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USING PIECEWISE FUNCTIONS TO NORMALIZE INPUT VARIABLES OF FUZZY INFERENCE SYSTEMS

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Abstract. This paper proposes a method for normalizing the input variables of fuzzy inference systems (FISs), which are used in assessing integrally the state of a complex object. The method involves piecewise functions: the variable's range is divided into several intervals (the length of each interval depends the variable's specifics), and a particular function is assigned to each interval. This function shows the patterns of the variable's variations on the normalized scale relative to its variations on the absolute scale. The set of these functions for the entire range of the variable forms the normalization operator. When implementing the normalization operator, the functions are selected so that after transformation, all input variables positively correlate with the output variable. This approach simplifies the construction of FISs: the same terms of the input variables have the same semantic meaning after transformation. According to the simulation results, FISs with the proposed normalization method are adequate to similar FISs without the normalization of the input variables. The proposed normalization method allows reducing the number of rules in the FIS knowledge base if the input variables have an optimum of their influence on the value of the output variable.

Keywords: fuzzy inference system, normalization, input variable, knowledge base, rule, integral assessment, information processing.

INTRODUCTION

Among other tasks, decision support systems are developed for integral assessment to rank a definite group of objects. Ranking serves for various purposes, e.g., to assess the creditworthiness of bank clients. Analysis of the publications [1–6] shows that fuzzy inference systems (FISs) are widely used to solve such problems.

According to [1–9], many FISs aggregate parameters with different measurement units, different ranges of assessment scales, different influences on the output variable, and different correlations with the values of the output variable. Therefore, to simplify the design of FISs, normalization methods were adopted in [1, 2, 7–12]: the values of input variables were reduced to a single scale. As demonstrated therein, such methods allow using identical membership functions (MFs) when describing the input variables and making the FIS invariant to changing the absolute value range of the input variables: if necessary, the normalization operator is subject to changes. Considering the results of [1, 2, 7–13], we divide all normalization methods into two classes:

– Class 1 contains the methods in which a mathematical function is used for transformation, and the normalized parameter interacts with the constants characterizing the normalized sample of values [1, 7–13].

– Class 2 contains the methods in which an interval of the normalized values is assigned to an interval of the initial parameter values [2].

Among the restrictions of class 1 methods, we mention the difficulty of reducing the absolute values of the input variable with a nonlinear influence on the output variable to the normalized values with a linear influence on the FIS final value. Class 2 methods have restrictions as well: in the publications discussed above, the mathematical methods for transforming the absolute values of variables into the normalized ones on a given interval were not formally implemented; the transformation itself preserved the correlation with the output variable. Such restrictions complicate the design of rules to transform the values of the input parameters into the value of the output variable in FISs.

Thus, this paper aims at improving the methods for normalizing the values of input variables in fuzzy inference systems.

1. A SURVEY OF RELATED WORKS AND PROBLEM STATEMENT

As described in [1, 2, 14], an FIS is implemented in the following stages:

- forming an array of aggregated variables $X=\{x_i\}: i=\overline{1,n}$, where each variable x_i has a scale sc_i;
- forming an array of output variables $Y=\{y_j\}: j=\overline{1,m}$.

Below, we will consider FISs with one output variable only.

After the array of the input variables x_i is formed, an array of terms $T_i=\{t_k^{x_i}\}: k=\overline{1,r}$ is specified for each variable. Each term $t_k^{x_i}$ has a semantic name characterizing the state of the described parameter x_i. For each term $t_k^{x_i}$, an MF $\mu_k^{x_i}(x_i)$ is formed, where $\mu_k^{x_i}$ denotes an operator for transforming an input variable from a crisp value x_i to a fuzzy one $x_{fuzz,i}$. The MF value shows the level of confidence that a crisp value of a variable corresponds to a specific term. The MP domain is the interval [0, 1]: unity indicates that the value of the variable fully matches the semantic meaning of the term, and zero indicates that the value of the variable does not match the semantic meaning of the term. Triangular and trapezoidal functions are often used to form MFs.

The variable aggregation operator is implemented by forming rules. Often a rule is a logical statement written as (*if* $A \Rightarrow B$, ξ, where A denotes the set of initial conditions (antecedent), B is a conclusion (consequent), and ξ gives the coefficient of confidence in the rule). (Further, let $\xi = 1$). When forming the set *A*, all value combinations for the terms of the input variables are enumerated, which form definite values of the terms of the output variables *B*. There are two structures of rules: Multiple Input Single Output (MISO) and Multiple Input Multiple Output (MIMO). Various logical operations "and," "or," and "not" can be used to aggregate input variables. This paper will study FISs based on MISO rules with the "and" operator to aggregate input variables. We form a knowledge base (KB) by enumerating all term combinations for the array of input variables and specifying the target value of the output variables. It displays the pattern of the influence of the input variables values on the output variables. Generally, an FIS with MISO rules can be described by

$$y = F_{\text{FIS}}(x_1,\ldots,x_i),$$

where F_{FIS} denotes a fuzzy inference operator for the variables array $X=\{x_i\}$. Figure 1*a* shows an example of an FIS aggregating the variables x_1 and x_2. According to the monograph [1], variables can be supplied to the FIS input in absolute or normalized measurement units. In the latter case, the values measured on a single scale are supplied to the FIS input. The transition from absolute values x_{abs} to the normalized units is performed using a normalization operator f_{norm}:

$$x_{\text{norm}} = f_{\text{norm}}(x_{\text{abs}}).$$

Figure 1*b* presents the FIS aggregating the variables x_1 and x_2 subjected to normalization [1]. Here, the normalizer block implements the operator f_{norm}.

Analysis of the publications [3–6, 14, 15] shows that the input variables often have the following influences on the integral assessment of the object's state:

– directly proportional (e.g., the income of a borrower who wants to get a loan from a bank: the input variable positively correlated with the output value characterizing the reasonability of issuing a loan);

– inversely proportional (e.g., the debt load of a bank client applying for the next loan: the input variable negatively correlates with the output value);

– optimal: there is an interval of input parameter values (or even a point) beyond which, on the left or right, the value of the final assessment becomes worse (e.g., the borrower's age).

Additionally, the aggregated variables can have different measurement units, different ranges of values, and different levels of influence on the final result. In turn, their degrees of significance can depend on the variable value. If the rules are formed using expertise,

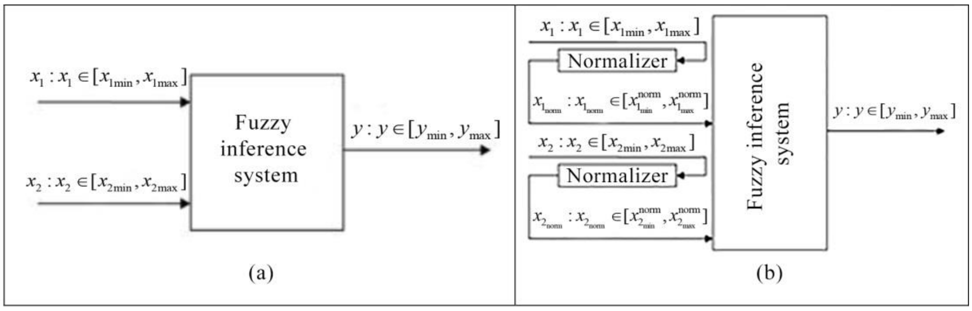

Fig. 1. The structure of models under study.

such a variety of properties of variables may cause technical errors in logical inference. To reduce the variety of properties, the authors [1, 2, 8, 9] proposed normalizing the variables to the interval [0, 1]. Considering the results of [1, 2, 7–13], we identify normalization methods based on the construction of a general pattern for transforming the variables (class 1 methods) or dividing the absolute value range of the variables into intervals and then assigning intervals on a normalized scale to each original interval (class 2 methods). When applying class 1 methods, the maximum and minimum values of the parameter and statistical indicators of the normalized sample can be considered. The papers [7, 10] described normalization methods based on the relation

$$x_{\text{norm}} = x_{\text{abs}} / x_{\max}^{\text{abs}}, \quad (1)$$

where $x_{\max}^{\text{abs}}$ is the parameter's maximum value in absolute units. The authors [1, 11] proposed to consider the minimum value $x_{\min}^{\text{abs}}$:

$$x_{\text{norm}} = \left(x_{\text{abs}} - x_{\min}^{\text{abs}}\right) / \left(x_{\max}^{\text{abs}} - x_{\min}^{\text{abs}}\right), \quad (2)$$

or

$$x_{\text{norm}} = \left(x_{\text{abs}} - x_{\max}^{\text{abs}}\right) / \left(x_{\max}^{\text{abs}} - x_{\min}^{\text{abs}}\right). \quad (3)$$

