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A MATHEMATICAL MODEL OF MANAGING A REGULATED MONOPOLY DISTRICT HEATING MARKET[1]

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Abstract. This paper formulates an approach to managing the district heating of consumers in a hierarchical two-level system. The upper level is a regulator (e.g., a regional tariff service) that adjusts the heat energy tariff for consumers. The lower level is a district heating system that technologically and organizationally combines heat energy production and transport within a unified heat supply organization. The interaction of all participants in the process of heat supply to consumers is described. Optimization criteria are proposed for the upper and lower levels. A bi-level mathematical model of the district heating system is developed using the theory of hydraulic circuits and bi-level programming. This model operates in the conditions of a regulated monopoly heat energy market. The developed approach is applied to the real district heating system of Angarsk.

Keywords: district heating systems, heat energy market, mathematical modeling, hierarchical management, optimization.

INTRODUCTION

District heating plays an important role in the heat energy markets of many countries. Today, there are about 80 thousand district heating systems (DHSs) in the world: 50 thousand DHSs are located in Russia [2], 6 thousand large DHSs are operating in Europe [3], and the other 24 thousand DHSs are located in China, the USA, Canada, and former USSR countries (Ukraine, Kazakhstan, Belarus, etc.).

According to Global Market Insights[2], the global district heating market was worth more than $150 billion in 2019, producing about 3300 million Gcal of heat, with nearly 1300 million Gcal (40%) in Russia.

In world practice, two organizational models are used to manage district heating markets: the competitive model and the natural monopoly model.

Like in other spheres, competition is very important in the district heating market: it contributes to the efficiency of heat production and its quality. As a consequence, the price for heat energy reduces, favorably affecting the development of the entire industry. The competitive model is technologically based on several independent heat sources connected to consumers via heating networks. Heating networks have to be organizationally separated from heat generation and combined into a single heating network company with an independent sphere of activity. Such an organizational model is commonly referred to as the Unified Purchaser [4, 5]. The competitive model in the district heating market successfully operates in some European countries, such as Germany [3], Finland [6], and Sweden [7].

The natural monopoly heating market model is most widespread for heat supply to consumers. This model includes tariff regulation for consumers and operates in many EU countries, e.g., the Netherlands [8],

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[2] URL: https://www.gminsights.com/industry-analysis/district-heating-market (Accessed September 26, 2021.)

Poland [9], Lithuania [10], Latvia [11, 12], Norway [13], and Estonia [14] as well as in Russia, China, etc. In a particular country, heat energy tariffs are usually controlled by an executive authority on the state regulation of prices (tariffs) or a local authority (if vested with the corresponding powers). For details, see Table 1.

The organizational heating management model in the form of a natural monopoly with tariff regulation for consumers can be represented as a hierarchical vertically integrated system with two levels (Fig. 1).

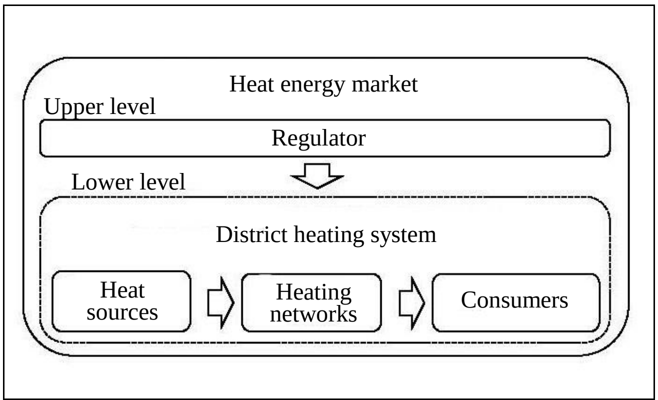

Fig. 1. Two-level heating management model.

The upper level here is the regulator responsible for adjusting the heat supply tariff for consumers. The lower level is a district heating system that technologically and organizationally combines heat energy production and transport within a unified heat supply organization (UHSO). The two-level management scheme for a monopoly district heating market separates the subsystems corresponding to particular market agents to model their behavior when implementing the objectives.

Heat energy market participants interact with each other as follows. Based on the forecasted heat demand from consumers, the UHSO produces and sells heat energy to them under several requirements. First, heat sources shall collectively produce a total heat volume to cover the demand from consumers and simultaneously obtain the maximum profit. Second, the available capacities of heat sources and the physical and technical limitations of heating networks shall be considered. In turn, protecting the rights of consumers, the regulator sets a heat energy tariff for motivating heat sources to satisfy the consumer demand (on the one hand) and maximize their profits while maintaining the optimal modes in heating networks.

Such a system is mathematically described using two-level modeling [16]. We pass to a one-level optimization problem by replacing the convex optimization problem of the second (lower) level with first-order optimality conditions. Note that these studies develop the existing approaches and methods and mostly rest on the paper [17], including the main provisions in mathematical modeling of district heating system objects (particularly heat sources, heating networks, and consumers). Nevertheless, the problem statement below fundamentally differs from that of [17] with the equal interaction of heat supply participants: it is hierarchical and, therefore, has completely different mathematical properties.

Table 1

Heat energy tariff regulators in different countries

Country	The Netherlands	Poland	Lithuania	Latvia
Regulator	Authority for Consumer and Market[3]	Energy Regulatory Office[4]	National Control Commission for Prices and Energy[5]	Sabiedrisko pakalpojumu regulēšanas Komisijas[6]
Country	Norway	Estonia	Russia	China
Regulator	Norwegian Water Resources and Energy Directorate[7]	Republic of Estonian Competition Authority[8]	Federal Anti-Monopoly Service, regional authorities[9]	Municipal authorities [15]

[3] URL: https://www.acm.nl/en/about-acm/our-organization/the-netherlands-authority-for-consumers-and-markets (Accessed October 1, 2021.)
[4] URL: http://www.ure.gov.pl/en/about-us/presidents-duties/22,Presidents-duties.html (Accessed October 1, 2021.)
[5] URL: http://www.regula.lt/en/Pages/default.aspx (Accessed October 1, 2021.)
[6] URL: https://www.sprk.gov.lv/content/siltumenergija (Accessed October 1, 2021.)
[7] URL: https://www.nve.no/energy-market-and-regulation/?ref=mainmenu (Accessed October 1, 2021.)
[8] URL: https://www.riigiteataja.ee/en/eli/ee/Riigikogu/act/520062017016/consolide (Accessed October 1, 2021.)
[9] Federal Law of the Russian Federation of July 27, 2010, no. 190-FZ "On Heat Supply."

