MODELS OF POWER HIERARCHIES AT THE GLOBAL LEVEL

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Abstract. This paper is devoted to global power hierarchies. A global power hierarchy is a three-level control system, with the upper level occupied by a Principal. The second (middle) level consists of influence agents subordinate to the Principal. At the third (lower) level, there are basic agents subordinate to influence agents. The model is considered in a dynamic discrete-time setting. An algorithm is formulated to solve the Germeier game Γ_{1t}. The model is studied via computer simulations using both model input parameters and data corresponding to global realities for the period from 1990 to 2025. The simulation results are analyzed. In particular, conditions for the emergence of a unipolar, bipolar, and multipolar power hierarchy at the global level are established. As shown, the main condition for a multipolar world is a large initial quantity of resources. If this quantity is sufficiently large, then regardless of the values of competition parameters, the world becomes multipolar. In practice, however, all resources are limited; therefore, a bipolar or unipolar world emerges, depending on the strength of power hierarchies and the values of competition parameters.

Keywords: power hierarchies, simulation modeling, Germeier games, Nash equilibrium, difference games.

INTRODUCTION

Contemporary research into power hierarchies rests on interdisciplinary approaches combining sociology, political science, and psychology with the use of mathematical models and information technology. Note Mikhailov's works [1–3], emphasizing the need for a comprehensive analysis that covers the diversity of factors affecting power structures. Khazin's theory of power hierarchies is of great interest [4, 5], albeit without mathematical models. Other relevant approaches to the problem were proposed in [6–9].

In this paper, we utilize the formulations and results of [10]. Compared to [10], the algorithm for studying the model has been refined under the information rules of the Germeier game Γ_{1t}, and available data have been adopted to model global-level power hierarchies (the basis of a multipolar world). Note that the agents of a multipolar world are civilizations [11, 12], and they have dual organization as an inherent property. On the one hand, a civilization consists of legally independent sovereign states with a certain degree of economic and cultural autonomy. On the other hand, a civilization is actually a power hierarchy where members delegate the primary authorities—the issues of interacting with other poles, ensuring defense, conducting transactions in a single currency, etc.—to a leading state or a certain coordinating body. Below, a civilization will be considered precisely from this perspective.

The remainder of this paper is organized as follows. In Section 1, we provide a rigorous mathematical problem statement for the case of three-level power hierarchies ("Principal–influence agents–basic agents"). Section 2 presents the results of computer simulations for model examples and real data, along with their interpretation and analysis. The key findings of the paper are summarized in the Conclusions.

1. PROBLEM STATEMENT

When constructing a mathematical model of global power hierarchies, we will follow the author's approach formulated in [10]. A global power hierarchy is a multilevel control structure. Below, global power hierarchies are investigated using three-level control systems.

In such systems, there are:

– several upper-level subjects (Principals or Centers of power structures, poles of a multipolar world), which compete with each other for power (the power resource);

– several middle-level control subjects (influence agents), which are subordinate to each Principal and compete with each other;

– several lower-level control subjects (basic agents), which are subordinate to each influence agent and compete with each other as well.

The resource is created through the joint efforts of the basic agents of all power structures. A fairly strong model assumption is that influence agents and Principals do not create the resource, but only regulate the creation process. Therefore, at each time instant, the share of resource controlled by a power group (its volume of power) is proportional to the aggregate efforts of the group's basic agents. Each agent (including each Principal) possesses a certain quantity of resources and allocates them between efforts to increase their group's share of the power resource and efforts to compete with other agents or Principals. Each agent's payoff consists of the utility from increasing the jointly created resource and from defeating competitors. This problem statement employs the ideology of the coordination of social and private interests (social and private interests coordination engines, SPICE models) [13, 14]. The control actions of basic agents are their efforts and resources (e.g., the time they spend on increasing their group's power resource). In turn, influence agents control the activities of their basic agents (by imposing lower bounds on the latter's efforts to create resources, i.e., by restricting the "selfishness" of the basic agents). Similarly, Principals control the activities of their influence agents. This is the primary regulatory function of Principals and influence agents, mentioned at the beginning of the paragraph. An example is the US requirement that the NATO allies spend no less than a given share of their GDP on military expenditures (in fact, to purchase weapons from the USA). Based on their influence, other Principals can also impose similar requirements on subordinate agents (e.g., to use only the Principal's currency in transactions or weapons systems).