The following modifications of the relations (2), (3) were presented in [12]:

$$x_{\text{norm}} = D_1 \frac{x_{\text{abs}} - x_{\min}^{\text{abs}}}{x_{\max}^{\text{abs}} - x_{\min}^{\text{abs}}} + D_2, \quad (4)$$

where D_1 and D_2 denote some constants. The paper [13] introduced a method for normalizing input variables with the operator

$$x_{\text{norm}} = \left(x_{\text{abs}} - \bar{x}\right) / \sigma, \quad (5)$$

where $\bar{x}$ and σ are the sample mean and the standard deviation in the normalized sample, respectively. Note that with the operator (5), the range of x_{norm} does not fall into [0, 1], and the variable's range depends on its values in absolute measurement units. This feature may cause additional difficulties when implementing FISs. According to the authors cited, such a relation is appropriate for transforming the data used to train fuzzy neural networks. The monograph [1] proposed methods based on the relations

$$x_{\text{norm}} = \left(x_{\text{abs}} - x_{\text{mean}}^{\text{abs}}\right) / \left(x_{\max}^{\text{abs}} - x_{\min}^{\text{abs}}\right), \quad (6)$$

where $x_{\text{mean}} = 0.5\left(x_{\max}^{\text{abs}} + x_{\min}^{\text{abs}}\right)$.

As noted in [1], the operators (1)–(4), (6) can be generalized to the straight-line equation

$$x_{\text{norm}} = kx_{\text{abs}} + b, \quad (7)$$

where *k* and *b* denote some constants.

Note that the methods described by (1)–(7) have the following common feature: it is difficult to consider the influence of the normalized parameter on the value of the output variable depending on the value of this parameter. Also, these methods do not yield the optimum of the influence of the input parameter value on the output variable if it lies between the maximum and minimum absolute values of the variable. The nonlinear influence can be taken into account using class 2 methods. In the monograph [2], certain intervals on the normalized scale were assigned to intervals on the absolute scale of the parameter. Often, the same number of intervals are used relative to the middle of the normalization axis. To illustrate this method, Table 1 shows an example where the parameter $x_{\text{abs}} \in [x_0^{\text{abs}}, x_5^{\text{abs}}]$ is normalized to the interval $\left[x_0^{\text{norm}}, x_5^{\text{norm}}\right]$.

Table 1

Variable normalization: example

Absolute value interval	Normalized value interval	Interval no.
$\left[x_0^{\text{abs}}, x_1^{\text{abs}}\right]$	$\left[x_0^{\text{norm}}, x_1^{\text{norm}}\right]$	–2
$\left[x_1^{\text{abs}}, x_2^{\text{abs}}\right]$	$\left[x_1^{\text{norm}}, x_2^{\text{norm}}\right]$	–1
$\left[x_2^{\text{abs}}, x_3^{\text{abs}}\right]$	$\left[x_2^{\text{norm}}, x_3^{\text{norm}}\right]$	0
$\left[x_3^{\text{abs}}, x_4^{\text{abs}}\right]$	$\left[x_3^{\text{norm}}, x_4^{\text{norm}}\right]$	1
$\left[x_4^{\text{abs}}, x_5^{\text{abs}}\right]$	$\left[x_4^{\text{norm}}, x_5^{\text{norm}}\right]$	2

According to the monograph [2], the normalization operator ensures the conditions $x_0^{\text{abs}} = x_{\min}^{\text{abs}}$, $x_5^{\text{abs}} = x_{\max}^{\text{abs}}$, $x_0^{\text{norm}} = x_{\min}^{\text{norm}}$, and $x_5^{\text{norm}} = x_{\max}^{\text{norm}}$: there is a positive correlation of the variables. Thus, the correlation between the input and output variables does not change after the normalization described above. In addition, the monograph [2] suggested no analytical methods for transforming absolute values into the normalized ones; provided no recommendations on choosing the number of intervals to divide the axis of normalized values and the limits of these intervals; gave no explanations on the influence of input variable normalization on the accuracy of the resulting FIS compared to the original FIS (in which the variables are measured in absolute units).

The method described in [16, 17], originally not designed for FISs, can be considered a development of the normalization procedure suggested in [2]. This method divides the normalized values of a variable into intervals with the indication of points (their limits) and assigns to the points (the limits of the normalized-scale intervals) some values of the variable on the absolute scale. The resulting pairs of points are used to construct a curve, implementing the transformation of a variable from one scale to another. The paper [17] proposed selecting points to divide the interval of normalized values into parts: $0 \sim x_{\text{norm}}^{0}$, $0.25 \sim x_{\text{norm}}^{0.25}$, $0.5 \sim x_{\text{norm}}^{0.5}$, $0.75 \sim x_{\text{norm}}^{0.75}$, and $1 \sim x_{\text{norm}}^{1}$. Some values of the variable on the absolute scale are assigned to them: x_{abs}^{0}, $x_{\text{abs}}^{0.25}$, $x_{\text{abs}}^{0.5}$, $x_{\text{abs}}^{0.75}$, and x_{abs}^{1}. The resulting transformation operator is represented by a broken line constructed from the pairs of points $\left(x_{\text{abs}}^{0}; x_{\text{norm}}^{0}\right)$, $\left(x_{\text{abs}}^{0.25}; x_{\text{norm}}^{0.25}\right)$, $\left(x_{\text{abs}}^{0.5}; x_{\text{norm}}^{0.5}\right)$, $\left(x_{\text{abs}}^{0.75}; x_{\text{norm}}^{0.75}\right)$, and $\left(x_{\text{abs}}^{1}; x_{\text{norm}}^{1}\right)$. However, the papers [16, 17] described no analytical methods for transforming the absolute values into the normalized ones. Moreover, similar to the method described in [2], the values positively correlate before and after transformation: the type of correlation between the input and output variables remains the same.

Thus, the methods for constructing FISs [1, 2, 14] and the ones for normalizing input variables [1, 2, 8–13] have several restrictions. They relate to the properties of the input variables aggregated using FISs. Among such properties of input variables, we mention different measurement units, different ranges of values, different and (or) uneven levels of influence on the output variable, and different types of correlation with the output variable. Due to such properties of the variables, experts face difficulties when forming rules for the FIS knowledge base.

Some of the restrictions are eliminated using normalization methods. Such methods mainly transform various values of the input variables to a single scale. The disadvantages of the normalization methods often proceed from statistical indicators of the normalized samples of input variables values (usually, the maximum, minimum, and average values). The most universal properties of variable normalization are observed for a method that assigns an interval on the normalized scale to intervals on the absolute values of the parameter. Therefore, we will improve this method below.

2. IMPROVING THE NORMALIZATION METHOD FOR INPUT VARIABLES OF FUZZY INFERENCE SYSTEMS

2.1 Proposed modifications

To improve the approach described in the monograph [2], we propose a normalization method based on an operator f_{norm} for transforming the absolute values of an input variable into the normalized ones so that (1) the value of the output variable will not decrease with an increase in the normalized value of the input variable and (2) the value of the output variable will not increase with a decrease in the normalized value of the input variable. Assume that trapezoidal MFs are used to describe the input variables. Such an assumption is well-grounded: according to [1, 2, 14], this type of MFs is widespread in applications, and the corresponding MFs have crisp limits of the support and the core. (The crisp limits of the core and the support of MFs will be used below.)