1. MATHEMATICAL MODELING OF A REGULATED DISTRICT HEATING MARKET

1.1. Modeling of district heating systems

The DHS topology is described by an incidence matrix A, where the number of rows coincides with the number of nodes (i = 1,..., m), and the number of columns coincides with the number of branches (j = 1,..., n). The elements a_{ji} of the matrix A are given by

$$a_{ji} = \begin{cases} 0 \text{ if arc } i \text{ is not connected to node } j, \\ 1 \text{ if arc } i \text{ has outgoing flow from node } j, \\ -1 \text{ if arc } i \text{ has incoming flow in node } j, \end{cases}$$

$$i \in I, j \in J,$$

where I and J denote the sets of heating network edges and heating network nodes, respectively.

A DHS is modeled on a time interval with an initial instant $\tau_0 = 1$ (corresponding to the total heat load calculated) and a terminal instant τ_{fin} (e.g., the calendar number of hours in a year, 8760). The set $T = \{1, 2, ..., \tau_{fin}\}$ consists of all instants.

1.2. Modeling of heat sources

The behavior of heat sources in market conditions is modeled under the following requirements: at each instant $\tau \in T$, they shall collectively produce a total heat volume to cover a given consumer demand and simultaneously obtain the maximum profit under their capacity constraints.

Let the variable $Q_{\tau j}^G$ describe the heat volume produced by heat source $j \in J_G$ at an instant $\tau \in T$, where J_G denotes the set of all heat sources. Also, we introduce the following notations: $w_{\tau j}^{HE}$ is the unit price of heat energy received by generator $j \in J_G$ at an instant τ; $w_j^P = \gamma_j / \overline{Q}_j^G$ is the fixed charge (rate) per unit of installed capacity, where γ_j is the semi-fixed costs of heat source j, and $\overline{Q}_j^G$ is the installed (maximum) capacity of heat source j. Similarly to [17], the profit of heat source j at an instant τ considering its heat capacity constraints and additional proceeds for the provided heat power is determined by

$$F_{\tau j}^G(Q_{\tau j}^G) = w_j^P \overline{Q}_j^G + w_{\tau j}^{HE} Q_{\tau j}^G - Z_{\tau j}^G(Q_{\tau j}^G) \to \max, \quad (1)$$

$$\underline{Q}_j^G \le Q_{\tau j}^G \le \overline{Q}_j^G. \quad (2)$$

In this optimization problem, $\underline{Q}_j^G$ is the minimum allowable used capacity of the heat source, and $Z_{\tau j}^G(Q_{\tau j}^G)$ is the costs of heat source j to produce the heat volume $Q_{\tau j}^G$. These costs are represented as a quadratic polynomial [17].

The price $w_{\tau j}^{HE}$ in problem (1), (2) is treated as an external parameter. The price w_j^P is a given constant. The profit function $F_{\tau j}^G(Q_{\tau j}^G)$ is strongly concave. Hence, problem (1), (2) has a unique solution $Q_{\tau j}^{G,*}(w_{\tau j}^{HE})$:

$$Q_{\tau j}^{G,*}(w_{\tau j}^{HE}) = \begin{cases} \underline{Q}_j^G, & w_{\tau j}^{HE} < \underline{w}_{\tau j}^{HE}, \\ \dfrac{w_{\tau j}^{HE} - \beta_j}{2\alpha_j}, & \underline{w}_{\tau j}^{HE} \le w_{\tau j}^{HE} \le \overline{w}_{\tau j}^{HE}, \\ \overline{Q}_j^G, & w_{\tau j}^{HE} > \overline{w}_{\tau j}^{HE}, \end{cases} \quad (3)$$

where $\underline{w}_{\tau j}^{HE} = 2\alpha_j \underline{Q}_j^G + \beta_j$ and $\overline{w}_{\tau j}^{HE} = 2\alpha_j \overline{Q}_j^G + \beta_j$ are the heat prices of source j corresponding to the minimum and installed (maximum) capacities, and α_j and β_j are approximation coefficients in the cost function $Z_{\tau j}^G(Q_{\tau j}^G)$ [17].

1.3. Modeling of heat consumers

The set of all heat consumers J_D can be written as the union $J_D = J_{DH} \cup J_{DIG} \cup J_{DIN}$, where: J_{DH} is the consumers of housing and communal services (HCS); J_{DIG} is the industrial consumers connected to heating networks not representing source nodes (collectors); J_{DIN} is industrial consumers located on the collectors of heat sources. Clearly,

$$J_{DIN} \subset J_G, \; J_G \cap J_{DIG} = \varnothing, \; J_{DIN} \cap J_{DIG} = \varnothing. \quad (4)$$

Let $Q_{\tau j}^D$ be the total demand of consumers in node $j \in J_D$ at an instant $\tau \in T$. For brevity, we introduce the following notations: $Q_{\tau j}^{DH}$ is the demand of HCS consumers, i.e., $Q_{\tau j}^D$, $j \in J_{DH}$, where J_{DH} is the set of HCS consumers; $Q_{\tau j}^{DIG}$ is the demand of industrial consumers connected to heating networks, i.e., $Q_{\tau j}^D$, $j \in J_{DIG}$; $Q_{\tau j}^{DIN}$ is the demand of industrial consumers located on source collectors, i.e., $Q_{\tau j}^D$, $j \in J_{DIN}$. Due to the properties (4), we have:

$$Q_{\tau j}^D = \begin{cases} Q_{\tau j}^{DH} + Q_{\tau j}^{DIG}, & j \in J_{DH} \cap J_{DIG}, \\ Q_{\tau j}^{DH} + Q_{\tau j}^{DIN}, & j \in J_{DH} \cap J_{DIN}, \\ Q_{\tau j}^{DH}, & j \in J_{DH} \setminus (J_{DIG} \cap J_{DIN}), \\ Q_{\tau j}^{DIG}, & j \in J_{DIG} \setminus J_{DH}, \\ Q_{\tau j}^{DIN}, & j \in J_{DIN} \setminus J_{DH}. \end{cases}$$

For any instant τ, the total heat demand from HCS consumers, $Q_{\tau j}^{DH}$, is the demand for heating $Q_{\tau j}^{DHH}$ (this value varies during the heating period depending on the ambient temperature) and the demand for hot water supply $Q_{\tau j}^{DHW}$ (a fixed value during the year):

$$Q_{\tau j}^{DH} = Q_{\tau j}^{DHH} + Q_{\tau j}^{DHW}, \quad j \in J_{DH}.$$

The demand for heating needs from HCS consumers is determined using the Rossander equation [17].