Thus, the system of three-level power hierarchies has the following model [10]:

$$J_i = \sum_{t=1}^{T} \delta^t \left[\left(A - \sum_{p=1}^{N} v_p^t \right) v_p^t + R_i^t \right] \to \max, \qquad (1)$$

$$0 \le q_{ij}^t \le 1, \qquad (2)$$

$$J_{ij} = \sum_{t=1}^{T} \delta^t \left[\left(A_i - \sum_{r=1}^{n_i} v_{ir}^t \right) v_{ij}^t + R_{ij}^t \right] \to \max, \qquad (3)$$

$$q_{ij}^t \le q_{ijk}^t \le 1, \qquad (4)$$

$$J_{ijk} = \sum_{t=1}^{T} \delta^t \left[\left(A_{ij} - \sum_{s=1}^{m_{ij}} v_{ijs}^t \right) v_{ijk}^t + R_{ijk}^t \right] \to \max, \qquad (5)$$

$$q_{ijk}^t \le u_{ijk}^t \le 1, \qquad (6)$$

$$R^t = \left(1 + \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right) R^{t-1}, \quad R^0 = R_0, \qquad (7)$$

$$R_i^t = \begin{cases} \dfrac{R^t \left(\sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right)}{\sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t} & \text{if } \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0 & \text{otherwise,} \end{cases} \qquad (8)$$

$$R_{ij}^t = \begin{cases} \dfrac{R_i^t \left(\sum_{k=1}^{m_{ij}} u_{ijk}^t \right)}{\sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t} & \text{if } \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0 & \text{otherwise,} \end{cases} \qquad (9)$$

$$R_{ijk}^t = \begin{cases} \dfrac{R_{ij}^t u_{ijk}^t}{\sum_{k=1}^{m_{ij}} u_{ijk}^t} & \text{if } \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0 & \text{otherwise,} \end{cases} \qquad (10)$$

$$i = 1, \ldots, N, \quad j = 1, \ldots, n_i,$$
$$k = 1, \ldots, m_{ij}, \quad t = 1, \ldots, T.$$

Here, the number i is associated with a Principal (a power hierarchy); the number j is associated with an influence agent (a power group) of a given hierarchy; the number k is associated with a basic agent of a given group; t denotes discrete time; N is the total number of power hierarchies; n_i is the total number of power groups in a given hierarchy; m_{ij} is the total number of basic agents in a given power group of a given hierarchy; J_i, J_{ij}, and J_{ijk} are the payoffs of a given Principal, a given influence agent, and a given basic agent, respectively; R^t, R_i^t, R_{ij}^t, and R_{ijk}^t are the total resource quantity, the resource quantity of a given power hierarchy, the resource quantity of a given power group, and the resource quantity of a given basic agent, respectively, at a time instant t; q_{ij}^t is the control action of a given Principal applied to a given influence agent at a time instant t; $\sum_{j=1}^{n_i} q_{ij}^t$ is the share of Principal's efforts to control its hierarchy; $v_i^t = \begin{cases} 1-\sum_{j=1}^{n_i} q_{ij}^t & \text{if } 1-\sum_{j=1}^{n_i} q_{ij}^t > 0 \\ 0 & \text{otherwise} \end{cases}$ is the share of Principal's efforts to compete with other Principals; q_{ijk}^t is the control action of a given influence agent applied to a given basic agent at a time instant *t*; $\sum_{k=1}^{m_{ij}} q_{ijk}^t$ is the share of efforts of a given influence agent to control its group at a time instant *t*; $v_{ij}^t = \begin{cases} 1-\sum_{k=1}^{m_{ij}} q_{ijk}^t & \text{if } 1-\sum_{j=1}^{n_i} q_{ij}^t > 0 \\ 0 & \text{otherwise} \end{cases}$ is the share of efforts of a given influence agent to compete with other influence agents within its hierarchy; $v_{ijk}^t = \begin{cases} 1-u_{ijk}^t & \text{if } 1-u_{ijk}^t > 0 \\ 0 & \text{otherwise} \end{cases}$ is the share of efforts of a given basic agent to compete with other basic agents within its group; u_{ijk}^t is the share of efforts of a given basic agent to increase the resource quantity; A, A_i, and A_{ij} are the competition parameters; $\delta \in (0, 1)$ is the discount factor; R_0 is an initial value of the total resource quantity; finally, T is the planning horizon.

