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A MATHEMATICAL MODEL OF MECHANICAL PENETRATION RATE WITH THREE CONTROL PARAMETERS TO OPTIMIZE OIL AND GAS WELL DRILLING

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Abstract. The types of rock destruction at the bottom hole under different loads on the drilling bit are considered, and well-known domestic and foreign models of the penetration rate are analyzed. As shown, they have no optima as power-type functions, being unsuitable for drilling optimization. In addition, they can be used for quick drilling control by adjusting only one parameter (the load on the bit). A mathematical model based on a sinusoid curve is constructed. This model allows the simultaneous control of three drilling mode parameters, namely, the axial load on the bit, its rotation frequency, and the mud flow rate for flushing the well. The adequacy of the model to the drilling process is verified, and its software implementation is performed. This model automatically recognizes the rock at the bottom hole during drilling, adapts to it, and calculates the optimal control parameters for destructing the traversed rock. The model is intended for an intelligent optimal adaptive control system for oil and gas well drilling.

Keywords: analysis of mathematical models of drilling rate, the optimum of a function, a model with three control parameters, optimal adaptive control, adequacy of the model.

INTRODUCTION

The main drilling process in well construction is the mechanical destruction of the rock with a bit at the bottom hole. This process is described by the equation of the mechanical penetration rate υ_m. Numerous factors affect the penetration rate; among them, note the load and torque on the bit, bit rotation frequency, mud flow rate and pressure, the rheological properties of the mud, and the lithological characteristics of the rock at the bottom hole.

On a large array of field and experimental drilling data, M.G. Bingham (the USA) studied in detail the function $\upsilon_m = f(\overline{G})$, where $\overline{G}$ is the specific axial load on the bit [1, 2] (the load reduced to the bottom hole area $S_{bot} = \pi D_{bit}^2/4$, where D_{bit} is bit diameter). As he concluded, this function is of power type, unimodal, and has the form of an *S*-shaped curve (Fig. 1). The qualitative relation of the function with physical and

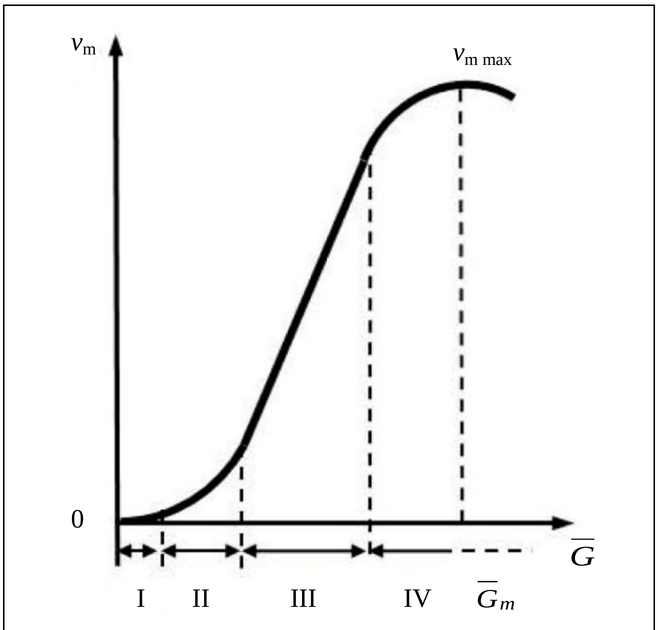


Fig. 1. Bingham's *S*-curve.

mechanical properties of the rock formation and parameters of flushing fluid was also established by Bingham. Domestic and foreign drilling practice confirms his conclusions; see [2–9] and other publications.

According to Bingham, the penetration rate function υ_m has several zones:

- zone I, where axial loads are low, the rock is destructed insignificantly (surface abrasion), and bit teeth pressure on the rock is smaller than its strength limit;
- zone II, where the contact pressure of bit teeth on the bottom hole increases and small pieces of the rock break off, causing a considerable increase in the penetration rate υ_m according to a nonlinear power-type law;
- zone III, where the load G exceeds the rock strength, causing the significant volumetric destruction of the rock according to an almost-linear law with a slope much greater than in zone I;
- zone IV, where the flushing fluid does not carry the drilled rock to the surface in due time; the cuttings are deposited on the bottom hole and are remilled. In addition, the penetration rate υ_m achieves maximum at the axial load G_m and then decreases.

