ESTIMATING THE FRAGILITY OF PI CONTROLLERS USING THE TECHNIQUE OF LINEAR MATRIX INEQUALITIES AND VISUALIZATION TOOLS

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Abstract. This paper considers the problem of estimating the fragility of a PI controller for a linear control system: it is required to construct admissible variation regions for the controller gains in which the closed-loop system remains stable. The method proposed below is based on the technique of linear matrix inequalities (LMIs); it reduces the original estimation problem to non-complex convex optimization problems. The non-fragility radius of a PI controller and a non-fragile bounding ellipsoid for rejecting nonrandom bounded exogenous disturbances are introduced. A simple method is described to estimate the stabilizability region as a whole. According to the numerical simulation results provided, the approach is effective. Note that this approach can be easily extended to more general problems, in particular, to the fragility analysis of PID controllers and PID controllers with an aperiodic filter.

Keywords: linear control system, PI controller, stability, fragility, non-fragility, linear matrix inequality (LMI), bounding ellipse, ellipse of admissible perturbations.

INTRODUCTION

As is well known, PI/PID controllers are the most common type of automatic controllers. The reasons are their suitability for solving most practical problems, their low cost, and their simplicity: tuning requires correctly choosing only a few gains, i.e., three for a PID controller, two for a PI controller, and (sometimes) four, as in the case of a PID controller with a filter. However, this choice is far from trivial, so in practice, controllers are often tuned manually, based on an intuitive understanding of the corresponding process.

At the same time, when implementing and tuning PI/PID controllers, one inevitably faces uncertainty due to the inaccurate technical realization of a controller or the need to tune its parameters during operation. Small variations in the controller's gain may significantly deteriorate control performance or even make the closed-loop system unstable; see illustrative examples in the seminal work [1]. This effect was called *fragility*; in subsequent publications, it was investigated in various problem statements [2–7].

The phenomenon of fragility is well known from engineering practice, and various frequency-domain methods have been developed to prevent it. Here, following the state-space approach, by endowing a controller with the non-fragility property, we mean robustness with respect to its parameter variations; this term allows distinguishing uncertainty in a controller from uncertainty in model parameters (in the system matrices). Sometimes the term *"rough controller"* is used in the literature, dating back to [8]. However, in this context, the concept of fragility is more appropriate: the matter is that roughness commonly implies a small change in the system's properties, whereas fragility does not necessarily assume small parameter variations.

This paper has a similar framework, addressing the issues of fragility of PI controllers and, primarily, the estimation of admissible variation regions for PI controller's gains in which the closed-loop system remains stable. The D-partition (D-decomposition) method is widely used for analyzing stability regions dependent on two parameters [9]. Meanwhile, we utilize a different approach below, based on the technique of linear matrix inequalities (LMIs) [10, 11].

It has proven effective for studying uncertain systems, and it was applied in [12–14] for PI/PID controller design. The choice of this technique can be explained by two factors as follows. First, D-partition yields the two-dimensional stability region of a PI controller in analytical form; however, for a PID controller, the construction of the corresponding three-dimensional region is fraught with insurmountable difficulties (with the exception of certain special cases; see the details in [9]). In contrast, the approach discussed in this paper can be easily extended to the case of PID controllers. Second, the technique of LMIs can also be applied to *design* non-fragile PI controllers.

Much attention in the paper is devoted to the visualization of the resulting estimates of non-fragility. More precisely, a series of numerical experiments is conducted to estimate the fragility of PI controllers; simple estimates of the admissible variation region of the stabilizing controller's gains are obtained and plotted graphically; and a simple method is described to estimate the stabilizability region as a whole. The results of these numerical experiments are discussed in detail. Finally, the concept of a non-fragile bounding ellipse is considered, with application to the problem of rejecting bounded exogenous disturbances.

Note that the approach developed below can also be extended to the fragility analysis of PID controllers. In this case, one deals with ellipsoidal estimates, while the conceptual framework remains largely unchanged.

