ENERGY CONTROL OF THREE IDENTICAL PENDULUMS UNDER ENERGY CONSTRAINTS

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Abstract. This paper addresses the problem of energy control for three identical independent pendulums driven by a single scalar input. The objective is to asymptotically steer the energy of the first pendulum to a prescribed level while keeping those of the other two below given bounds. A penalty-type Lyapunov function combining a quadratic target term and barrier terms for the constraints is introduced. A numerically implementable explicit control law is derived based on the speed-gradient method. Additional consideration is given to the physical meaning of the basic model equations and the selective energy redistribution mechanism under a common control action. Numerical simulations are provided to confirm the convergence of the first pendulum's energy to the target level while satisfying the imposed constraints.

Keywords: energy control, identical pendulums, speed-gradient method, penalty functions, Hamiltonian systems, numerical simulation.

INTRODUCTION

Energy control problems for oscillatory mechanical systems with constraints arise in various applications, e.g., in systems for harvesting and storing energy based on pendulum arrays [1], vibration protection systems for spacecraft [2], and wearable devices for harvesting and storing energy from human motion [3]. In such applications, it is necessary to selectively excite the energy of a certain subsystem (a single motion mode) without causing other subsystems to exceed permissible energy levels. For underactuated systems of this type, an effective control tool is the speed-gradient (SG) method and its variants [4–6], supplemented by penalty or barrier functions [7–10].

In this paper, we generalize a previously studied problem for two pendulums to the case of three identical mechanical subsystems driven by the same scalar control. This formulation preserves the structural simplicity of the model but significantly complicates targeted motion separation: three degrees of freedom have to be controlled simultaneously using a single control channel.

Unlike the classical stabilization of a single coordinate, here it is required to redistribute energy appropriately: the first pendulum has to be energized to a prescribed level while keeping the energies of the second and third pendulums within given limits. The model's symmetry makes the problem particularly sensitive to the choice of initial conditions: if the initial states of the subsystems coincide, the overall control cannot form a selective mode.

The SG-method [4, 5] is well-suited for solving such problems, as a control law is constructed directly from the derivative of a specially chosen objective function. We include barrier terms in this function to prevent the energies of the second and third pendulums from approaching their upper bounds. Such an approach is a natural extension of the results for a two-pendulum system [8, 9] and yields a compact analytical control law.

A well-known control technique combining the requirements of stability and safe operation is based on *barrier Lyapunov functions* (BLFs). The concept of barrier functions dates back to the 1940s; see studies on the invariance of sets and M. Nagumo's results [11]. The name itself may be associated with barrier functions used in optimization, i.e., the terms added to an objective function to keep the solution variables beyond a forbidden region. The modern conceptual framework of BLFs emerged in the 2000s by uniting *control Lyapunov functions* (CLFs) with barrier functions [12, 13]. A BLF is a CLF tending to infinity as its arguments approach points of some "forbidden" set. The boundedness of a Lyapunov barrier function in a closed loop ensures that the solution will not enter this set. Since a BLF remains a Lyapunov function (positive definite with a negative definite derivative), it also serves to prove the stability of the limit set of a closed-loop system. Currently, BLFs have become a widespread controller design tool for numerous applications. Just one popular database reports more than 1500 citations using the term "BLF" over the past three years. Among them, motion control applications are richly represented.

Most early research on BLFs employed the logarithmic BLF for control design [13]. Over time, other types of BLFs were introduced, such as square tangent [14], variable square tangent [15], or inverse hyperbolic tangent [16]. Currently, several types of BLFs are in use. Although some comparisons are available [15], the systematic choice of an appropriate BLF for a particular application remains a topical problem. According to a contemporary literature review, the BLF approach continues to be popular. Various aspects of BLF application have been considered in the recent publications [17–20].

Note that in this paper, BLF and the SG-method organically complement each other. With the SG-method, the control law is designed directly from the derivative of an objective function, and the resulting controller has an analytical explicit form. In turn, BLFs are incorporated into the objective function as penalty terms (barrier terms) tending to infinity as the energies of the second and third pendulums approach given upper bounds. Thus, the SG-method forms the control law, while BLFs serve to satisfy the energy constraints throughout the transients. This combination yields a unified adaptive algorithm simultaneously stabilizing the energy of the first pendulum and satisfying the energy constraints for the other two pendulums.

1. PROBLEM STATEMENT AND DESCRIPTION OF SYSTEM DYNAMICS

Consider a system of three identical independent pendulums controlled by a common input $u(t)$. It is required that:

– the energy of the first pendulum asymptotically approach a given level E_1;

– the energies of the second and third pendulums be kept below levels E_2 and E_3, respectively.

This formulation describes the problem of selectively exciting a given mode under strict constraints on the other modes.