Within the proposed method, the normalization operator f_{norm} is implemented using a set of functions: a certain pattern $f_{\text{norm},z}$ is assigned to each zth interval of the absolute values x_{abs} of an input variable. In other words,

$$x_{\text{norm}} = \begin{cases} f_{\text{norm},1}\left(x_{\text{abs}}\right) \text{if } x_{\text{abs}} \in \left[x_{\text{abs},i_{\min}}, x_{\text{abs}.i_1}\right), \\ f_{\text{norm},2}\left(x_{\text{abs}}\right) \text{if } x_{\text{abs}} \in \left[x_{\text{abs}.i_1}, x_{\text{abs}.i_2}\right), \\ \dots \\ f_{\text{norm},z}\left(x_{\text{abs}}\right) \text{if } x_{\text{abs}} \in \left[x_{\text{abs}.i_{\max}-1}, x_{\text{abs}.i_{\max}}\right). \end{cases} \quad (8)$$

If the patterns $f_{\text{norm},z}$ for neighbor intervals are the same, the intervals are combined. The set of functions (8) is formed so that the minimum and maximum values of the normalized variable, $x_{\text{norm}}^{\min}$ and $x_{\text{norm}}^{\min}$, have the worst (maximum negative) and best (maximum positive), respectively, influences on the value of the output variable of the FIS. We propose implementing the operators $f_{\text{norm},z}$ as follows:

1. Define the maximum and minimum absolute value of the variable $x_{\text{abs}.i}$.

2. Define the maximum $x_{\text{norm}}^{\max}$ and minimum $x_{\text{norm}}^{\min}$ values of this variable in relative units (in this paper, $x_{\text{norm}}^{\min} = 0$ and $x_{\text{norm}}^{\max} = 100$).

3. On the absolute scale, define the interval limits z (for the set of functions (8), the points $x_{\text{abs}.i_1}$, $x_{\text{abs}.i_2}$, …, $x_{\text{abs}.i_{\max}-1}$); on the relative scale, define their counterparts (the point x_{norm,i_1} corresponds to the point $x_{\text{abs}.i_1}$, etc.) in the following way:

3.1. Considering the recommendations of [1], experts give the primary representation of a fuzzy input variable in the traditional form using trapezoidal MFs (in this paper, triangular MFs are studied as a special case of trapezoidal MFs: the core degenerates into a point).

3.2. The interval limits are the limits of the MF cores of the variable's primary fuzzy representation.

3.3. Similar types of MFs are used to describe the variables after normalization. The interval limits are the points corresponding to the limits of the MF cores of the variable's primary fuzzy representation.

3.4. If the core of the term's MF belongs to an interval with the optimality domain, it is divided into two equal intervals.

4. For each interval of z, define the function $f_{\text{norm},z}\left(x_{\text{abs}.i}\right) = x_{\text{norm}.i}$.

5. Based on the set of functions $f_{\text{norm},z}$, construct the normalization operator (8).

Following the monograph [1], we propose implementing the functions $f_{\text{norm},z}$ as straight lines passing through two points: the limits of the intervals on the normalized and absolute scales. For example, let points 1 and 2 for constructing the operator $f_{\text{norm},1}$ have the coordinates $\left(x_{\text{abs},i_{\min}}, x_{\text{norm}}^{\min}\right)$ and $\left(x_{\text{abs},i_1}, x_{\text{norm},i_1}\right)$, respectively. Then the operator is given by

$$f_{\text{norm},1}\left(x_{\text{abs}}\right) = \left[\left(x_{i_1}^{\text{norm}} - x_{i_{\min}}^{\text{norm}}\right) \times \times \left(x_{i_{\text{abs}}} - x_{i_{\min}}\right) / \left(x_{i_1} - x_{i_{\min}}\right)\right] + x_{i_{\min}}^{\text{norm}}. \quad (9)$$

Figure 2*a* shows the normalization operator (8) implemented for the input variables positively correlating with the values of the output variable. The case of their negative correlation is illustrated in Fig. 2*b*. Finally, Fig. 2*c* corresponds to the situation in which the input variables have an optimum of their influence on the value of the output variable.

The proposed modifications allow:

– determining the number of intervals for dividing the normalized value axis based on the methods used to construct the MF of the FIS (it equals the number of pairs of the MF core limits if normalization applies to the variables positively or negatively correlating with the output variable, or plus 1 if one optimum affects the value of the output variable);

– ensuring that after normalization, the input variables have the same correlation with the output variable;

– considering changes in the degree of significance of the input variable's influence on the output value of the FIS, depending on the variable's value.

2.2. Verification of the proposed modifications

The proposed modifications of the normalization method were verified by mathematical modeling. Several FISs based on the zero-order Takagi–Sugeno model were constructed. According to [18], a feature of such models is using constants to describe the MF of the output variable. The FISs under study had two input variables, x_1 and x_2, and one output variable y. Six experiments were carried out:

– The variables x_1 and x_2 positively correlate with the variable y.

– The variables x_1 and x_2 negatively correlate with the variable y.

– The variables x_1 and x_2 have an optimum with respect to the variable y.

– The variable x_1 has a positive correlation with the variable y; the variable x_2, a negative correlation with the variable y.

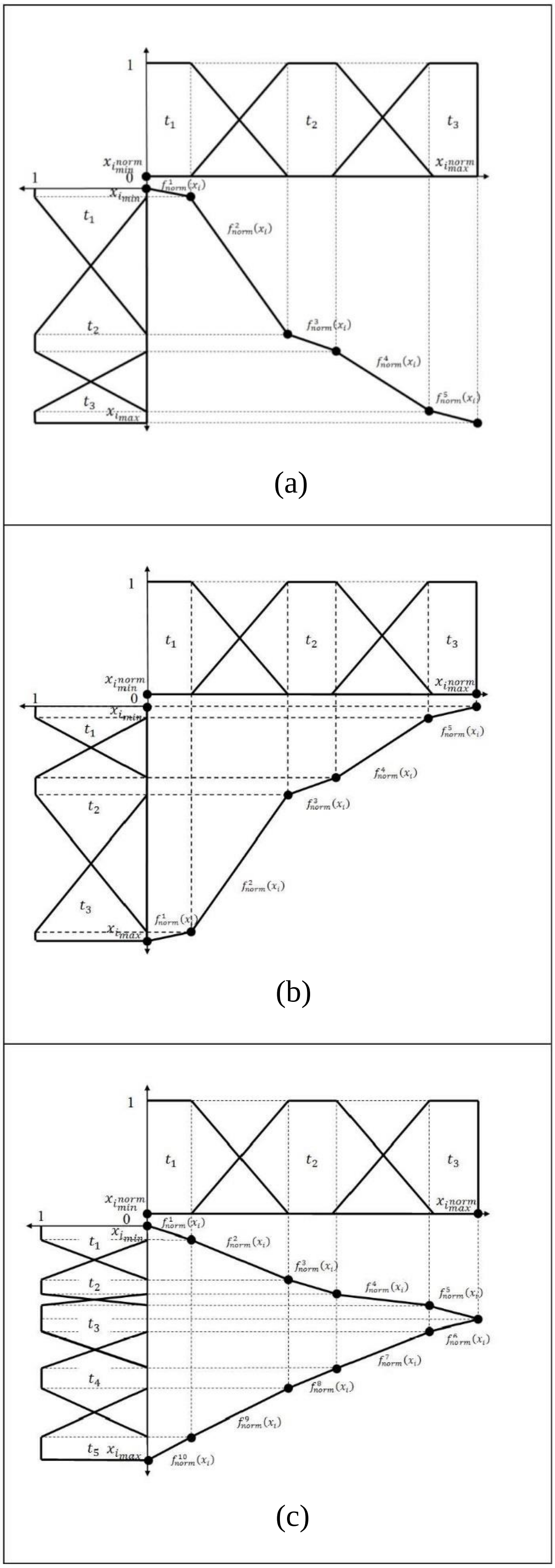

Fig. 2. Normalization operators for input variables of FISs: some examples.

– The variable x_1 has a positive correlation with the variable y; the variable x_2, an optimum with respect to the variable y.

– The variable x_1 has a negative correlation with the variable y; the variable x_2, an optimum with respect to the variable y.

Within each experiment, two FISs were investigated: the FIS of type 1 (the variables x_1 and x_2 normalized by the proposed method) and the FIS of type 2 (the variables x_1 and x_2 processed without normalization). The absolute values of the variables x_1 and x_2 were measured in the range from 0 to 10 points; the values of the variables x_1 and x_2 after normalization, in the range from 0 to 100 points; the value of the variable y, in the range from 0 to 100 points.

