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ESTIMATING INDUSTRIAL PROCESS STABILITY BY WHITNEY'S SINGULARITY THEORY WHEN CHOOSING A SUFFICIENT TIME-SAMPLING FREQUENCY OF THE CONTROL SIGNAL

M.A. Rabotnikov[1], B.G. Stafeichuk[2], and A.G. Shumikhin[3]

Perm National Research Polytechnic University, Perm, Russia

[1]✉ rabotnikovma@gmail.com, [2]✉ bgstaf@mail.ru, [3]✉ shumichin@gmail.com

Abstract. In this paper, we estimate the stability of continuous-type automated industrial processes and choose a sufficient time-sampling frequency of the control signal using Whitney's singularity theory. The proposed stability analysis approach is based on constructing typical bifurcations for the historical data of a technological object under different time-sampling frequencies of its control signal. The singularity equation serves for obtaining the equation of the equilibrium state curves of the system and a sufficient time-sampling frequency of the control signal corresponding to the vertex of the resulting curve. As an illustrative example, the developed method is applied to the control system of the mass balance stripping section in the purification process of a styrene distillation column of the ethylbenzene, styrene, and polystyrene plants. Based on the quantitative analysis results, we construct a bifurcation and determine a sufficient time-sampling frequency of the control signal to ensure system stability.

Keywords: catastrophe theory, bifurcation, dynamical systems, stability criteria.

INTRODUCTION

Catastrophe theory has been widely applied to analyze the behavior of economic and social systems, assess the design properties of structures and apparatuses over time, and study the properties and quantitative characteristics of dynamical systems [1–6]. The existing mathematical framework numerically describes jump-like transients of the output variable due to a smooth change in the input parameters of a dynamical system [7]. In particular, this approach can be used to estimate the stability of continuous-type automated industrial processes [8].

The problem under consideration—the stable operation of industrial processes—consists in complying with the regulatory limits of the detected technological parameters to minimize the risk of stopping production and any violations of the quality composition of commercial products (on the one hand) and maximize the technological efficiency under various disturbances, often of sporadic nature (on the other hand); for details, see [9]. The modern technical base and the existing mathematical approaches and algorithmic solutions used in automated control allow reducing this problem (technological mode stabilization and compliance with the regulatory limits) to multidimensional model predictive control [10]. For systems of this class, the time-sampling frequency of the control device is one of the indicators characterizing the inertia of the controlled process and the effect of disturbing factors on the controlled technological parameters. Under fixed values of other characteristics, a sufficient value of this frequency ensures the stable operation of a technological object within the specified regulatory limits of the corresponding industrial process.

1. CONSTRUCTING BIFURCATIONS OF CONTROL SYSTEMS

How does the stability of an industrial process depend on the control signal frequency? What is the range of frequencies ensuring system operation within given constraints? To answer these questions, we consider the dynamics of a controlled process parameter as a typical bifurcation.

By conditions, the bifurcation depends on the time-sampling frequency of the control device and the effect of disturbing factors on the system, possessing a codimension of 2. Due to the continuity and natural origin of the processes under study, the bifurcation dependence must be a continuously differentiable function on the entire definitional domain.

For the problem under consideration, it is convenient to represent the bifurcation as a second Whitney singularity, the cusp [11]. Its functional mapping is given by

$$f(x, g(\omega)) = x^3 + g(\omega)x,$$

with the following notations: $f(x)$ is the normalized distribution of the controlled parameter; $x \in [-1, 1]$ is the normalized spectrum of the equivalent disturbance characterizing the resulting effect of all external factors of the system on the controlled variable; $g(\omega)$ is the frequency function; finally, ω is the time-sampling frequency characterizing the periodicity of the control signal.

According to the bifurcation properties, the control system is in an unstable state if $g(\omega) > 0$: the function $f(x, g(\omega))$ has no nondegenerate singularities and increases continuously over the entire definitional domain [12]. For $\omega = \omega_s$, we obtain $g(\omega) = 0$, where ω_s is a sufficient time-sampling frequency of the control device algorithms: the system passes through a bifurcation point S, being in an equilibrium state. In the case $g(\omega) < 0$, the system comes to a stable equilibrium, forming a bifurcation with two stationary points.