The preliminary results of this research were presented at the 21st All-Russian School-Conference of Young Scientists "Large-Scale Systems Control" [15].

The following notation is used hereinafter: $\|\cdot\|$ is the Euclidean norm of a vector; $^{\mathrm{T}}$ is the transpose symbol; I is an identity matrix of appropriate dimensions; $\lambda_i(A)$ are the eigenvalues of a matrix A; and $\mathbb{S}^n$ is the space of symmetric matrices of order n. All matrix inequalities are understood in the sense of positive/negative definiteness of corresponding matrices.

1. PROBLEM STATEMENT AND SOLUTION APPROACHES

Consider a linear Single-Input Single-Output (SISO) system described by

$$\begin{aligned} &\dot{x} = Ax + bu + Dw, \quad x(0) = x_0, \\ &y = c^{\mathrm{T}} x, \\ &z = Cx, \end{aligned} \tag{1}$$

Where $A \in \mathbb{R}^{n\times n}$, $b \in \mathbb{R}^n$, $D \in \mathbb{R}^{n\times m}$, $c \in \mathbb{R}^n$, $C \in \mathbb{R}^{r\times n}$, with the state vector $x(t) \in \mathbb{R}^n$, the observed output $y(t) \in \mathbb{R}$, the controlled output $z(t) \in \mathbb{R}^r$, a bounded exogenous disturbance $w(t) \in \mathbb{R}^m$ satisfying the constraint

$$\|w(t)\| \le 1 \quad \forall t \ge 0,$$

and the control input $u(t) \in \mathbb{R}$ in the form of a PI controller

$$u = -k_p y - k_i \int_0^t y(\tau)\, d\tau. \tag{2}$$

Let a given (nominal) PI controller (2) with some gains

$$K_0 = \begin{pmatrix} k_p \\ k_i \end{pmatrix} \in \mathbb{R}^2$$

stabilize system (1). Perturbing the components of the vector K_0, we consider the vector $K_{\text{stab}} = K_0 + \delta$ of the perturbed controller's gains. Assume that the perturbations are ellipsoidally bounded, i.e., they belong to an ellipse with a certain matrix $R \succ 0$ and center at the origin:

$$\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix} \in \mathcal{E}(R) = \left\{ \delta \in \mathbb{R}^2 : \ \delta^{\mathrm{T}} R^{-1} \delta \le 1 \right\}.$$

In addition, we introduce the designations

$$A_0 = \begin{pmatrix} A & 0 \\ c^{\mathrm{T}} & 0 \end{pmatrix}, \quad F = \begin{pmatrix} -b \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} c^{\mathrm{T}} & 0 \\ 0 & 1 \end{pmatrix},$$

$$A_c(K) = A_0 + F K^{\mathrm{T}} H,$$

$$D_c = \begin{pmatrix} D \\ 0 \end{pmatrix}, \quad C_c = \begin{pmatrix} C & 0 \end{pmatrix}.$$

The problem is to find an elliptical region of admissible perturbations $\delta: \|\delta\| \le r$ of the gains K_0 of the PI controller (2) that preserve the stability of the closed-loop system (1). The value of r is called the *non-fragility radius* of the PI controller with the gains K_0. One approach to solving this problem is to maximize the length of the minor axis of the ellipse $\mathcal{E}(R)$ of admissible uncertainties. In other words, it is necessary to maximize the radius of the circle containing the admissible perturbations of the controller's gains. In particular, the following result is valid.

Lemma 1 ([16]). *Let $\hat{\gamma}$, $\hat{R}_{\text{axis}}$ be the solution of the semidefinite programming problem*

$$\max \gamma$$

subject to the constraints

$$\begin{pmatrix} A_c^{\mathrm{T}}(K_0)Q + QA_c(K_0) + H^{\mathrm{T}}RH & QF \\ F^{\mathrm{T}}Q & -I \end{pmatrix} \prec 0,$$

$$Q \succ 0,\ R \succeq \gamma I,$$

where optimization is performed with respect to the matrix variables $Q \in \mathbb{S}^{n+1}$ and $R \in \mathbb{S}^2$ and the scalar variable γ.