The mathematical model of the drilling rate should reliably reflect these rock destruction zones and have an optimum for calculating the optimal values of the mode parameters. It is also important to determine the model's control parameters.

1. DRILLING MODELS

Many mathematical models of the penetration rate have been developed to describe the rock destruction process, both in Russia and abroad; see [1–3, 5–8, 10–12]. The basic (and typical) models and curves for the mechanical penetration rate are combined in Table 1.

Table 1

Mathematical models and graphs of the mechanical drilling rate

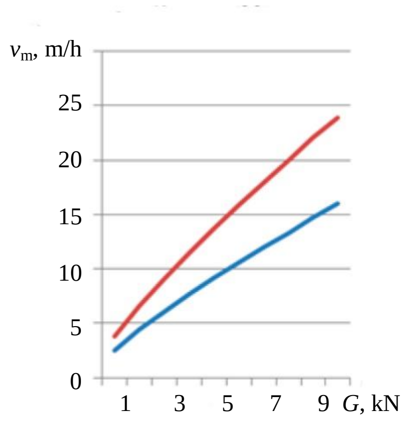
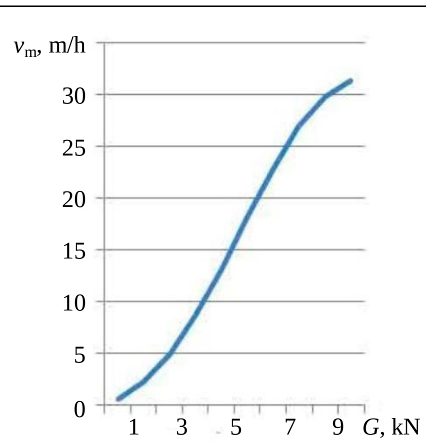
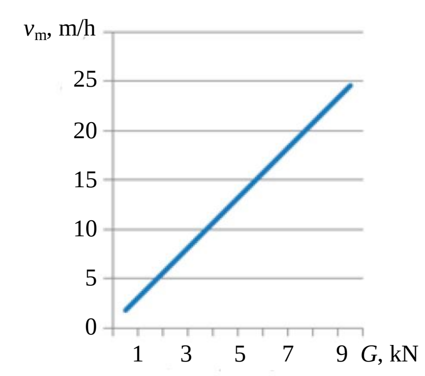
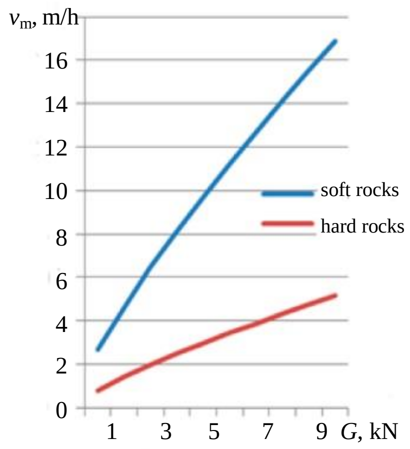
Models	Graphs
The VNIIBT model [6, 7]: $\upsilon_m = k_{dril}\, G^{\delta} n^{\alpha}$, where k_{dril} is the rock drillability coefficient, which depends on bit type and rock and mud properties; n is the bit rotation frequency, rpm; finally, δ and α are the slope parameters of the curve.	
The Pogarskii model [5–7]: $\upsilon_m = \dfrac{a\, n^{\alpha} \bar{G}^2}{1 + b^4 \bar{G}^4}$, where a is the drillability coefficient; α is an index affecting the curve steepness; finally, b is a coefficient depending on the mud flow rate.	