Note that the frequency function $g(\omega)$ has no strict mathematical formalization. For the quantitative description of observed real processes, it can be constructed by piecewise linear approximation based on the historical operation data of the controlled object. Considering the inverse dependence of the stability margin on the frequency ω ensuring system stability within the range $\omega \in (\omega_s, +\infty)$, the frequency function for this system operation range takes the form

$$g(\omega) = -\omega + \omega_s .$$

In this case, the bifurcation is transformed to

$$f(x, \omega) = x^3 + (\omega_s - \omega)x.$$

Let us reduce the distribution $f(x, \omega)$ from the normalized variables to those expressed in the original units of measurement:

$$y(x, \omega) = \alpha f(x, \omega) + M_y,$$

where α is the normalization coefficient and M_y denotes the expectation of $y(x, \omega)$. As a result, the surface is given by

$$y(x,\omega) = \alpha\left(x^3 + (\omega_s - \omega)x\right) + M_y. \qquad (1)$$

The projection of the cusp $y(x, \omega)$ onto the plane (y, ω) yields the set of singularities y_s with the vertex at the initial bifurcation point S. The bifurcation set under study forms the boundaries of the branches of the equilibrium state curves. According to the definition [13], these curves are obtained by equating the derivative $y'_x(x, \omega)$ to zero:

$$3x^2 + \omega_s - \omega = 0 .$$

Substituting this condition into formula (1), we determine the general equation of the bifurcation set of the control system:

$$y_s(\omega) = M_y \pm 2\alpha\left(\frac{\omega - \omega_s}{3}\right)^{3/2}, \ \omega \ge \omega_s . \qquad (2)$$

The graph of the resulting bifurcation is shown in Fig. 1.

The parameters ω_s and α, characterizing the stability of the control system, are determined empirically by the historical data of the closed-loop dynamics under different frequencies ω. According to the correspondence principle for the degrees of freedom and the necessary number of equations to identify the parameters ω_s and α, a statistical sample must contain historical data sets for two operation modes under different frequencies ω_1 and ω_2 of the control system: $y_1(x_1, \omega_1)$ and $y_2(x_2, \omega_2)$, where y_1 and y_2 are the detected states of the controlled parameter under external disturbances x_1 and x_2, respectively. In this case, we have the system of equations

$$\begin{cases} y_1 = \alpha\left(x_1^3 + (\omega_s - \omega_1)x_1\right) + M_y, \\ y_2 = \alpha\left(x_2^3 + (\omega_s - \omega_2)x_2\right) + M_y. \end{cases}$$

After trivial transformations, this system yields expressions for the parameters ω_s and α:

$$\omega_s = \frac{(y_1 - M_y)(x_2\omega_2 - x_2^3) - (y_2 - M_y)(x_1\omega_1 - x_1^3)}{(y_1 - M_y)x_2 - (y_2 - M_y)x_1}, \qquad (3)$$

$$\alpha = \frac{(y_1 - M_y)x_2 - (y_2 - M_y)x_1}{x_1 x_2\left(x_1^2 - x_2^2 - \omega_1 + \omega_2\right)}. \qquad (4)$$