For the kth pendulum, the total mechanical energy is given by

$$H_0^k(p_k, q_k) = \frac{p_k^2}{2ml^2} + mgl(1 - \cos q_k),\quad k = 1, 2, 3. \tag{1}$$

Here, H_0^k is the eigenenergy of the kth pendulum; q_k is its deviation angle from the lower equilibrium; p_k is its generalized momentum; m is the load mass; l is the suspension length; finally, g is the acceleration due to gravity.

The total energy of the three-pendulum system is the sum of the energies of the individual subsystems:

$$H_0(p, q) = \sum_{k=1}^{3} H_0^k(p_k, q_k). \tag{2}$$

In formulas (1) and (2), the vector $q = (q_1, q_2, q_3)$ contains the angular coordinates, and the vector $p = (p_1, p_2, p_3)$ contains the corresponding momenta. The additivity of H_0 reflects the absence of direct mechanical links between the pendulums.

In view of the common scalar control input $u(t)$, the controlled Hamiltonian takes the form

$$H(p, q, u) = H_0(p, q) + H_1(q)u,\quad H_1(q) = -l\sum_{k=1}^{3} q_k. \tag{3}$$

Here, $u(t)$ is the control input acting identically on all three subsystems; the function $H_1(q)$ characterizes the system's geometric sensitivity to the control input. The sign of $H_1(q)u$ is determined by the chosen representation of the potential–force interaction.

The motion equations are immediate from the Hamiltonian description (3):

$$\frac{dq_k}{dt} = \frac{p_k}{ml^2},\ \frac{dp_k}{dt} = -mgl\sin q_k + ul\cos q_k,\ k = 1,2,3. \tag{4}$$

The first equation in (4) establishes the kinematic relationship between the angle and the momentum, while the second describes the balance between the gravitational torque $mgl \sin q_k$ and the control torque $ul \cos q_k$. With the factor $\cos q_k$, the control effectiveness depends on the current position of the pendulum.

Differentiating the eigenenergy H_0^k along the solutions of system (4) yields

$$\frac{dH_0^k}{dt} = u\frac{p_k \cos q_k}{ml},\quad k = 1, 2, 3. \tag{5}$$

Formula (5) has an important physical meaning: for each pendulum, the energy variation is determined by the sign of the product $up_k \cos q_k$. Consequently, the same control signal can simultaneously deliver energy to one subsystem and extract it from another. This mechanism underlies selective energy control.

2. CONTROL LAW DESIGN

The required mode is specified by the conditions

$$\lim_{t\to\infty} H_0^1(p_1,\ q_1) = E_1,\ \ H_0^2(p_2,\ q_2) < E_2,\quad H_0^3(p_3,\ q_3) < E_3,\ \ t \ge 0. \tag{6}$$

Here, E_1 is the target energy level of the first pendulum, and E_2 and E_3 are the permissible upper bounds for the second and third pendulums, respectively. These inequalities must hold during the entire motion, not just in steady state.

To combine the target condition and the constraints, we introduce a penalty objective function of the form

$$V(p,\ q,\ \alpha,\ \beta) = \frac{1}{2}\Big(H_0^1(p_1,\ q_1) - E_1\Big)^2 + \frac{\alpha}{E_2 - H_0^2(p_2,\ q_2)} + \frac{\beta}{E_3 - H_0^3(p_3,\ q_3)}. \tag{7}$$

The parameters $\alpha > 0$ and $\beta > 0$ determine the weight of the barrier terms. As H_0^2 approaches E_2 or H_0^3 approaches E_3, the corresponding term in (7) increases rapidly, amplifying the control law's response as the distance to the permissible bounds becomes dangerously small [21].

Based on the SG-method, the control is selected by the rule

$$u(t) = -\Gamma \frac{\partial \dot{V}}{\partial u},\quad \Gamma > 0, \tag{8}$$

with

$$\dot{V} = u\left[\frac{\Big(H_0^1(p_1,\ q_1) - E_1\Big)p_1 \cos q_1}{ml} + \frac{\alpha\ p_2 \cos q_2}{ml(E_2 - H_0^2)^2} + \frac{\beta\, p_3 \cos q_3}{ml(E_3 - H_0^3)^2}\right].$$

Here, Γ is the algorithm's gain. The control law (8) determines the system's response rate to the current gradient value and tunes the trade-off between response speed and control signal amplitude.

After calculating the derivative $\dot{V}$ with respect to the variable u, we obtain the explicit control law

$$u(t) = -\Gamma\left[\frac{(H_0^1(p_1,\ q_1) - E_1)\, p_1 \cos q_1}{ml} + \frac{\alpha\ p_2 \cos q_2}{ml(E_2 - H_0^2)^2} + \frac{\beta\, p_3 \cos q_3}{ml(E_3 - H_0^3)^2}\right]. \tag{9}$$

According to formula (9), the control action consists of three components: the first term delivers energy to the first subsystem in the direction of reaching the level E_1, whereas the second and third terms prevent the bounds E_2 and E_3 from being exceeded. The square in the denominators of the barrier terms ensures a sharp increase in the corrective impact when approaching critical values.

