t \;\cdots\; u_{t-s+1}}_{\text{input sample}} \Big]^{\mathrm{T}},$$

where t is the current discrete time instant.

Unlike the classical counterpart, fuzzy clustering [20, 28, 29] allows any element to belong to several clusters simultaneously.

Another approach to fuzzy modeling is the fuzzy finite-state automaton model[2], which is used to describe discrete-event systems [22].

[2] In the below description of this approach, we have departed from the convention of part II of the survey to consider only dynamic systems with models (2) and (3) [30]: the symptom formation method is also applicable to the systems specified above.

2.2. Fault Diagnosis

The difference between the system outputs measured and their values obtained using the implicit model (2) can be taken as the residual:

$$r_t = y_t - \hat{y}_t.$$

Figure 5 shows the simplified block diagram of the residual formation based on an adaptive fuzzy model. Here, a fuzzy prototype and a recursive computing unit that implements some identification algorithm for the parameters use the same input and output data independently.

If several different faults have to be detected, the block diagram is reduced to another form with the multiplication of prototypes (models). Such a diagram will be discussed in part IV of the survey.

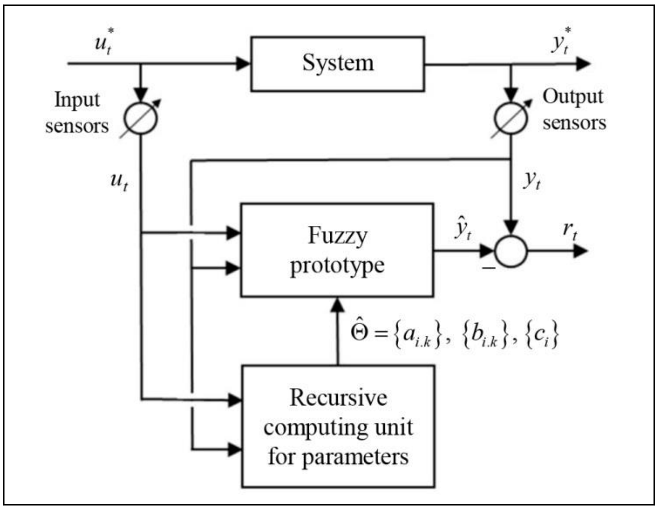

Fig. 5. Residual computation based on a fuzzy model.

A separate but very important problem is the fuzzy clustering of the system's state space [31]. Most fuzzy clustering algorithms optimize the *c*-means objective function $J(Z, U, V)$ as follows:

- The data are represented in the matrix form

$$Z = \begin{bmatrix} z_{11} & \cdots & z_{1N} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nN} \end{bmatrix},$$

where n and N denote the data dimension (the number of measurements taken per observation) and the number of available observations, respectively.

- The fuzzy partition matrix $U = [\mu_{ik}]$ is compiled from the values of the membership function, where i and k indicate the ith measurement and the kth cluster, respectively.

- The centers of the so-called prototypes $V = [v_1 \;\; \cdots \;\; v_{n_c}]$ are determined, i.e., the points used to estimate the distance between the cluster and the current state of the system.

The formula for minimizing the widespread *c*-means objective function [32] is

$$v_k = \arg\min J(Z, U, V)$$
$$= \arg\min \sum_{k=1}^{n_c} \sum_{i=1}^{N} (\mu_{ik})^m (z_i - v_k)^{\mathrm{T}} A (z_i - v_k)$$

with a weight $m > 1$, where the matrix A specifies the shape of the cluster.

To diagnose failures under measurement noises and modeling uncertainties, it is necessary to satisfy the noise separation principle: in this case, the residual generator is not affected by modeling uncertainty and input disturbances. This can be achieved using generalized observation schemes, consistency conditions [33], or particular design approaches [34].

For discrete-event systems, which often behave randomly [35, 36] and are described by a fuzzy finite-state automaton model, a fault diagnosis method based on the mathematical apparatus of fuzzy logic was proposed in [22]. The key element therein is the use of the determinizer of a fuzzy finite-state automaton, i.e., a finite-state automaton that simultaneously describes both the normal and abnormal behavior of the original automaton.

Compared to the algebraic approaches [37, 38] to the determinization of fuzzy finite-state automata, which preserve the dimension of the original automata, the approach under consideration involves additional useful information about the degree of confidence in the implementation of each possible transition during diagnosis, which potentially increases (if necessary) the depth of the fault search. The price paid is a significant dimension of the determinizer's transition table.

The fault diagnosis scheme in [22] contains several channels (exceeding by one the number of possible system faults). Each channel includes an observer in the form of the determinizer of a fuzzy finite-state machine. A decision block based on a comparison of the outputs of the discrete-event system and observers decides on the (in)operability of the system.

A distinctive feature of this method is that its implementation does not require the preliminary formation of a tabular description of diagnostic tools; all calculations are performed directly during the fault diagnosis process using compact analytical relationships, which is very important for technical condition monitoring.

The use of fuzzy logic for diagnosing a specific three-tank system was presented in [39]. Note that this system as a test object was taken in numerous publications, in particular, with consideration of various observers [40–42] (see part II of the survey [30]), parity equations with neuro-fuzzy identification [43], residuals from physical nonlinear equations [44] (as in part II of the survey [30], albeit with nonlinear equations), estimation of physical parameters using fuzzy NNs [45], and a fuzzy model based on the B-spline[3] of a network [46].

In [39], a *fault isolation system* (FIS) was defined as a quadruple

$$\text{FIS} = \langle F,\ S,\ V,\ \phi \rangle,$$

with the following notation: F is the set of faults; S is the set of symptoms with the two-level {0, 1} or three-level $\{-1,\ 0,\ +1\}$ scale; V is the set of symptom values from the interval [0, 1] or $[-1...+1]$, respectively; finally, ϕ is a function defined as the Cartesian product of the sets F and S, i.e., the set of all pairs $(f_i,\ s_j)$. Figure 6 illustrates the peculiarities of symptom computation with the following notation: r_j is a residual; $s_j \in S$ is a symptom; $v_{ij} \in V$ is the continuous value of the *j*th symptom for the *i*th fault; P and $N-$, $N+$ are the positive and negative symptom assessment domains, respectively.

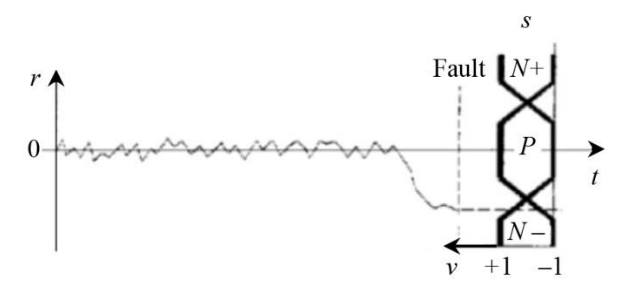

Fig. 6. Three-level symptom assessment.

