A UNIFIED DETECTION PROBABILITY FIELD FOR A GROUP OF STATIONARY OBSERVERS

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Abstract. This paper addresses the problem of calculating a unified detection probability field for a group of stationary observers monitoring a given area and operating in active mode. Stationary observers are located on a plane equipped with the Cartesian coordinate system, and their coordinates are known. For an object with given coordinates, a unified detection probability field is calculated for all stationary observers in the area. As shown, the greatest complexity arises for the objects necessitating the consideration of the terrain relief. For such objects, it is required to calculate a multilayer map with the altitude of the object's movement, in contrast to a single-layer (flat) detection probability map, which is sufficient for detecting objects without taking the terrain relief into account. Examples are provided to demonstrate that the probability of object detection depends on the location and altitude of the observers, the altitude of the object's movement, and the terrain relief. With a unified detection probability field calculated for a given area in the form of a multilayer map, it is much easier to optimize the location of observers (on the one hand) and control moving objects in a conflict environment (on the other).

Keywords: probability of object detection, detection probability field for an object, trajectory of an object, terrain relief.

INTRODUCTION

When detecting an object in active mode, the laws of radio wave propagation in space are used. The main laws are as follows: the constancy of the signal propagation speed; the rectilinear nature of signal propagation; the directionality of signal radiation and reception, which is based on the phenomenon of radio wave interference; and the Doppler effect. In this case, the probing and reflected signals propagate along a rectilinear trajectory without distorting their shape [1]. Considering the curvature of the Earth's surface, for an object at an altitude of 50 m, the line-of-sight range of an observer located on a perfectly flat surface is approximately 39 km and will increase with the object's altitude. The detection range of an object moving at an altitude of 100 m is approximately 52–55 km. A moving object is successfully detected only within the line of sight. For objects whose detection is not affected by the terrain relief, it is possible to create a so-called continuous detection field. This is much more difficult to do when taking the terrain relief into account. Note that in the USSR, a continuous detection field was never created, even in the European part of the country. If an object moves at very low altitudes (VLA), from several tens to hundreds of meters [2], then terrain, forests, buildings, and structures create so-called clutter notches. Today, 99% of objects move at altitudes of 200–300 m, and even higher above rugged terrain, since flight altitude is measured either by satellite global positioning systems or by barometric altimeters. The adoption of laser altimeters will reduce this altitude to 30–50 m, making them even more difficult to detect.

An observer processes the signals reflected from an object in three stages [1]:

- *Primary processing* includes operations for detecting and measuring (estimating) the parameters of received signals. Primary processing is performed directly by the observer. The set of signal parameter estimates forms an object's mark.
- *Secondary processing* is performed on the set of marks and provides trajectory information.

• *Tertiary processing* is intended to combine and identify information from individual observers within the system or information from individual systems.

This paper addresses the problem of object detection considering the terrain relief in an area monitored by several stationary observers (i.e., primary and tertiary processing).

The region of potential object detection depends on the terrain relief, i.e., it has a contrasting structure, with alternating line-of-sight (LOS) regions (object in direct visibility) and shadow regions (object outside direct visibility). The contrasting structure of the observation region can be described by any indicators: either as the intensity of the useful signal or as the probability of detecting the useful signal, as long as they represent a correct convolution of all the main influencing factors and are available for estimation. This possibility is directly characterized by a **probabilistic criterion**, i.e., the probability of an event that, during the object's movement along the route, it will be detected by at least one of $L \geq 1$ observers located in the area. Let this probability of object detection be denoted by P_{det}. In the case of independent observers, the probability of object detection by at least one observer is estimated using the integral (cumulative) probability formula [3]:

$$P_{det} = 1 - \prod_{i=1}^{L} (1 - P_{det\,i}), \quad (1)$$

where L is the number of observers, and $P_{det\,i}$ is the probability of detection by the ith observer. This task belongs to tertiary processing. With known coordinates of the observers, one can use formula (1) to calculate the integral probability $P_{det}(x, y)$ for all points in the area monitored by these observers.

In [4], stationary hydroacoustic observation devices operating in passive mode, with known coordinates, were considered on a plane equipped with the Cartesian coordinate system. A unified detection probability field of an object, $P_{det}(x, y)$, was calculated for the above devices over the area, taking the anisotropy of the hydroacoustic field into account.