Table 1 (continued)

The Tenneco Oil Company model (the USA) [2, 7]: $$\upsilon_{\text{m}} = \frac{k_{\text{dril}}\,(G - G_0)\,n^{\alpha}}{f(h)},$$ where G_0 is the critical load on the bit for the teeth to penetrate into the rock; $f(h)$ is a function characterizing the bit condition.	
The Galle–Woods–Lubinski model (the USA) [2, 7]: $$\frac{dh}{dt} = k_{\text{dril}} \frac{\overline{G}^{\beta}\, r}{[a(D_3)]^{b}},$$ where β is the power index at the axial load; r is a function depending on the bit rotation frequency n, calculated separately for rocks of different hardness; $a(D_3)$ is the wear function of the bit armament; D_3 is the relative wear degree of the bit armament; finally, b is the power index for the function $a(D_3)$.	

According to analysis results, these models describe bit operation with different accuracy mainly within the linear zone III of Bingham's curve and have no maximum. Therefore, they are unsuitable for optimization. Moreover, in drilling practice, penetration rate control based on these models often adjusts the axial load G only: the parameters n and Q remain fixed during the trip. As a result, drilling modes are not optimal.

The contribution of the axial load G to the penetration rate reaches 43%; for the bit rotation frequency n and the mud flow rate Q, the corresponding figures are up to 14% and 7%, respectively [13]. Hence, they should be considered when calculating the optimal drilling parameters.

2. A DRILLING MODEL WITH THREE PARAMETERS

As a regression equation, Bingham's curve $\upsilon_{\text{m}} = f(G)$ can be represented as a fragment of a sinusoid shifted to quadrant I of the coordinate plane (Fig. 2):

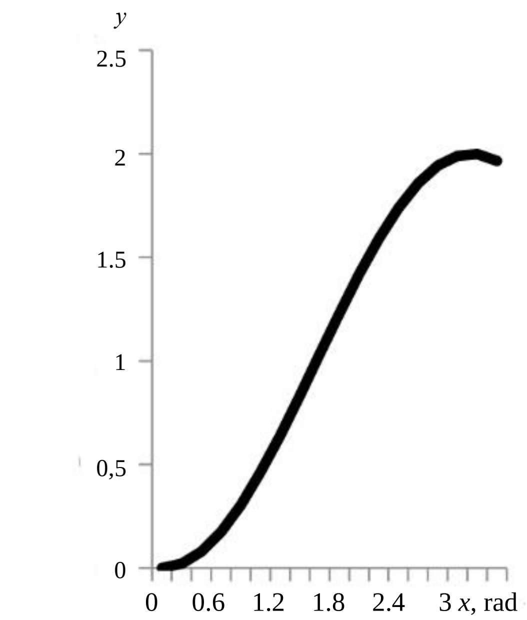


Fig. 2. A fragment of sin(x - π/2) + 1 shifted to quadrant I.

the graph should be raised by one on the y axis and shifted to the right by 1.57 rad on the x axis.

In drilling modes, depending on rig power and well depth, the axial load G on the bit varies from 0 to 40 N, and the drilling rate may reach 10–14 m/h and higher [4–8, 10–12, 14]. To match the argument x with the load G and the function y with the real drilling rate υ_m, we have to rescale them by introducing appropriate coefficients into the equation:

– the constant C_G to convert the radian measure of the argument x into the units of the load G, N;

– the proportionality factor k_υ to scale the function y vertically.

As a result, the dependence $\upsilon_m = f(G)$ takes the form

$$\upsilon_m = k_G \sin(C_G G - 1.57) + 1.$$

According to [3, 10–12] and others, the functions $\upsilon_m = f(n)$ with G = const and Q = const and $\upsilon_m = f(Q)$ with G = const and n = const are also unimodal. Their graphs are presented in Fig. 3 and can also be approximated by sinusoid fragments.