If a variable passed normalization, its MFs are shown in Fig. 3*a*. If a variable positively correlated with the variable y, its MFs are shown in Fig. 3*b*. If a variable negatively correlated with the variable y, its MFs are shown in Fig. 3*c*. If a variable had an optimum with respect to the variable y, its MFs are shown in Fig. 3*d* (variant 1) and Fig. 3*e* (variant 2).

The terms in Fig. 3 are described as follows. In Fig. 3*a*: *NB* indicates a worst influence on the output variable; *Z*, a medium influence on the output variable; *PB*, a best influence on the output variable. In Fig. 3*b*–3*e*: *NB* indicates a low value; *NM*, a value closer to the average; *Z*, the average; *PM*, a value closer to a high value; *PB*, a high value. The output variable y is described by three terms in the form of the constants $t_{NB}^{y}=0$, $t_{Z}^{y}=50$, and $t_{PB}^{y}=100$ points (low, average, and high values, respectively). In view of the monograph [1], we assume that the partition-of-unity condition holds when constructing the MFs: for any crisp value of the variable x_i, the terms covering the corresponding segment of the crisp value axis have the grades of membership to the MFs adding up to 1. This condition can be written as $\sum_k \mu_k^{x_i}\left(x_i\right) \equiv 1 \forall x_i \subset X$.

Due to the expressions (8) and (9), the normalization operators of the input variables are the following:

- under a positive correlation with the variable y,

$$x_{\text{norm}}^{\text{direct}}=\begin{cases} f_{\text{norm},1}\left(x_{\text{abs}}\right)=5x_{\text{abs}} \mid 0 \le x_{\text{abs}} \le 2, \\ f_{\text{norm},2}\left(x_{\text{abs}}\right)=7.5x_{\text{abs}}-5 \mid 2 < x_{\text{abs}} \le 6, \\ f_{\text{norm},3}\left(x_{\text{abs}}\right)=10x_{\text{abs}}-20 \mid 6 < x_{\text{abs}} \le 8, \\ f_{\text{norm},4}\left(x_{\text{abs}}\right)=30x_{\text{abs}}-180 \mid 8 < x_{\text{abs}} \le 9, \\ f_{\text{norm},5}\left(x_{\text{abs}}\right)=10x_{\text{abs}} \mid 9 < x_{\text{abs}} \le 10; \end{cases} \quad (10)$$

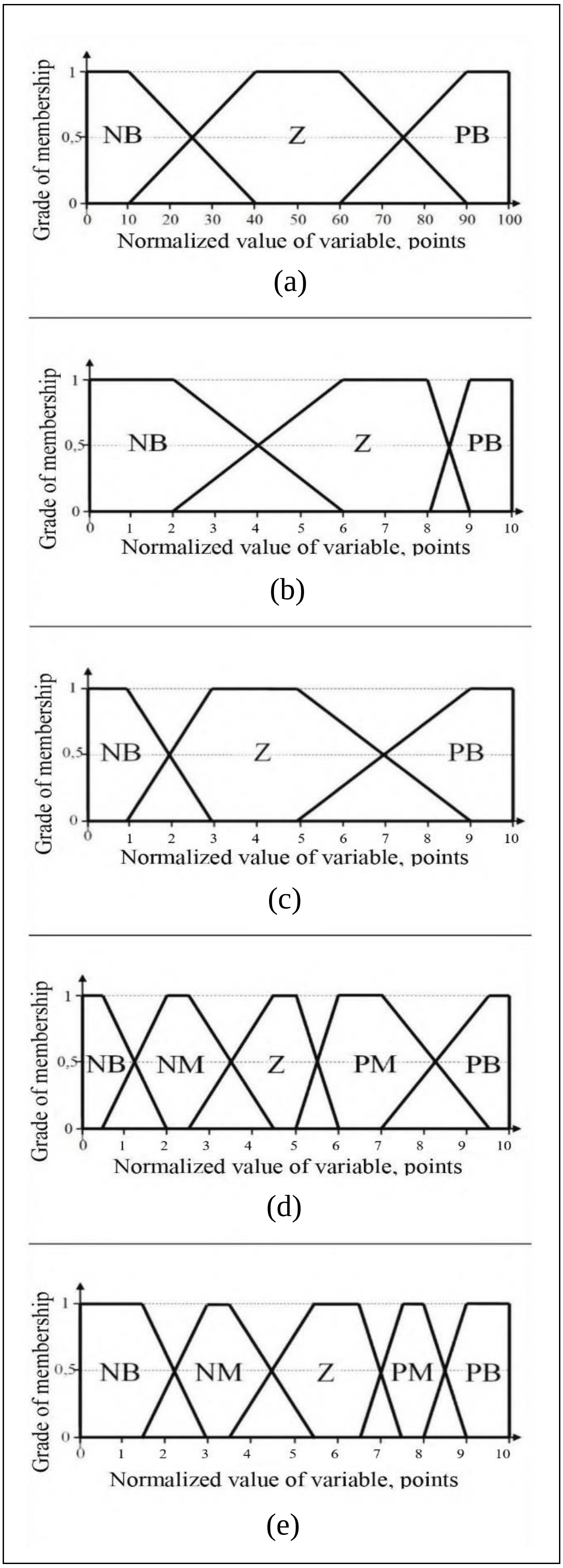

Fig. 3. Membership functions of input variables.

• under a negative correlation with the variable y,

$$x_{\text{norm}}^{\text{invers}}=\begin{cases} f_{\text{norm},1}\left(x_{\text{abs}}\right)=-10\,x_{\text{abs}}+100\,|\,0\le x_{\text{abs}}\le 1,\\ f_{\text{norm},2}\left(x_{\text{abs}}\right)=-15\,x_{\text{abs}}+105\,|\,1< x_{\text{abs}}\le 3,\\ f_{\text{norm},3}\left(x_{\text{abs}}\right)=-10\,x_{\text{abs}}+90\,|\,3< x_{\text{abs}}\le 5,\\ f_{\text{norm},4}\left(x_{\text{abs}}\right)=-7.5\,x_{\text{abs}}+77.5\,|\,5< x_{\text{abs}}\le 9,\\ f_{\text{norm},5}\left(x_{\text{abs}}\right)=-10\,x_{\text{abs}}+100\,|\,9< x_{\text{abs}}\le 10; \end{cases} \quad (11)$$

• under an optimum with respect to the variable y (variant 1),

$$x_{\text{norm}}^{\text{opt.var1}}=\begin{cases} f_{\text{norm},1\div 2}\left(x_{\text{abs}}\right)=20\,x_{\text{abs}}\,|\,0\le x_{\text{abs}}\le 2,\\ f_{\text{norm},3}\left(x_{\text{abs}}\right)=40\,x_{\text{abs}}-40\,|\,2\le x_{\text{abs}}\le 2.5,\\ f_{\text{norm},4}\left(x_{\text{abs}}\right)=15\,x_{\text{abs}}+22.5\,|\,2.5\le x_{\text{abs}}\le 4.5,\\ f_{\text{norm},5}\left(x_{\text{abs}}\right)=40\,x_{\text{abs}}-90\,|\,4.5\le x_{\text{abs}}\le 4.75,\\ f_{\text{norm},6}\left(x_{\text{abs}}\right)=-40\,x_{\text{abs}}+290\,|\,4.75\le x_{\text{abs}}\le 5,\\ f_{\text{norm},7}\left(x_{\text{abs}}\right)=-30\,x_{\text{abs}}+240\,|\,5\le x_{\text{abs}}\le 6,\\ f_{\text{norm},8}\left(x_{\text{abs}}\right)=-20\,x_{\text{abs}}+180\,|\,6\le x_{\text{abs}}\le 7,\\ f_{\text{norm},9}\left(x_{\text{abs}}\right)=-12\,x_{\text{abs}}+124\,|\,7\le x_{\text{abs}}\le 9.5,\\ f_{\text{norm},10}\left(x_{\text{abs}}\right)=-20\,x_{\text{abs}}+200\,|\,9.5\le x_{\text{abs}}\le 10; \end{cases} \quad (12)$$