This paper deals with stationary observers operating in active mode, with known coordinates, on a plane equipped with the Cartesian coordinate system XOY. A unified detection probability field of an object, $P_{det}(x, y)$, is calculated for all observers over the area depending on the altitude of the observers, the altitude of the object, and the terrain relief.

1. BASIC RELATIONSHIPS

Consider the maximum detection range [5]

$$R_{\max} = \sqrt[4]{\frac{E_{tra} G A \delta_c}{E_{rec\,\max} (4\pi)^2}},$$

where E_{tra} is the transmitted power; $E_{rec\,\max}$ is the power of the signal received by the observer; G is the directivity of the transmitting antenna; δ_c is the radar cross section (RCS) of the object; A is the effective aperture of the receiving antenna; finally, $R_{\max}$ is the maximum detection range, i.e., for a distance $R \leq R_{\max}$, the target will be detected with a probability of correct detection (P_{det}) not smaller than the permissible one for the given false alarm probability P_{fal} and minimum permissible detection probability $P_{det\,\min}$. This definition generally refers to a single (instantaneous) observation under established dependencies between the observation time T_0, the signal-to-noise ratio ρ, P_{det}, and P_{fal}.

The parameters E_{tra}, G, and A are associated with the observer and, therefore, do not depend on R. The parameter δ_c is also independent of the distance to the object R [1]. The energy of the received signal, like its power, is inversely proportional to the fourth power of the distance to the object [5]. Thus, the power of the signal received by the observer can be written as

$$E_{rec} = \beta / R^4, \quad (2)$$

where β combines all the parameters independent of R.

The criterion for the possibility of detecting a signal against a noisy background is the signal-to-noise ratio ρ, defined as [6]

$$\rho = 2\int_{\tau} P_{rec} dt / P_{noi}, \quad (3)$$

where P_{rec} is the power of the useful signal at the input of the observer's receiving device, and P_{noi} is the spectral power density of the noise. Integration is performed over the signal transmission time τ. That is, the problem is reduced to a classical energy receiver.

Thus, the detection problem is to test two hypotheses: H_0 (only noise is received) and H_1 (a mixture of a useful signal and noise is received). The decision on the presence of a useful signal is made according to the Neumann–Pearson criterion: the optimal detection system shall maximize the probability of correct detection P_{det} under a fixed value of the false alarm probability P_{fal}, i.e., P_{det} => max with P_{fal} = const [1].

2. CALCULATING A UNIFIED DETECTION PROBABILITY FIELD FOR MULTIPLE OBSERVERS

In active mode, detection is performed based on the processing results of the signal transmitted and received by an observer in the presence of interference. The decision on the presence or absence of a signal from an object is made periodically, after the preliminary processing of the realization of a Gaussian random process $X(t)$ with zero mean received during an observation (accumulation) interval of duration T_0. (This can be a single pulse or a pulse train.) In the absence of a signal from the object, the random process $X(t)$ has variance σ_{noi}^2; in the presence of a useful signal from the object, variance $\sigma_{sig}^2 + \sigma_{noi}^2$.

As is known [7], the density of a statistic y for the energy receiver (3), as the sum of squares of Gaussian random variables with zero mean and variance σ^2, is described by the central χ^2-distribution:

$$f(y) = \frac{1}{(2\sigma^2)^{n/2}\Gamma(n/2)} y^{n/2-1} e^{-y/2\sigma^2}, \; y \geq 0,$$

where $\Gamma(\cdot)$ denotes the gamma function, and n is the number of degrees of freedom. In the case of digital analysis, it is determined by the number N of averaged sample energy estimates: $n = 2N$; and in the case of analog processing, $n = 2T_0\Delta f$, where T_0 is the duration of the transmitted signal (pulse), and Δf is the filter bandwidth.