By substituting the control formula (9) into the expression for $\dot{V}$, we find that the objective function is bounded above by its initial value. Therefore, the constraints (6) are satisfied.

3. SOME REMARKS

From a mechanical standpoint, the control law (9) redistributes the energy appropriately rather than directly stabilizes the coordinates. In this case, the role of the common input is not to control each subsystem independently, but to utilize the differences between the factors $p_k \cos q_k$ in formula (5). These differences allow separating the desired pendulum and forming the required mode even with a common scalar control.

The barrier structure of the function (7) is particularly important for practical implementation. If the energy of the second or third pendulum approaches the permissible upper bound, the derivative of the penalty function will increase sharply, and the corresponding control component will dominate. Thus, the constraints are incorporated directly into the control law structure, rather than being verified post hoc after control calculation [21].

Note that, due to the complete identity of the subsystems, the problem cannot be solved in a meaningful way under identical initial conditions for all subsystems. In this case, the common input excites the same response in all channels, and no separation of motions occurs. Therefore, the presence of even a slight initial asymmetry is a natural condition for selective energy exchange, which agrees with the results established for the two-pendulum system [8, 9].

The proposed algorithm has the following implementation features:

• Initial asymmetry. To separate motions, the initial states must not coincide; otherwise, identical subsystems will respond to the common input in almost the same way.

• Parameter selection. The coefficients Γ, α, and β should be selected in a coordinated manner: increasing Γ accelerates the transients, while increasing α and β makes the constraints stricter but may lead to a higher control amplitude.

• Numerical integration. Near the bounds $H_0^2 = E_2$ and $H_0^3 = E_3$, the barrier terms grow rapidly; therefore, it is advisable to use a sufficiently small integration step for computations and to monitor the stability of the numerical scheme [22].

4. SIMULATION RESULTS

To assess the effectiveness of the proposed control law, two representative numerical examples were considered. In both cases, the same parameters of the mechanical model were used:

• $m = 1.0$, $l = 1$, and $g = 9.8$;
• $q_1(0) = 0.8$, $q_2(0) = 0.5$, and $q_3(0) = 0.3$;
• $p_1(0) = 1.542$, $p_2(0) = 2.378$, and $p_3(0) = 2.678$;
• $E_1 = 12$, $E_2 = 6$, and $E_3 = 3$;
• $\alpha = 0.1$, $\beta = 0.1$, and $\Gamma = 0.05$;
• a simulation time of 80 s.

Figure 1 shows the behavior of the system under the following parameters:

– the initial conditions $q_1(0) = 0.8$, $q_2(0) = 0.5$, and $q_3(0) = 0.3$ rad;

– the control parameters $\alpha = 0.1$, $\beta = 0.1$, and $\Gamma = 0.05$.

The upper graph presents the energy trajectories H_1, H_2, and H_3 (solid lines) and the target levels $E_1 = 12$, $E_2 = 6$, and $E_3 = 3$ J (dotted lines). The energy of the first pendulum quickly reaches the target level, while the energies of the second and third pendulums remain below the permissible bounds. The lower graph presents the control signal $u(t)$, which has a brief spike at the beginning of the process.

The time-dependent control signals $u(t)$ shown in Figs. 1a–6a are provided for direct comparison with the corresponding energy variations of the pendulums shown in Figs. 1a–6a.

In Fig. 2, the parameter α is reduced to 0.01 while $\beta = 0.1$ and $\Gamma = 0.05$.

Decreasing the parameter α weakens the barrier term for the second pendulum. As a result, the energy H_2 approaches the upper bound $E_2 = 6$ J more closely than in the basic scenario. The control signal becomes more intense at the instants when H_2 approaches the bound.

