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INTER-ORBITAL SPACECRAFT TRANSFER: TRAJECTORY DESIGN BY ITERATING PARAMETER VALUES WITHIN A DATA GRID

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Abstract. This paper considers the problem of designing an optimal inter-orbital spacecraft transfer. We present a computational algorithm and modeling results of the optimal transfer trajectory between near-Earth elliptical orbits for a spacecraft with a chemical booster and fixed thrust. The trajectory design procedure includes four stages as follows: a) formation of the primary ranges of initial approximations for typical optimization problems; b) iterative integration to find the domains of convergence for a typical variational problem; c) determination of the optimum for each problem statement within the accepted ranges and its implementation by calculating the final conditions residuals; d) analysis of the results obtained. We use numerical methods of mathematical analysis and mathematical programming. The risk of "overstepping" the potentially optimal result is minimized by varying the accuracy at different stages of calculations. Based on the results, we improve the primary solution of the reference problem statement, identify the domains of convergence of solutions, and obtain the sets of initial approximation vectors ensuring convergence in the considered problems for further analysis. The results of this study can be used to develop further and refine the algorithm for selecting optimal initial approximations for different optimization problems (including spacecraft trajectory optimization as a typical one).

Keywords: optimal control, spacecraft trajectory optimization, maximum principle, mathematical modeling, nonlinear programming.

INTRODUCTION

Currently, there exist many methods for solving optimal control problems. However, Pontryagin's maximum principle [1] is one of the most widespread approaches to dynamic optimization. This method yields optimality conditions, including the cases when optimal control is on the admissible domain boundary. Also, it allows deriving all necessary conditions for the variational calculus problem, reducing the original problem to a boundary value problem of differential equations [1]. Like other methods [2], the maximum principle requires an initial approximation for the parameter values, and the correct choice of initial approximations provides faster convergence and successful determination of an optimum. At the same time, the choice problem is connected with the branching of optimal solutions and the high sensitivity of the residuals of the boundary value problem to its parameter variations [3]. Moreover, despite the possibility to create formal estimation algorithms under a given initial approximation, one should follow intuition and a priori knowledge to select good approximations [2, 4].

Modern literature offers some practical methods facilitating the choice of initial approximations, such as the homotopy of maximum thrust [5], edge normalization [6], edge estimation by a reference trajectory [7], and others. In special cases, some of the developed methods turn out useful. For example, we mention approximate rephrasing solutions, which were developed to obtain initial approximations for indirect methods [7–10]. Relevant studies focus on particular problems; solving some of them gives new (auxiliary) methods for finding initial approximations for relevant problems. Nevertheless, most of the modern approaches still involve the trial-and-error method, often more effective than subtle counterparts. It consists in choosing initial approximations based on a priori knowledge and intuition [2–4, 8].

Thus, more and more efforts of the global research community are applied to find algorithms that will be effective in choosing correct initial approximations for optimal control problems; for example, see [2, 4–6, 8].

This paper implements the first stage of research, intended to identify the relationships between the vector components ensuring convergence for typical inter-orbital spacecraft transfer optimization problems. The first part of the study identifies the domains of convergence for typical optimization problems within accepted ranges. It reduces to forming initial approximation vectors ensuring the best solutions within a data grid. Expectedly, this study will contribute to refining initial approximation choice algorithms for typical problems.

1. PROBLEM STATEMENT

This paper considers the problem of designing an optimal inter-orbital transfer trajectory for a spacecraft.

1.1. The general formulation and parameters of the optimization problem

We consider a spacecraft on a given initial near-Earth orbit. The spacecraft includes a chemical booster with some known characteristics. This booster must transfer the spacecraft to a working near-Earth orbit with specified characteristics.

It is required to find a rational transfer scheme between the orbits. As an optimality criterion, we choose the spacecraft mass inserted into the working orbit: the mass is maximized. The transfer time is not limited.

We fix the following parameters and conditions for all cases under consideration: the spacecraft mass on the initial orbit is 5000 kg; the thrust of the unregulated rocket engine of the booster is 5 kN; the number of engine ignitions is arbitrary; the specific impulse is 330 s; the spacecraft transfer scheme is limited to one revolution; the orbits belong to the same plane; the apsidal lines of the orbits coincide; the gravity field is Newtonian.

We vary the following parameters: the perigee altitude of the initial orbit and its apogee altitude; the perigee altitude of the final orbit and its apogee altitude.