The numerical characteristics (the first two moments) of the χ^2-distribution are given by [8]

$$m_\chi = n\sigma^2, \; \sigma_\chi^2 = 2n\sigma^4.$$

For the hypothesis H_0, we have $m_0 = n\sigma_{noi}^2$ and $\sigma_0^2 = 2n\sigma_{noi}^4$; for the hypothesis H_1, $m_1 = n(\sigma_{noi}^2 + \sigma_{sig}^2)$ and $\sigma_1^2 = 2n(\sigma_{noi}^2 + \sigma_{sig}^2)^2$. Let us define the signal-to-noise ratio as $\rho = \sigma_{sig}^2 / \sigma_{noi}^2$, then

$$m_1 = n\sigma_{noi}^2 (1+\rho),$$
$$\sigma_1^2 = 2n[\sigma_{noi}^2(1+\rho)]^2 = 2n\sigma_{noi}^4(1+\rho)^2. \quad (4)$$

For sufficiently large n, the χ^2-distribution is well approximated by the Gaussian distribution $N(m_\chi, \sigma_\chi^2)$.

Based on formulas (2)–(4), under a fixed P_{fal}, it is possible to construct the dependence of P_{det} on the distance R to the target, i.e., $P_{det}(R)$, or the dependence of P_{det} on the target's RCS, i.e., $P_{det}(\delta_c)$. The qualitative nature of these dependencies for two fixed values δ_c is shown in Fig. 1, where $\delta_{c1} > \delta_{c2}$. With regard to surveillance observers, by conventional assumption, reliable detection is ensured at $\rho > 25$ [6].

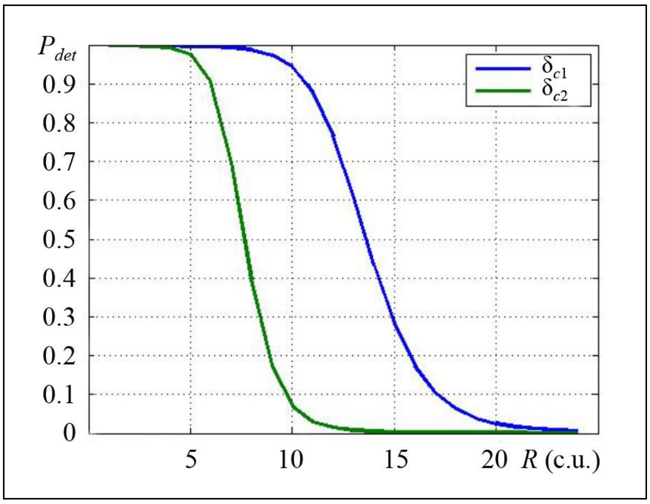

Fig. 1. The dependence of P_{det} on the distance to the target under fixed P_{fal} for different δ_c.

This dependence describes the primary processing stage. For a given false alarm probability $P_{fal} = \alpha$, the probability of correct detection P_{det} grows with increasing ρ, and ρ decreases monotonically with increasing the distance R to the target.

When solving the detection problem, it is necessary to consider the influence of the terrain relief. In other words, $P_{det}(R)$ is the conditional density

$$\{P_{det}(R) \mid P_{los}\}, \quad (5)$$

where $P_{los} = 1$ in the LOS region and $P_{los} = 0$ outside it (in the shadow region).

As a rule, the observer's receiving system operates periodically, accumulating and processing the signal for a fixed time T (a single observation period), during which the position of the detected object and its speed remain almost invariable. After this time, a final decision is made on the presence/absence of the object.

Since each observer generally operates with a particular accumulation time, the integral probability formula (1) cannot be applied. If the accumulation times of all observers were reduced to the same (basic) value T_{bas} = const, it would be possible to construct a unified probability map $P_{det}(x, y)$ in XOY coordinates for all observers located in a given area.

According to the paper [4], for a group of observers with different single observation times T_i, the probabilities of non-detection during the time T_i, $P_{ndet\,i}(T_i)$, can be recalculated into the probabilities of non-detection during the basic time T_b, $P_{ndet\,i}(T_b)$, as follows:

$$P_{ndet\,i}(T_{bas}) = \left[P_{ndet\,i}(T_i)\right]^{T_{bas}/T_i}. \qquad (6)$$

Consequently, the probabilities of non-detection for two observers with different observation times can be combined using formula (6) into the probability of non-detection with a unified time (recalculated into the observation time of one observer):

$$P_{ndet\Sigma}(T_2) = P_{ndet2}(T_2)P_{ndet1}(T_2) = P_{ndet2}(T_2)\left[P_{ndet1}(T_1)\right]^{T_2/T_1}.$$

Let an observer in the area have coordinates (x_l, y_l, H_l), where H_l is the altitude of the observer's location. Then, using the above dependence $P_{det}(R)$ for a given altitude h and the expression (5), it is possible to calculate the probability of non-detection for all area points (x, y), i.e., to form a field (matrix) of the conditional probabilities of non-detection $P_{ndet}(x, y \mid h)$.