• under an optimum with respect to the variable y (variant 2):

$$x_{\text{norm}}^{\text{opt.var2}}=\begin{cases} f_{\text{norm},1}\left(x_{\text{abs}}\right)=\dfrac{20}{3}\,x_{\text{abs}}\,|\,0\le x_{\text{abs}}\le 1.5,\\ f_{\text{norm},2}\left(x_{\text{abs}}\right)=20\,x_{\text{abs}}-20\,|\,1.5\le x_{\text{abs}}\le 3,\\ f_{\text{norm},3}\left(x_{\text{abs}}\right)=40\,x_{\text{abs}}-80\,|\,3\le x_{\text{abs}}\le 3.5,\\ f_{\text{norm},4}\left(x_{\text{abs}}\right)=15\,x_{\text{abs}}+7.5\,|\,3.5\le x_{\text{abs}}\le 5.5,\\ f_{\text{norm},5}\left(x_{\text{abs}}\right)=20\,x_{\text{abs}}-20\,|\,5.5\le x_{\text{abs}}\le 6,\\ f_{\text{norm},6}\left(x_{\text{abs}}\right)=-20\,x_{\text{abs}}+220\,|\,6\le x_{\text{abs}}\le 6.5,\\ f_{\text{norm},7}\left(x_{\text{abs}}\right)=-30\,x_{\text{abs}}+285\,|\,6.5\le x_{\text{abs}}\le 7.5,\\ f_{\text{norm},8}\left(x_{\text{abs}}\right)=-40\,x_{\text{abs}}+360\,|\,7.5\le x_{\text{abs}}\le 8,\\ f_{\text{norm},9}\left(x_{\text{abs}}\right)=-30\,x_{\text{abs}}+280\,|\,8\le x_{\text{abs}}\le 9,\\ f_{\text{norm},10}\left(x_{\text{abs}}\right)=-10\,x_{\text{abs}}+100\,|\,9\le x_{\text{abs}}\le 10. \end{cases} \quad (13)$$

In all experiments, the FISs of type 1 had the same surface (Fig. 4*a*). The other FIS parameters were as follows:

– In experiment 1, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operator (10). The variables x_1 and x_2 in the FIS of type 2 positively correlated with the variable y (Fig. 4*b*).

– In experiment 2, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operator (11). The variables x_1 and x_2 in the FIS of type 2 negatively correlated with the variable y (Fig. 4*c*).

– In experiment 3, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operators (12) and (13). The variables x_1 and x_2 in the FIS of type 2 had an optimum with respect to the variable y (Fig. 4*d*).

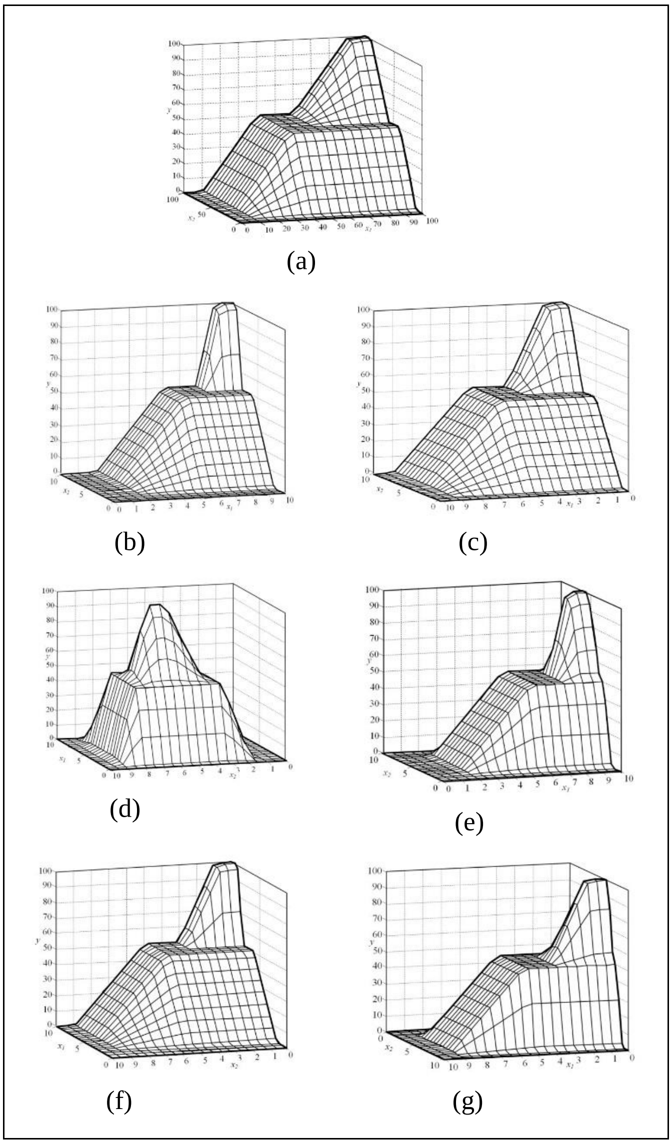

Fig. 4. Surfaces of FISs under comparison.

– In experiment 4, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operators (10) and (13), respectively. The variable x_1 in the FIS of type 2 positively correlated with the variable y, whereas the variable x_2 in the FIS of type 2 had an optimum with respect to the variable y (Fig. 4*e*).

– In experiment 5, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operators (10) and (11), respectively. The variables x_1 and x_2 in the FIS of type 2 positively and negatively correlated, respectively, with the variable y (Fig. 4*f*).

– In experiment 6, the variables x_1 and x_2 in the FIS of type 1 were normalized using the operators (11) and (13), respectively. The variable x_1 in the FIS of type 2 negatively correlated with the variable y, whereas the variable x_2 in the FIS of type 2 had an optimum with respect to the variable y (Fig. 4*g*).

During the experiments, the variables were supplied to the FIS input in the following values combinations in the range from 0 to 10 points:

– equal values of the variables with a step of 0.5 points (22 values of each variable in total),

– the values of the variable x_1 with a step of 0.5 points, and the fixed values of the variable x_2 starting from 0, increased by 0.5 points for each subsequent series of the experiment (462 values of each variable in total).

The error of the results was determined as the difference between the FISs of types 1 and 2:

$$\Delta_{\exp N,w} = \left| y^{\mathrm{FIS}m1}_{\mathrm{out},\exp N,w} - y^{\mathrm{FIS}m2}_{\mathrm{out},\exp N,w} \right|, \qquad (14)$$

where N denotes the experiment number; w is the number of the vector of values supplied to the FIS input during the experiment; $y^{\mathrm{FIS}m1}_{\mathrm{out},\exp N,w}$ and $y^{\mathrm{FIS}m2}_{\mathrm{out},\exp N,w}$ indicate the outputs of the FISs of types 1 and 2, respectively. In experiments 1–6, it was established that there are no errors between the output values of the FIS of type 1 with the proposed normalization method and the FIS of type 2 without normalization. For explaining the reasons, the features of the FIS operation were analyzed.

The FIS output is the accumulated result of activating the rules to construct the knowledge base. The result of each rule is determined by the grades of membership of the crisp values of the input variables to each term and the aggregation operators of their fuzzified values. In the case under consideration, the results of rule execution were processed without any changes, and the final FIS value was formed without any changes as well. The changes were introduced when preparing the variables (before supplying them to the FIS input).

Therefore, the FISs will operate without errors if the variables have the same values after the fuzzification procedure, regardless of using the proposed normalization method. This assertion was verified in a separate series of experiments. Below, we compare the values of the input variables after fuzzification for the MFs shown in Fig. 3*a* (the FISs with variables normalization) and MFs shown in Fig. 3*d* (the FISs with a variable having an optimum of its influence on the final result).