Then system (1) closed by the stabilizing PI controller (2) with the gains K_0 remains stable under all their perturbations $\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix}$ such that

$$\delta^{\mathrm{T}} \hat{R}_{\text{axis}}^{-1} \delta < 1,$$

and, in particular, those satisfying

$$\|\delta\| < \sqrt{\hat{\gamma}}\,.$$

Thus, the non-fragility radius r of a given PI controller is defined by the value $\sqrt{\hat{\gamma}}$.

The region of admissible perturbations δ can be also estimated by maximizing the *area* of the ellipse $\mathcal{E}(R)$ as follows.

Lemma 2 ([16]). *Let $\hat{R}_{\text{area}}$ be the solution of the convex optimization problem*

$$\min(-\text{logdet}\, R)$$

subject to the constraints

$$\begin{pmatrix} A_c^{\mathrm{T}}(K_0)Q + QA_c(K_0) + H^{\mathrm{T}}RH & QF \\ F^{\mathrm{T}}Q & -I \end{pmatrix} \prec 0,$$

$$Q \succ 0,\ R \succ 0,$$

where optimization is performed with respect to the matrix variables $Q \in \mathbb{S}^{n+1}$ and $R \in \mathbb{S}^2$.

Then system (1) closed by the stabilizing PI controller (2) with the gains K_0 remains stable under all their perturbations $\delta = \begin{pmatrix} \delta_1 \\ \delta_2 \end{pmatrix}$ such that

$$\delta^{\mathrm{T}} \hat{R}_{\text{area}}^{-1} \delta < 1\,.$$

2. NON-FRAGILE BOUNDING ELLIPSOID

This section considers the problem of rejecting bounded exogenous disturbances affecting system (1). It consists in minimizing the effect of such disturbances on the system's output:

$$\max_{t \ge 0} \max_{\|w\| \le 1} \|z(t)\|\,.$$

It is difficult to obtain an exact solution. However, an LMI-based approach proposed in [11] yields upper bounds for the maximax value. Within this approach, a key concept is that of *an invariant ellipsoid*; for convenience, we recall it here. Consider a system described by

$$\begin{aligned} &\dot{x} = Ax + Dw,\ \ x(0) = x_0, \\ &z = Cx, \end{aligned} \tag{3}$$

with a Hurwitz matrix A and an exogenous disturbance w. An ellipsoid of the form

$$\mathcal{E} = \left\{ x \in \mathbb{R}^n :\ \ x^{\mathrm{T}} P^{-1} x \le 1 \right\},$$

centered at the origin, is said to be invariant if

$$x_0 \in \mathcal{E} \Rightarrow x(t) \in \mathcal{E} \quad \forall t \ge 0.$$

In other words, if the initial point x_0 belongs to an invariant ellipsoid, the entire state trajectory of system (3) will remain inside this ellipsoid under all admissible disturbances w affecting the system. A closely related concept is that of a *bounding ellipsoid*, defined as

$$\mathcal{E}_z = \left\{ z \in \mathbb{R}^r :\ \ z^{\mathrm{T}} \left(CPC^{\mathrm{T}}\right)^{-1} z \le 1 \right\}.$$

Thus, a bounding ellipsoid contains the output z of system (3) under all admissible disturbances affecting it. In this context, it is natural to minimize the bounding ellipsoid by some criterion. A common one is $\operatorname{tr} CPC^{\mathrm{T}}$, representing the sum of the squared semi-axes of the bounding ellipsoid.

Now we formulate the following problem: it is required to construct the minimal ellipsoid containing the output z of system (1) under all admissible perturbations δ of the controller's gains K such that $\|\delta\| \le \gamma < r$. It will be called a *non-fragile bounding ellipsoid* corresponding to the non-fragility level γ. One method for finding this ellipsoid is given below; for details, see the paper [16].