With the probabilities of non-detection obtained for each observer by formula (6), one can calculate a unified (total) field for all observers:

$$P_{ndet}(x, y) = \prod_{l=1}^{L} P_{ndet\,l}(x_l, y_l, H_l)^{T_M/T_l}, \qquad (7)$$

where L is the number of observers; T_l is the observation duration for the lth observer; $T_M = \max[T_l]$; (x_l, y_l) are the coordinates of the lth observer; finally, $P_{ndet\,l}$ is the probability of non-detection for the lth observer. Thus,

$$P_{det}(x, y) = 1 - P_{ndet}(x, y).$$

In addition to setting a single observation time for all observers, there is another obvious requirement to calculate a unified detection probability field: all observers shall have **the same probability of false alarms**, $P_{fal} = \alpha$.

3. SIMULATION RESULTS

The simulation was performed in MATLAB.

Figure 2 shows the map used for the simulation (the right-hand side is a color scale of altitudes; hereinafter, the scales of altitudes and distances are specified in conventional units (c.u.)).

Two stationary observers with coordinates (x, y) and altitude H (5 c.u. above the ground) were considered:

– the first observer with (x_1 = 30, y_1 = 60, H_1 = 263);

– the second observer with three possible coordinates (see the label "+" in Fig. 4b), i.e., (x_2 = 85, y_2 = 56, H_2 = 193), (x_2 = 83, y_2 = 75, H_2 = 104), and (x_2 = 76, y_2 = 92, H_2 = 119).

The distance between the first and second observers is the same in all three variants.

Two variants of the object's flight altitude were considered: h_1 = 600 c.u. (a high-altitude object) and h_2 = 200 c.u. (a VLA object).

Figure 3 shows an example of calculating the unified detection probability field (DPF) for two observers monitoring a high-altitude object (h_1). The right-hand side of the figure is the color scale of the detection probabilities.

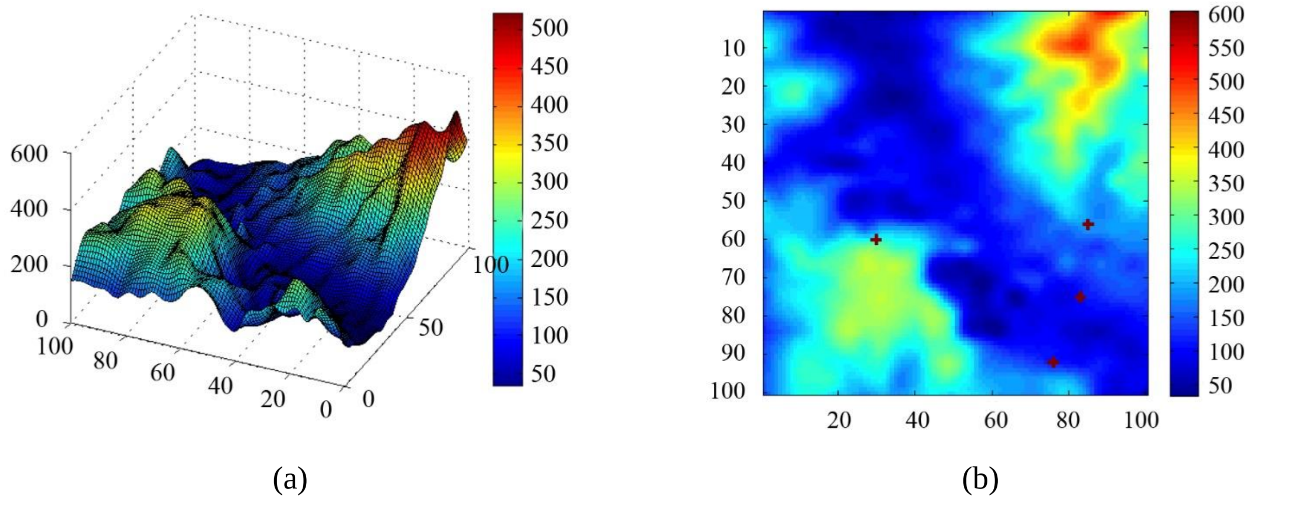

Fig. 2. The map used for simulation: (a) the 3D projection and (b) the 2D projection.