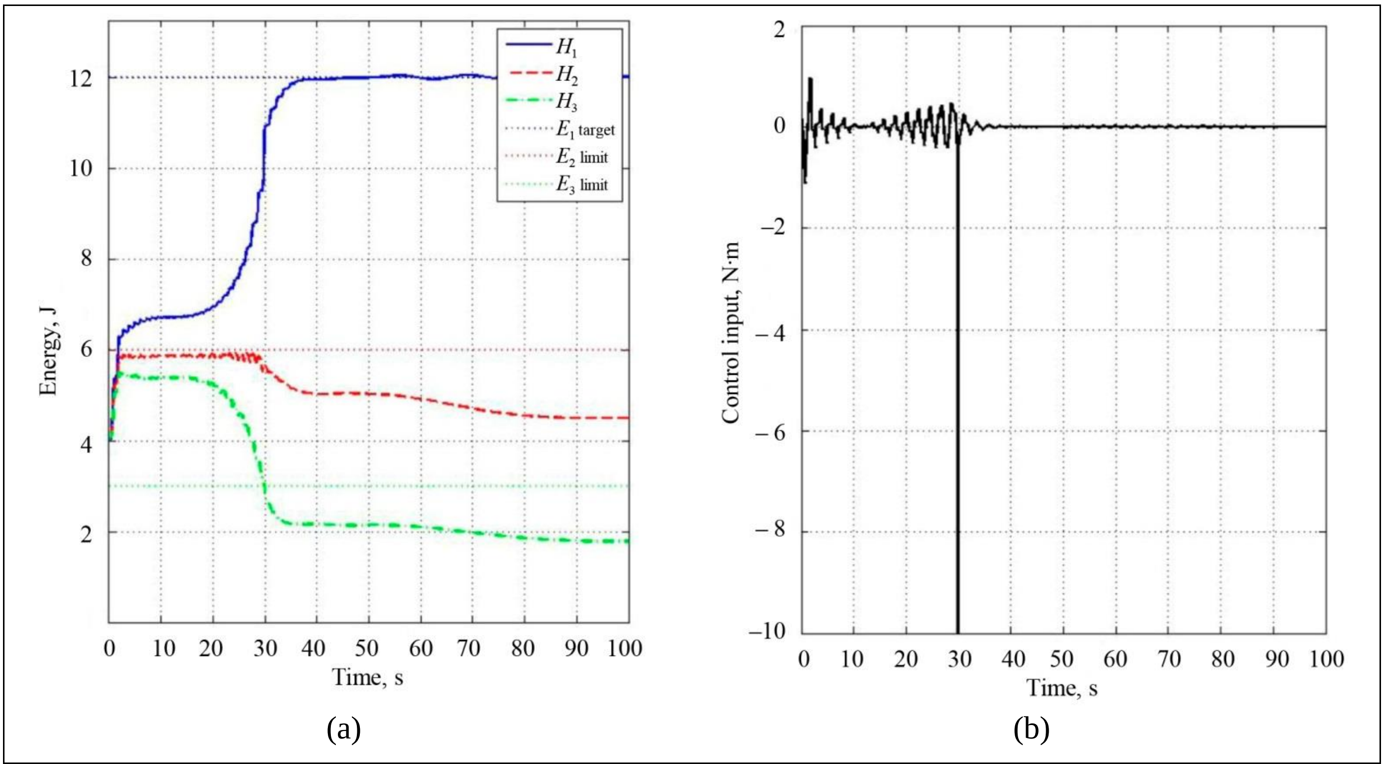

Fig. 1. The basic scenario (nominal parameters): (a) energy variations and (b) control input.

In Fig. 3, the parameter β is reduced to 0.01 while α = 0.1 and Γ = 0.05.

The weakening of the barrier term for the third pendulum causes the energy H_3 to periodically approach the bound $E_3 = 3$ J. The control signal responds to these approaches with additional spikes. This scenario demonstrates the importance of choosing sufficiently large values of β to satisfy the constraints.