The transfer scheme characteristics are as follows:

– the start point of the spacecraft on the initial orbit,

– the number of active and passive sections on the transfer trajectory,

– the duration of active and passive sections of the trajectory and their location on the transfer trajectory (in other words, the time instants of engine ignition and cutoff),

– the pitch angle program on each active section,

– the end point on the final orbit.

1.2. Spacecraft transfer model

The mathematical model of the spacecraft motion includes the vector of its phase coordinates with the following components: the radial velocity V_r, the normal velocity V_n, the radius r, and the polar angle β. Recall that the sequence of active and passive sections is not fixed, and we design an optimal law of engine ignition and cutoff. Therefore, it is reasonable to add the spacecraft mass m to the listed variables, and the resulting vector of the phase variables (further called the phase vector) takes the form

$$z = \begin{pmatrix} V_r \\ V_n \\ r \\ \beta \\ m \end{pmatrix}. \tag{1}$$

The spacecraft motion is described by the system of differential equations

$$\begin{aligned} \dot{V}_r &= \frac{P\sin(\vartheta)}{m}\delta - \frac{\mu}{r^2} + \frac{V_n^{\,2}}{r}, \\ \dot{V}_n &= \frac{P\cos(\vartheta)}{m}\delta - \frac{V_r V_n}{r}, \\ \dot{r} &= V_r, \\ \dot{\beta} &= \frac{V_n}{r}, \\ \dot{m} &= -q\delta. \end{aligned} \tag{2}$$

The notations are as follows:

P is the engine thrust (unregulated, a known value);

ϑ is the true anomaly;

μ is the Earth's gravitational parameter;

q is the mass flow rate of the engine (a known value);

δ is the thrust function taking only two values: δ = 1 (ignition) and δ = 0 (cutoff);

φ is the pitch angle of the spacecraft (the angle between the thrust vector and the local horizon);

δ(t) and φ(t) are the control functions to be optimized.

In the first stage of the analysis, we fix the motion conditions in the initial orbit perigee as the initial conditions:

$$\begin{aligned} V_r(t_0) &= 0, \\ V_n(t_0) &= \sqrt{\frac{\mu}{p_0}}(1+e_0), \\ r(t_0) &= \frac{p_0}{1+e_0}, \\ \beta(t_0) &= 0, \\ m(t_0) &= m_0. \end{aligned} \tag{3}$$

Here, t_0 is the start time, which can be set to 0 (the time is counted from the start), and p_0 and e_0 are the focal parameter and the eccentricity of the initial orbit, respectively. The subscript "f" in the relations below denotes belonging to the final motion conditions. We count the angular range (the polar angle) from the apsidal line of the initial orbit.

The spacecraft transport problem arising in the first stage is as follows: for the set of initial conditions (3), find the control functions $\delta(t)$ and $\varphi(t)$ and the transfer time t_f under which the spacecraft will reach the phase space point

$$V_r(t_f) = 0,$$

$$V_n(t_f) = \sqrt{\frac{\mu}{p_f}(1 - e_f)},$$

$$r(t_f) = \frac{p_f}{1 - e_f},$$

$$\beta(t_f) = \pi$$

with the minimum fuel consumption, $m(t_f) \to \max$.

1.3. The mathematical optimization problem

We introduce an auxiliary function (Hamiltonian). It can be treated as the scalar product of two vectors: the right-hand sides of the motion equations and the conjugate variables. The vector of conjugate variables, further called the conjugate vector, has the same dimension as the phase vector; each component of the conjugate vector corresponds to some phase variable. In other words, the dimension of the phase vector is 5; see (1).

The conjugate vector has the form

$$\lambda = \begin{pmatrix} \lambda V_r \\ \lambda V_n \\ \lambda r \\ \lambda\beta \\ \lambda m \end{pmatrix}.$$

The Hamiltonian is given by

$$H = \lambda V_r \left(\frac{P \sin(\vartheta)}{m}\delta - \frac{\mu}{r^2} + \frac{V_n^2}{r} \right)$$
$$+ \lambda V_n \left(\frac{P \cos(\vartheta)}{m}\delta - \frac{V_r V_n}{r} \right) + \lambda r V_r + \lambda\beta \frac{V_n}{r} - \lambda m q \delta.$$

According to the maximum principle, the chosen control law maximizes the Hamiltonian, i.e., the optimal control functions ($\delta(t)$ and $\varphi(t)$) can be found from the maximum conditions for the Hamiltonian.