The curve $\upsilon_m = f(n)$ is described by the equation $\upsilon_m = k_n \sin(C_n n)$, and the curve $\upsilon_m = f(Q)$ by the equation $\upsilon_m = k_Q (\sin(C_Q Q - 0.7) + 0.645)$, where the values k_n, k_Q, C_n, and C_Q have the same meaning as k_G and C_G for the curve of the load on the bit. The operating ranges for wells are as follows: the bit rotation frequency, from 10 to 120 rpm; the mud flow rate, from 20 to 80 l/s [4–8, 10–12, 14].

The full mathematical model of the drilling rate $\upsilon_m = f(G, n, Q)$ as a function of the three parameters for optimal control of the drilling process has the form of their product:

$$\upsilon_m = k_\upsilon (\sin(C_G G - 1.57) + 1) \times \sin(C_n n) \times (\sin(C_Q Q - 0.7) + 0.645), \qquad (1)$$

where $k_\upsilon = k_G k_n k_Q$ is the total coefficient of the curve shape, equivalent to the drillability coefficient of the rock traversed by the bit at the bottom hole.

We verified the reliability of this model and its adequacy to real drilling conditions using drilling report data for completed wells in the Krasnodar region: Vostochno-Pribrezhnaya no. 9, Peschanaya no. 7, and Krupskaya no. 1 (wells nos. 1–3 in Fig. 4, respectively). The average deviations of the experimental data from the data based on model (1) were 12%, 13%, and 23%, respectively, which is a good outcome: the wells were drilled according to the drilling project documentation (not in optimal modes). The closest-to-optimal results were obtained for Vostochno-Pribrezhnaya no. 9 (well no. 1).

The graphs of the function (1) and its components and the drilling data for the three wells are shown in Fig. 4.

To plot the four-dimensional function $\upsilon_m = f(G, n, Q)$ on the two-dimensional coordinate plane, we represented the argument x in Fig. 4 in relative units, with $x = G$ for the function $\upsilon_m = f(G)$, $x = n/6$ for the function $\upsilon_m = f(n)$, and $x = Q/4$ for the function $\upsilon_m = f(Q)$.

As a result, the following conclusions can be made.

- The data obtained from the drilled wells confirm that the drilling model (1) accurately enough, with average errors of 12–23%, describes the mechanical destruction of rocks. Note that the wells were drilled on the parameter values recommended by the projects, which are compiled according to the results of the