Theorem 1. *Let* $\hat{P}$ *be the solution to the semidefinite programming problem*

$$\min \operatorname{tr} C_c P C_c^{\mathrm{T}}$$

subject to the constraints

$$\begin{pmatrix} A_c(K_0)P + PA_c^{\mathrm{T}}(K_0) + \alpha P + \frac{1}{\alpha}D_c D_c^{\mathrm{T}} + \varepsilon\gamma^2 FF^{\mathrm{T}} & PH^{\mathrm{T}} \\ HP & -\varepsilon I \end{pmatrix} \preceq 0,$$

$$P \succ 0,$$

where optimization is performed with respect to the matrix variable $P \in \mathbb{S}^{n+1}$, *the scalar variable* ε, *and the scalar parameter* $\alpha > 0$.

Then the ellipsoid with the matrix

$$C_c \hat{P} C_c^{\mathrm{T}}$$

is a non-fragile bounding ellipsoid for the output z of system (1) *with* $x_0 = 0$ *closed by the stabilizing PI controller* (2) *with the gains* K_0, *corresponding to the admissible non-fragility level* γ.

The optimization problem in Theorem 1 is a parameterized semidefinite programming problem. According to numerical simulations, minimization with respect to the parameter α is always convex. (Although a rigorous proof of this fact remains an open problem.) Hence, minimization with respect to α can be performed by using much faster methods of one-dimensional convex optimization (e.g., the golden section method) rather than enumerative search on a one-dimensional grid.

3. NUMERICAL EXPERIMENTS AND VISUALIZATION

Example 1. Consider a control system described by

$$\dot{x} = \begin{pmatrix} -1.3690 & 1 \\ -0.1488 & 0 \end{pmatrix} x + \begin{pmatrix} -0.3214 \\ 0.4018 \end{pmatrix} u + \begin{pmatrix} -0.3214 \\ 0.4018 \end{pmatrix} w,$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x,$$

$$z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x,$$

with the state vector x, the control input u, the observed output y, the controlled output z, and a bounded exogenous disturbance $\|w\| \le 1$. Let this system be closed by a stabilizing PI controller with the gains

$$K_{pi} = \begin{pmatrix} 0.832 \\ 0.120 \end{pmatrix};$$

we find the matrices of the ellipses of admissible uncertainties

$$\hat{R}_{\text{axis}} = \begin{pmatrix} 0.1021 & 0 \\ 0 & 0.0144 \end{pmatrix}, \ \hat{R}_{\text{area}} = \begin{pmatrix} 0.6343 & 0 \\ 0 & 0.0135 \end{pmatrix}$$

and the non-fragility radius

$$r = 0.12\,.$$

In all examples of this section, the computations were carried out in MATLAB using the CVX package [17].

In Fig. 1, the dashed line corresponds to the ellipse with the matrix $\hat{R}_{\text{axis}}$; the solid line, to the ellipse with the matrix $\hat{R}_{\text{area}}$; the gray solid line, to the circle of radius *r*. Also, the region of stabilizing controllers constructed using D-partition is shown by the bold solid line. Naturally, these elliptic estimates belong to the region of stabilizing controllers.

Using the approach discussed, one can obtain simple inner approximations for the set of stabilizing controllers by iteratively repeating the ellipsoidal estimation procedure. (The point corresponding to a nominal PI controller is taken as the starting point, and the next point is generated randomly on the boundary of the ellipse built.) The results of 40 iterations are presented in Fig. 2.

Now, for the system under consideration, we utilize Theorem 1 to find a non-fragile bounding ellipse for the output *z* corresponding to the admissible non-fragility level $\gamma = 0.02$, and a bounding ellipse without the non-fragility requirements. The first of these has the matrix

$$C_c P C_c^{\mathrm{T}} = \begin{pmatrix} 4.9971 & 5.0735 \\ 5.0735 & 9.0761 \end{pmatrix}$$

(its semi-axes are 1.25 and 3.54), see the solid line in Fig. 3; and the minimal bounding ellipse has the matrix

$$C_c \tilde{P} C_c^{\mathrm{T}} = \begin{pmatrix} 3.5414 & 3.5712 \\ 3.5712 & 6.2697 \end{pmatrix}$$