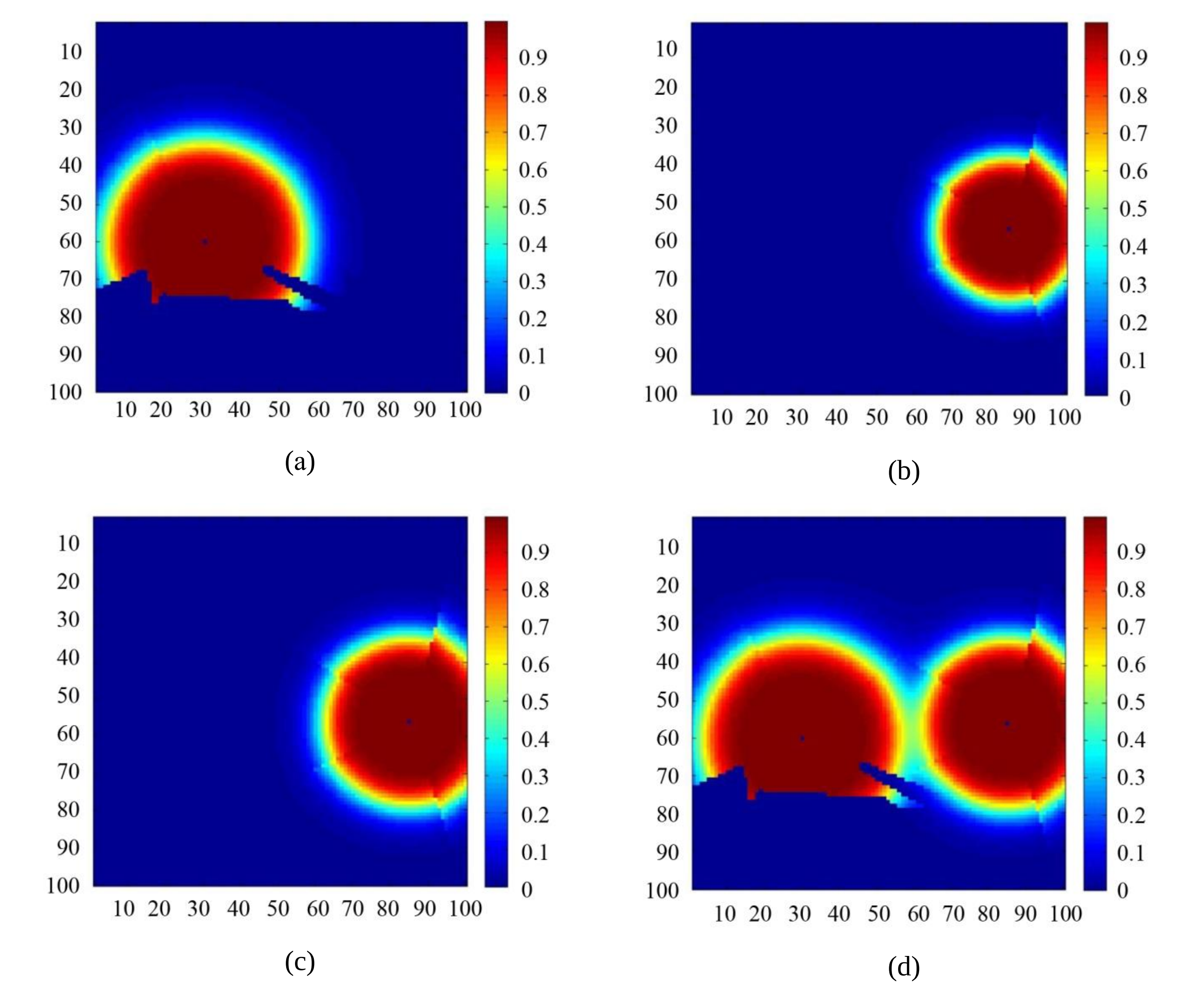

Fig. 3. The unified DPF for two radars monitoring a high-altitude object (h_1): (a) the DPF P_{det1} for the first observer (the observation time T_1); (b) the DPF P_{det2} for the second observer (the observation time T_2); (c) the DPF P_{det3} for the second observer recalculated by (6) (the observation time T_1); and (d) the unified DPF $P_{det} = 1 - P_{ndet}$ for two observers calculated by (7).