For $i = 1,..., N$, the relations (1)–(10) define a normal-form game of *N* players.

A system of power hierarchies can be unipolar, bipolar, or multipolar. A unipolar system arises when one hierarchy clearly dominates, surpassing all others in terms of a combination of economic, military, and political indicators. The conditions for its emergence are a significant weakening of potential competitors and an economic breakthrough ensuring technological superiority and military dominance. A bipolar system arises when there are two Principals of comparable strength competing for influence. For its formation, the two hierarchies shall possess almost equal economic and military potentials that significantly exceed those of the other hierarchies. A multipolar system of power hierarchies is characterized by the presence of several centers of power, none of which possesses sufficient superiority to dominate alone. The conditions for its formation are the relative equality of the economic, political, and military potential of several hierarchies.

Let us formalize the above considerations in mathematical terms.

Definition [10]. A system of power hierarchies at time instant *t* is said to be:

– *unipolar* if $\exists i \in \{1,..., N\}\ R_i^t \ge 0.75R^t$;

– *bipolar* if $\exists i,\ j \in \{1,..., N\}\ R_i^t + R_j^t \ge 0.75R^t \wedge \left|R_i^t - R_j^t\right| \le 0.15R^t$;

– *multipolar* otherwise. ♦

Of course, unipolarity is defined subjectively: a coefficient of 0.75 is taken based on "qualified majority" considerations; a "pole comparability" coefficient of 0.15 is also arbitrary. Anyway, conditions for the emergence of systems of power hierarchies with different numbers of poles are of great interest. These conditions are analyzed below.

2. SIMULATION EXPERIMENTS

Model (1)–(10) is studied numerically via computer simulations by the following algorithm.

1. The parametric nonlinear optimization problem (5), (6), (10) is solved; it represents a normal-form game of $\sum_{i=1}^{N}\sum_{j=1}^{n_i} m_{ij}$ players (the basic agents of all power groups). The resulting Nash equilibrium parametrically depends on the control actions of the influence agents.

2. The resulting strategies $\{u_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$ of the basic agents of all power groups (found at Step 1 of the algorithm) are substituted into formulas (7)–(9). The n_i (*i* = 1, 2,…, *N*) optimization problems (3), (4) of the influence agents are solved subject to the constraints (9); they are the middle-level game (3), (4), (9). Each such problem is a parametric optimal control problem. The resulting optimal strategies $\{q_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$ of the influence agents parametrically depend on the control action of the corresponding Principal. For each influence agent, the optimal strategy is the one generated by a certain Nash equilibrium from the game of the basic agents.

3. The strategies $\{u_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$ of the basic agents of all power groups (yielded by Step 1 of the algorithm) are substituted into equations (1), (2), and (8). The upper-level game (1), (2), and (8) is solved. The resulting Nash equilibrium is denoted by $\{q_{ij}^t\}_{t=1}^{T}{}_{j=1}^{n_i}$.

4. The final set $\{q_{ij}^t,\ q_{ijk}^t,\ u_{ijk}^t\}_{t=1}^{T}{}_{j=1}^{n_i}{}_{k=1}^{m_{ij}}$ is a solution to the Germeier game Γ_{1t} (1)–(10) for fixed *i*, and the

aggregate of these solutions for all $i=1,..., N$ is a solution of the general difference game.