In addition, it is possible to show that

$$\cos(\vartheta_{\text{opt}}) = \frac{\lambda V_n}{\lambda V},\ \sin(\vartheta_{\text{opt}}) = \frac{\lambda V_r}{\lambda V},$$

where

$$\lambda V = \sqrt{\lambda V_r^{\,2} + \lambda V_n^{\,2}},$$

$$\delta_{\text{opt}} = \begin{cases} 1 \text{ if } \Psi > 0 \\ 0 \text{ if } \Psi < 0, \end{cases}$$

$$\Psi = \frac{P}{m}\lambda V - \lambda m q \quad \text{or} \quad \Psi = \frac{W}{m}\lambda V - \lambda m.$$

Here, Ψ denotes the engine switching function and W is the exhaust velocity. The subscript "opt" means that the corresponding relations are derived by maximizing the Hamiltonian.

The obtained pitch angle program is as follows:

$$\text{tangag} = \left\{ \begin{array}{ll} \arccos\left(\dfrac{\lambda V_n}{\sqrt{(\lambda V_n)^2 + (\lambda V_r)^2}} \right) & \text{if } \lambda V_r > 0 \\ -\arccos\left(\dfrac{\lambda V_n}{\sqrt{(\lambda V_n)^2 + (\lambda V_r)^2}} \right) & \text{if } \lambda V_r \le 0 \end{array} \right\}.$$

Considering the optimal control laws (the pitch angle program and the optimal thrust function), the equations of the phase variables on the optimal trajectory take the form

$$\begin{aligned} \dot{V}_r &= \frac{P\lambda V_r}{m\lambda V}\delta_{\text{opt}} - \frac{\mu}{r^2} + \frac{V_n^2}{r}, \\ \dot{V}_n &= \frac{P\lambda V_n}{m\lambda V}\delta_{\text{opt}} - \frac{V_r V_n}{r}, \\ \dot{r} &= V_r, \\ \dot{\beta} &= \frac{V_n}{r}, \\ \dot{m} &= -q\delta_{\text{opt}}. \end{aligned} \tag{4}$$

Due to the maximum principle, the conjugate variables satisfy the system of differential equations

$$H = \lambda V_r \left(\frac{P \sin(\vartheta)}{m}\delta - \frac{\mu}{r^2} + \frac{V_n^2}{r} \right)$$
$$+ \lambda V_n \left(\frac{P \cos(\vartheta)}{m}\delta - \frac{V_r V_n}{r} \right) + \lambda r V_r + \lambda\beta \frac{V_n}{r} - \lambda m q \delta,$$
$$\frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial z_i}.$$

Consequently,

$$\frac{d\lambda V_r}{dt} = -\frac{\partial H}{\partial V_r} = \lambda V_n \frac{V_n}{r} - \lambda r,$$

$$\frac{d\lambda V_n}{dt} = -\frac{\partial H}{\partial V_n} = -\lambda V_r \frac{2V_n}{r} + \lambda V_n \frac{V_r}{r} - \lambda\beta \frac{1}{r},$$

$$\frac{d\lambda r}{dt} = -\frac{\partial H}{\partial r} = \lambda V_r \left(-\frac{2\mu}{r^3} + \frac{V_n^2}{r^2} \right) \quad (5)$$

$$+\lambda V_n \left(-\frac{V_r V_n}{r^2} \right) + \lambda\beta \frac{V_n}{r^2},$$

$$\frac{d\lambda\beta}{dt} = -\frac{\partial H}{\partial \beta} = 0.$$

The corresponding boundary value problem of the maximum principle is as follows: find values of the conjugate vector components at the start point, $\lambda_{Vr}(t_0)$, $\lambda_{Vn}(t_0)$, $\lambda_r(t_0)$, $\lambda_\beta(t_0)$, and $\lambda_m(t_0)$, and the transfer time t_f (six unknowns in total) such that

$$H(t_0) = 0,$$

$$V_r(t_f) = 0,$$

$$V_n(t_f) = \sqrt{\frac{\mu}{p_f}} (1 - e_f),$$

$$r(t_f) = \frac{p_f}{1 - e_f},$$

$$\beta(t_f) = \pi,$$

$$\lambda m(t_f) = 1.$$

In the second stage of the analysis, the start and end points of the spacecraft transfer trajectory are floating. To implement this requirement, we introduce the transversality conditions.