(its semi-axes are 1.04 and 2.95), see the dashed line therein. The computations were executed on a laptop with an Intel Core i7-1165G7 CPU (the 11th generation, 2.8 GHz) and 16 GB RAM; the computation time for the bounding ellipse was 1.5–2 s)

Also, Fig. 3 demonstrates the trajectories of the controlled output *z* under admissible perturbations in the nominal controller K_{pi} of level $\gamma = 0.02$. ♦

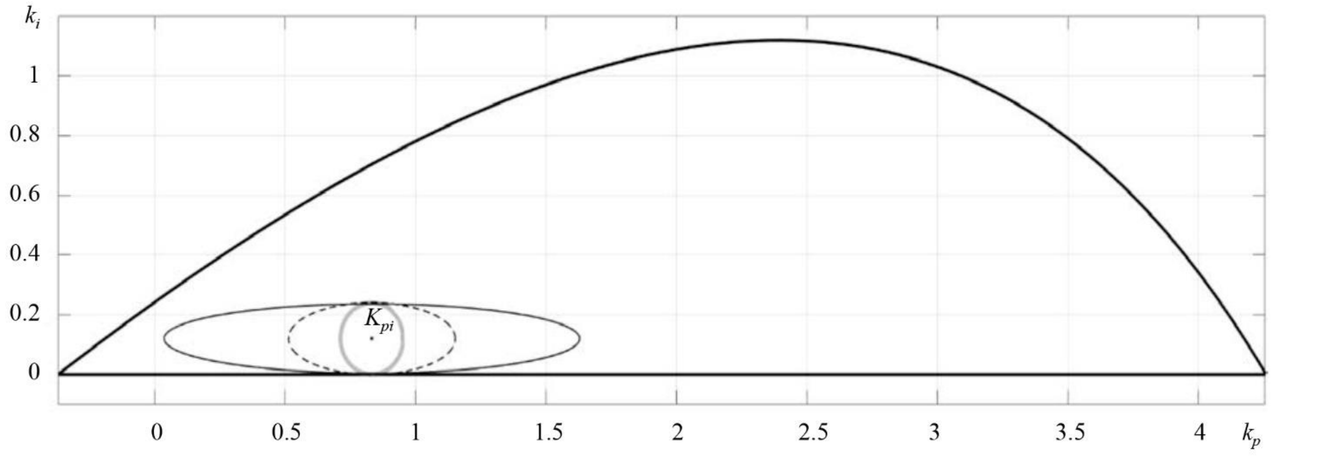

Fig. 1. The fragility estimate for the PI controller with the gains K_{pi} (Example 1).