In Fig. 3*a*, the MFs are given by:

$$\mu^{3a}_{NB}(x) = \begin{cases} \mu^{3a}_{NB}(x) = 1 \mid 0 \le x < 10, \\ \mu^{3a}_{NB}(x) = -\dfrac{1}{30}x + \dfrac{4}{3} \mid 10 \le x < 40, \\ \mu^{3a}_{NB}(x) = 0 \mid x > 40, \end{cases} \qquad (15)$$

$$\mu^{3a}_{Z}(x) = \begin{cases} \mu^{3a}_{Z}(x) = 0 \mid 0 \le x < 10, \\ \mu^{3a}_{Z}(x) = \dfrac{1}{30}x - \dfrac{1}{3} \mid 10 \le x < 40, \\ \mu^{3a}_{Z}(x) = 1 \mid 40 \le x < 60, \\ \mu^{3a}_{Z}(x) = \dfrac{1}{30}x + 3 \mid 60 \le x < 90, \\ \mu^{3a}_{Z}(x) = 0 \mid x > 90, \end{cases} \qquad (16)$$

$$\mu^{3a}_{Z}(x) = \begin{cases} \mu^{3a}_{PB}(x) = 0 \mid x < 60, \\ \mu^{3a}_{PB}(x) = \dfrac{1}{30}x - 2 \mid 60 \le x \le 90, \\ \mu^{3a}_{PB}(x) = 0 \mid x > 90. \end{cases} \qquad (17)$$

In Fig. 3*d*, the MFs are given by:

$$\mu^{3г}_{NB}(x) = \begin{cases} \mu^{3г}_{NB}(x) = 1 \mid 0 \le x < 0{,}5, \\ \mu^{3г}_{NB}(x) = -\dfrac{2}{3}x + \dfrac{4}{3} \mid 0{,}5 \le x < 2, \\ \mu^{3г}_{NB}(x) = 0 \mid x > 2, \end{cases} \qquad (18)$$

$$\mu^{3г}_{NM}(x) = \begin{cases} \mu^{3г}_{NM}(x) = 0 \mid 0 \le x < 2, \\ \mu^{3г}_{NM}(x) = \dfrac{2}{3}x - \dfrac{1}{3} \mid 2 \le x < 6, \\ \mu^{3г}_{NM}(x) = 1 \mid 2 \le x < 2{,}5, \\ \mu^{3г}_{NM}(x) = -\dfrac{1}{2}x + 2\dfrac{1}{4} \mid 2{,}5 \le x < 4{,}5, \\ \mu^{3г}_{NM}(x) = 0 \mid x > 4{,}5, \end{cases} \qquad (19)$$

$$\mu^{3г}_{Z}(x) = \begin{cases} \mu^{3г}_{Z}(x) = 0 \mid x \le 2{,}5, \\ \mu^{3г}_{Z}(x) = \dfrac{1}{2}x - 1\dfrac{1}{4} \mid 2{,}5 \le x < 4{,}5, \\ \mu^{3г}_{Z}(x) = 1 \mid 4{,}5 \le x < 5, \\ \mu^{3г}_{Z}(x) = -x + 6 \mid 5 \le x \le 6, \\ \mu^{3г}_{Z}(x) = 0 \mid x > 6, \end{cases} \qquad (20)$$

$$\mu^{3г}_{PM}(x) = \begin{cases} \mu^{3г}_{PM}(x) = 0 \mid x < 5, \\ \mu^{3г}_{PM}(x) = x - 5 \mid 5 \le x < 6, \\ \mu^{3г}_{PM}(x) = 1 \mid 6 \le x < 7, \\ \mu^{3г}_{PM}(x) = -\dfrac{2}{5}x + 3\dfrac{4}{5} \mid 7 \le x < 9{,}5, \\ \mu^{3г}_{PM}(x) = 0 \mid x > 9{,}5, \end{cases} \qquad (21)$$

$$\mu^{3d}_{PB}(x) = \begin{cases} \mu^{3d}_{PB}(x) = 0 \mid x < 7, \\ \mu^{3d}_{PB}(x) = \dfrac{2}{5}x - 2\dfrac{4}{5} \mid 7 \le x \le 9.5, \\ \mu^{3d}_{PB}(x) = 1 \mid x > 9.5. \end{cases} \qquad (22)$$

Also, note the semantic equivalence of the following terms in Figs. 3*a* and 3*d*: $NB_{3a} \Leftrightarrow NB_{3d}$, $NB_{3a} \Leftrightarrow PB_{3d}$, $Z_{3a} \Leftrightarrow NM_{3d}$, $Z_{3a} \Leftrightarrow PM_{3d}$, and $PB_{3a} \Leftrightarrow Z_{3d}$. The values of the input variables are given in Table 2.

For clarity, the fuzzified values of the input variables normalized by the proposed method are highlighted in gray.

With the semantic equivalence of the terms, the errors using a formula similar to (14) were determined as follows. First, columns 3 and 7, as well as 4 and 6, were summed elementwise. Then:

– From the elementwise sum of columns 3 and 7, the corresponding elements of column 8 were subtracted.

– From the elementwise sum of columns 4 and 6, the corresponding elements of column 9 were subtracted.

– From the elements of column 5, the corresponding elements of column 10 were subtracted.

These operations yielded arrays with zero elements. The results of such experiments with the MFs shown in Figs. 3*b*, 3*c*, and 3*e* were similar. Under the experiment restrictions, such an accuracy was achieved because the interval limits on the normalization operator graph coincided with the points of the MF cores for the terms of the normalized input variables.

Due to the proposed modifications, six types of different FISs (Figs. 4*b*–4*g*) were replaced by one type of FIS (Fig. 4*a*). Experiment 3 showed a 64% reduction in the number of rules (9 rules in the FIS of type 1 vs. 25 rules in the FIS of type 2). Experiments 5 and 6 showed a 40% reduction in the number of rules (9 rules in the FIS of type 1 vs. 15 rules in the FIS of type 2). In experiment 3, both variables aggregated had one optimum of the influence on the value of the output variable; in experiments 5 and 6, only one variable had an optimum of the influence on the value of the output variable.

In addition, the time to form the rule bases was compared for the FISs of types 1 and 2 in experiments 2–6. Such a comparison was not carried out in experiment 1: the knowledge bases of the FISs of types 1 and

Table 2

Values for the terms of MFs in Figs. 3*a* and 3*d*

Input values		Fuzzified values of input variables:							
1	2	3	4	5	6	7	8	9	10
Without normalization	Normalized using operator (12)	(18) – $\mu_{NB}^{3d}(x)$	(19) – $\mu_{NM}^{3d}(x)$	(20) – $\mu_{Z}^{3d}(x)$	(21) – $\mu_{PM}^{3d}(x)$	(22) – $\mu_{PB}^{3d}(x)$	(15) – $\mu_{NB}^{3a}(x)$	(16) – $\mu_{Z}^{3a}(x)$	(17) – $\mu_{PB}^{3a}(x)$
0.5	10	1	0	0	0	0	1	0	0
1	20	0.667	0.333	0	0	0	0.667	0.333	0
1.5	30	0.333	0.667	0	0	0	0.333	0.667	0
2	40	0	1	0	0	0	0	1	0
2.5	60	0	1	0	0	0	0	1	0
3	67.5	0	0.75	0.25	0	0	0	0.75	0.25
3.5	75	0	0.5	0.5	0	0	0	0.5	0.5
4	82.5	0	0.25	0.75	0	0	0	0.25	0.75
4.5	90	0	0	1	0	0	0	0	1
5	90	0	0	1	0	0	0	0	1
5.5	75	0	0	0.5	0.5	0	0	0.5	0.5
6	60	0	0	0	1	0	0	1	0
6.5	50	0	0	0	1	0	0	1	0
7	40	0	0	0	1	0	0	1	0
7.5	34	0	0	0	0.8	0.2	0.2	0.8	0
8	28	0	0	0	0.6	0.4	0.4	0.6	0
8.5	22	0	0	0	0.4	0.6	0.6	0.4	0
9	16	0	0	0	0.2	0.8	0.8	0.2	0
9.5	10	0	0	0	0	1	1	0	0
10	0	0	0	0	0	1	1	0	0

2 contain the same rules. For each group of the FISs of types 1 and 2, five rule bases were formed. As a result, the rule base formation time when using the FIS of type 1 instead of the FIS of type 2 was:

- in experiments 2 and 5, by approximately 17% smaller, due to various types of correlations between the input variables and the output variable (no need for the expert to compare the semantic value of the term describing the input variable range and its influence on the final result);
- in experiments 4 and 6, by approximately 37% smaller, due to the number of rules in the knowledge base of the FIS of type 1 (9 rules) and the FIS of type 2 (15 rules);
- in experiment 3, by approximately 58% smaller, due to the number of rules in the knowledge base of the FIS of type 1 (9 rules) and the FIS of type 2 (25 rules).