1.4. The transversality conditions at the start and end points of the transfer trajectory

The transversality condition expresses the perpendicularity of the conjugate vector to all tangent vectors of the boundary manifold.

If the phase variables at a boundary point (first, the start point) are a function of some chosen parameter (in the case under consideration, the true anomaly of the initial orbital point, υ_0), the tangent vector of the initial manifold has the components

$$\left[\frac{d}{d\upsilon_0}(V_{r0}), \frac{d}{d\upsilon_0}(V_{n0}), \frac{d}{d\upsilon_0}(r_0), \frac{d}{d\upsilon_0}(\beta_0), \frac{d}{d\upsilon_0}(m_0) \right].$$

Calculating the derivatives, we obtain the vector

$$\left[\frac{e_0 \cos(\upsilon_0)}{\sqrt{p_0}}, \frac{-e_0 \sin(\upsilon_0)}{\sqrt{p_0}}, \frac{p_0 e_0 \sin(\upsilon_0)}{(1 + e_0 \cos(\upsilon_0))^2}, 1, 0 \right].$$

The optimality conditions for the start point on the initial orbit are given by the perpendicularity of the conjugate vector and this tangent vector. The perpendicularity condition can be written as

$$\lambda V_r(t_0) \frac{e_0 \cos(\upsilon_0)}{\sqrt{p_0}} - \lambda V_n(t_0) \frac{e_0 sin(\upsilon_0)}{\sqrt{p_0}}$$

$$+ \lambda r(t_0) \frac{p_0 e_0 sin(\upsilon_0)}{(1 + e_0 \cos(\upsilon_0))^2}$$

$$+\lambda\beta(t_0) \cdot 1 + \lambda m(t_0) \cdot 0 = 0.$$

Hence,

$$\lambda\beta(t_0) = -\lambda V_r(t_0) \frac{e_0 \cos(\upsilon_0)}{\sqrt{p_0}}$$

$$+\lambda V_n(t_0) \frac{e_0 \sin(\upsilon_0)}{\sqrt{p_0}} - \lambda r(t_0) \frac{p_0 e_0 \sin(\upsilon_0)}{(1 + e_0 \cos(\upsilon_0))^2}.$$

Similarly, the optimality condition for the transfer end point (the optimality of the final angular distance) has the form

$$\lambda\beta(t_f) = -\lambda V_r(t_f) \frac{e_f \cos(\beta_f)}{\sqrt{p_f}}$$

$$+\lambda V_n(t_f) \frac{e_f \sin(\beta_f)}{\sqrt{p_f}} - \lambda r(t_f) \frac{p_f e_f \sin(\beta_f)}{(1 + e_f \cos(\beta_f))^2}.$$

2. A DATA GRID TO FIND INITIAL APPROXIMATIONS

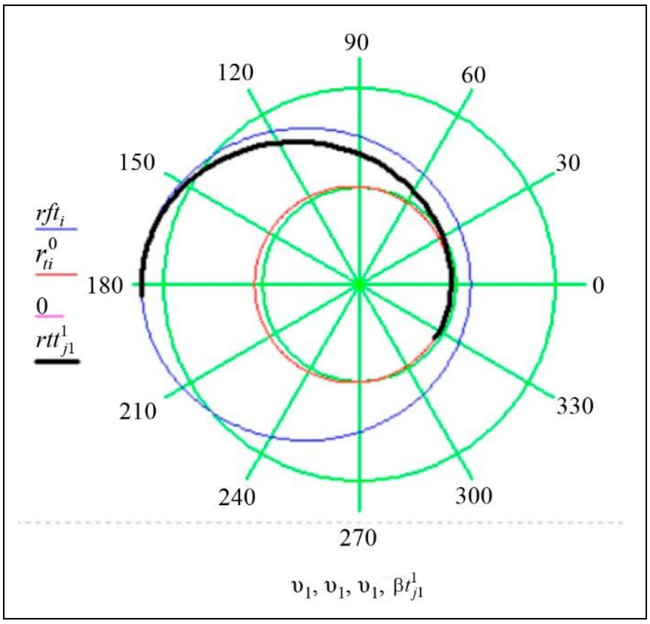
The algorithm for further study (the trajectory design procedure) consists of four stages:

- calculation of optimal intra-orbital transfer trajectories with different parameters for the spacecraft with a chemical booster and a fixed thrust to identify and refine the ranges of initial approximations for typical problems. It produces a data grid for an exhaustive search over the initial values vector;
- iterative integration to find the domains of convergence and reach an optimum of the variational problem;
- determination of the optimum for each problem statement within the accepted ranges and its implementation by calculating the final conditions residuals.
- analysis of the results obtained.