The market principles of Russia's energy industry, particularly in district heating systems, led to the appearance of a new variable (the price of heat energy). From the economic point of view, demand for any good or service is characterized by a demand curve (the demand-price dependence).

Most district heating markets have inelastic demand: during the heating season, demand in the heat energy markets is a fixed value, being therefore rigidly bound to a particular system. Heat prices are not adjusted in real time depending on the heat source load but are calculated and approved for the medium or long term. At the same time, a market of modern and efficient equipment of small and medium capacity (usually boiler houses) has already formed in heat supply. With its expansion, an economically feasible approach is often to build heat sources (on the sites of industrial objects or individual heat sources in private residential buildings) of small capacity with acceptable one-time costs and minimum payback period. As a result, the demand for heat energy may become elastic.

Consumer behavior in the industrial sector is modeled using the inverse demand function. As a rule, this function is constructed based on real calculations for separate industrial consumers by approximating retrospective data considering the forecasted heat consumption volumes and prices. For industrial consumers connected to heating networks, the inverse demand function $w_{\tau j}^{DIG} = \Phi^{-1}(Q_{\tau j}^{DIG})$, where $w_{\tau j}^{DIG}$ is the consumer's purchase price for the heat volume $Q_{\tau j}^{DIG}$ (in RUB/Gcal), has a linear form [17].

For industrial consumers located on heat source collectors, the inverse demand function takes the form

$$w_{\tau j}^{DIN} = \xi_j^{DIN} - \vartheta_j^{DIN} Q_{\tau j}^{DIN}, \; j \in J_{DIN},$$

with the following notations: ξ_j^{DIN} and ϑ_j^{DIN} are the constants obtained by approximating the actual heat volume purchased by industrial enterprise $j \in J_{DIN}$ depending on its price; $w_{\tau j}^{DIN}$ is the purchase price determined only by the cost of heat production (in RUB/Gcal); $Q_{\tau j}^{DIN}$ is the heat volume purchased by industrial enterprise $j \in J_{DIN}$ located on heat source collectors (in Gcal/h).

Fluctuations in heat demand depending on the time of day and weather conditions are among the main problems of the heat energy market. Therefore, it is proposed to study the interaction between producers and consumers during each hour on a given period. Such a discrete modeling approach is practically important due to considering both the daily and seasonal heat demand factors. These factors significantly affect the volumes of heat demand and production for each heat source (consequently, its profit).

1.4. Modeling of heating networks

Unlike the competitive heat energy market model (optimization-based statement) [17], the mathematical model describing heating networks with the UHSO is a system of linear and nonlinear equations. The material balance (the first Kirchhoff law), acting as a constraint in the cost optimization of heating networks [17], is supplemented by the second Kirchhoff law (5) and closing relations (6):

$$y_\tau - A^{\mathrm{T}} p_\tau = 0, \tag{5}$$

$$y_{\tau i} - s_i x_{\tau i} \left| x_{\tau i} \right| = -H_{\tau i}, \; i \in I. \tag{6}$$

Formulas (5) and (6) have the following notations: $y_\tau = (y_{\tau 1}, ..., y_{\tau n})^{\mathrm{T}}$, where $y_{\tau i}$ is the pressure drop on network edge i at an instant τ, in mAq; A^{T} is the transposed incidence matrix; $p_\tau = (p_{\tau 1}, ..., p_{\tau m})^{\mathrm{T}}$, where $p_{\tau j}$ is the pressure in node j at an instant τ, in mAq; s_i is the hydraulic resistance coefficient of all branches on network edge i, in mh^2/t^2; $x_\tau = (x_{\tau 1}, ..., x_{\tau n})^{\mathrm{T}}$, where $x_{\tau i}$ is the flow rate of the heat-transfer agent on network edge i at an instant τ, in t/h; finally, $H_{\tau i}$ is the effective head on network edge i at an instant τ, in mAq.

The vectors x_τ, y_τ, and p_τ are the variables determining the optimal flow distribution in the heating network. If the material balance

$$\sum_{j \in J_G} Q_{\tau j}^{G} = \sum_{j \in J_{DH}} Q_{\tau j}^{DH} + \sum_{j \in J_{DIG}} Q_{\tau j}^{DIG} + \sum_{j \in J_{DIN}} Q_{\tau j}^{DIN}$$

holds at each instant $\tau \in$ T, the system of equations has a solution (x_τ^*, y_τ^*, p_τ^*) [18]. This solution will always be unique in the variables (x_τ^*, y_τ^*) and nonunique in the variables p_τ^*. The uniqueness in p_τ^* can be ensured by fixing the pressure in one node [18]. The optimal flow distribution in heating networks is obtained by solving the system of linear and nonlinear equations. The corresponding methodology is well developed [18] and can be applied without considerable difficulties.

However, finding the optimal flow distribution in heating networks becomes complicated (in comparison with the traditional technical and economic calculation) in market conditions: the fixed loads $Q_{\tau j}^{DH}$ of

HCS consumers are the only available data in the material balance relations. The heat volumes $Q_{\tau j}{}^{G}$ produced by heat sources and the loads $Q_{\tau j}{}^{DIG}$ and $Q_{\tau j}{}^{DIN}$ of industrial consumers are unknown. Therefore, the problem becomes underspecified.

Assume that the heat production prices $w_\tau{}^{H}$ = $\{w_{\tau j}{}^{HE}: j\in J_G\}$ are given. In this case, formula (3) yields the production volume for each source,

$$Q_{\tau j}{}^{G} = Q_{\tau j}{}^{G,*}(w_{\tau j}{}^{HE}),\ j\in J_G,$$

and the total heat supply $\sum_{j\in J_G} Q_{\tau j}^{G} = \sum_{j\in J_G} Q_{\tau j}^{G,*}(w_{\tau j}^{HE})$.