Fault diagnosis is performed on such a set of symptoms using threshold elements and rules of the form (3) for each fault. For example, for three-digit symptoms, the rules for the number of faults are as follows:

$$R_i : \text{IF } s_1 = 0, s_2 = +1 \text{ or } -1,$$

$$s_3 = +1 \text{ or } -1, \ldots \text{THEN fault } f_i.$$

Thus, FIS is a table assigning a value or a subset of symptom values s_j to each fault f_i.

To isolate faults, the continuous values of symptoms v_{ij} are additionally calculated using formulas similar to (4), and the significance (grade of membership) of symptoms is thereby taken into account.

This interpretation of residuals allows considering the main uncertainty in the fault diagnosis process, i.e., the uncertainty of symptoms s_j. As claimed [40], this method provides much better robustness of the generated diagnoses to measurement noise compared to algorithms with threshold residual checking and logical inference.

3. STRUCTURAL FAULT DIAGNOSIS METHODS

Structural analysis involves a structural representation of a model, which is a rough description taking into account only the appearance of variables in each equation. Hence, large-scale problems can be analyzed efficiently and without numerical difficulties. However, the price paid is the excessive generality of the results. A fault diagnosis approach based on structural analysis was proposed in numerous works [47–50].

Structural analysis has good computer support, both in MATLAB and in Python.

A graph-theoretical algorithm called the Dulmage–Mendelsohn decomposition is fundamental to the analysis of the diagnostic properties of models [47, 49]. It consists in permuting the rows and columns of the so-called structural matrix of the system, containing special symbols that reflect the appearance of variables in different equations of the mathematical model of this system.

3.1. Structural Models

Let E be the set of constraints or equations in a model, and let V be the set of its variables. Then the structural model can be represented by a bipartite graph $G = (E \bigcup V, A)$, where A denotes the set of edges between nodes in two node sets E and V. An edge $(e_i, v_j) \in A$ if and only if the variable $v_j \in V$ appears in the model relation $e_i \in E$. A common way to visualize a structural model is the so-called biadjacency matrix of the graph, i.e., a matrix with rows and columns corresponding to constraints and variables. An element $(i,\ j)$ of the biadjacency matrix is empty if the variable v_j does not appear in the constraint (equation) e_i. A biadjacency matrix representing the structure of a model is also simply called a structural matrix.

The set of variables V can be classified as unknown $X \subset V$, known $Z \subset V$, or faulty $F \subset V$ under the condition $V = X \bigcup Z \bigcup F.$ For the purposes of analysis, the most important part of the structure is the

[3] Abbreviation for "basis spline," i.e., a spline with the smallest carrier for specifying the degree.

one related to unknown variables, and the subgraph containing only the constraints (equations) and unknown variables is called the reduced structural graph.

Structural analysis is often explained on a demonstration example. As such an example, we take the mathematical model of an electric motor from the monograph [49]. This model is described by the equations

$$
\begin{aligned}
&e_1: U = h(R + f_R) + Lh' + K_a h\omega,\\
&e_2: h = \sqrt{\frac{T_m}{K_a\omega}},\\
&e_3: J\omega' = T - b\omega,\\
&e_4: T = T_m - T_l - f_{\text{load}},\\
&e_5: y_h = h + f_h,\\
&e_6: y_\omega = \omega + f_\omega,\\
&e_7: y_T = T + f_T,\\
&e_8: \frac{dh}{dt} = h',\\
&e_9: \frac{d\omega}{dt} = \omega',
\end{aligned}
\qquad (6)
$$

with the following notation: U is supply voltage; T_m is torque; T_l is rated load torque; h is the winding current of the motor; ω is the angular speed of the motor; R is the rated resistance of the motor winding; L is the inductance of the motor winding; K_a is the electromechanical coupling coefficient; J is the total rotational inertia of the rotor and load; finally, b is the friction coefficient.

The motor is equipped with sensors measuring the current y_h, the shaft torque y_T, and the angular velocity y_ω. Faults under consideration include the faults of all sensors, f_h, f_ω, and f_T, the winding resistance fault f_R, and the neglected additional motor load f_{load}.

In this case, with the above designations, the sets of variables V include

$$X = \{h, h', \omega, \omega', T, T_m, T_l\} = \{x_i\}_{i=\overline{1,7}},$$

$$Z = \{y_h, y_\omega, y_T, y_U\} = \{y_j\}_{j=\overline{1,4}},$$

$$F = \{f_R, f_h, f_\omega, f_T, f_{\text{load}}\} = \{f_k\}_{k=\overline{1,5}}.$$

In addition, the values of the following parameters are known: R, L, K_a, b, and J.

The structural matrix is represented as a table (e_i is the ith row, v_j is the jth column, here $v_j = \{x_{k=\overline{1,7}}, y_{l=\overline{1,4}}, f_{m=\overline{1,5}}\}$), and its entries are filled with the following symbols: "•" if the variable v_j directly appears in e_i; "D" if the variable v_j is represented by its derivative; "I" if the variable v_j is represented by its integral.

Figure 7 shows the initial structural matrix corresponding to model (6).

The result of the Dulmage–Mendelsohn decomposition is presented in Fig. 8: by permuting the constraints (equations) $e_1, \ldots, e_5$ and the variables $x_1, \ldots, x_4$, the biadjacency matrix of the reduced structural graph is transformed into a block triangular form. Three blocks are highlighted on the diagonal: the first two represent the well-defined part M^0 (the number of "internal" variables is equal to the number of equations), and the third represents the overdefined part M^+ (the number of "internal" variables is less than the number of equations) of the system model.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	f_1	f_2	f_3	f_4	f_5	y_1	y_2	y_3	y_4
e_1	•	•	•					•								•
e_2	•					•										
e_3			•	•	•											
e_4					•	•	•					•				
e_5	•								•				•			
e_6			•							•				•		
e_7					•						•				•	
e_8	I	D														
e_9			I	D												

Fig. 7. The structural matrix of system (6).

	x_7	x_6	x_1	x_2	x_3	x_4	x_5
e_4	•	•					•
e_2		•	•				
e_1			•	•	•		
e_8			I	D			•
e_3					•	•	•
e_9					I	D	
e_7							•
e_5			•				
e_6					•		

Fig. 8. The reduced structural matrix (diagnostic model) of system (6).

According to Fig. 8, the model has no underdefined part M^-. The well-defined part is used to uniquely determine the input unknowns, and the overdefined part forms the basis for fault diagnosis.