According to Fig. 3, both of the observers have a circular directional pattern, but the first observer is located on a hillside (see Fig. 2); therefore, the slope creates a shadow region for it (see the lower part of Fig. 3a). The detection probability fields for the first and second observers do not overlap; in view of $T_1 > T_2$, after the reduction to a single observation time by formula (6), their unified DPF almost forms a continuous detection probability field (Fig. 3d).

Figure 4 presents an example of calculating the unified DPF for two observers when working with the terrain relief (h_2). The right-hand side of Fig. 4a is the color scale of altitudes, and the right-hand sides of Figs. 4b–4e are the color scales of the detection probability.

Figure 4a shows the geographical map sectioned by the object's altitude and the relief exceeding h_2, which is the region where the object cannot appear. This relief defines the "mask of prohibited regions" for the object since the relief altitude in the regions is greater than the object's altitude h. The DPF on the maps is shaded in green.

The shadow region clearly seen in Fig. 4b is formed by the slope of the hill on which the first observer is located. The shadow appears due to $H_1 > h_2$: the slope of the hill forms a clutter notch completely covering the field of view of the first observer in this sector at the altitude h_2.

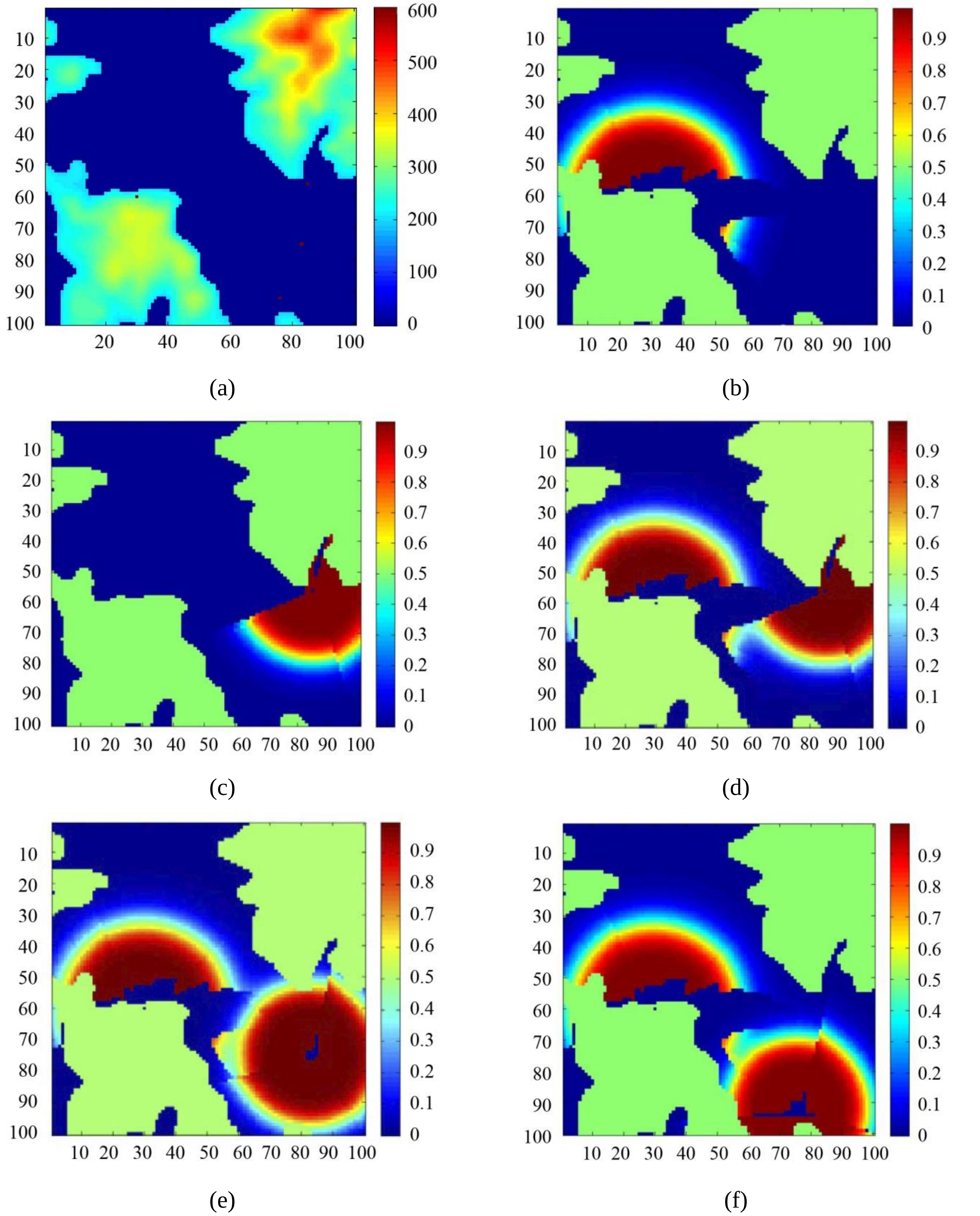

Fig. 4. The unified DPF for two observers when working with the terrain relief (h_2): (a) the map section at altitude h_2; (b) the DPF P_{det1} for the first observer (the observation time T_1); (c) the DPF P_{det3} for the second observer recalculated by (6) (the observation time T_1); (d) the unified DPF $P_{det} = 1 – P_{ndet}$ for two observers calculated by (7) (the coordinates of the second radar are ($x_2 = 85$, $y_2 = 56$, $H_2 = 193$)); (e) the unified DPF $P_{det} = 1 – P_{ndet}$ for two observers calculated by (7) (the coordinates of the second radar are ($x_2 = 83$, $y_2 = 75$, $H_2 = 104$)); (f) the unified DPF $P_{det} = 1 – P_{ndet}$ for two observers calculated by (7) (the coordinates of the second radar are ($x_2 = 76$, $y_2 = 92$, $H_2 = 119$)).

Figure 5 presents an example of forming LOS regions (green) and shadow regions (red) in a given direction for the map coordinates (x_1 = 30, y_1 = 60, H_1 = 266) and (x_2 = 100, y_2 = 40) in the case h = 200.

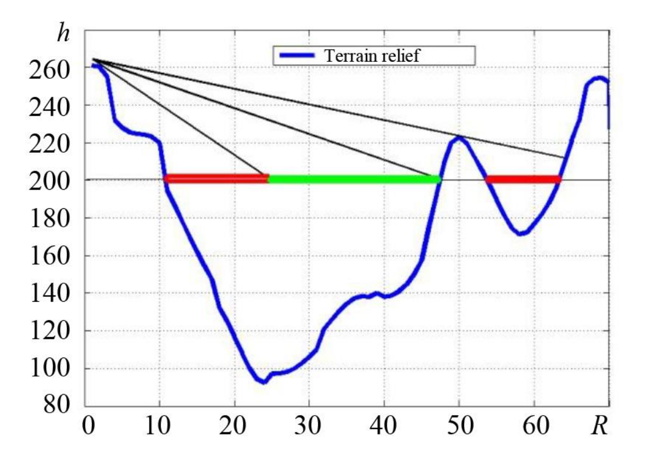

Fig. 5. LOS and shadow regions.

Direct comparison of Figs. 4 and 5 indicates that for object detection considering the terrain relief, the mutual position of the observers is crucial when forming a unified DPF.

Taking the terrain relief into account, the circular diagram of the DPF is significantly distorted and depends on four parameters:

– the distance R between the object and the observer,

– the object's altitude h,

– the observer's antenna altitude H, and

– the direction to the object.

Nevertheless, under a fixed altitude h of the object, it is also possible to calculate the probability of detection for all points in the area, $P_{det}(x, y)$, i.e., construct a detection probability map for h = const. Such a set of probability maps can be described by a three-dimensional matrix in the coordinates (x, y, h) for a series of fixed altitudes h_j $(1 \le j \le J)$ under fixed coordinates and altitudes H_l $(1 \le l \le L)$ of each observer. This procedure yields a so-called multilayer map.

Obviously, the above calculation of a unified DPF using formulas (5)–(7) can be easily generalized to an arbitrary number of observers.