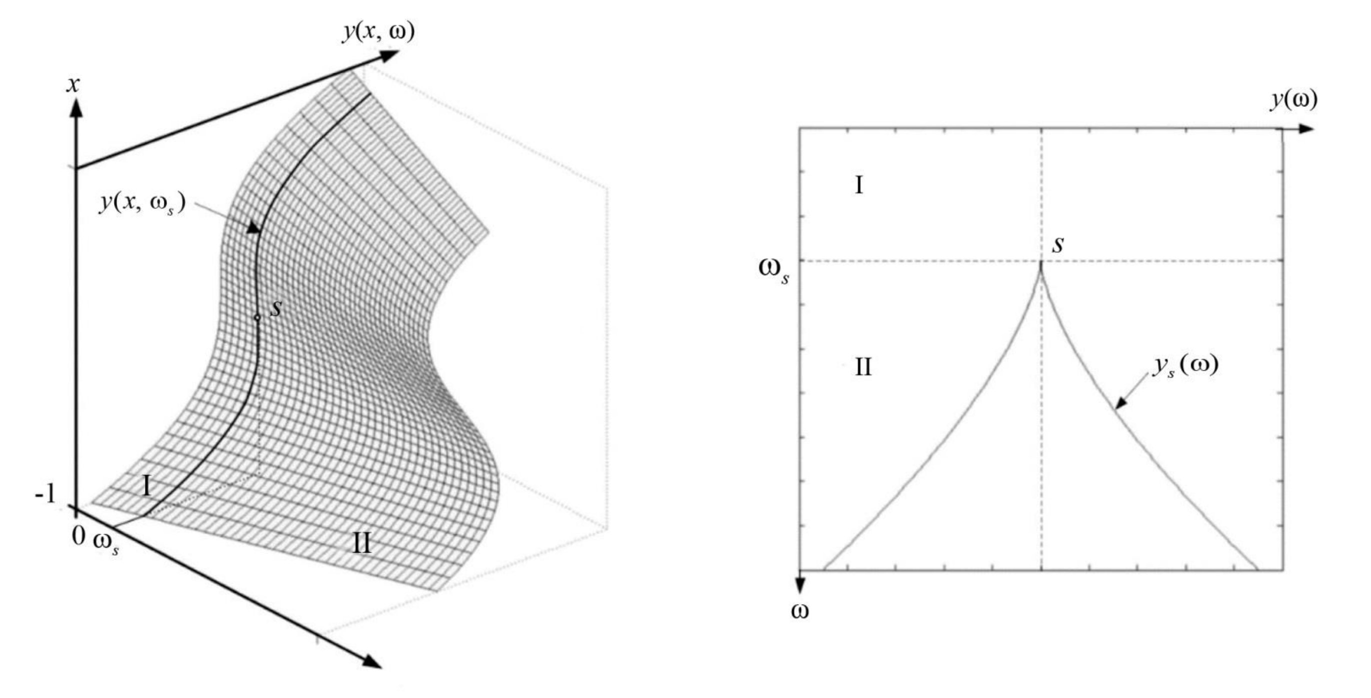

Fig. 1. Control system bifurcation: *s*—bifurcation point, I—unstable state domain, and II—stable state domain.

It is convenient to calculate ω_s and α at the maximum spike points of the controller parameter: y_1 and y_2 for the frequencies ω_1 and ω_2, respectively. These responses of the output signal correspond to the limits of the undetected spectrum of the resulting disturbance of the system in the normalized form: $|x_{1,2}| = 1$. The sign of $x_{1,2}$ is determined depending on the location of the spike point of the controlled variable relative to the sampling mean M_y.

2. BIFURCATIONS OF REAL TECHNOLOGICAL OBJECTS: ONE EXAMPLE OF CONSTRUCTION

As an industrial process example, we consider the stripping section of the distillation column C-2 in the separation unit of ethylbenzene, styrene, and polystyrene production (PESP): construct a bifurcation and choose a sufficient time-sampling frequency for the control signal of the automated process control system. This unit consists of three columns (C-1, C-2, and C-3) with the following functions: separating hydrocarbon condensate supplied from the ethylbenzene dehydrogenation block into raw styrene and the benzene-toluene-ethylbenzene fraction; purifying commercial styrene from heavier fractions (residue of styrene rectification); separating the benzene-toluene-ethylbenzene fraction into recycled ethylbenzene and the benzene-toluene fraction (benthol). Figure 2 shows the general diagram of the main material flows of the distillation unit for ethylbenzene, styrene, and polystyrene production.

The technological mode of the stripping section of column C-2 is maintained by multidimensional control: it is required to stabilize the level in the column (*LI001*) under a given constraint (the limit value of the column temperature profile (*TI001*)) due to beginning the styrene polymerization reaction. The main control action is the superheated steam supply into the heat exchanger H-2 (*FIC001*), intended for heating the bottom fraction. The evacuation of high boiling components from the column (*FIC002*) is fixed at the minimum value due to commercial styrene losses in the mixture. The key disturbance for this automated process control system is the quantitatively undetected change in the composition of the incoming hydrocarbon feedstock.

The dynamics of the technological object are described by the system of differential equations

$$\begin{cases} \sum_{i=0}^{n_1} a_{1i} \frac{\partial^i \Delta L(t)}{\partial t^i} \\ = \sum_{j=0}^{m_1} b_{1j} \frac{\partial^j \Delta F(t-\tau_{11})}{\partial t^j} + \sum_{k=0}^{q_1} c_{1k} \frac{\partial^k \Delta Q(t-\tau_{12})}{\partial t^k}, \\ \sum_{i=0}^{n_2} a_{2i} \frac{\partial^i \Delta T(t)}{\partial t^i} \\ = \sum_{j=0}^{m_2} b_{2j} \frac{\partial^j \Delta F(t-\tau_{21})}{\partial t^j} + \sum_{k=0}^{q_2} c_{2k} \frac{\partial^k \Delta Q(t-\tau_{22})}{\partial t^k}, \end{cases} \quad (5)$$