3.2. Fault Diagnosability Analysis

Fault diagnosability is understood as a combination of two properties:

– fault detectability (the potential capability to establish the fact of its existence),

– fault isolability[4] (the potential capability to determine the type, location, and time of occurrence of a detected fault).

The set of detectable and isolable (mutually distinguishable) faults in a system depends on the physical properties of the system and its relationships as well as on the available measurements. Fault detectability and isolability are system properties limiting the diagnostic performance that can be achieved by any fault diagnosis system.

Without loss of generality, by assumption, a single fault f_i can violate a single equation e_j of the model. If this is not the case, then additional equations are introduced to obtain an appropriate form.

For a model M, let O_f and O_{NF} denote the set of all its observations during operation with a fault f and without any faults, respectively. Obviously, the fault f in the model M is detectable under the condition[5] $O_f \setminus O_{\mathrm{NF}} \neq \varnothing$.

[4] The term “localizability” is occasionally used in the literature as well [51, 52].

[5] It reads: the set of observations minus those corresponding to normal (fault-free) operation is non-empty.

Therefore, the structurally detectable fault f affects the equation e_f located in the overdefined part M^+ of the model:

$$e_f \in M^+ .$$

As applied to model (6), the fragment of the Dulmage–Mendelsohn decomposition takes the form shown in Fig. 9: the overdefined part M^+ (highlighted in gray) of the model includes the faults $f_1, \ldots, f_4$, whereas the fault f_5 (f_{load}) is not detectable ($e_{f_5} \in M^0$).

A fault f_i is considered to be isolable (and distinguishable from a fault f_j) if there exists an observation corresponding to f_i (allowing it to be detected) and simultaneously not associated with f_j. This is expressed by the formula $O_i \setminus O_j \neq \varnothing$ or

$$e_{f_i} \in \left(M \setminus \{e_{f_j}\}\right)^+ .$$

Detection is a necessary condition for isolation, i.e., the isolability of a fault always implies its detectability, and the converse is false:

$$e_{f_i} \in \left(M \setminus \{e_{f_j}\}\right)^+ \quad \Rightarrow \quad e_f \in M^+ .$$

To illustrate isolability using the example (6), we continue the Dulmage–Mendelsohn decomposition to the result shown in Fig. 10.

According to the analysis of Fig. 10, the detectable faults f_1 and f_2 (f_R and f_h) belong to one overdefined block (denoted by M_1^+) whereas the faults f_3 and f_4 (f_ω and f_T) to another overdefined block M_2^+. Thus, the specified groups of faults are isolated from each other, but they are not isolated within the groups.

In general, the isolability of a single fault can be calculated by analyzing ordered pairs of faults using the formula

$$\bigcap_{f_j \in F} \left\{ (f_i, f_j) \mid e_{f_i} \in \left(M \setminus \{e_{f_j}\}\right)^+ \right\},$$

with one Dulmage–Mendelsohn decomposition performed for each fault in the model. Thus, for the entire model, it is necessary to perform as many Dulmage–Mendelsohn decompositions as there are single system faults being analyzed.

Appropriate observations are required to implement potential detectability and isolability. Structural

	x_7	x_6	x_1	x_2	x_3	x_4	x_5	f_1	f_2	f_3	f_4	f_5
e_4	•	•					•					•
e_2		•	•									
e_1			•	•	•			•				
e_8			I	D			•					
e_3					•	•	•					
e_9					I	D						
e_7							•				•	
e_5			•						•			
e_6					•					•		

Fig. 9. The structural matrix (fault detectability) of system (6).

	x_7	x_6	x_1	x_2	x_4	x_5	x_3	f_1	f_2	f_3	f_4	f_5
e_4	•	•				•						•
e_2		•	•									
e_1			•	•			•	•				
e_5			•						•			
e_8			I	D								
e_3					•	•	•					
e_7						•					•	
e_9					D		I					
e_6							•			•		

Fig. 10. The structural matrix (fault isolability) of system (6).

analysis methods allow handling available observations, such as those in model (6) and in Fig. 6. In addition, it is possible to place sensors for achieving a given or possible level of fault detectability and isolability as well as to form recommendations for the development of diagnostic tests.

3.3. Fault Detection Based on Structural Analysis

There exist various modifications of structural analysis [53] and descriptions of their implementation algorithms [54]. Basically, mechanisms for forming and analyzing structurally overdefined sets (SOSs) M^+ or, more precisely, the minimal SOS are used. (Any subset of this SOS is not an SOS.) The minimal SOS can be achieved by partitioning the set M into equivalence classes [16] according to the appearing equations e_i to avoid searching for the same SOS in different ways.

As a result, each SOS contains a set of equations where at least one equation can be used for analytically expressing the residual. A set of equations resolved for the unknowns is used to design a residual generator.

As an illustrative example, we consider the SOS $M = \{e_1, e_3, e_5, e_7, e_8, e_9\}$ for an electric motor described by model (6). This set has five variables in six equations, and any of the equations can be used to form the residual. Figure 11 shows the diagrams of computation flows when selecting e_5, e_1, or e_3 as the redundant equation, respectively. For instance, Fig. 11a shows the computation flow from the differentiated variable ω' to the undifferentiated variable ω (i.e., integration). Figure 11c presents the opposite case, from h to h' (i.e., differentiation). Finally, Fig. 11b demonstrates both cases, integration and differentiation.

The diagrams in Fig. 11 are organized as follows: the input variables are indicated on the left edge, and the vertical line means the resolution of an appropriate equation with respect to the subsequent variable.

Each diagram mentioned has particular computational characteristics and sensitivity to faults.