Hence, the heat volume $Q_\tau{}^{TSI}(w_\tau{}^{HE})$ offered to the industrial sector is calculated as

$$Q_\tau^{TSI}(w_\tau^{HE}) = \sum_{j\in J_G} Q_{\tau j}^{G,*}(w_{\tau j}^{HE}) - \sum_{j\in J_{DH}} Q_{\tau j}^{DH}. \qquad (7)$$

Finally, we have to distribute the total supply $Q_\tau{}^{TSI}(w_\tau{}^{HE})$ among the industrial heat consumers, i.e., determine the nodal values $Q_{\tau j}{}^{DIN}$ and $Q_{\tau j}{}^{DIG}$ satisfying

$$\sum_{j\in J_{DIG}} Q_{\tau j}^{DIG} + \sum_{j\in J_{DIN}} Q_{\tau j}^{DIN} = Q_\tau^{TSI}(w_\tau^{HE}).$$

To find them, we apply an approach based on redundant circuits of district heating systems [18]. A redundant circuit is constructed from the design one by introducing a dummy node with number (m + 1) and dummy arcs outgoing from nodes $j \in J_{DIN} \cap J_{DIG}$ (the locations of industrial consumers) and incoming to the dummy node (m + 1). The dummy arcs are assigned numbers from (n + 1) to (n + r), where r denotes the number of industrial consumers (the elements of the set $J_{DIN} \cup J_{DIG}$). The dummy node is assigned the total demand of industrial consumers $Q_\tau{}^{TSI}(w_{\tau j}{}^{HE})$ (7), and the demand of industrial consumers in nodes $j \in J_{DIN} \cup J_{DIG}$ is set to zero. The heat flow from each node $j \in J_{DIN} \cup J_{DIG}$ to node (m + 1) is denoted by $x_{\tau\eta(j)}$, where $\eta(j) \in \{n + 1,..., n + r\}$ is the number of the dummy arc outgoing from node j. The resistances and heads of the dummy arcs are also assumed to be zero: $s_{\eta(j)} = 0$ and $H_{\tau\eta(j)} = 0$, $j \in J_{DIN} \cup J_{DIG}$. Due to the closure relations (6), the pressure drops are also zero: $y_{\tau\eta(j)} = 0$, $j \in J_{DIN} \cup J_{DIG}$. Next, formula (5) yields $(A)^{\mathrm{T}}{}_{\tau\eta(j)}\, p_\tau = 0$, where $(A)^{\mathrm{T}}{}_{\tau\eta(j)}$ is the $\eta(j)$th row of the transposed incidence matrix. The relations $(A)^{\mathrm{T}}{}_{\tau\eta(j)}\, p_\tau = 0$, $j \in J_{DIN} \cup J_{DIG}$, are equivalent to $p_{\tau j} = p_{\tau(m+1)}$, $j \in J_{DIN} \cup J_{DIG}$ (the pressure in the industrial sector nodes equals the pressure in the dummy node). As mentioned above, it suffices to fix the pressure in one DHS nodes to ensure the unique solution in the variable p_τ. Without loss of generality, we traditionally [18] let $p_{\tau(m+1)} = 0$. Then the analogs of equations (5) and (6) corresponding to dummy branches disappear. The final system of equations of the redundant circuit with the balance (material) relations (the first Kirchhoff law (8)–(16)) takes the form

$$A_j x_\tau + x_{\tau\eta(j)} = Q_{\tau j}^{G,*}(w_{\tau j}^{HE}) - Q_{\tau j}^{DH},\ j\in J_{DH}\bigcap J_{DIN}, \qquad (8)$$

$$A_j x_\tau = Q_{\tau j}^{G,*}(w_{\tau j}^{HE}) - Q_{\tau j}^{DH},\ j\in J_G\bigcap J_{DH}\setminus J_{DIN}, \qquad (9)$$

$$A_j x_\tau + x_{\tau\eta(j)} = Q_{\tau j}^{G,*}(w_{\tau j}^{HE}),\ j\in J_{DIN}\setminus J_{DH}, \qquad (10)$$

$$A_j x_\tau = Q_{\tau j}^{G,*}(w_{\tau j}^{HE}),\ j\in J_G\setminus J_{DH}\bigcup J_{DIN}, \qquad (11)$$

$$A_j x_\tau = -Q_{\tau j}^{DH},\ j\in J_{DH}\setminus(J_G\bigcup J_{DIG}), \qquad (12)$$

$$A_j x_\tau + x_{\tau\eta(j)} = 0,\ j\in J_{DIG}\setminus J_{DH}, \qquad (13)$$

$$A_j x_\tau + x_{\tau\eta(j)} = -Q_{\tau j}^{DH},\ j\in J_{DH}\setminus J_{DIG}, \qquad (14)$$

$$A_j x_\tau = 0,\ j\in J_0, \qquad (15)$$

$$-\sum_{j\in J_{DIN}\bigcup J_{DH}} x_{\tau\eta(j)} = -Q_\tau^{TSI}(w_{\tau j}^{HE}), \qquad (16)$$

$$y_\tau - \mathrm{A}^{\mathrm{T}} p_\tau = 0, \qquad (17)$$

$$y_{\tau i} - s_i x_{\tau i}\left|x_{\tau i}\right| = -H_{\tau i},\ i\in I. \qquad (18)$$

Equation (16) corresponds to the dummy node. Due to the balance relation (7), this system always has a solution ($x_\tau^*(w_\tau^{HE})$, $y_\tau^*(w_\tau^{HE})$, $p_\tau^*(w_\tau^{HE})$). The heat volumes of industrial consumers are given by

$$\begin{aligned} Q_{\tau j}^{DIG}(w_{\tau j}^{HE}) &= x_{\tau\eta(j)}^{*}(w_\tau^{HE}),\ j\in J_{DIG},\\ Q_{\tau j}^{DIN}(w_{\tau j}^{HE}) &= x_{\tau\eta(j)}^{*}(w_\tau^{HE}),\ j\in J_{DIN}. \end{aligned} \qquad (19)$$

We illustrate the method of redundant circuits on an example of the district heating system in Fig. 2a. In Figs. 2a and 2b, the numbers indicate the nodes, and the numbers in circles indicate the branches. The other notations are as follows: $Q_1{}^{G}$ and $Q_5{}^{G}$ are the loads of heat sources; $Q_1{}^{DIN}$ is the load of the consumer located on the heat source collector; $Q_3{}^{DIG}$ is the load of the industrial consumer connected to the heating networks; $Q_2{}^{DH}$ and $Q_4{}^{DH}$ are the loads of HCS consumers; $Q_6{}^{TSI}$ is the total load of industrial consumers. The district heating system (Fig. 2a) consists of m = 5 nodes and n = 8 branches: J = {1, 2, 3, 4, 5} and I = {1, 2, 3, 4, 5, 6, 7, 8}. The correspondence between the branches and nodes is presented in Table 2.