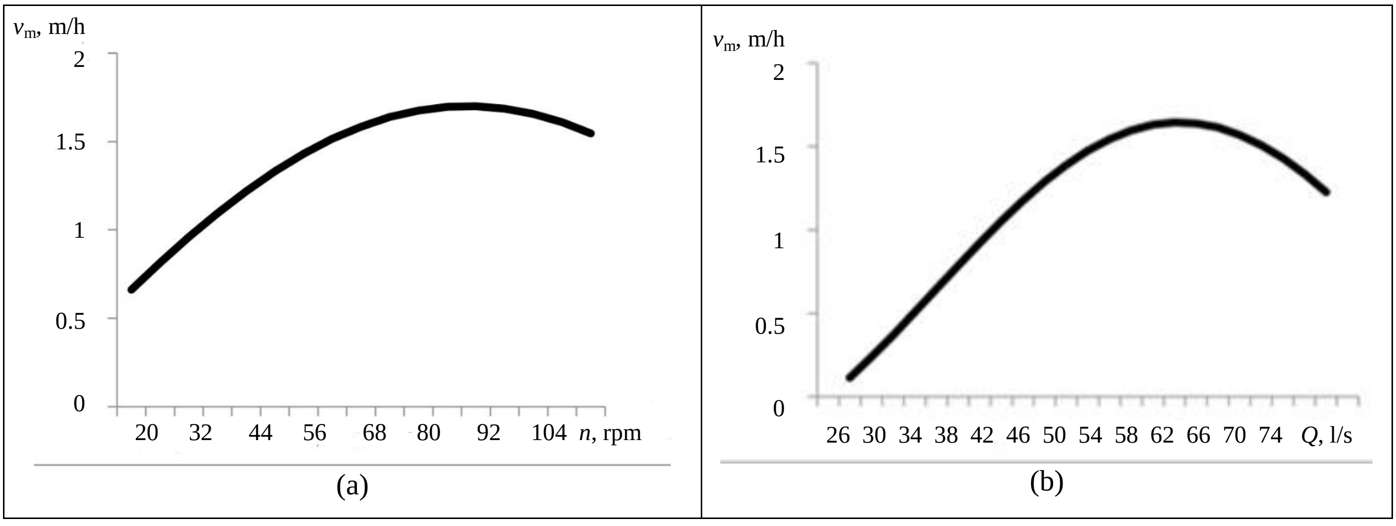


Fig. 3. The graphs of functions: (a) $\upsilon_m = f(n)$ and (b) $\upsilon_m = f(Q)$.

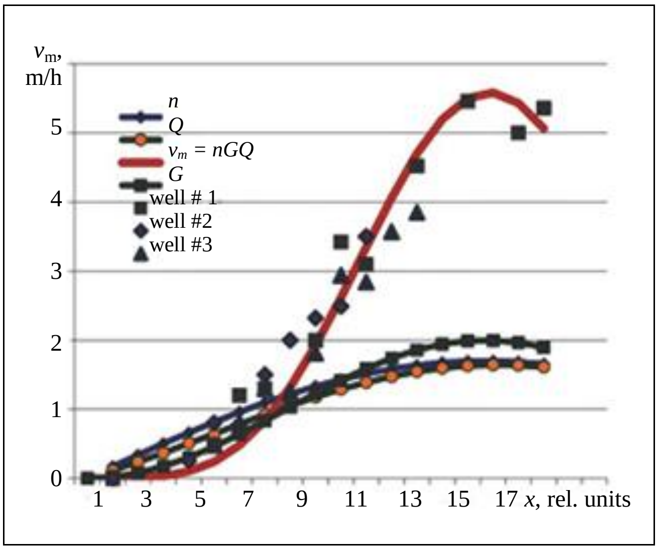


Fig. 4. The graphs of functions $\upsilon_m = f(G)$, $\upsilon_m = f(n)$, $\upsilon_m = f(Q)$, and $\upsilon_m = f(n, G, Q)$.

neighboring wells. For a new well, they are practically not optimal.

- During drilling, the optimal modes were achieved only at some depth intervals, mainly for well no. 1.
- The experimental and model-based data confirm that Bingham's curve is *S*-shaped.

3. ASSESSING THE ADEQUACY OF THE MODEL

As recommended in [15], generally accepted statistical criteria should be used for assessing the adequacy and quality of mathematical models and quickly estimating their main parameters. These recommendations were developed for transport networks. However, statistical criteria are universal and can be applied to models of any processes and objects.

Following the recommendations [15], we employed five criteria to assess the models:

– the absolute mean error δ_a,
– the relative mean error δ_p,
– the standard deviation ϑ_a,
– the relative standard deviation ϑ_p,
– the coefficient of correlation *r*.

At present, there are no precise values of these criteria under which a model is considered reliable. For applications, however, the relative criteria should not exceed 10%, and the coefficient of correlation should not be smaller than 0.9; for details, see [15].

The values of the adequacy criteria for model (1) are presented in Table 2.

According to the results, the model correlates well with real drilling processes and is suitable for optimal well control; the model's coefficient of correlation with drilling data is close to 1.

Table 2

Values of the adequacy criteria

Criterion	Well no. 1	Well no. 7	Well no. 9
δ_a	0.33	0.16	0.38
δ_p	10.01%	9.08%	18.40%
ϑ_a	2.02	0.93	1.21
ϑ_p	6.13%	5.59%	5.85%
r	0.98	0.74	0.89

4. PARAMETER OPTIMIZATION

The model was tested using the method of Bryansk partisans [16], i.e., an intelligent global optimization method for functions of several variables. This method includes two stages as follows. At the first stage (reconnaissance), the domain of the function is divided in half for each argument, and up to 30 agents are randomly initialized in each zone; then, the optimum of each zone is found, and the zone with the best optimum is selected. At the second stage (diversion), up to 500 agents are initialized in the selected zone, their optima are calculated, and the best optimum of the function is selected. We developed a Python program for optimum search and launched it with the following parameters: the number of partitions at the first stage, from 1 to 4; the number of reconnaissance agents, from 10 to 50; the number of diversion agents, from 200 to 500. The numerical results coincide; see Fig. 5 for one scenario of calculating the maximum drilling rate.