It is difficult to solve problem (1)–(10) analytically due to the nonlinearity of the model and the large number of decision-makers. Therefore, the model was solved numerically using the above algorithm.

2.1. Model Examples

Due to the limited resources of this study, all experiments were carried out for the case $N = 3$, $n_i = 2$; $m_{ij} = 2$ $(i=1, 2, 3;\ j=1, 2)$, and $T = 4$. The computational experiments were planned as follows.

1. The model's input parameters R_0, A, A_1, A_2, A_3, A_{11}, A_{12}, A_{21}, A_{22}, A_{31}, and A_{32} were selected.
2. All values were enumerated with a step of 2.
3. The value of R_0 was varied from 1 to 200; A, from 2 to 60; A_1, A_2, A_3, A_{11}, A_{12}, A_{21}, A_{22}, A_{31}, and A_{32}, from 0.1 to 60 each.
4. For each model run, the equilibrium of the Germeier game Γ_{1t} was determined, and the uni/bi/multipolarity of the world was identified.

A total of approximately 1000 computational experiments were carried out; some results are presented in Table 1. According to the calculations, for different values of the competition parameters A, A_i, and A_{ij}, there always exists a value of the initial resource quantity R_{00} with the following properties: if $R_0 > R_{00}$, the world is multipolar; if $R_0 \le R_{00}$, we have several nonidentical Nash equilibria, some resulting in a unipolar world, others in a bipolar world, and exactly one in a multipolar world.

For instance, in the case $A = 10$, $A_1 = 40$, $A_2 = 5$, $A_3 = 0.3$, $A_{11} = 1$, $A_{12} = A_{2j} = 2$, $A_{3j} = 0.1$ ($j = 1, 2$), this value turns out to be $R_{00} = 2.5$ conditional units (c.u.).

If $R_0 > 2.5$ c.u., there is only one Nash equilibrium. In all cases, the world turns out to be multipolar: for $R_0 = 8$ c.u. as an example, we obtain a Nash equilibrium with $J_1 = J_2 = J_3 = 53.3$ c.u., $R^1 = 32$, $R^2 = 128$, $R^1_i = 10.66$, and $R^2_i = 42.66$ ($i = 1, 2, 3$); for $R_0 = 3$ c.u. and the remaining input data from the previous example, $J_1 = J_2 = J_3 = 20$ c.u., $R^1 = 12$, $R^2 = 48$, $R^1_i = 4$, and $R^2_i = 16$ ($i = 1, 2, 3$).

For $R_0 = 2.5$ c.u., there are four Nash equilibria, with a bipolar world in three and a multipolar world in one equilibrium.

If $R_0 < 2.5$ c.u., there are seven Nash equilibria: in three of them, the world is bipolar; in three others, it is unipolar; and in one equilibrium, the world is multipolar.

Generally speaking, in all the simulation experiments, it was possible to determine a value of R_0 with the above properties.

Thus, under a sufficiently large value of the initial resource quantity R_0, the world is multipolar regardless of the competition parameters; if the resource quantity is limited, a bipolar or unipolar world emerges, depending on the strength of the hierarchies and the values of the competition parameters.