For the circuit in Fig. 2a, the incidence matrix is

$$A=\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0\\ -1 & 0 & 0 & 1 & 0 & -1 & 0 & 0\\ 0 & -1 & 0 & -1 & -1 & 0 & -1 & 0\\ 0 & 0 & -1 & 0 & 1 & 0 & 0 & -1\\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}.$$

We have the following sets of nodes: the heat sources, J_G = {1, 5}; the HCS consumers, J_{DH} = {2, 4}; the industrial consumers without heat sources, J_{DIG} = {3}; the industrial consumers with heat sources, J_{DIN} = {1}; simple branching nodes, $J_0 = \varnothing$. For the

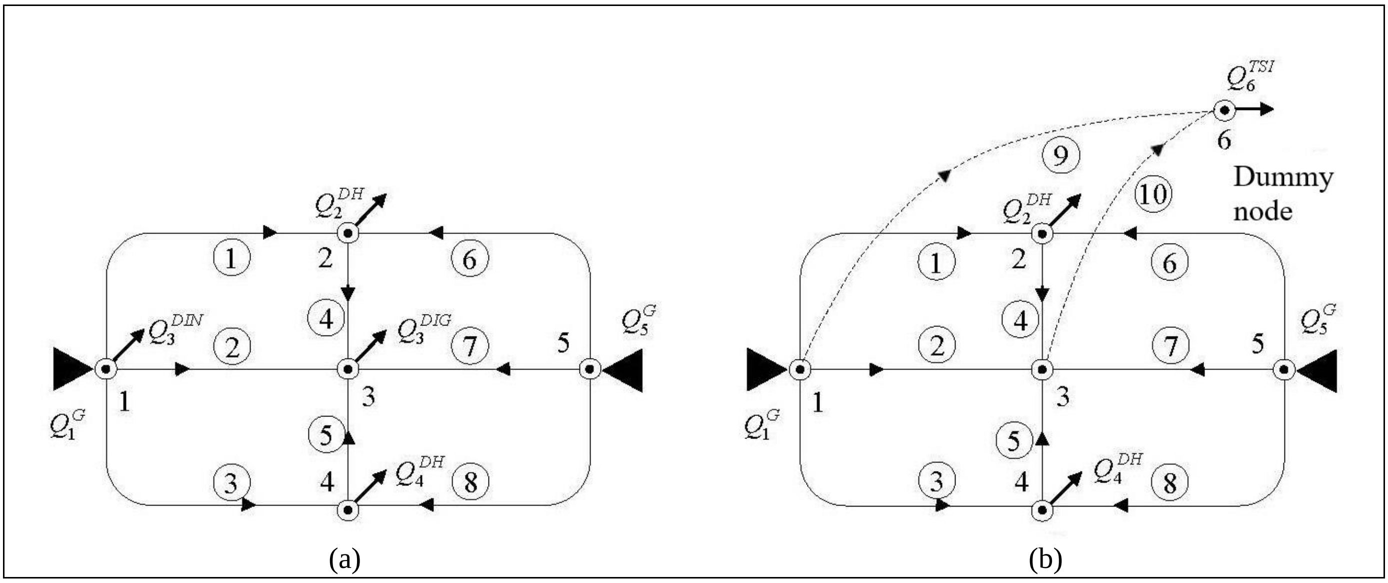

Fig. 2. A heat supply circuit: (a) design and (b) redundant.

Table 2

Correspondence between branches and nodes

	Branch 1	Branch 2	Branch 3	Branch 4	Branch 5	Branch 6	Branch 7	Branch 8
Initial node	1	1	1	2	4	5	5	5
End node	2	3	4	3	3	2	3	4

heat supply circuit in Fig. 2*a*, the first Kirchhoff law takes the form

$$x_{\tau 1}+x_{\tau 2}+x_{\tau 3}=Q_{\tau 1}^{G}-Q_{\tau 1}^{DIN},$$

$$-x_{\tau 1}+x_{\tau 4}-x_{\tau 6}=-Q_{\tau 2}^{DH},$$

$$-x_{\tau 2}-x_{\tau 4}-x_{\tau 5}-x_{7}=-Q_{\tau 3}^{DIG},$$

$$-x_{\tau 3}+x_{\tau 5}-x_{\tau 8}=-Q_{\tau 4}^{DH},$$

$$x_{\tau 6}+x_{\tau 7}+x_{\tau 8}=Q_{\tau 5}^{G}.$$

Let the heat volumes $Q_{\tau 1}{}^{G}$ and $Q_{\tau 5}{}^{G}$ and the demands $Q_{\tau 2}{}^{DH}$ and $Q_{\tau 4}{}^{DH}$ be known. Then the total demand from industrial consumers is given by

$$Q_{\tau}^{TSI}=Q_{\tau 1}^{DIN}+Q_{\tau 3}^{DIG}=Q_{\tau 1}^{G}+Q_{\tau 5}^{G}-Q_{\tau 2}^{DH}-Q_{\tau 4}^{DH}.$$

Assume that $Q_{\tau}{}^{TS} > 0$. (Otherwise, the situation becomes trivial.) The scheme in Fig. 2*a* is represented by two industrial consumers, $J_{DIN} \cup J_{DIG} = \{1, 3\}$, $r = 2$. We introduce the dummy node 6, the dummy branch 9 connecting nodes 1 and 6, and the dummy branch 10 connecting nodes 3 and 6, with $\eta(1) = 9$ and $\eta(3) = 10$ (Fig. 2*b*). In this case, the first Kirchhoff law for the redundant circuit is the system of equations

$$x_{\tau 1}+x_{\tau 2}+x_{\tau 3}+x_{\tau 9}=Q_{\tau 1}^{G},$$

$$-x_{\tau 1}+x_{\tau 4}-x_{\tau 6}=-Q_{\tau 2}^{DH},$$

$$-x_{\tau 2}-x_{\tau 4}-x_{\tau 5}-x_{7}+x_{10}=0,$$

$$-x_{\tau 3}+x_{\tau 5}-x_{\tau 8}=-Q_{\tau 4}^{DH},$$

$$x_{\tau 6}+x_{\tau 7}+x_{\tau 8}=Q_{\tau 5}^{G}.$$

$$-x_{\tau 9}-x_{\tau 10}=-Q_{\tau 6}^{TSI}.$$

We solve the system of equations of the optimal flow distribution in the heating network. Its solution—the flow vector $x_{\tau}^{*}(w_{\tau j}^{HE})$—corresponds to the minimum heat transport costs in the network [18]. Then, the heat volume supplied to industrial consumers (see the expression (7)) is determined by minimizing the network costs under the optimal flow distribution in the heating network. For details, see the book [18]. Industrial consumers will pay for their heat volumes at the prices determined by the inverse demand functions $w_{\tau j}^{DIN}=\Phi^{-1}(Q_{\tau j}^{DIN})$ and $w_{\tau j}^{DIG}=\Phi^{-1}(Q_{\tau j}^{DIG})$.