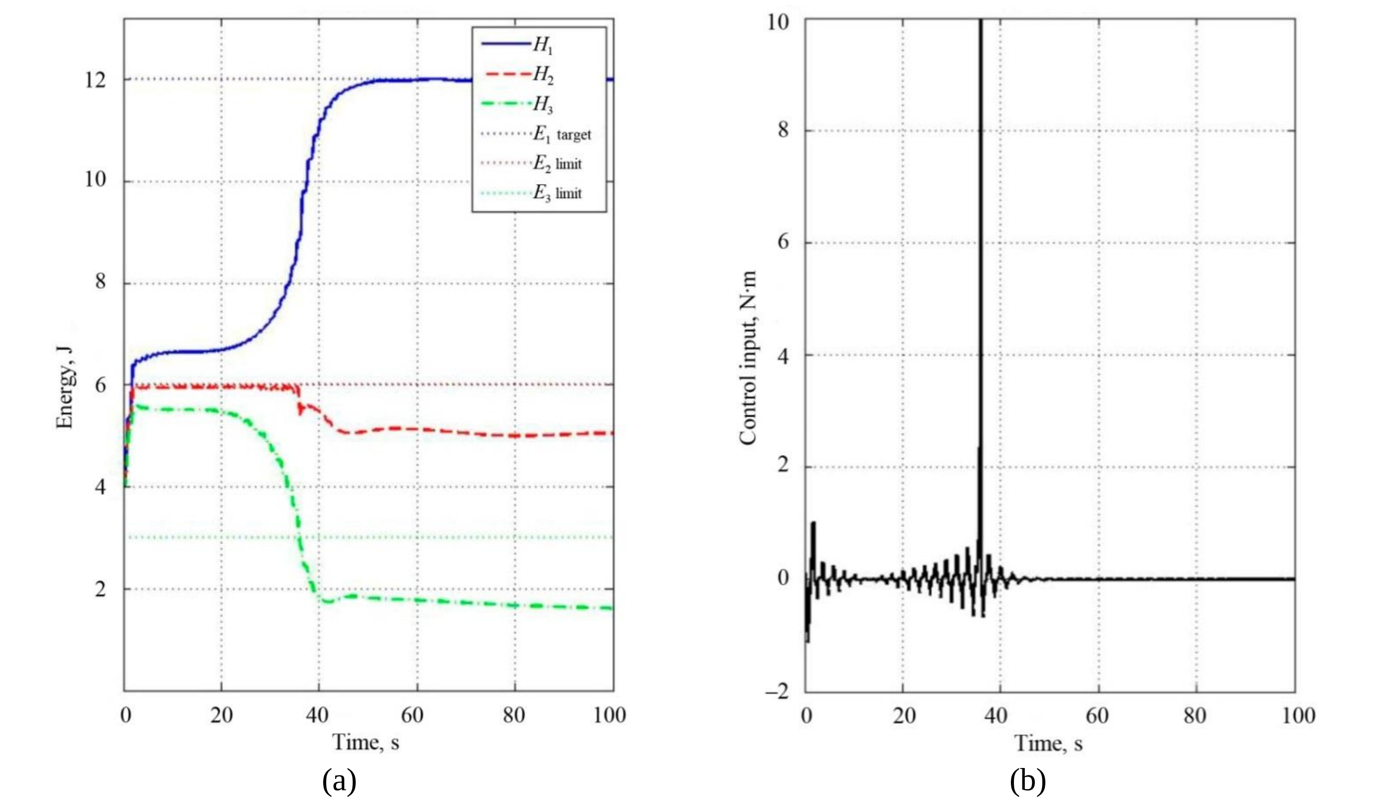

Fig. 2. The impact of a small penalty parameter α: (a) energy variations and (b) control input.

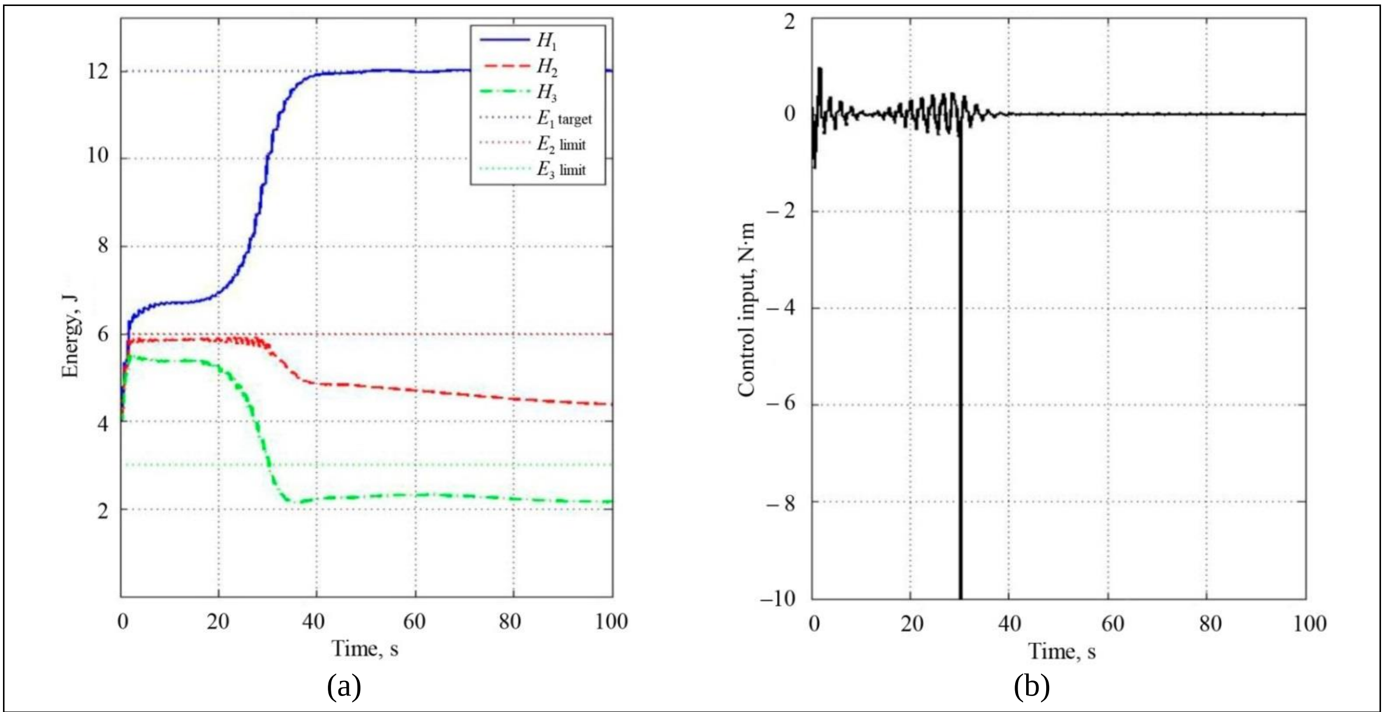

Fig. 3. The impact of a small penalty parameter β = 0.01: (a) energy variations and (b) control input.

In Fig. 4, the gain is increased to $\Gamma = 0.5$ while $\alpha = 0.1$ and $\beta = 0.1$.