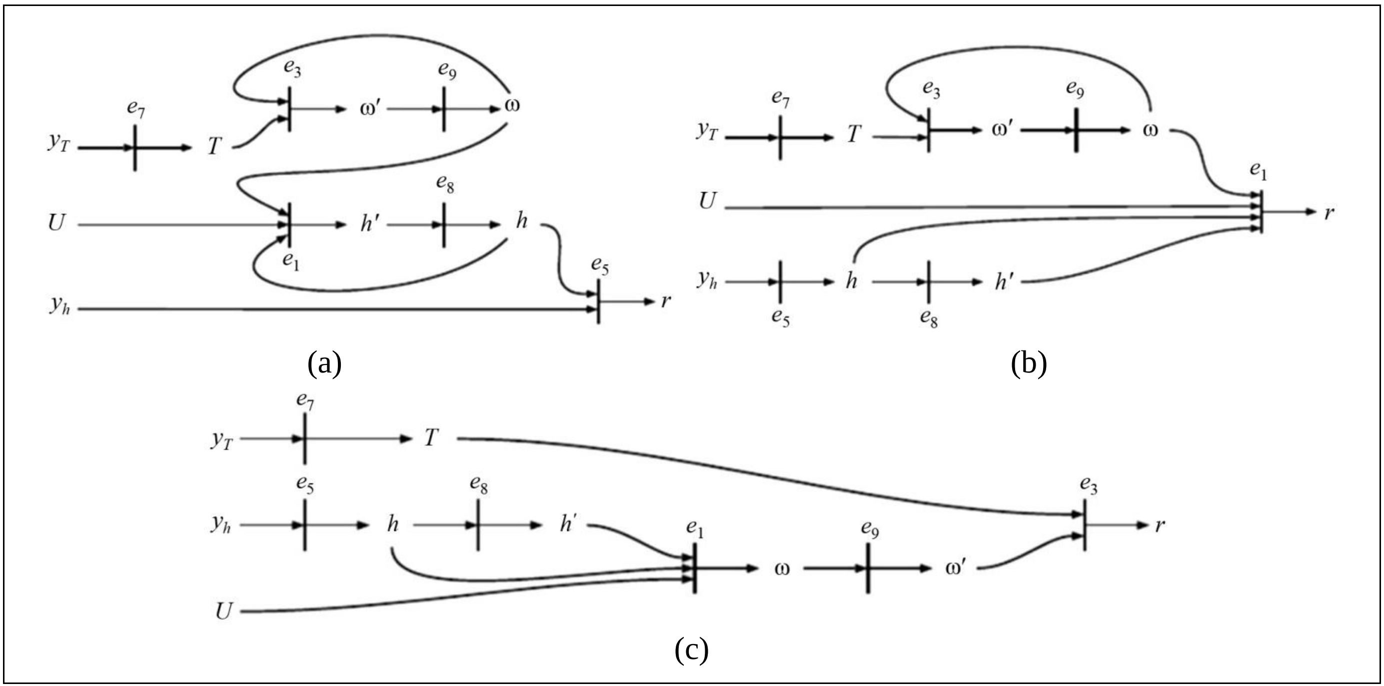

Fig. 11. SOS diagrams with different residual formation methods: (a) integral, (b) mixed, and (c) differential.

Now we derive the final computation formula for fault diagnosis based, e.g., on the diagram in Fig. 11c. This formula determines the content of any algorithm implemented. By making the corresponding substitutions, introducing the differentiation operator $p = d / dt$, and performing simple transformations, we arrive at the "exact" expression for the residual:

$$r = \frac{Jp + b}{K_a}\left(\frac{U}{y_h - f_h} - Lp\ln(y_h - f_h) - R - f_R\right) - y_T + f_T.$$

However, by definition, the faults f_h, f_R, and f_T are unavailable for direct measurement; thus, the formula for the residual in the neighborhood of the point $f_h = 0$, $f_R = 0$, $f_T = 0$, reduced to the signal of the motor shaft torque sensor, becomes

$$r(f_h, f_R, f_T) = \frac{Jp + b}{K_a}\left(\frac{U}{y_h} - Lp\ln(y_h) - R\right) - y_T. \quad (7)$$

According to this formula, the measurable signals U, y_h, and y_T are used to detect at least the separate faults f_h, f_R, and f_T. In the absence of faults, equality (7) takes a zero value. The values of the system parameters J, b, K_a, L, and the nominal (fault-free) value of R must be known.

The sensitivity to the "permanent" faults ($pf_h = 0$, $pf_R = 0$, $pf_T = 0$) is characterized by the values

$$\frac{\partial r}{\partial f_h} = \frac{Jp + b}{K_a}\left(Lp\frac{1}{y_h} - U\frac{1}{y_h^2}\right), \quad \frac{\partial r}{\partial f_R} = -\frac{b}{K_a}, \quad \frac{\partial r}{\partial f_T} = 1.$$

Hence, if the sensor's signal y_h of the motor winding current changes in accordance with the nonlinear differential equation

$$JLp^2\frac{1}{y_h} + bLp\frac{1}{y_h} - JUp\frac{1}{y_h^2} - bU\frac{1}{y_h^2} = 0,$$

the sufficient condition $\partial r / \partial f_h \neq 0$ of the sensitivity (7) to the fault f_h will not hold.

This circumstance suggests that, on certain phase trajectories of the system's motion, fault diagnosis may not be performed.[6]

4. FAULT DIAGNOSIS METHODS BASED ON SET THEORY

When using dynamic system models for monitoring purposes, there is always some deviation between the modeling results and the actual behavior of the systems. This phenomenon is due to both the neglect of some known "insignificant" relationships and the presence of unknown or inaccurately known relationships in the object modeled. The resulting modeling errors introduce uncertainty, which most often lies within the estimated ranges.

There are several ways to deal with model-related uncertainty, depending on whether it appears in the parameters (structured uncertainty) or in the model structure (unstructured uncertainty). The most developed group of approaches, known as active, is based on generating residuals insensitive to given uncertain-

[6] This applies not only to structural methods.

ties but sensitive to system faults. At the same time, there is another family of approaches, known as passive, that increase the reliability of a fault detection system by extending uncertainty to the residual values with creating appropriate adaptive thresholds.[7] The second approach was proposed in the fundamental works [55] (the time domain) and [27] (the frequency domain).

4.1. Initial Conditions and Problem Statement

The idea behind the set-membership approach is to use a geometric set to bind uncertain states. Well-known geometric sets include intervals, ellipsoids, polyhedra, zonotopes, etc. Among all the set-based approaches, zonotopes are remarkable for the lowest computational complexity of their implementation.

The approach under consideration involves the specific concept and notation of a zonotope—a mathematical construct[8] of the form

$$Z = \langle c, H \rangle = \left\{ c + Hz, \ \|z\|_\infty \le 1 \right\}, \tag{8}$$

where c and H are the center and segment matrix of the zonotope. The expression (8) is used for the formal designation of an interval in a multidimensional space. On the plane, this is a rectangle with sides $2h_1$ and $2h_2$; in the 3D space, a parallelepiped with edges $2h_1$, $2h_2$, $2h_3$, etc.

Zonotopes are characterized by linearity in the sense of

$$\langle c_1, H_1 \rangle \oplus \langle c_2, H_2 \rangle = \langle c_1 + c_2, [H_1, H_2] \rangle,$$
$$L\langle c, H \rangle = \langle Lc, LH \rangle,$$

where $\oplus$ denotes the Minkowski sum;[9] L is an arbitrary matrix of compatible dimensions. Here, $[H_1, H_2]$ means the formation of a new segment matrix from the initial ones H_1 and H_2. As a rule, it leads to the "growth" of the zonotope limits.