For the primary choice of the range of initial approximations, we solve an optimization problem with the following orbit data: 500-km perigee altitude and 1500-km apogee altitude (the initial orbit), and 2000-km perigee altitude and 10 000-km apogee altitude (the final orbit). For these data, the Bard method [2] yields the following initial approximations (with a final spacecraft mass of 3203.788 kg):

$$\begin{pmatrix} \lambda m(t_0) \\ \lambda V_r(t_0) \\ \lambda V_n(t_0) \\ \lambda r(t_0) \\ tf \\ \upsilon_o \\ \beta_f \end{pmatrix} = \begin{pmatrix} 0.6 \\ -0.065851324495908 \\ 1.3618066619686777 \\ 1.4193211140743078 \\ 6.803580030230557 \\ -0.6054269338992805 \\ 3.14159265358979 \end{pmatrix}.$$

The aircraft transfer scheme after calculating the residuals is presented in the figure below. The notations are as follows: rtt_{j1}^1 is the transfer trajectory obtained with integrating by the Runge–Kutta method of the fourth order with a variable step; βt_{j1}^1 is the change in the polar angle βt^1 on this trajectory.



The optimal transfer scheme (choice of initial values).

This solution is taken as a reference to form the range for each of the unknown variables. To determine such a range, we add the values for the left and right boundaries at the initial time instant; they are calculated so that, according to the generated initial data vectors, the sizes of the domains of convergence will be sufficient to identify the relationships between the parameter values and the degree of convergence of the results. Note that the ranges should be not very large to reduce the risk of "overstepping" the possible solution during integration (to avoid critical steps).

Thus, we choose the following ranges to find good approximations for the unknown variables at the initial time instant:

$$\lambda V_{r0} = -0.079, ..., -0.058,$$

$$\lambda V_{n0} = 1.291, ..., 1.431,$$

$$\lambda r_0 = 1.361, ..., 1.491,$$

$$\lambda m_0 = 0.3, ..., 0.7,$$

$$\upsilon_0 = -0.61, ..., -0.60405.$$

Over 4 145 000 variations of the initial approximation vectors were checked for each of the five problems in the selected range to identify the relationships between the parameter values and convergence.

The data grid (see Table 1) was loaded into Python.

3. THE ITERATIVE ALGORITHM FOR SOLVING THE SET OF TYPICAL OPTIMIZATION PROBLEMS

3.1. The iterative algorithm for solving the optimization problem

The algorithm was implemented in MathCad in two stages as follows.

The first stage is to find the preliminary domains of convergence areas by iterating parameter values (performing an exhaustive search) within the data grid. Note that the solution accuracy is set to 10^{-3}. This moderate value reduces the risk of "overstepping" the potential solution and allows avoiding the multiply increasing number of vectors to be checked (in the case of four and more decimal places). In the second stage, we return to the required accuracy of 10^{-14} and refine the preliminary solution by calculating the residuals.

These stages are discussed in detail below.

3.1.1. The first stage (preliminary solution)

The first step of the algorithm is to enter the data and reduce them to a common dimensionless form. We select five sets of input parameters to demonstrate the algorithm:

1. 400-km perigee altitude and 1400-km apogee altitude (the initial orbit), and 1900-km perigee altitude and 9900-km apogee altitude (the final orbit).

2. 400-km perigee altitude and 1400-km apogee altitude (the initial orbit), and 2000-km perigee altitude and 10 000-km apogee altitude (the final orbit).