Thus, the average reduction in the knowledge base formation time over the entire series of experiments 3–6 was about 35%.

3. AN EXAMPLE OF IMPLEMENTING THE PROPOSED MODIFICATIONS

As an example, consider a simplified FIS for ranking bank clients by creditworthiness. The decision on issuing a loan is based on three parameters:

- x_1, the borrower's monthly income, ranging from 15 to 100 thousand RUB. This variable positively correlates with the output variable: the higher the person's income is, the more likely he/she will get a loan;
- x_2, the borrower's share of monthly payments for serving the current loans. It is measured as a percentage of his/her earnings, ranging from 0 to 60%. The greater the person's share of current payments is, the less likely he/she will get a new loan;
- x_3, the borrower's age, ranging from 14 to 85 years. The best interval of this variable is between 35 to 45 years: at this age, a person has the highest "life" stability in terms of income and health.

The purpose of the experiment was to compare the results yielded by model 1 (with the proposed normalization method), model 2 (without normalization of the input variables), and model 3 (with the normalization method described in [2]: the values of the input variables were divided into intervals, and an interval on the normalized scale was assigned to each interval on the absolute axis). The models were FISs with three inputs (the variables x_1, x_2, and x_3) and one output (the variable y). The structure of models 1 and 3 is shown in Fig. 5*a*; the structure of model 2, in Fig. 5*b*.

In model 1, the variables x_1, x_2, and x_3 were described by the MFs shown in Fig. 3*a*. In model 2, the variables x_1, x_2, and x_3 were described by the MFs shown in Figs. 6*a*–6*c*.

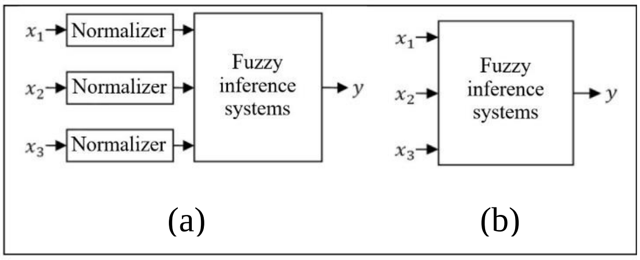

Fig. 5. The structure of studied models.

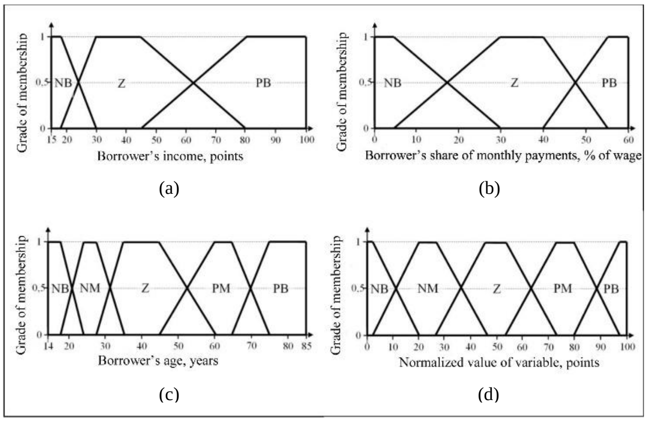

Fig. 6. Membership functions of input variables for studied models.

In model 3, the variables x_1 and x_2 were described by the MFs shown in Fig. 3*a*; the variable x_3, by the MFs shown in Fig. 6*d*. The semantic meanings of the terms in model 1 corresponded to those in Fig. 3*a*. The semantic meanings of the terms in models 2 and 3 corresponded to those in Fig. 3*b*–3*d*. The term set $\{NB, NM, Z, PM, PB\}$, where $NB \sim 0$, $NM \sim 25$, $Z \sim 50$, $PM \sim 75$, and $PB \sim 100$ points, was used to describe the output variable y. The term values of the output variable in all models were similar to those of the models of Section 2. The meaning of the term NM was "closer to the average"; PM, "closer to a high value."

The normalization operators for the input variables of model 1 were described by the following relations:

– for the variable x_1,

$$x_{1,\text{norm}} = \begin{cases} f_{\text{norm},1}(x_1) = (10/3)x_1 - 50 \,|\, 15 \le x_1 \le 18, \\ f_{\text{norm},2}(x_1) = 2.5x_1 - 35 \,|\, 18 < x_1 \le 30, \\ f_{\text{norm},3}(x_1) = (4/3)x_1 \,|\, 30 < x_1 \le 45, \\ f_{\text{norm},4}(x_1) = (6/7)x_1 + (150/7) \,|\, 45 < x_1 \le 80, \\ f_{\text{norm},5}(x_1) = 0.5x_1 + 50 \,|\, 80 < x_1 \le 100; \end{cases}$$

– for the variable x_2,

$$x_{2,\text{norm}} = \begin{cases} f_{\text{norm},1}(x_2) = -2x_2 + 100 \,|\, 0 \le x_2 \le 5, \\ f_{\text{norm},2}(x_2) = -1.2x_2 + 96 \,|\, 5 < x_2 \le 30, \\ f_{\text{norm},3,4,5}(x_2) = -2x_2 + 120 \,|\, 30 < x_2 \le 60; \end{cases}$$

– for the variable x_3,

$$x_{3,\text{norm}} = \begin{cases} f_{\text{norm},1}(x_3) = 2.5x_3 - 35 \mid 14 \le x_3 \le 18, \\ f_{\text{norm},\,2,3}(x_3) = 5x_3 - 80 \mid 18 \le x_3 \le 28, \\ f_{\text{norm},4}(x_3) = \dfrac{30}{7}x_3 - 60 \mid 28 \le x_3 \le 35, \\ f_{\text{norm},5}(x_3) = 2x_3 + 20 \mid 35 \le x_3 \le 40, \\ f_{\text{norm},6,7}(x_3) = -2x_3 + 180 \mid 40 \le x_3 \le 60, \\ f_{\text{norm},8}(x_3) = -4x_3 + 300 \mid 60 \le x_3 \le 65, \\ f_{\text{norm},9}(x_3) = -3x_3 + 235 \mid 65 \le x_3 \le 75, \\ f_{\text{norm},10}(x_3) = -1x_3 + 85 \mid 75 \le x_3 \le 85. \end{cases}$$

The normalization operators for the input variables of model 3 were implemented considering the method described in [2]. Table 3 compares the intervals of the parameter values in absolute and normalized units.

Table 3

Variable normalization for model 3

Variable and its measurement unit	Absolute-value intervals	Normalized-value intervals
x_1 (income, thousand RUB)	$15 \le x_1 \le 18$	[0, 10]
	$18 < x_1 \le 30$	[10, 40]
	$30 < x_1 \le 45$	[40, 60]
	$45 < x_1 \le 80$	[40, 90]
	$80 < x_1 \le 100$	[90, 100]
x_2 (the share of monthly payments on current loans, % of income)	$0 \le x_2 \le 5$	[0, 10]
	$5 < x_2 \le 30$	[10, 40]
	$30 < x_2 \le 40$	[40, 60]
	$40 < x_2 \le 55$	[40, 90]
	$55 < x_2 \le 60$	[90, 100]
x_3 (age, years)	$14 \le x_3 \le 18$	[0, 2.5]
	$18 < x_3 \le 24$	[2.5, 20]
	$24 < x_3 \le 28$	[20, 27]
	$28 < x_3 \le 35$	[27, 46]
	$35 < x_3 \le 45$	[46, 54]
	$45 < x_3 \le 60$	[54, 73]
	$60 < x_3 \le 65$	[73, 80]
	$65 < x_3 \le 75$	[80, 97.5]
	$75 < x_3 \le 85$	[97.5, 100]

Various value combinations of the variables x_1, x_2, and x_3 were supplied to the input of the studied models: 100 different combinations of the absolute values in total. The values were random numbers between the maximum and minimum values of the corresponding parameter. The results yielded by models 1, 2, and 3 are shown in Figs. 7*a*, 7*b*, and 7*c*, respectively.