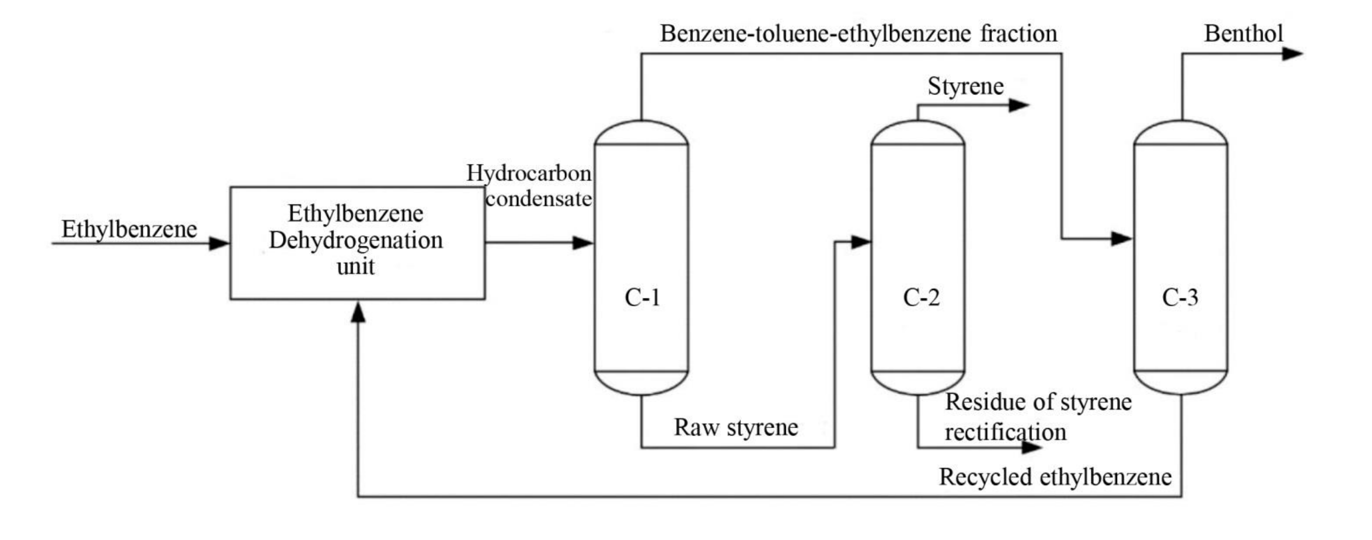

Fig. 2. Distillation unit diagram.

with the following notations: L is the liquid phase level in column C-2; F is the steam flow rate in the heat exchanger H-2; Q is the quantitative composition of column feeding; T is the bottom temperature in column C-2; a, b, and c are the coefficients of differential equations; n, m, and k are the orders of polynomials ($n > m$, $n > k$); finally, τ is the time lag of dynamic channels.

The technological mode is maintained within the specified regulatory limits by an automated *model predictive control* (MPC) system [14]. The model is presented as a matrix transfer function approximating the real behavior of the industrial process according to the empirical transient characteristics of the transmission channels (5). Figure 3 shows the process control diagram for the stripping section of column C-2.

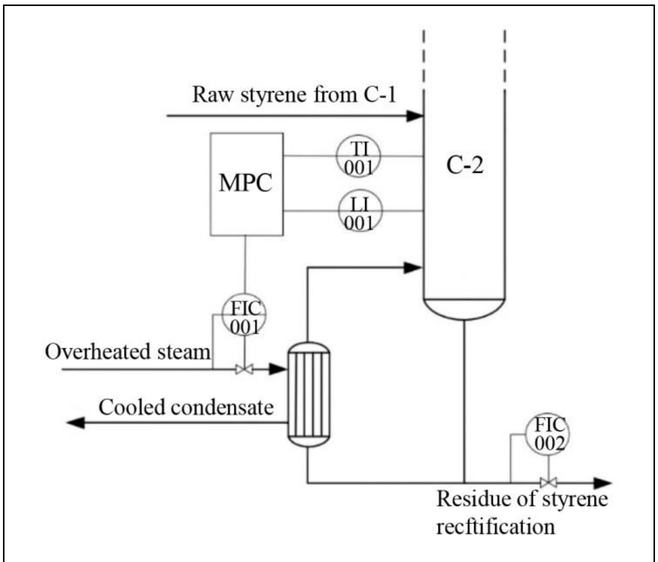

Fig. 3. Process control for the stripping section of column C-2.