Table 1

The data grid for the exhaustive search when integrating by the Runge–Kutta method of the fourth order

V_r	V_n	R	λV_r	λV_n	λr	υ_0	λm
–0.0394875	1.05887075	0.942166729	–0.079	1.291	1.361	–0.61	0.3
–0.0394875	1.05887075	0.942166729	–0.079	1.291	1.361	–0.61	0.4
–0.0394875	1.05887075	0.942166729	–0.079	1.291	1.361	–0.61	0.5
–0.0394875	1.05887075	0.942166729	–0.079	1.291	1.361	–0.61	0.6
–0.0394875	1.05887075	0.942166729	–0.079	1.291	1.361	–0.61	0.7
–0.0394677	1.05888457	0.942154435	–0.079	1.291	1.361	–0.6097	0.3
–0.0394677	1.05888457	0.942154435	–0.079	1.291	1.361	–0.6097	0.4
–0.0394677	1.05888457	0.942154435	–0.079	1.291	1.361	–0.6097	0.5
–0.0394677	1.05888457	0.942154435	–0.079	1.291	1.361	–0.6097	0.6
–0.0394677	1.05888457	0.942154435	–0.079	1.291	1.361	–0.6097	0.7
–0.0394479	1.05889838	0.942142147	–0.079	1.291	1.361	–0.6093	0.3
–0.0394479	1.05889838	0.942142147	–0.079	1.291	1.361	–0.6093	0.4
–0.0394479	1.05889838	0.942142147	–0.079	1.291	1.361	–0.6093	0.5
–0.0394479	1.05889838	0.942142147	–0.079	1.291	1.361	–0.6093	0.6
–0.0394479	1.05889838	0.942142147	–0.079	1.291	1.361	–0.6093	0.7

3. 400-km perigee altitude and 1400-km apogee altitude (the initial orbit), and 2100-km perigee altitude and 10 100-km apogee altitude (the final orbit).

4. 500-km perigee altitude and 1500-km apogee altitude (the initial orbit), and 2000-km perigee altitude and 10 000-km apogee altitude (the final orbit).

5. 600-km perigee altitude and 1600-km apogee altitude (the initial orbit), and 2000-km perigee altitude and 10 000-km apogee altitude (the final orbit).

All varying characteristics are entered iteratively in a loop with a counter from 0 to 4 (five problem statements). Due to a very significant load on the computational system (exhaustive search within the data grid and iterative calculations), we divided the program by stages into the following subprograms:

- a program with preliminary calculations, which outputs the results in a separate Excel file;
- a program with basic calculations in the loop;
- a program that visualizes the results.

The input data for the problem include:

- the Earth's gravitational parameter (398 600 $\frac{\text{km}^3}{\text{s}^2}$) and the Earth's radius (6371 km);
- the initial spacecraft mass (5000 kg), the thrust of the chemical rocket engine (5000 N), and its specific impulse (330 s·g);
- the perigee and apogee altitudes of the initial and final orbits as well as the angle between their apsidal lines;
- the elements of the initial and final orbits (the perigee radius rp and the apogee radius ra, the semi-major axis A, the energy constant h, the eccentricity e, and the focal parameter p);
- the orbital equation, i.e., the length of the spacecraft radius vector as a function of the true anomaly:

$$r = \frac{p}{1 + e \cdot \cos(\vartheta)}.$$

Next, the entered dimensional quantities are converted to dimensionless form. Dimensionless quantities reduce the CPU load by replacing thousands of kilometers with the normalized values. Also, the number of input arguments is reduced.

The mathematical model is the equations of the planar motion of the spacecraft. The orbital coordinate system is used to analyze the spacecraft velocity (the radial V_r and transversal V_n components). The spacecraft position is considered in the polar coordinate system: the principal axis x is directed along the radius vector of the apsidal point of the initial orbit. In this case, r is the length of the radius vector and b is the polar angle.

The angle θ in the equations is the pitch angle of the spacecraft measured from the local horizon line. All variables in the system of differential equations under consideration are dimensionless.

See Section 1, system (2), for the initial mathematical model. The final model based on the maximum principle is represented by the system of ten first-order ordinary differential equations (4) and (5).

Integration involves the Rkadapt tool of MathCad. The initial approximations for Rkadapt are found by exhaustive search within the data grid; see Section 2. The data grid is presented in Table 1. In this stage of calculations, the solution accuracy is set to 10^{-3}.

In the first stage, the preliminary check of the solutions (insertion into the final orbit) is visual by the graphs of the results since the final conditions are not considered and the required accuracy is not observed. The algorithm fixes all sets of vectors without convergence and distributes all solutions with convergence into three groups for further analysis:

– insertion into the final orbit (for each case),

– insertion into an orbit above the final one,

– insertion into an orbit below the final one.

Note that the final spacecraft mass on the orbit (the transfer optimality criterion) is calculated but not considered due to insufficient accuracy.