According to the experiment results, the total absolute error between models 1 and 2 is approximately 0 points;

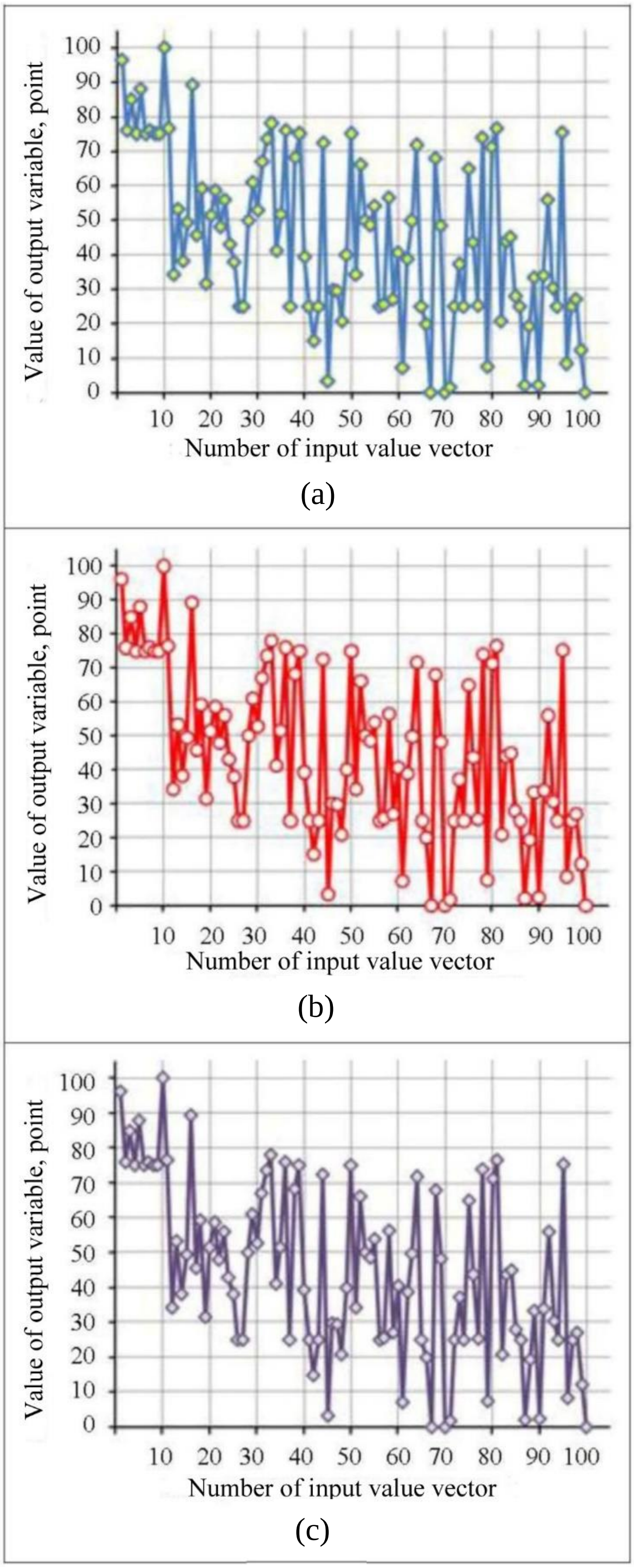

Fig. 7. Client's creditworthiness: experiment results.

between models 2 and 3, approximately 0 points. "Approximately" means that the maximum error value in the experiments did not exceed 10^{-13}. Thus, the models yield almost the same output values. Such an accuracy was achieved due to normalizing the variables of models 1 and 2 similar to the experiment of subsection 2.2. (The results of this experiment are shown in Figs. 4*a*–4*g*.) The experiment carried out confirms the reliability of the proposed normalization method in the formation of model 1. The knowledge bases of models 2 and 3 contain 45 rules each; the knowledge base of model 1, 27 rules. Consequently, with the proposed modifications, model 1 has the same accuracy of calculations under a smaller (by 40%) number of rules in the knowledge base.

Similar to the previous experiment (see Section 2), the knowledge base formation time was compared for the FISs in models 1 and 2. (Model 3 was not considered due to the same rules as in model 2.) By analogy, five knowledge bases were formed for the FISs of models 1 and 2. As a result of the comparison, it was found that the knowledge base formation time of model 1 is by approximately 47% smaller than that of model 2. This difference can be explained as follows. The knowledge base of model 2 contains more rules compared to model 1 (45 vs. 27). Also, all input variables in model 2 have different types of correlation with the output variable.

4. DISCUSSION OF THE RESULTS

The proposed approach for transforming the values of input variables can be implemented using software methods: it reduces to constructing an array of straight-line equations with two given points. According to the experimental evidence of this paper, if the variable under normalization has an optimum of the influence on the final result, the number of production rules in the FIS knowledge base will be reduced. As is known, the formation of production rules consumes much effort, often manually performed by an expert. Thus, the proposed modifications allow decreasing the labor costs of expert groups when forming knowledge bases. Here, an additional difficulty for experts is caused by different properties of the input variables due to the variety of ranges, measurement units, and influences on the value of the output variable. Under such a variety, an expert has to remember the specifics of each variable, analyzing the input variables more carefully during rules formation. According to the paper [19], the increased concentration of person's attention leads to rapid fatiguability, negatively affecting the number of mistakes allowed. Eliminating mistakes is time-consuming. With the proposed modifications, the normalized variables have a single range and the same correlation with the output variable. Such homogeneous properties of the input variables simplify the process of forming production rules.

Despite the introduction of additional mathematical operations, the experiments have not revealed a significant decrease in the computing performance of the hardware means of the FISs under consideration. This is due to the large computing power reserve of modern computers. The proposed modifications have a wide application area and are intended to implement expert systems for the integral assessment of a complex object. Therefore, the increasing computational cost can be considered insignificant compared to the resulting simplification of the knowledge base formation process. As is known, this process is mainly performed directly by experts. In the experiments of Sections 2 and 3, less time was spent on FIS knowledge base formation using the proposed modifications (on average, by approximately 37% and 47%, respectively) than on FIS knowledge base formation without normalization.

As follows from the monograph [1], the curse of dimensionality is a problem of FIS construction. More specifically, the number of rules in the knowledge base strongly depends on the number of variables n_{var} and the number of terms n_{term} describing each variable. For example, if the same number of terms is used to describe all input variables, then the number of MISO rules is given by $n_{rule} = n_{term}^{n_{var}}$. The negative influence of the curse of dimensionality can be decreased by developing modifications that will reduce the number of rules in the knowledge base while maintaining the accuracy of FISs. According to the experimental evidence of this paper, the proposed modifications allow reducing the number of rules in the knowledge base depending on the experiment conditions: by 40% in the experiment on the borrower's creditworthiness (Section 3) and in experiments 5 and 6 (Section 2); by 64% in experiment 3 (Section 2).

CONCLUSIONS

This paper has proposed a method for normalizing the input variables of fuzzy inference systems. The method transforms the absolute values of input variables to a single range of values in normalized units.

Note that the minimum value of the normalized variable has the worst influence on the output parameter, whereas the maximum value of the normalized variable has the best influence on it. The method divides the variable's range into a sequence of intervals. Then a pattern is formed for each interval to transform the absolute values of the parameter to the normalized ones. The normalization operator on a given interval is implemented by constructing a straight line with two given points. According to the modeling results, under the specified restrictions, the FIS models with the proposed normalization method are adequate to the FIS models without normalization of the input variables. The proposed normalization method allows reducing the number of rules in the knowledge base depending on the experiment conditions: by 40% in the experiment on the borrower's creditworthiness (Section 3) and in experiments 5 and 6 (Section 2); by 64% in experiment 3 (Section 2). A common property of fuzzy inference systems in these experiments has been the presence of input variables with an optimum of the influence on the value of the output variable. In the experiments presented above, less time has been spent on FIS knowledge base formation using the proposed modifications (on average, by approximately 37% and 47%, respectively) than on FIS knowledge base formation without normalization.

The proposed modifications open up opportunities for developing information processing methods for decision support systems of various purposes.

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