Due to the relatively small dimensions of the column and the high rate of mass exchange processes, the distillation column C-2 has low inertia. Figure 4 demonstrates the level in the column under different time-sampling frequencies of the control signal. For $\omega_1 = 0.1$/min, we observe jump changes of this level with a root-mean-square (RMS) deviation of 7.29%, which determine the possibility of an emergency shutdown of the plant: the pumping equipment will switch off under the full release of the bottom fraction from the column. For $\omega_2 = 1$/min (all other system parameters remain unchanged), the liquid phase level in the column has admissible fluctuations with an RMS deviation of 4.33%: the industrial process is in a steady state.

To construct a bifurcation of the system on a given historical data set (a representative sample), we selected necessary initial data. For this purpose, we considered the maximum deviations of the liquid phase level in column C-2 relative to the mean $M_y = 88.75\%$ for the system operating with the frequencies $\omega_1 = 0.1$/min and $\omega_2 = 1$/min: $y(x_1, \omega_1) = 55.29\%$, $x_1 = -1$; $y(x_2, \omega_2) = 82.32\%$, $x_2 = -1$.

According to formulas (3) and (4), the quantitative properties of this bifurcation are given by the parameters $\omega_s = 0.22$/min and $\alpha = 30.03$. Then the bifurcation of the automated process control system for the stripping section of the distillation column C-2 and the corresponding function of the bifurcation take the following form (Fig. 5):

$$y(x, \omega) = 30.03\left(x^3 + (0.22 - \omega)x\right) + 88.75,$$

$$y_s(\omega) = 88.75 \pm 60.07\left(\frac{0.22 - \omega}{3}\right)^{3/2}, \ \omega \geq 0.22.$$