Thus, the main result of the first stage is the set of initial value vectors ensuring insertion into the final orbit (for each specific case) and the sets of initial value vectors for further analysis.

3.1.2. The second stage (optimal solution)

In the second stage, the algorithm returns to the required accuracy 10^{-14} and operates the sets of candidate vectors (the ones ensuring insertion into the final orbit).

To find the exact solution and then the optimum, we developed a program calculating the final conditions residuals at a floating point of inserting into the final orbit. The program requires the transversality conditions derived in Section 2.

The program outputs the vector of residuals, which is used to find an exact solution: equating their values to 0 allows obtaining the factual initial variables, the final transfer time, and the polar angle characterizing the transfer end point.

In this stage, the spacecraft mass at the transfer end point is analyzed and the optimum (the solution of the problem) is identified. In addition, the effect of start point variations within the data grid on the final spacecraft mass (on the final orbit) is assessed.

The result obtained within the data grid is the vectors ensuring the optimal transfer between the orbits. For problem statements 1–5, they are combined in Table 2. The notations are as follows: T_F is the transfer time; β_F is the polar angle characterizing the transfer end point; m_F is the spacecraft mass on the final orbit. Here, the subscript “1” indicates the parameters at the initial time instant with the best result by the final aircraft mass criterion.

Thus, the implemented approach improved the result for the “reference” problem (statement 4) by 365 g compared to the Bard method [2].

3.2. Analysis and discussion of the results

Five typical problems were calculated in a loop. The domains of convergence were identified and the best solutions satisfying the optimality condition were obtained within the grid (the ones with the maximum aircraft mass on the final orbit). The risk of “overstepping” the potentially optimal result was reduced using variable accuracy in different stages of the calculations.

The results—the vectors of the desired variables for each case—were written in Excel tables using MathCad. They will be estimated and analyzed by statistical methods in a Python program.

With the proposed approach to calculations and accuracy, the initial result for the “reference” problem was improved by 365 g without leaving the ranges for each value.

Table 2

The optimal transfer characteristics in problem statements 1–5 (analysis within the data grid)

Characteristics	Statement 1	Statement 2	Statement 3	Statement 4	Statement 5
ϑ_0^1	–0.608528181266	–0.687262955509	–0.58694862445	–0.58063271537992	–0.67460422765
λV_r^1	–0.065265825615	–0.075257944004	–0.06564653582	–0.06466045327036	–0.06339073139
λV_n^1	1.3623340777782	1.3653869345017	1.364158466294	1.36474044140864	1.357648817782
λr^1	1.4200011568480	1.4207257561587	1.421069918027	1.42243156356652	1.40764863211
T_F	6.7321648922778	7.0228415355241	12.30599404742	12.48386539989280	5.855669428846
β_F	3.1311300732552	3.166796288751	5.127686975088	5.33807657900554	2.859452408788
m_F	3227.339 kg	3182.552 kg	3181.12 kg	3204.153 kg	3247.748 kg

Thus, this paper demonstrates the effectiveness of exhaustive search within the data grid when refining and improving the initial result in optimal inter-orbital spacecraft transfer problems. This paper is the first (preliminary) stage of research aimed at identifying the mathematical relationships between the vector components ensuring convergence for typical problems. Calculations were carried out in parallelized programs in MathCad 15 and Python 3.9 mainly based on a Core i5 1035G1 processor. The total running time of the programs was 8 hours.

CONCLUSIONS

In this paper, we have studied and extended the capabilities of mathematical programming with application to typical optimization problems (optimization of the spacecraft transfer trajectory between near-Earth elliptical orbits). In addition, we have demonstrated an effective approach to finding optimal initial approximations for the variational problem of inter-orbital spacecraft transfer optimization with the minimum mass flow criterion.

The results are as follows:

- Over 4 145 000 variations of the initial approximations were checked for each problem in MathCad and Python programs to identify relationships between the parameter values and convergence.
- The primary solution of the reference variational problem of inter-orbital spacecraft transfer optimization was improved within the considered ranges.
- Five typical problems were iteratively solved in a loop in MathCad and their parameter sets were examined.

The results of this study can be used to develop further and refine an algorithm for selecting optimal initial approximations for different optimization problems (including spacecraft trajectory optimization as a typical one). Expectedly, they will simplify the solution of such problems and will contribute to the refinement and development of the corresponding mathematical apparatus.

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