For a zonotope $Z = \langle c, H \rangle$, the weighted reduction operator $\downarrow_{q,W}^{H}$ [56] satisfies the property

$$\langle c, \downarrow_{q,W}^{H} \rangle \subseteq \langle c, H \rangle,$$

where $q \ge n$ indicates the maximum number of the columns of $\downarrow_{q,W}^{H}$ and W is a weight matrix of compatible dimensions. This operator reduces the segment matrix determining the "spread" of possible values in the neighborhood of the zonotope center.

Consider a linear system with discrete time $t = 0, 1, 2, \ldots$:

$$x_{t+1} = Ax_t + Bu_t + D_w w_t, \ y_t = Cx_t + D_v v_t, \tag{9}$$

where $x_t \in \Re^{n_x}$, $u_t \in \Re^{m_u}$, and $y_t \in \Re^{n_y}$ are the system state vectors, known inputs, and outputs, respectively; $w_t \in \Re^{m_w}$ and $v_t \in \Re^{m_v}$ are the disturbance and measurement noise vectors, respectively.

The values of the vectors w_t and v_t and the initial state x_0 are bounded by the zonotopes[10]

$$w_t \in \langle 0, I_{m_w} \rangle, \ v_t \in \langle 0, I_{m_v} \rangle, \ x_0 \in X_0 = \langle c_0, H_0 \rangle \tag{10}$$

and are supposed to be unknown at each time instant.

4.2. Zonotopic Observer

In this subsection, we briefly present one fault diagnosis approach using zonotopes [16, 49].

For system (9), an observer with a gain $G(k) \in \Re^{n_x \times n_y}$ is introduced:

$$\hat{x}_{t+1} = A\hat{x}_t + Bu_t + D_w w_t + G_t(y_t - C\hat{x}_t - D_v v_t). \tag{11}$$

As claimed [58], for any $t+1 \in N$, the observer (11) satisfies the inclusion

$$\hat{x}_{t+1} \in \aleph_{t+1} = \langle c_{t+1}, H_{t+1} \rangle, \tag{12}$$

where the center and segment matrix are given by

$$\begin{aligned} c_{t+1} &= (A - G_t C)c_t + Bu_t + G_t y_t, \\ H_{t+1} &= \left[(A - G_t C) \downarrow_{q,W}^{H}, D_w, G_t D_v \right]. \end{aligned} \tag{13}$$

According to formulas (13), the inclusion (12) is expanded as follows:

$$\begin{aligned} \hat{x}_{t+1} &\in \langle c_{t+1}, H_{t+1} \rangle \\ &= \left((A - G_t C) \langle c_t, \downarrow_{q,W}^{H} \rangle \right) \\ &\oplus \left(B\langle u_t, 0 \rangle \right) \oplus \left(C_t \langle y_t, 0 \rangle \right) \\ &\oplus \left(D_w \langle 0, I_{m_w} \rangle \right) \oplus \left(-G_t D_v \langle 0, I_{m_v} \rangle \right). \end{aligned} \tag{14}$$

Formula (14) defines the temporal transformation of the zonotope limiting the subsequent values of the vector $\hat{x}_t$ $\forall t+1 \in N$.

[7] Although the approach uses traditional models of the form (1) [30] (see part II of the survey), they do not constitute its core.

[8] A polyhedron representing the Minkowski sum of a set of vectors.

[9] The Minkowski sum of sets *A* and *B* of a linear space *V* is the set *C* consisting of the sums of all possible vectors from *A* and *B*: $C = \{c \mid c = a + b, \ a \in A, \ b \in B\}$, see the paper [57].

[10] The first two zonotopes are called unitary.

4.3. Fault Detection

Consider a system where formulas (9) are replaced by the equations

$$x_{t+1} = Ax_t + Bu_t + D_w w_t + Q^u f_t^u,\\ y_t = Cx_t + D_v v_t + Q^y f_t^y, \quad (15)$$

where $f_t \in \Re^{m_f}$ and $Q \in \Re^{n_y \times m_f}$ are the fault effect vector and the fault effect matrix of the actuator (the superscript u) and sensor (the superscript y), respectively. To detect faults in this system, the following actions are performed:

– Input $U_t = \{u_t\}$ and output $Y_t = \{y_t\}$ data are acquired and stored.

– The set of estimated states $Y_t = \{y_t\}$ is calculated using equation (14) and the interval constraints (10)

$$X_0, u_t, y_t, w_t, v_t \to X_t^e.$$

– The set of estimated outputs Y_t^e is calculated using the second equation (9) and the interval constraints (10)

$$X_t^e, v_t \to Y_t^e.$$

– The intersection of the sets of estimated and measured outputs is calculated:

$$X_t^{f=0} = Y_t^e \cap Y_t. \quad (16)$$

If for each t the intersection (16) is non-empty, then the actual value of the system output is considered to coincide with its forecast; therefore, the absence of faults is concluded. The emptiness of this intersection is a sign of system faults.

In this case, the sensitivity condition of the test (16) to sensor faults in equations (15) is the non-strict inequality

$$\sum_{t=0}^{\infty} (c_t^{rf})^{\mathrm{T}} c_t^{rf} \geq \beta_s^2 \sum_{t=0}^{\infty} f_t^{\mathrm{T}} f_t \quad (17)$$

given the existence of matrices[11] $P \in S_{\succ 0}^{n_x}$, $K \in \Re^{n_x \times n_y}$, and $N \in \Re^{n_y \times n_y}$ as well as scalars $\alpha > 0$ and $\beta > 0$ such that[12]

$$\begin{bmatrix} \alpha P & * & * \\ \alpha A^{\mathrm{T}} P - \alpha C^{\mathrm{T}} K^{\mathrm{T}} & \alpha P + C^{\mathrm{T}} NC & * \\ -\alpha Q^{\mathrm{T}} K^{\mathrm{T}} & Q^{\mathrm{T}} N^{\mathrm{T}} C & Q^{\mathrm{T}} NQ - \beta^2 I_{m_f} \end{bmatrix} \succ 0.$$

In (17), c_t^{rf} is the center of the zonotope $R_t = \langle c_t^r, H_t^r \rangle, \forall t \in N$, of the residual $r_t^f = y_t - Cx_t$ with the parameters

$$c_t^r = M(y_t - Cc_t),\ H_t^r = \begin{bmatrix} MCH_t & MD_v \end{bmatrix},$$

where $M \in \Re^{n_y \times n_y}$ is the gain matrix of the zonotope of the residual; as an option, the equalities $G = P^{-1}K$ and $M = N^2$ can be used.

The corresponding proofs and examples of using zonotopes can be found in [51, 59].