4. APPLICATION OF A UNIFIED DETECTION PROBABILITY FIELD

There exists a fairly wide range of problems where the calculation of a unified detection probability field for several observers is of interest. These problems can be divided into two groups.

The first group. When detecting a high-altitude object, calculating the unified probability field $P_{\text{det}}(x, y)$ for the entire area allows combining information from observers with different technical characteristics (see Fig. 3). In this case, the map is one-dimensional.

A unified probability field $P_{\text{det}}(x, y, h)$ calculated for the entire area can be superimposed on a geographical map. Then a human operator can visually identify weak spots (the regions of small P_{det}) and, e.g., send additional resources there or change the location of observers to cover the weak spots. (As is well known, the human eye copes very well with such a task.) Calculation of a unified DPF in the form of a multilayer map simplifies the solution of optimization problems. For example, in the case of object detection, it simplifies the optimal placement of a fixed number of radars in the area according to a given criterion. Such a problem has not been considered so far.

The second group includes problems of penetrating through an area monitored by several stationary observers or evading detection by these observers. In the literature, this class of problems is referred to as control problems for mobile objects in a conflict environment. Thus, the problem is reduced to route planning by a probabilistic criterion. Route planning on a single-layer map with obstacles is a well-studied problem, both theoretically and practically. Classical graph search algorithms, such as Dijkstra's algorithm, are typically used to solve it. They are widely and successfully applied to a single layer of the map. However, it is challenging to extend these algorithms to multilayer maps [9].

In the case of a probabilistic criterion, route selection consists in minimizing the accumulated probability according to formula (1) for a single-layer map [10–12], but a unified DPF is not used. In [13], the problem of selecting the trajectory of a maneuvering object and the law of its velocity in a three-dimensional anisotropic signal propagation environment was considered, provided that several observers located in a given area are trying to detect it. (For each observer, P_{det} was calculated separately.) In the problems mentioned, the preliminary calculation of a unified DPF would significantly facilitate dynamic programming.

For example, when an object avoids the terrain at a given altitude (a nap-of-the-earth flight), an available multilayer map of the unified DPF of the area can be easily reduced to a single-layer one; on the latter, a route is planned much more easily. In addition, on a single-layer map, the operator can visually select a route, as shown in Fig. 6 (the map of Fig. 4d is used).

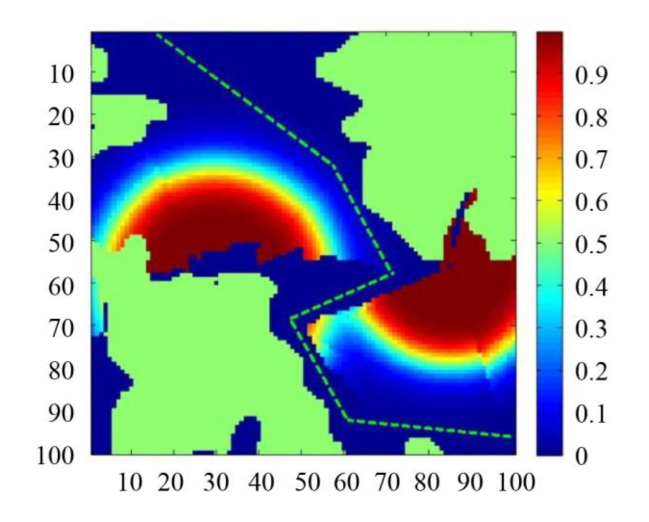

Fig. 6. The route planned by the operator.

CONCLUSIONS

The availability of a unified detection probability field for all observers monitoring a given area provides the following capabilities:

– It can be generalized to the detection of a moving object, taking the terrain relief into account, by adding two coordinates (the observer's altitude and the object's altitude).

– It can be used for visualization, i.e., presented to a human operator.

– It can be selected as a basis for combining detection probabilities from other sources of information (e.g., from observers located on moving objects [14]). Passive location systems can also be applied to detect and track moving objects [14]. In addition to location-based detection methods, other groups of methods for detecting moving objects are currently being investigated [15], in particular, infrared (thermal) detection, radio frequency scanning, detection using optical cameras, detection of acoustic signals, and approaches involving classical machine learning methods with feature extraction as preprocessing (artificial intelligence). DPFs from such information sources can be included in a unified detection probability field if there are models for calculating the probabilities of object detection by these means.

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