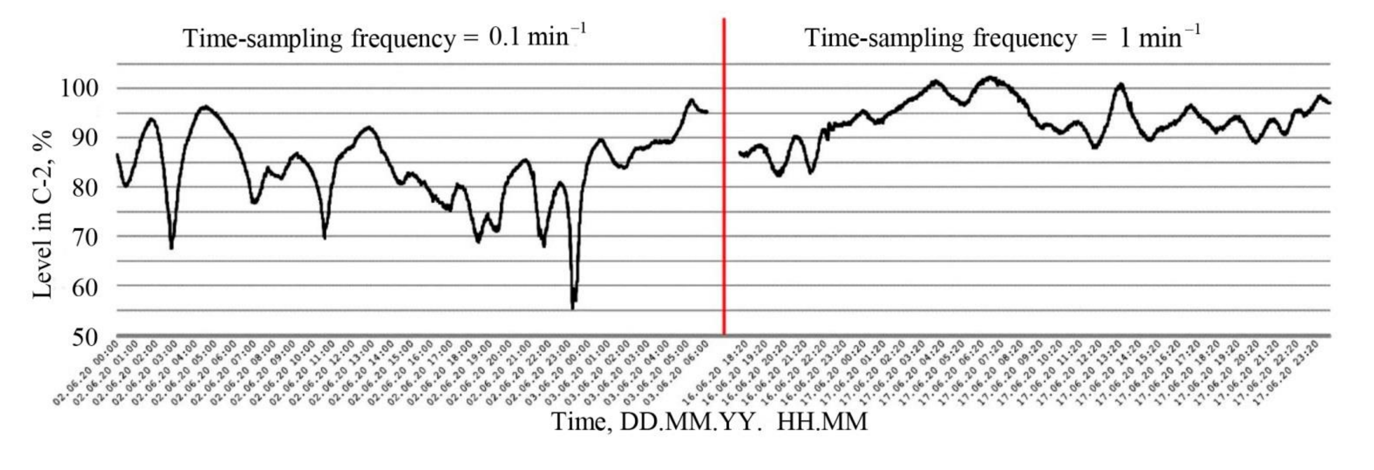

Fig. 4. The level in column C-2

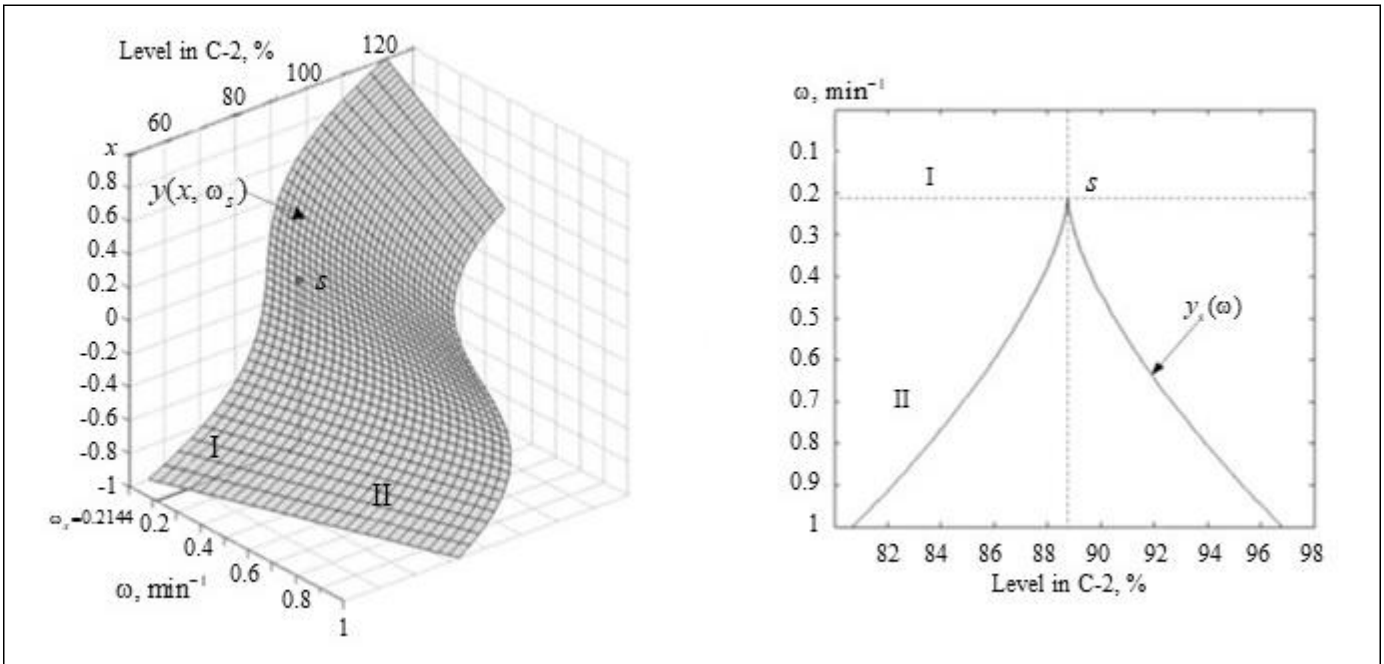

Fig. 5. Bifurcation of the stripping section of column C-2: *s*—bifurcation point, I—unstable state domain, and II—stable state domain.

The constructed bifurcation for choosing the sufficient time-sampling frequency of the control signal is adequate: it agrees with the dynamical characteristics of the real material balance stabilization system for the stripping section of the distillation column C-2. The control action channel (Fig. 3) has the delay $\tau_e \approx 5$ min; therefore, the admissible time-sampling frequency is empirically defined by $\omega_e = \tau_e^{-1} \approx 0.2$ min^{-1}. The analytical solution obtained by the method of bifurcation diagrams gives the sufficient time-sampling frequency $\omega_s \geq 0.22$ min^{-1}.

CONCLUSIONS

This paper has presented a method for constructing bifurcations of dynamical systems based on historical data. It has been applied to estimate the stability of the control system of the mass balance stripping section in the purification process of a styrene distillation column of the ethylbenzene, styrene, and polystyrene plants. According to the analysis results, the control signal ensures stable system operation within the admissible regulatory limits of the technological process parameters under a sufficient time-sampling frequency of 0.22 min^{-1}.

This approach to constructing bifurcations of control systems and calculating the stable state domain of the chemical engineering object is estimative: it serves for preliminarily determining the optimal time step of the control system considering the dynamical properties of the controlled object.

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Author information

Rabotnikov, Mikhail Alekseevich. Assistant, Perm National Research Polytechnic University, Perm, Russia
✉ rabotnikovma@gmail. com

Stafeichuk, Boris Grigor'evich. Cand. Sci. (Eng.), Perm National Research Polytechnic University, Perm, Russia
✉ bgstaf@mail.ru

Shumikhin, Aleksandr Georgievich. Dr. Sci. (Eng.), Perm National Research Polytechnic University, Perm, Russia
✉ shumichin@gmail.com

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Translated into English by *Alexander Yu. Mazurov*,
Cand. Sci. (Phys.–Math.),
Trapeznikov Institute of Control Sciences,
Russian Academy of Sciences, Moscow, Russia
✉ alexander.mazurov08@gmail.com