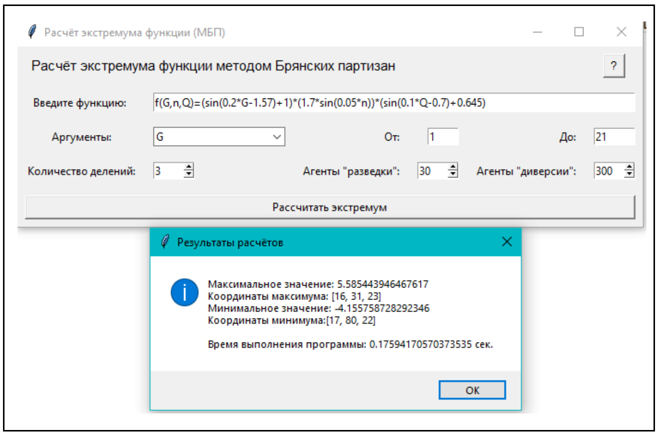


Fig. 5. The interface of the optimum search program.

The maximum mechanical penetration rate $\upsilon_{m\ max}$ = 5.58 m/h is achieved for G_{opt} = 16 N, n_{opt} = 31 rpm, and Q_{opt} = 23 l/s, which corresponds to the real parameters of drilling process control.

5. AN ADAPTIVE DRILLING METHOD

Model (1) is intended for the adaptive procedure of optimal drilling control.

The paper [17] described a methodology for adapting computer systems to exogenous impacts (intrusions) through their classification. It includes five modules (stages): input data (impact) processing, input data transformation (autocoding), searching for analogs in the database, classification, and feedback (developing the system response to the exogenous impacts). For the drilling process, this adaptation principle was modified as follows:

– With a chosen step of the penetration interval (e.g., every 0.3 m), the current values of the drilling parameters *G*, *n*, and *Q* and the resulting penetration rate υ_m are entered into the model.

– The model coefficients *k* and *C* are recalculated for the current values of *G*, *n*, *Q*, and υ_m. Therefore, the model is adapted to the rock at the bottom hole. The model automatically recognizes the type of rock traversed by the bit.

– The optimal values of the parameters G_{opt}, n_{opt}, and Q_{opt} are calculated on the adapted model. (The optimality criterion is υ_m = max.)

– The parameter values G_{opt}, n_{opt}, and Q_{opt} are set on the oil rig, and the next interval of 0.3 m is executed in the optimal mode.

This cycle (entering the new values of *G*, *n*, *Q*, and υ_m; recognizing the rock; adapting the model to it; calculating the optimal parameters; drilling in the optimal mode) is repeated until the well depth is reached, or the bit is worn. The described procedure has an obvious advantage: there is no need to identify the rock drilled at the bottom hole with the one in the lithological database of the well and classify it. (Note that the rock is not necessarily included in the database.)

CONCLUSIONS

As shown by the analysis, the widespread drilling models mainly involve the linear zone of Bingham's curve, adjust only one control parameter, have no optimum, and therefore are not suitable for optimization.

The new drilling model based on the sinusoidal curve allows the simultaneous optimal control of three drilling parameters (the load on the bit, the bit rotation frequency, and the mud flow rate) and has a common optimum for them. Moreover, the reliability of this model has been confirmed by the practical results obtained on the drilled wells: the model's coefficient of correlation with the drilling data is close to 1.

The optimal parameters calculated using the optimum search program have confirmed the suitability of the model for the optimal control of oil and gas well drilling.

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