The network costs are calculated using an original formula from [17] after finding the steady-state flow distribution.

1.5. Management model for heat supply to consumers

When constructing a management model for heat supply on a regulated monopoly district heating market, we have to formulate the problem statement. It is required to find a state of the regulated monopoly district heating market in which heat sources collectively produce a total heat volume to cover the demand from consumers and simultaneously obtain the maximum profit under the available capacities of heat sources and the physical and technical limitations of heating networks. In addition, the price of heat for industrial consumers is determined using their inverse demand functions, and the regulator adjusts a fair heat energy tariff for HCS consumers.

To model the regulator's behavior, we formalize its objective. Assume that the regulator defends the interests of HCS consumers, seeking to minimize their heat energy tariff w_τ^{DH}. The economic balance of the DHS is written as

$$\sum_{j\in J_G}\left(w_{\tau j}^{HE}Q_{\tau j}^{G}+w_j^{P}\overline{Q}_j^{G}\right)+Z_\tau^{NET}(x_\tau)=$$
$$w_\tau^{DH}\sum_{j\in J_{DH}}Q_{\tau j}^{DH}+\sum_{j\in J_{DIG}}w_{\tau j}^{DIG}Q_{\tau j}^{DIG}+\sum_{j\in J_{DIN}}w_{\tau j}^{DIN}Q_{\tau j}^{DIN},\quad(20)$$

where $Z_\tau^{NET}(x_\tau)$ denotes the costs of the heating network (in rubles); see the formula in [17].

We determine the total demand from the balance relations (20). The price of heat for HCS consumers is expressed as an affine function of the other variables:

$$w_\tau^{DH}=f_\tau\left(w_\tau^{HE},\,Q_\tau^{G},\,Z_\tau^{NET},\,w_\tau^{DIG},\,Q_\tau^{DIG},\,w_\tau^{DIN},\,Q_\tau^{DIN}\right)=$$
$$\frac{1}{\sum_{j\in J_{DH}}Q_{\tau j}^{DH}}\left[\sum_{j\in J_G}\left(w_{\tau j}^{HE}Q_{\tau j}^{G}+w_j^{P}\overline{Q}_j^{G}\right)+Z_\tau^{NET}(x_\tau)-\right.$$
$$\left.\sum_{j\in J_{DIG}}w_{\tau j}^{DIG}Q_{\tau j}^{DIG}-\sum_{j\in J_{DIN}}w_{\tau j}^{DIN}Q_{\tau j}^{DIN}\right].\qquad(21)$$

The heat sources, heating network, and industrial consumers are guided by their economic interests and are not directly subordinate to the regulator. As described above, the heat sources maximize their profits; the heating network optimizes the heat transport costs from sources to consumers by calculating the optimal flow distribution in it; the industrial consumers behave in accordance with their inverse demand functions. The price vector w_τ^{DH} defines the behavior of heat sources, heating networks, and industrial consumers. Therefore, by varying the price vector w_τ^{HE}, the regulator can minimize the heat energy tariff w_τ^{DH} in (21) considering the interests of heat sources, heating networks, and industrial consumers.

These considerations lead to the following two-level model. For each instant $\tau \in T$, the regulator (the upper level) solves the problem

$$\begin{gathered}w_\tau^{DH}=f_\tau\left(w_\tau^{HE},Q_\tau^{G},Z_\tau^{NET},\right.\\ \left.w_\tau^{DIG},Q_\tau^{DIG},w_\tau^{DIN},Q_\tau^{DIN}\right)\to\min,\end{gathered}\qquad(22)$$

$$\underline{w}_\tau^{HE}\le w_\tau^{HE}\le\overline{w}_\tau^{HE},\ \ j\in J_G.\qquad(23)$$

The regulator transmits the price vector w_τ^{HE} to the lower level consisting of heat sources, heating networks, and consumers. Heat sources $j \in J_G$ maximize their profits (1), (2), producing the heat volume $Q^{G}_{\tau j} = Q^{G*}_{\tau j}(w^{HE}_{\tau j})$ (3). The demand for heat energy from the HCS sector is constant and independent of w_τ^{HE}. Given the heat supply from sources and the demand from the HCS sector, the heating network uses the method of redundant circuits to find the optimal flow distribution, determining the network costs and heat volumes

$$Q_\tau^{DIG}=Q_{\tau j}^{DIG}(w_{\tau j}^{HE}),\ Q_\tau^{DIN}=Q_{\tau j}^{DIN}(w_{\tau j}^{HE}),$$

supplied to the industrial consumers (19). Given these volumes, the industrial consumers set the prices in accordance with their reverse demand functions. After that, all variables of the regulator's function f in (22) become known; the regulator determines the heat energy tariff for HCS consumers corresponding to the vector w_τ^{HE}. The interaction between the upper and lower levels of the system is shown in Fig. 3.

We present **the algorithm FV (w_τ^{HE})** for calculating the upper-level objective function.

The input is the price vector w_τ^{HE}.

The output is the value $FV(w_\tau^{HE})$.

Step FV.1. Calculating the heat volume $Q_{\tau j}^{G}=Q_{\tau j}^{G,*}(w_{\tau j}^{HE}),\ j\in J_G$, by formula (3).

Step FV.2. Calculating the heat volume $Q_\tau^{TSI}(w_{\tau j}^{HE,k})$ by formula (7).

Step FV.3. Determining the flows $x_\tau^k=x_\tau^*(w_\tau^{HE,k})$ and $\tilde{x}_\tau^k=\tilde{x}_\tau^*(w_\tau^{HE,k})$ from the system of equations (8)–(18).

Step FV.4. Calculating the network costs Z_τ^{NET} as described in [17].

Step FV.5. Determining the heat volumes Q_τ^{DIG} and Q_τ^{DIN} by formulas (19).