5. STATISTICAL FAULT DIAGNOSIS

The pattern recognition-based diagnostic approach is based on the statistical decision method, which consists in the following.

Consider a system with a discrete feature space (measurements) of dimension k with axes ξ_1, ξ_2,..., ξ_k and n gradations along the axes π_1, π_2,..., π_n, respectively. The gradations provide for both the operable technical condition of the system and the presence of various faults. By assumption, there are sufficiently many such systems,[13] and depending on the relevant characteristics, they are divided into classes K_{ij}, $i = \overline{1, k}$, $j = \overline{1, n}$, including variants without faults $\mathrm{K}_{\text{no fault}}$ or with different faults $\mathrm{K}_{m\text{th fault}}$, where $m \leq kn - 1$.

The main parameter is an empirical (statistical) estimate of the likelihood ratio. In a simplified version (a "local" estimate) of the method, it is calculated by the formula [60]

$$L(\xi) = \frac{f_l(\xi)}{f_h(\xi)} \approx \frac{r_l + 1}{r_h + 1} \frac{N_h + 2}{N_l + 2}, \quad (18)$$

where $f_l(\xi)$ and $f_h(\xi)$ are the Bayesian estimates of the probabilities that the system from the lth and hth class, respectively, will fall into a certain point ξ (or its neighborhood); r_l and r_h are the number of systems of the lth and hth classes falling into the point ξ according to the a priori information; finally, N_l and N_h are the number of systems of the lth and hth classes, respectively, in the training sample. More accurate, albeit complex, formulas with probability densities can be found in [60, 61].

[11] The first matrix mentioned is positive definite and symmetric, $P \succ 0$.

[12] Here, asterisks serve to simplify the form of the symmetric matrix.

[13] Due to the ergodicity hypothesis of random processes [62], for a priori data, the matter concerns the number of systems used for training systems; for a posteriori data, the number of time-separated observations of the system diagnosed.

To decide on the system's belonging to a particular class, the following threshold rule is applied:

- the system is operable if $L(\xi) \in \mathrm{K}_{\text{no fault}}$;

- the system has *m*th fault if $L(\xi) \in \mathrm{K}_{m\text{th fault}}$.

The training stage lies in memorizing the systems from the training sample, and the statistical estimate (18) is calculated during the current recognition (fault diagnosis).

Formally, the solution of the fault diagnosis (prediction) problem using the mathematical apparatus of statistical classification does not differ in principle from that of other technical problems requiring recognition. However, there are several peculiarities that must be taken into account. The main stages of solving the problem include:

• selection of a recognition model, which can be deterministic or probabilistic and depends on the degree of mixing of the sets belonging to different classes;

• description of the patterns R_λ of different classes (operable and different faults) based on a priori information; evaluation of the informativeness of different parameters (their significance for recognition) allows optimizing the description of patterns;

• comparison of current information about the system monitored with a priori specified patterns R_λ of the classes;

• deciding on the degree of operability of the system or its parts based on the monitoring data.

The main peculiarity of the statistical approach is that, for acceptable performance, it requires a significant amount of both a priori data (training) and a posteriori data (current monitoring measurements). Nevertheless, the "local" estimate (18) allows solving problems with $k \le 50$ parameters (measurements) and $\pi \le 8$ gradations for each parameter (related to the number of possible faults). In other words, the matter concerns not too large samples, which is essential for monitoring tasks.

Statistical fault diagnosis based on pattern recognition has become widespread and developed in Russian scientific literature and technology; for example, see [63–65].

CONCLUSIONS

The use of nonclassical representations or descriptions for dynamic systems has significantly expanded the possibilities for their fault diagnosis, primarily in terms of overcoming the traditional difficulties for engineering applications: noise in real measurements and distortions inevitably introduced into formal models. As a result, diagnostic and monitoring systems with fundamentally new capabilities and areas of practical application have been created and widely adopted.

At the same time, this path poses additional problems: besides complicating the apparatus used, the results obtained may be opaque, vague, ambiguous, or overly general. A lack of specifics can seriously restrict the application of the relevant approaches.

Part IV of the survey will analyze new approaches to fault diagnosis and the integration of various models and methods.

REFERENCES

1. Forssell, U. and Ljung, L., Closed-Loop Identification Revisited, *Automatica*, 1999, vol. 35, no. 7, pp. 1215–1241.
2. Haber, R. and Unbehauen, H., Structure Identification of Nonlinear Dynamic Systems – A Survey on Input/Output Approaches, *Automatica*, 1990, vol. 26, no. 7, pp. 651–677.
3. Isermann, R., *Fault-Diagnosis Systems. An Introduction from Fault Detection to Fault Tolerance*, Berlin–Heidelberg: Springer, 2006.
4. Kalyavin, V.P. and Davydov, N.A., *Nadezhnost' i diagnostika avtomototransportnykh sredstv* (Reliability and Diagnosis of Motor Vehicles), St. Petersburg: Elmor, 2014. (In Russian.)
5. Palyukh, B.V., Shprekher, D.M., and Bogatikov, V.N., Electromechanical Systems Diagnosis Based on Neural Network Technologies, *Software & Systems*, 2015, no. 3, pp. 5–11. (In Russian.)
6. Pushkarev, D.O., Application of Neural Network Expert Systems for Monitoring, Diagnosis and Forecasting of Technical Condition of Aircraft Engines, *Inform.-Tekhnol. Vestn.*, 2020, no. 3(25), pp. 51–59. (In Russian.)
7. Kulida, E.L. and Lebedev, V.G., Modern Approaches to Prognostics and Health Management of an Aircraft Electromechanical Actuator, *Control Sciences*, 2024, no. 3, pp. 3–15.
8. Vasilescu, M.-V., Epikhin, A.I., and Toriya, T.G., Diagnosis of the Technical Condition of a Boiler Plant Using Bayesian Networks, *Operation of Maritime Transport*, 2023, no. 3, pp. 113–120.
9. Loskutov, A.I., Stolyarov, A.V., and Ryakhova, E.A., Method for Control of Technical State of Onboard Radioelectronic Equipment of Spacecrafts Based On Interaction of Elements of Distributed Technical Diagnosis System, *Testing. Diagnosis*, 2021, vol. 24, no. 1 (271), pp. 44–50. (In Russian.)
10. Haykin, S.S., *Neural Networks and Learning Machines. Vol. 3*, Upper Saddle River, NJ: Pearson Education, 2009.
11. Diaconescu, E., The Use of NARX Neural Networks to Predict Chaotic Time Series, *WSEAS Trans. Comp. Research*, 2008, vol. 3, pp. 182–191.
12. Liu, G.P. and Patton, R.J., *Eigenstructure Assignment for Control System Design*, New York: John Wiley & Sons, 1998.
13. Medsker, L. and Jain, L.C., *Recurrent Neural Networks: Design and Applications*, Cleveland: CRC Press, 1999.
14. Kruglov, V.V. and Borisov, V.V., *Iskusstvennye neironnye seti. Teoriya i praktika* (Artificial Neural Networks: Theory and Practice), Moscow: Goryachaya liniya – Telekom, 2002. (In Russian.)
15. Reizlin, V.I., *Chislennye metody optimizatsii* (Numerical Optimization Methods), Tomsk: Tomsk National Research Technical University, 2013. (In Russian.)