Table 1

Input data and calculation results

No.	A	A_1	A_2	A_3	A_{11}	A_{12}	A_{21}	A_{22}	A_{31}	A_{32}	R_{00}
1	10	5	0.3	1	1	2	2	2	0.1	0.1	2.4
2	5	2	1	2	2	2	5	5	1	0.1	1.1
3	2	1	0.3	5	10	2	2	5	1	1	0.8
4	10	0.1	1	10	5	1	5	28	0.1	0.1	2.7
5	5	10	2	15	2	2	5	2	0.2	0.1	1.7
6	2	5	5	20	2	1	2	1	0.2	0.5	0.3
7	10	2	10	25	5	1	1	2	0.5	0.5	2.6
8	20	2	15	30	5	0.5	1	1	0.5	0.1	6.2
9	30	0.1	15	25	10	0.5	0.2	0.5	0.5	1	8.7
10	2	1	20	20	10	1	0.2	0.5	1	0.5	5.2
11	5	2	20	15	5	1	0.5	1	1	0.5	1.1
12	10	5	1	10	5	0.5	0.5	0.2	1	0.1	2.8
13	20	10	2	5	1	0.5	2	1	0.5	1	5.8
14	2	15	0.2	2	5	0.5	1	1	0.5	1	4.2
15	5	20	5	1	5	2	1	5	0.2	0.1	4.5
16	10	25	5	0.1	5	0.2	1	5	0.2	0.2	2.6
17	20	30	10	5	1	1	2	2	0.2	0.2	6.8
18	5	20	10	10	2	1	2	2	0.1	0.1	3.9
19	2	25	2	15	2	0.1	0.5	1	0.1	0.2	0.3
20	10	40	5	0.3	1	2	2	2	0.1	0.1	2.5

Figure 1 shows the dependence of R_{00} on the competition parameter A for the input data of Examples 1, 2, and 3 (the solid, dotted, and dash-dotted lines, respectively). The example numbers are indicated in the first column of Table 1. The value of R_{00} grows linearly or exponentially when increasing the parameter A.

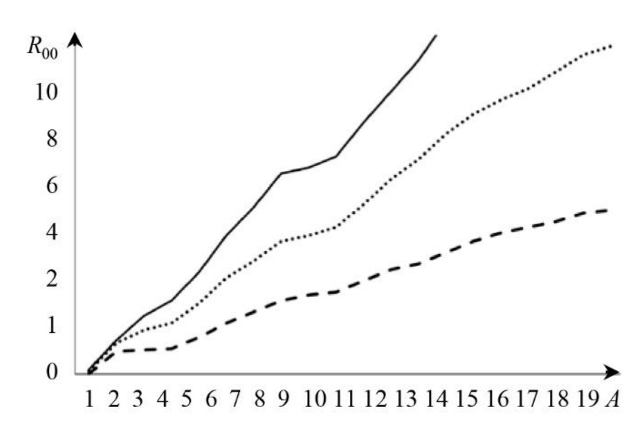

Fig. 1. The dependence of R_{00} on the competition parameter A.

The competition parameter A determines the value of R_{00}. For any values of the competition parameters A_i and A_{ij}, there exists a value of the initial resource quantity R_{00} that distinguishes the multipolar world case from that of seven nonidentical Nash equilibria corresponding to a uni/bi/multipolar world. However, the value of R_{00} depends only on the upper-level competition parameter A, being independent of the competition parameters A_i and A_{ij} at the lower and middle levels of the hierarchy.

Next, Fig. 2 presents the dependence of the number of Nash equilibria (M) on the initial resource quantity R_0 for the input data of Examples 1, 8, and 9 (the solid bold, dash-dotted, and dotted lines, respectively). All lines start at the point (0, 7) and initially coincide with the horizontal line $y = 7$; then they have a break, jumping over the point (x_1, 4) to the horizontal line $y = 1$; the breakpoint (x_1) depends on the input parameters of the example.

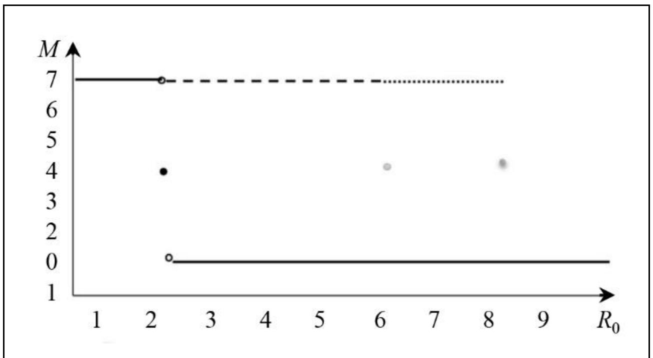

Fig. 2. The dependence of the number of Nash equilibria on the initial resource quantity R_0.