Increasing the gain Γ accelerates the transients: the energy H_1 reaches the target level E_1 much faster than in the basic scenario. However, the control signal becomes larger in amplitude and contains high-frequency oscillations. This illustrates the trade-off between response speed and control amplitude.

Next, Fig. 5 shows a scenario with close initial angles and the nominal parameters $\alpha = 0.1$, $\beta = 0.1$, and $\Gamma = 0.05$.

Due to the slight initial asymmetry, the active phase of energy transfer begins late: approximately at t = 70 s. Up to this instant, the energies of all three pendulums remain almost the same and do not exceed $E_2 = 6$ J. After $t = 70$ s, the energy H_1 begins to in-

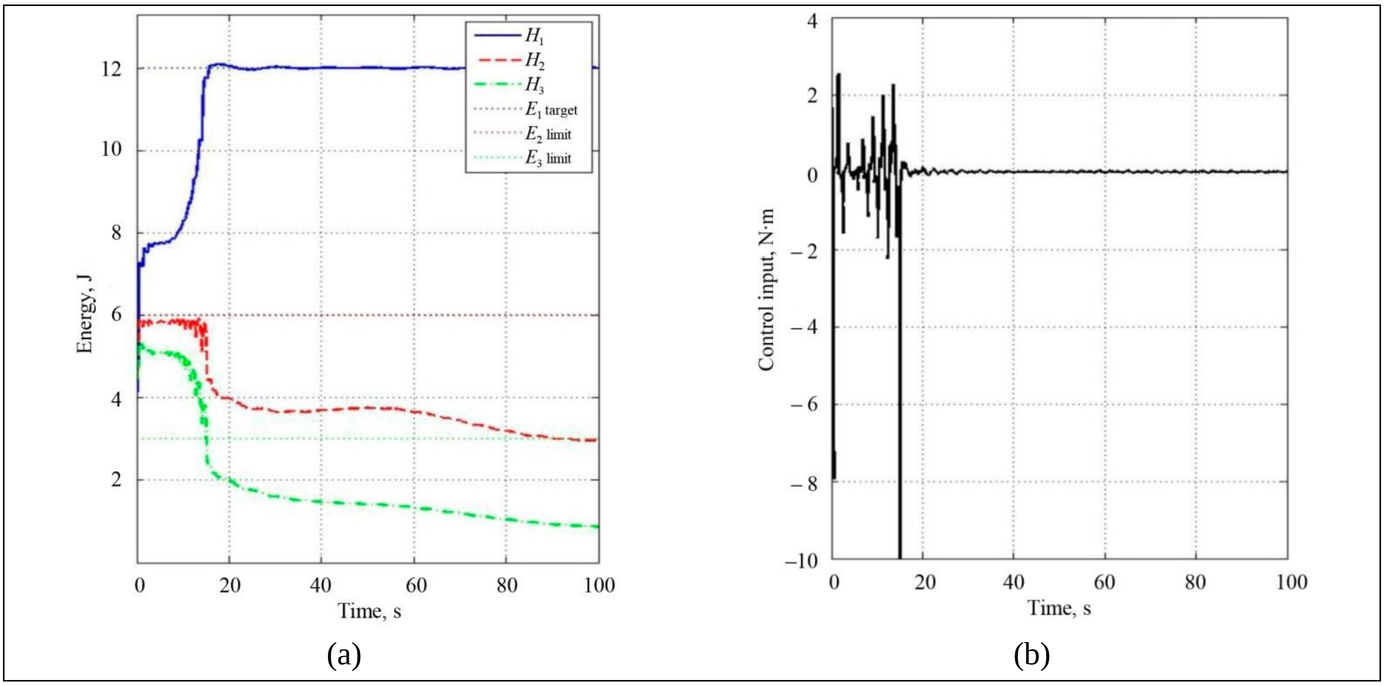

Fig. 4. The impact of an increased gain $\Gamma = 0.5$: (a) energy variations and (b) control input.