16.Krysander, M., Åslund, J., and Nyberg, M., An Efficient Algorithm for Finding Minimal Overconstrained Subsystems for Model-Based Diagnosis, *IEEE Transactions on Systems, Man, and Cybernetics – Part A: Systems and Humans*, 2008, vol. 38, no. 1, pp. 197–206.
17.Korbicz, J., Patan, K., and Obuchowicz, A., Dynamic Neural Network for Process Modelling in Fault Detection and Isolation Systems, *Applied Mathematics and Computer Science*, 1999, vol. 9, no. 2, pp. 519–546.
18.Dexter, A.L. and Benouarets, M., Model-Based Fault Diagnosis Using Fuzzy Matching, *IEEE Trans. on Sys. Man. and Cyber. Part A: Sys. & Humans*, 1997, vol. 27, no. 5, pp. 673–682.
19.Isermann, R., On Fuzzy Logic Applications for Automatic Control, Supervision and Fault Diagnosis, *IEEE Trans. on Sys. Man. and Cyber. Part A: Sys. & Humans*, 1998, vol. 28, no. 2, pp. 221–235.
20. Polkovnikova, N.A. and Kureichik, V.M., Neural Network Technologies, Fuzzy Clustering and Genetic Algorithms in an Expert System, *Izvestiya SFedU. Eng. Sci.*, 2014, no. 7 (156), pp. 7–15. (In Russian.)
21. Akhmetkhanov, R.S., Dubinin, E.F., and Kuksova, V.I., Application of Methods and Models of Fuzzy Logic in Technical Diagnosis Systems, *Machine Drives and Parts*, 2018, no. 1–2 (27), pp. 6–11. (In Russian.)
22. Shumsky, A.E. and Zhirabok, A.N., A Fault Diagnosis Method for Discrete-Event Systems Based on the Fuzzy Finite State Automaton Model, *Control Sciences*, 2025, no. 2, pp. 31–41.
23.Klyuchkin, A.K. and Khamatov, A.A., Synthesis of the Diagnostic Model Based on Fuzzy Logic Apparatus, *Aerospace Instrument-Making*, 2023, no. 7, pp. 34–41. (In Russian.)
24.Ablyazov, E.K. and Ablyazov, K.A., Assessment of the Technical Condition of the Port Lifting and Transport Equipment (LTE) Using Fuzzy Logic, *Marine Intellectual Technologies*, 2024, no. 2-1 (64), pp. 242–251. (In Russian.)
25.Fantuzzi, C. and Rovatti, R., On the Approximation Capabilities of the Homogeneous Takagi-Sugeno Model, *Proceedings of the 5th IEEE International Conference on Fuzzy Systems*, vol. 2, NJ: IEEE, Piscataway, 1996, pp. 1067–1072.
26.Rovatti, R., Takagi-Sugeno Models as Approximators in Sobolev Norms: The SISO Case, *Proceedings of the 5th IEEE International Conference on Fuzzy Systems, vol. 2*, NJ: IEEE, Piscataway, 1996, pp. 1060–1066.
27.Takagi, T. and Sugeno, M., Fuzzy Identification of Systems and Its Applications to Modeling and Control, *IEEE Transactions on Systems, Man and Cybernetics*, 1985, no. 1, pp. 116–132.
28.Jun, W., Shitong, W., and Chung, F.-L., Positive and Negative Fuzzy Rule System, Extreme Learning Machine and Image Classification, *Int. J. of Machine Learning and Cybernetics*, 2011, no. 2 (4), pp. 261–271.
29.Graaff, A.J. and Engelbrecht, A.P., Clustering Data in Stationary Environments with a Local Network Neighborhood Artificial Immune System, *Int. J. of Machine Learning and Cybernetics*, 2012, vol. 3, no. 1, pp. 1–26.
30.Bukov, V.N., Bronnikov, A.M., Popov, A.S., and Shurman, V.A., Technical Condition Monitoring Methods to Manage the Redundancy of Systems. Part II: Classical Models, *Control Sciences*, 2025, no. 3, pp. 2–11.
31.*Diagnosis and Fault-Tolerant Control 2: From Fault Diagnosis to Fault-Tolerant Control*, coord. by V. Puig and S. Simani. London: ISTE Ltd.; Hoboken: John Wiley & Sons, 2021.
32.Bezdek, J.C., *Pattern Recognition with Fuzzy Objective Function Algorithms*, Berlin: Springer Science & Business Media, 2013.
33.Chow, E.Y. and Willsky, A.S., Analytical Redundancy and the Design of Robust Detection Systems, *IEEE Trans. Automatic Control*, 1984, vol. 29, no. 7, pp. 603–614.
34.Duan, G., How, D., and Patton, R., Robust Fault Detection in Descriptor Systems via Generalised Unknown Input Observers, *Int. J. Systems Science*, 2002, vol. 32, no. 2, pp. 7724–7729.
35.Zaytoon, J. and Lafortune, S., Overview of Fault Diagnosis Methods for Discrete Event Systems, *Annual Reviews in Control*, 2013, vol. 37, no. 2, pp. 308–320.
36.Cassandras, Ch. and Lafortune, S., *Introduction to Discrete Event Systems*, 2nd ed., New York: Springer Science+Business Media, 2008.
37.Zhirabok, A.N., Kalinina, N.A., and Shumskii, A.E., Technique of Monitoring a Human Operator's Behavior in Man-Machine Systems, *Journal of Computer and Systems Sciences International*, 2018, vol. 57, no. 3, pp. 443–452.
38.Zhirabok, A.N., Kalinina, N.A., and Shumskii, A.E., Method for the Functional Diagnosis of Nondeterministic Finite State Machines, *Journal of Computer and Systems Sciences International*, 2020, vol. 59, no. 4, pp. 565–574.
39.Kościelny, J.M., Application of Fuzzy Logic for Fault Isolation in a Three-Tank System, *Proc. of the 14th IFAC World Congress*, Beijing, 1999, vol. 7, pp. 73–78.
40.Hengy, D. and Frank, P.M., Component Failure Detection Using Local Second-Order Observers, in *Fault Detection & Reliability*, Singh, M.G., Ed., Oxford: Pergamon Press, 1987, pp. 147–154.
41.Frank, P.M., Fault Diagnosis in Dynamic Systems via State Estimation – A Survey, in *System Fault Diagnosis, Reliability and Related Knowledge-Based Approaches*, Tzafestas. E.A., Ed., Dordrecht: Dr. Reidel Publ., 1987, vol. 1, pp. 35–98.
42.Koenig, D., Nowakowski, S., and Cecchin, T., An Original Approach for Actuator and Component Fault Detection and Isolation, *Proceedings of the CAA Symposium on Fault Detection Supervision and Safety for Technical Processes (SAFEPROCESS '97)*, Kingston upon Hull, 1997, vol. 1, pp. 95–105.
43.Garcia, F.L., Izquierdo, V., de Miguel, L., and Peran, J., Fuzzy Identification of Systems and Its Applications to Fault Diagnosis Systems, *Proceedings of the CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS '97)*, Kingston upon Hull, 1997, vol. 2, pp. 705–712.
44.Koscielny, J.M., Sedziak, D., and Sikora, A., The "DIAG" System for Fault Detection and Isolation in Industrial Processes, *Proceedings of the CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS '94)*, Helsinki, 1994, vol. 2, pp. 790–795.
45.Han, Z. and Frank, P.M., Physical Parameter Estimation Based FDI with Neural Networks, *Proceedings of the CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS '97)*, Kingston Upon Hull, 1997, vol. 1, pp. 294–299.
46.Benkbedda, H. and Patton, R., Information Fusion in Fault Diagnosis Based on B-spline Networks, *Proceedings of the CAA Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS '97)*, Kingston upon Hull, 1997, vol. 2, pp. 681–686.
47.Cassar, J.P. and Staroswiecki, M., A Structural Approach for the Design of Failure Detection and Identification Systems, *IFAC Proceedings Volumes*, 1997, vol. 30, no. 6, pp. 841–846.
48.Ploix, S., Désinde, M., and Touaf, S., Automatic Design of Detection Tests in Complex Dynamic Systems, *IFAC Proceedings Volumes*, 2005, vol. 38, no. 1, pp. 478–483.