Step FV.6. Calculating the prices w_τ^{DIG} and w_τ^{DIN} using the inverse demand functions.

Step FV.7. Calculating the value $FV(w_\tau^{HE})=f_\tau\left(w_\tau^{HE},Q_\tau^{G},Z_\tau^{NET},w_\tau^{DIG},Q_\tau^{DIG},w_\tau^{DIN},Q_\tau^{DIN}\right)$ by formula (22).

With the function FV, calculated by this algorithm, problem (22), (23) can be reformulated as follows:

$$FV(w_\tau^{HE})\to\min,$$

$$\underline{w}_\tau^{HE}\le w_\tau^{HE}\le\overline{w}_\tau^{HE},\ j\in J_G.$$

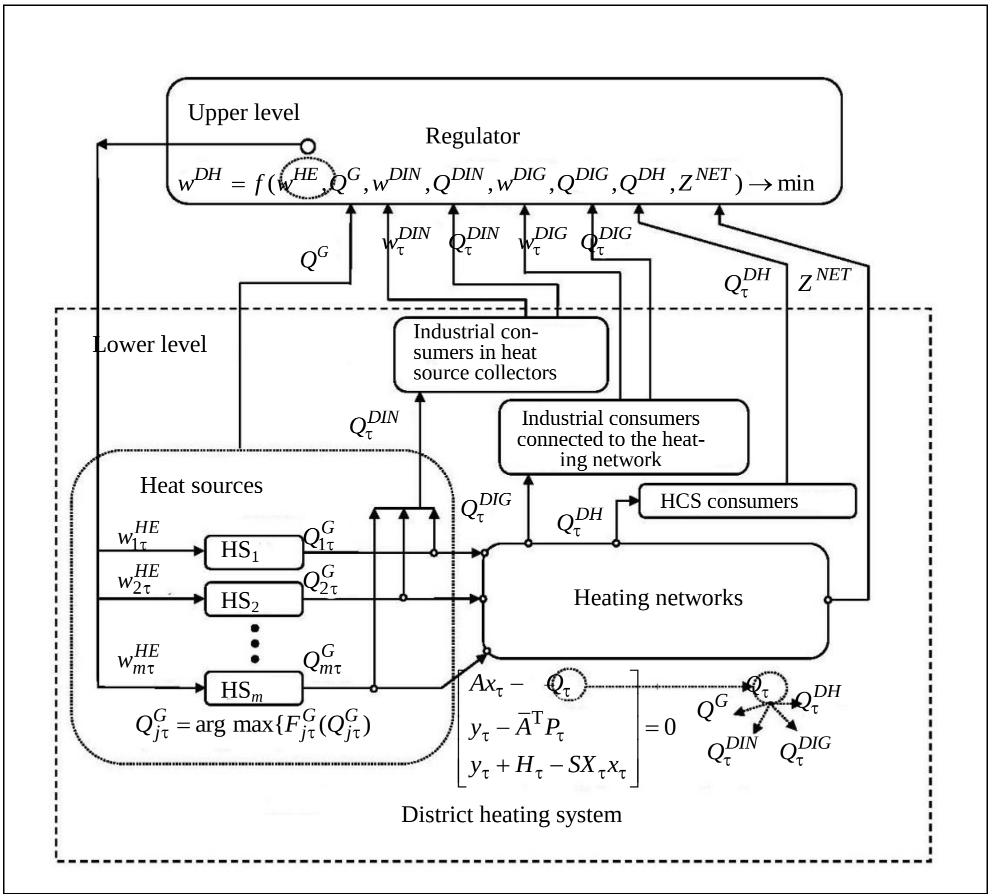

Fig. 3. Input and output parameters in the two-level management heat supply to consumers.

The proposed algorithm for minimizing the objective function (22) is an adaptation of the coordinate descent method to the problem under consideration [19].

A graphical interpretation of this algorithm as a step-by-step computational process is presented in Fig. 4.

In particular, Fig. 4*a* illustrates the change in the heat price function for HCS consumers depending on the heat production price of heat sources in a district heating system with two sources (Fig. 2). Figure 4*b* shows the contour lines of the objective function (the heat price domain for HCS consumers under different values of the heat energy production prices of heat sources).