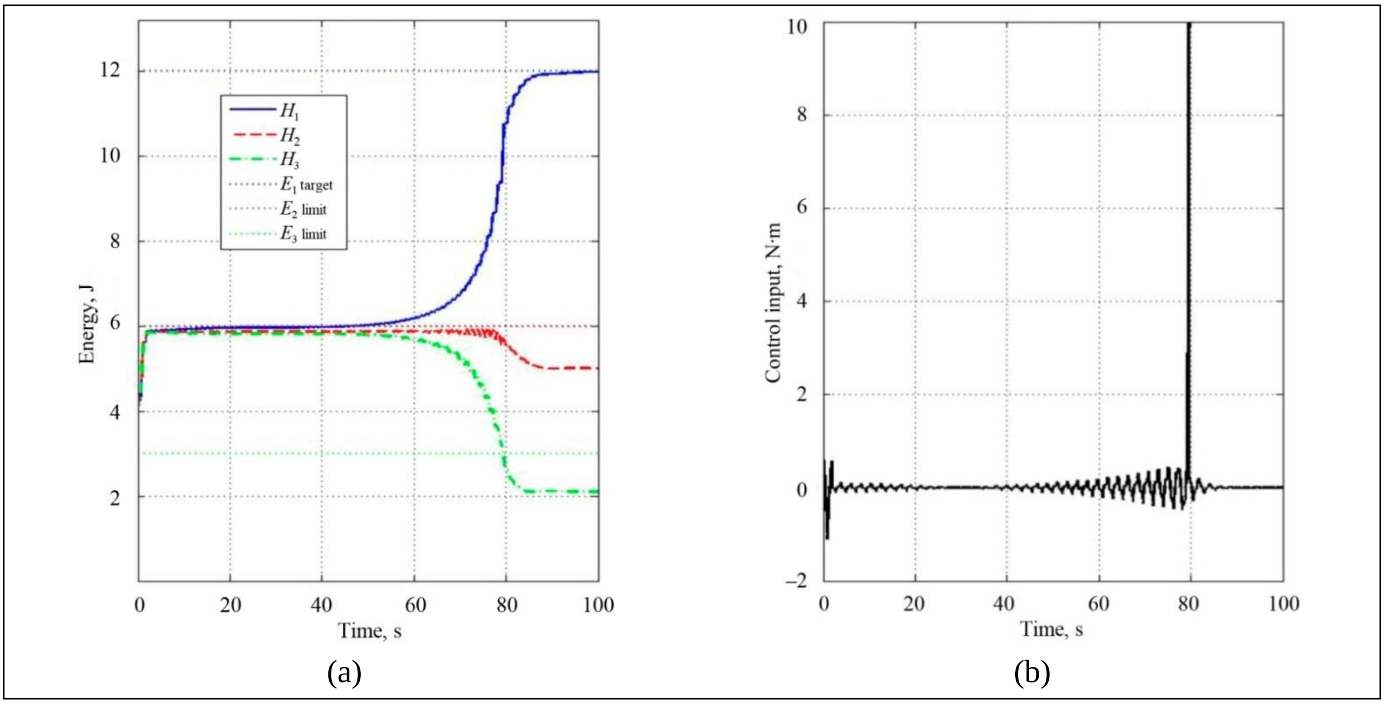

Fig. 5. The scenario with close initial conditions $q_1(0) = 0.4$, $q_2(0) = 0.3$, and $q_3(0) = 0.25$: (a) energy variations and (b) control input.

crease and reaches the level E_1 by the end of the simulation interval. The control signal exhibits a delayed and less pronounced spike.

In Fig. 6, the initial angles are $q_1(0) = 0.45$ rad, $q_2(0) = 0.3$ rad, and $q_3(0) = 0.25$ rad.

This scenario is characterized by an even smaller asymmetry compared to Fig. 5. As a result, the active energy delivery starts later. The energy H_1 begins to increase noticeably only after $t = 100$ s, and the target level E_1 is reached only by the end of the simulation interval. This result confirms the algorithm's high sensitivity to the degree of the system's initial asymmetry [23].

5. DISCUSSION

The above computational experiments yield the following findings.

- The basic scenario (Fig. 1) demonstrates the effectiveness of the proposed control law under sufficient initial asymmetry.
- Reducing the penalty parameters α or β leads to a closer approach of the energies of the corresponding pendulums to their upper bounds (Figs. 2 and 3). This confirms the need to select sufficiently large values for the penalty parameters.
- Increasing the gain Γ accelerates the transients, albeit at the cost of a higher amplitude and oscillations in the control signal (Fig. 4).
- The control law is sensitive to initial conditions (Figs. 5 and 6): the closer the initial states of the pendulums are, the later the selective energy delivery will occur. When the initial conditions are very close, the time to reach the target can grow significantly.

Direct comparison with the case of two pendulums studied previously [8, 9] shows that the three-pendulum problem is substantially more complex. First, the presence of two independent constraints (E_2 and E_3) leads to competition between the barrier terms in the control law (9), causing additional high-frequency oscillations in the signal $u(t)$ compared to the smoother control in the two-pendulum system. Second, the sensitivity to initial asymmetry increases: for the same relative proximity of the initial states, the target level E_1 in the three-pendulum system is reached significantly later than in the two-pendulum counterpart; see Figs. 5 and 6 and the corresponding results in [8, 9]. Third, the case of complete symmetry ($q_1(0) = q_2(0) = q_3(0)$) makes the problem fundamentally unsolvable. This is a more stringent condition compared to the two-pendulum system, where symmetry merely hinders the selective mode, without complete blocking the latter under small perturbations. Thus, the generalization to three subsystems is not trivial and requires a separate analysis, which has been presented in this paper.