2.2. Examples of Global Power Hierarchies

Global power hierarchies represent associations of states whose relations are based on a hierarchy. Power hierarchies connect states, creating a complex mechanism of interactions among them, where some countries act as leaders, and others as followers.

When modeling such hierarchies, it is important to identify the model's input data correctly. To describe a state's power and influence, one should consider its economic strength, resource capabilities for building economic and defense potential, and military power for exerting coercive pressure on other countries.

Determining a state's power is a complex and multifaceted task involving numerous interrelated factors. A state's power is not limited to military strength or economic prosperity, but represents a comprehensive assessment of its potential and influence on the international stage, encompassing its military and economic power, political stability, demographic potential, and cultural influence on other countries. All these indicators are interconnected and influence one another.

One indicator of a state's power is the Global Power Index (GPI), developed by Gullibility Intelligence. GPI is a comprehensive indicator for assessing and comparing the power of states on the international stage. It takes into account a wide range of factors, from economic indicators and military power to demographic characteristics and technological development.

GPIs for the 193 UN member states can be found on the official website [15]. Table 2 combines the most powerful countries according to this index (as of January 1, 2025).

Based on Table 2, the input data were obtained for model (1)–(10) to calculate the strength of power hierarchies.

Table 2

GPIs for the world's leading countries (as of June 1, 2025)

Country	GPI value
China	12.95
USA	10.19
Russia	5.68
India	5.3
Brazil	3.31
Canada	2.37
Germany	2.09
Japan	1.89
Indonesia	1.74
France	1.7
Australia	1.64
Iran	1.63
Republic of Korea	1.52
Saudi Arabia	1.52

The following global power hierarchies (coalitions) were adopted, with some degree of convention:

– the first coalition: the USA (with Canada and Germany as influence agents, and Japan, France, Australia, and the United Kingdom as basic agents);

– the second coalition: China (with Pakistan and Brazil as influence agents, and Nigeria, Vietnam, Egypt, and DR Congo as basic agents);

– the third coalition: Russia (with India and Iran as influence agents, and Indonesia, the Republic of Korea, Venezuela, and Kazakhstan as basic agents).

The following input data were used for the calculations: $N = 3$, $n_i = 2$ ($i = 1, 2, 3$), $m_{ij} = 2$ ($i = 1, 2, 3$; $j = 1, 2$), $R_0 = 100$ c.u., $A = 100$, $A_1 = 10.19$, $A_2 = 12.05$, $A_3 = 5.68$, $A_{11} = 2.37$, $A_{12} = 1.89$, $A_{21} = 1.04$, $A_{12} = 3.29$, $A_{31} = 5.89$, and $A_{32} = 1.63$.

During the experiment, only one Nash equilibrium corresponding to a multipolar world was found.

In this case, $J_1 = J_2 = J_3 = 666.7$ c.u., $R^1 = 400$, $R^2 = 600$, $R^1{}_i = 133$, and $R^2{}_i = 533$ ($i = 1, 2, 3$).

Indeed, the world is currently becoming multipolar, with centers in China, the USA, and Russia, taking their allies into account.

Table 3 provides a comparison of the aggregate macroeconomic indicators for the USA, Russia, and China from 1990 to 2025; for details, see [16–18]. The indicators include gross domestic product, inflation and unemployment rates, labor productivity, etc.

Based on Table 3 and the findings for the other countries [16–18], calculations were performed for all input parameters of global power hierarchies (with the USA, China, and Russia as the centers) for the period 1990–2025, $N = 3$, $n_i = 2$ ($i = 1, 2, 3$), $m_{ij} = 2$ ($i = 1, 2, 3$; $j = 1, 2$), and the input data from Table 4.