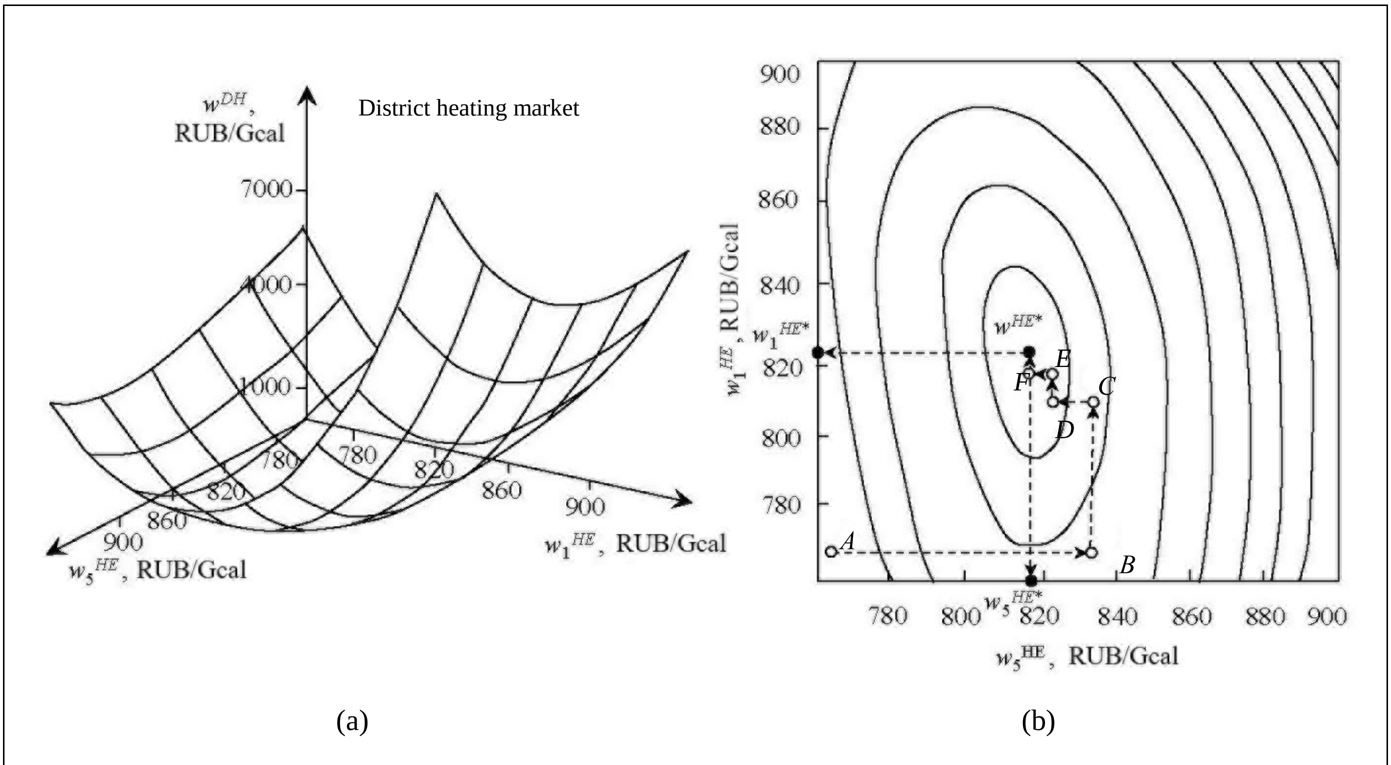

Fig. 4. A graphical interpretation of the computational algorithm.

Each heat source sequentially optimizes its heat production price under a fixed heat production price of the other heat source. The optimal heat production price of heat sources that minimizes the heat price for HCS consumers in this district heating system is achieved at the fifth iteration (see the point $w^{HE}*$). The broken line A-B-C-D-E-F-$w^{HE}*$ is the trajectory of the computational process (Fig. 4*b*).

2. APPLICATION TO A REAL DISTRICT HEATING SYSTEM

The proposed management model was applied to the district heating system of Angarsk. In an enlarged form, this system consists of 1273 edges and 1242 nodes (Fig. 5).

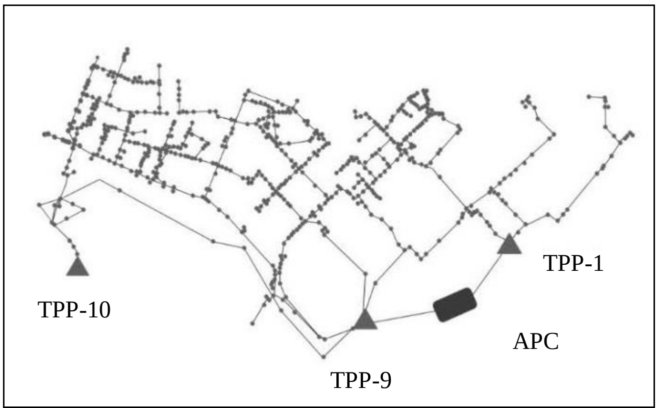

Fig. 5. The heat supply circuit of Angarsk.

The number of generalized consumers is represented by 534 nodes, of which 533 nodes correspond to consumers with fixed heat loads (HCS consumers) and one node to PJSC Angarsk Petrochemical Company (APC), located on the collectors of TPP-1 and TTP-9. We used the following initial data for calculations: the heat supply circuit of Angarsk with the technical characteristics of heating network edges (diameters, lengths, and resistances); the locations of heat sources in the heating system; the cost functions of heat sources; the temperature graph and climatic characteristics of the region; the estimated heat loads of HCS consumers; the heat demand function of PJSC APC.

The calculations were performed for a one-year time interval with a step of one hour. The calculated integral indices of the heating system of Angarsk are combined in Table 3.

According to Table 3, when supplying heat within the UHSO, TPP-9 covers 57.3% of the total heat load, and the shares of TPP-1 and TPP-10 are 22.6% and 20.1%, respectively. The main heat consumer in Angarsk is the HCS sector (69.3% of all heat produced by the district heating system). The remaining 30.7% of heat is consumed by PJSC APC.

Table 3

Estimated integral technical-and-economic indices of the heating system of Angarsk

Indicator	Value
Heat production volume, in million Gcal, including:	**6.85**
TPP-1	1.55
TPP-9	3.92
TPP-10	1.38
Heat production (fuel) costs, in billion RUB, including:	**3.30**
TPP-1	0.78
TPP-9	1.91
TPP-10	0.61
Semi-fixed (operating) costs, in billion RUB, including:	**2.19**
TPP-1	0.43
TPP-9	1.05
TPP-10	0.71
The cost of heat production, in RUB/Gcal:	
TPP-1	647.6
TPP-9	646.8
TPP-10	649.7
The price per unit of heat energy, in RUB/Gcal	
TPP-1	29.4
TPP-9	36.0
TPP-10	42.3
Profit, in billion RUB, including:	**1.02**
TPP-1	0.22
TPP-9	0.62
TPP-10	0.18
Costs of heating networks, in billion RUB	1.22
Heat price for HCS consumers, in RUB/Gcal	862.7
Heat price for PJSC APC, in RUB/Gcal	1350.5
Heat consumption by PJSC APC, in million Gcal	2.10
Heat consumption by HCS consumers, million Gcal	4.75

The average annual minimum tariff for HCS consumers is 862.7 RUB/Gcal (excluding VAT); for PJSC APC, 1350.5 RUB/Gcal. The total sales proceeds of the UHSO amount to 5.65 billion RUB, and the total profit of heat sources for the period is 1.22 billion RUB (0.22 billion RUB for TPP-1, 0.62 billion RUB for TPP-9, and 0.18 billion RUB for TPP-10). Heat transport costs (heating network costs) amount to 1.22 billion RUB (about 178.1 RUB/Gcal).

CONCLUSIONS

This paper has presented:

– new problem statements on developing district heating systems within the organizational regulated monopoly market model;

– requirements to mathematical models and methods used to solve them;

– the conceptual interaction of participants in the process of heat supply to consumers in the form of a hierarchical vertically integrated system.

As a result, a mathematical model of the district heating system within the two-level management system has been developed. In this hierarchical system, the regulator adjusts tariffs for HCS consumers, industrial consumers buy heat in accordance with their demand functions, and heat sources cover a given total demand from consumers to obtain the maximum profit. An optimization criterion has been proposed for the regulator. The two-level management approach has been applied to the real district heating system of Angarsk.

The proposed two-level mathematical model reflects well the real conditions in the local heat energy markets. This model reasonably considers the established "rules of play" in the heat energy market as well as the physical and technical and economic limitations of district heating systems.

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