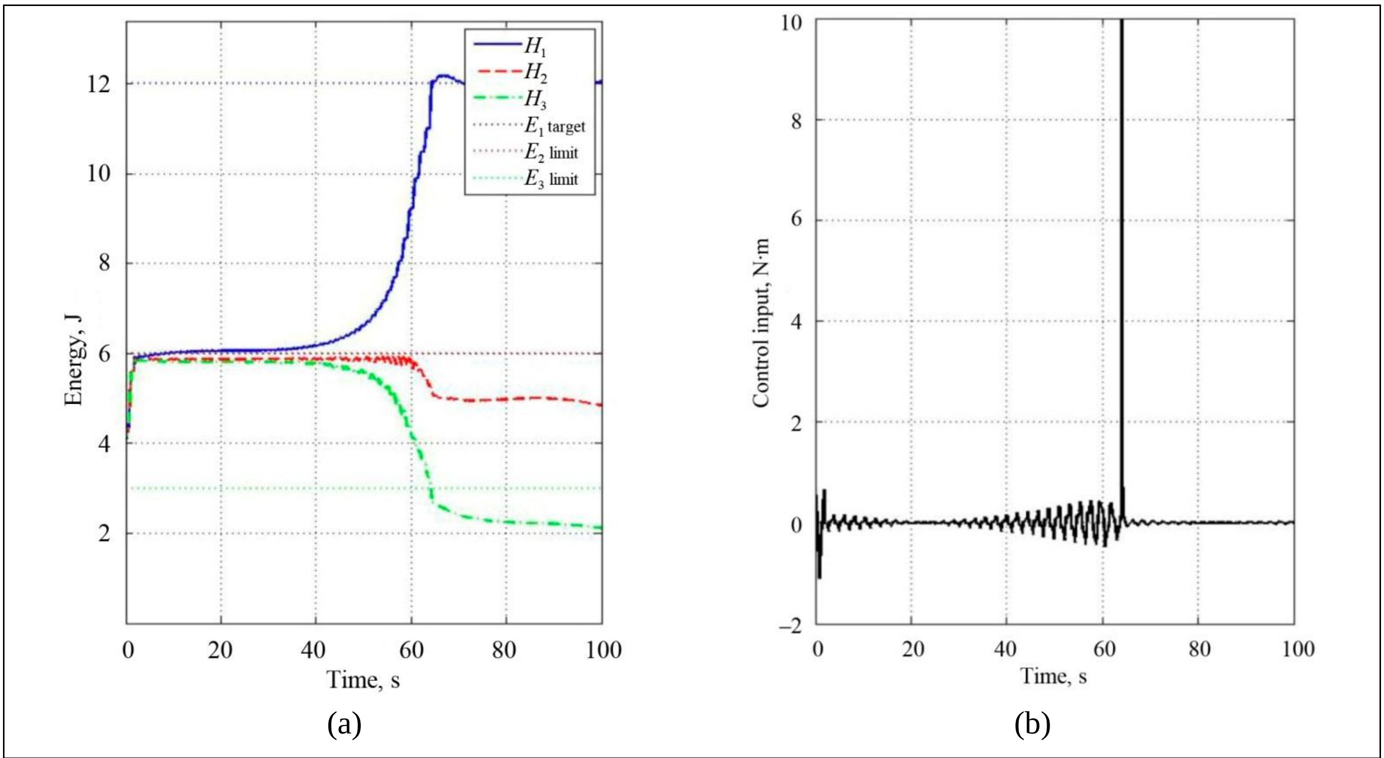

Fig. 6. The scenario with even closer initial conditions $q_1(0) = 0.45$, $q_2(0) = 0.3$, and $q_3(0) = 0.25$: (a) energy variations and (b) control input.

CONCLUSIONS

This paper has considered a system of three identical pendulums. An algorithm with a common scalar input has been proposed to control the energy of the first pendulum under one-sided energy constraints imposed on the second and third pendulums. The control law has been designed using the speed-gradient method with an objective function that includes the squared error of the target energy and barrier-type penalty terms for the constraints.

The resulting control law has an explicit analytical form. It can be implemented simply and is based on a physically transparent mechanism of energy redistribution via the factors $p_k \cos q_k$.

Numerical experiments have shown that the algorithm ensures the target mode under both significantly different and close initial conditions. In this case, the settling time and the shape of the control signal depend on the degree of the system's initial asymmetry.

This problem is complex: for example, if the pendulum parameters and initial conditions are identical, it becomes unsolvable for both linear and nonlinear pendulum models. Indeed, the trajectories of identical dynamic systems under identical initial conditions and with identical control signals coincide over the entire interval of existence. The difference between this problem and the case of two pendulums considered earlier is that the target system and the constraint system are not identical. The presented results can be viewed as a step toward developing selective energy control methods under constraints for mechanical systems with many degrees of freedom.

This paper has the following scientific contribution:

– For the first time in the literature, a control law has been proposed for the energy of three identical pendulums with a common scalar input and one-sided energy constraints.

– An explicit algorithm based on a penalty-type Lyapunov function has been obtained.

– A strong dependence of the settling time on the initial asymmetry has been discovered and quantitatively analyzed.

– Under completely symmetric initial conditions, the problem has been shown to be fundamentally unsolvable.

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