49. *Diagnosis and Fault-Tolerant Control 1: Data-Driven and Model-Based Fault Diagnosis Techniques*, coord. by V. Puig and S. Simani, London: ISTE Ltd.; Hoboken: John Wiley & Sons, 2021.
50. Kopkin, E.V. and Ivanyu, A.Yu. Structural-Stochastic Computational Models for Monitoring the Technical Condition of Onboard Spacecraft Systems, *Izv. Tula State Univ. Eng. Sci.*, 2022, no. 10, pp. 9–14. (In Russian.)
51. *Recommendations for Standardization R 50.1.048-2004: Information and Telecommunication Gaming Systems. Terms and Definitions*, Moscow: Gosstandart of Russia, 2004. (In Russian.)
52. Bruskin, S.N., Druzhayev, A.A., Maron, A.I., and Maron, M.A., Effective Methods for Creation of Malfunction Search Algorithms in Information Systems, *Intellekt. Sist. Proizv.*, 2017, vol. 15, no. 3, pp. 88–93. (In Russian.)
53. Armengol, J., Bregyn, A., Escobet, T., et al., Minimal Structurally Overdetermined Sets for Residual Generation: A Comparison of Alternative Approaches, *IFAC Proceedings Volumes*, 2009, vol. 42, no. 8, pp. 1480–1485.
54. Krysander, M., Åslund, J., and Frisk, E., A Structural Algorithm for Finding Testable Sub-models and Multiple Fault Isolability Analysis, *Proceedings of the 21st International Workshop on Principles of Diagnosis (DX-10)*, Portland, USA, 2010. DOI: https://doi.org/10.36001/phmconf.2010.v2i1.1940
55. Horak, D., Failure Detection in Dynamic Systems with Modelling Errors, *AIAA J. of Guidance, Control and Dynamics*, 1988, vol. 11, no. 6, pp. 508–516.
56. Song, Y., Sai, M.P.D., and Yu, H., Zonotope-Based Nonlinear Model Order Reduction for Fast Performance Bound Analysis of Analog Circuits with Multiple-Interval-Valued Parameter Variations, *Proceedings of IEEE Design, Automation & Test in Europe: Conference & Exhibition*, Dresden, Germany, 2014. DOI: 10.7873/DATE2014.024
57. Panyukov, A.V., The Linear Inequalities Set Representation of Minkowski’s Sum for Two Polyhedrons, *Bulletin of the South Ural State Univ. Math. Model. Program.*, 2012, no. 14, pp. 108–119. (In Russian.)
58. Combastel, C., Zonotopes and Kalman Observers: Gain Optimality under Distinct Uncertainty Paradigms and Robust Convergence, *Automatica*, 2015, no. 55, pp. 265–273.
59. Himmelblau, D.M., *Fault Diagnosis in Chemical and Petrochemical Processes*, Amsterdam: Elsevier, 1978.
60. Gaskarov, D.V., Golinkevich, T.A., and Mozgalevskii, A.V., *Prognozirovanie tekhnicheskogo sostoyaniya i nadezhnosti radioelektronnoi apparatury* (Predicting the Technical Condition and Reliability of Radio-Electronic Equipment), Moscow: Sovetskoe Radio, 1974. (In Russian.)
61. Mozgalevskii, A.V. and Koida, A.N., *Voprosy proektirovaniya sistem diagnostirovaniya* (Design Issues for Diagnosis Systems), Leningrad: Energoatomizdat, 1985. (In Russian.)
62. Ventsel’, E.S., *Teoriya veroyatnostei* (Probability Theory), Moscow: KnoRus, 2010. (In Russian.)
63. Obozov, A.A. and Tarichko, V.I., Stochastic Image Recognition Theory As a Means to Improve Technical Diagnosis of Car Engines, *Dvigatelestroenie*, 2013, no. 3 (253), pp. 16–22. (In Russian.)
64. Usik, B.A., Luchin, A.V., and Obynochnyi, I.A., Selection of Parameters for Diagnosing Converter Devices with Redundant Elements by Pattern Recognition Methods, *Izv. Inst. Inzhen. Fiz.*, 2022, no. 3 (65), pp. 14–16. (In Russian.)
65. Shamolin, M.V., Generalized Checking Problem in Diagnosis Problems, *Itogi Nauki Tekh. Sovremenn. Mat. Prilozh. Tematich. Obz.*, 2022, vol. 209, pp. 117–126. (In Russian.)

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