In 1990, 2020, and 2025, according to the calculations, one Nash equilibrium was obtained, and the world is multipolar; in 1995, 2000, 2005, 2010, and 2015, seven Nash equilibria were found, three corresponding to bipolarity, three to unipolarity, and one to multipolarity. In a unipolar world, there exists a single dominant center of power, influence, and decision-making. This is usually due to the fact that one country or bloc of countries (in the example under consideration, the USA) has significantly greater influence on global political, economic, and military processes compared to other states. In such a system, the interests of other countries are possibly disregarded, causing the emergence of conflicts. Over time, the world has become multipolar with the growing influence of such countries as China and Russia, leading to a more complex and competitive international political landscape.

The existence of several Nash equilibria for the same period indicates that, under given input data, the world may be unipolar, bipolar, or multipolar with varying probabilities. In this case, there are several stable strategies that different countries can choose, and each of these strategies is optimal from the standpoint of each country, considering the strategies of the others. In this case, control subjects face a dilemma in choosing an appropriate strategy. As a result, both uncertainty and instability grow in their behavior, as each can expect others to choose different strategies. Therefore, control subjects need coordination mechanisms to select one of the equilibria. Overall, the existence of several Nash equilibria underlines the complexity of strategic interaction between countries and the importance of considering the context and mechanisms that may influence strategy selection. All these features are currently observed in the behavior of individual countries.

Table 5 presents the values of R_{00} by year for the period 1990–2015.

Table 3

GPI values for the world's three leading countries

Country	Year						
	1990	1995	2000	2010	2015	2020	2025
USA	6.2	8.1	6.3	8.5	8.6	7.1	10.19
China	4.5	6.2	4.7	10.7	11.5	12.0	12.95
Russia	3.1	2.7	3.5	5.3	5.5	5.6	5.68

Table 4

Input parameters for 1990–2025

Year	R_0	A	A_1	A_2	A_3	A_{11}	A_{12}	A_{21}	A_{22}	A_{31}	A_{32}
1990	8	20	6	5	3	1.2	1.1	0.8	0.6	0.76	1.03
1995	11	27	8	6	2	1.8	1.6	1.0	0.9	0.65	0.91
2000	7	21	6.3	5.4	2.9	1.7	1.9	1.8	1.9	1.1	0.9
2005	5	26	7.1	8.7	2.9	2.0	2.5	1.9	2.3	1.1	0.86
2010	13	29	7.5	10.4	3.2	2.3	2.9	2.4	2.7	1.3	1.0
2015	15	30	8.1	12.5	3.4	3.1	3.2	3.2	2.9	1.5	1.2
2020	18	32	8.8	13.3	3.7	3.3	3.5	3.4	3.4	1.7	1.5
2024	22	34	9.1	13.9	3.9	3.7	3.9	3.8	3.8	1.9	1.8

Table 5

The values of R_{00} by year, 1990–2015

	Year							
	1990	1995	2000	2005	2010	2015	2020	2025
R_{00}	6.8	11.6	8.1	6.8	14.3	16.2	19.2	23.1

Note that since 2005, the resource quantity for which the number of Nash equilibria changes has been steadily increasing.

CONCLUSIONS

Based on the above model, it has been shown that the main condition for a multipolar world is a large value of the initial resource quantity. If this value is sufficiently large, then regardless of the competition parameters, the world becomes multipolar. However, in practice, all resources are limited, so a bipolar or unipolar world emerges, depending on the strength of power hierarchies and the values of the competition parameters. This conclusion is confirmed by the entire human history. A key prerequisite for multipolarity lies in changes in the global economic structure. The growing economies of several countries, such as China, India, and Brazil, are becoming more and more influential on the international stage. These states continue to improve their economic performance and seek to take a more active role in world politics.

As confirmed by the simulation experiments, prior to the collapse of the USSR, the world was characterized by a bipolar system of power hierarchies, with the USSR and the USA as the centers. Following the collapse of the USSR, a unipolar system centered in the USA emerged; subsequently, it evolved into a bipolar system centered in the USA and China. Finally, in recent years, the global system has been transforming into a multipolar system of power hierarchies, with the USA, China, and Russia as the centers. Other states may also become centers of power hierarchies.

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