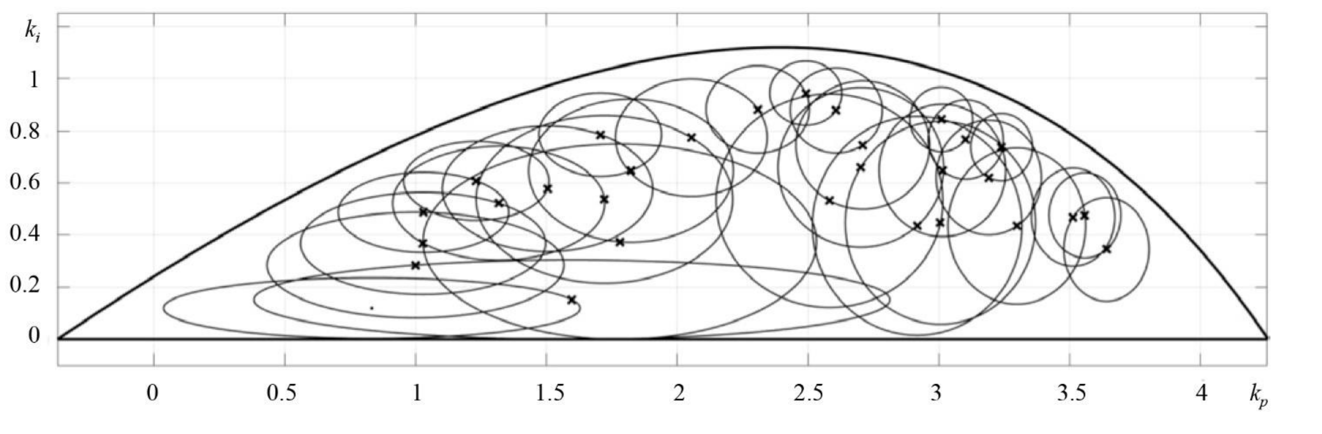

Fig. 2. The inner approximation of the set of stabilizing controllers (Example 1).

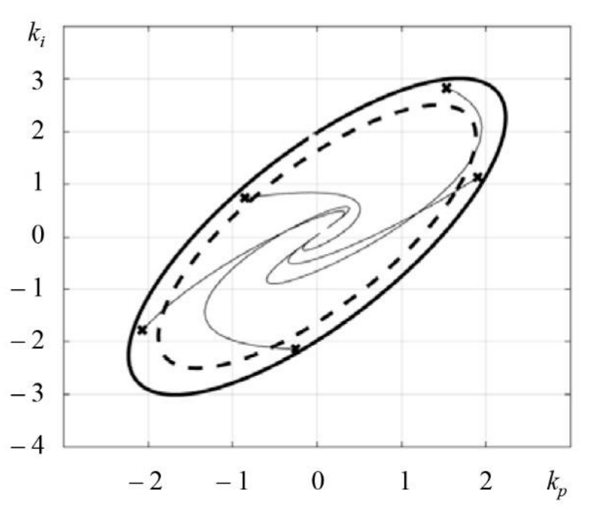

Fig. 3. The non-fragile bounding and minimal bounding ellipses (Example 1).

Example 2. Consider a control system described by

$$\dot{x} = \begin{pmatrix} -2 & 1 \\ -1 & 0 \end{pmatrix} x + \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix} u + \begin{pmatrix} -0.4 \\ -0.3 \end{pmatrix} w,$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} x,$$

$$z = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x,$$

with the state vector x, the observed output y, the controlled output z, the control input u, and a bounded exogenous disturbance $\|w\| \le 1$. Let this system be closed by randomly selected stabilizing PI controllers with the gains

$$K_0 = \begin{pmatrix} 1.713 \\ 2.556 \end{pmatrix}, \quad K_1 = \begin{pmatrix} 3.999 \\ 96.265 \end{pmatrix}, \quad K_2 = \begin{pmatrix} 44.388 \\ 61.927 \end{pmatrix},$$

$$K_3 = \begin{pmatrix} 57.561 \\ 92.464 \end{pmatrix}, \quad K_4 = \begin{pmatrix} 86.593 \\ 5.046 \end{pmatrix}, \quad K_5 = \begin{pmatrix} 100 \\ 100 \end{pmatrix}.$$

For each controller above, the ellipses of admissible uncertainties were obtained by maximizing the smallest semi-axis (the dashed line in Fig. 4) and the area (the solid line therein), and the corresponding non-fragility radii were determined:

$$r_0 = 2.4745,\ \ r_1 = 5.9376,\ \ r_2 = 45.5202,$$

$$r_3 = 58.8467,\ \ r_4 = 5.0460,\ \ r_5 = 100.$$

(The corresponding circles are plotted in gray.)

Figure 5a shows the bounding ellipse for the output z corresponding to the controller with the gains K_0. As before, the dashed line corresponds to the minimum bounding ellipse (for $\gamma = 0$). Also, the output trajectories of the closed-loop system with the non-fragility level $\gamma = 1$ are provided.

Figure 5b shows the bounding ellipses for the output z corresponding to the controller with the gains K_5 for the non-fragility levels $\gamma = 0$ (the dashed line) and $\gamma = 10$ (the solid line). ♦

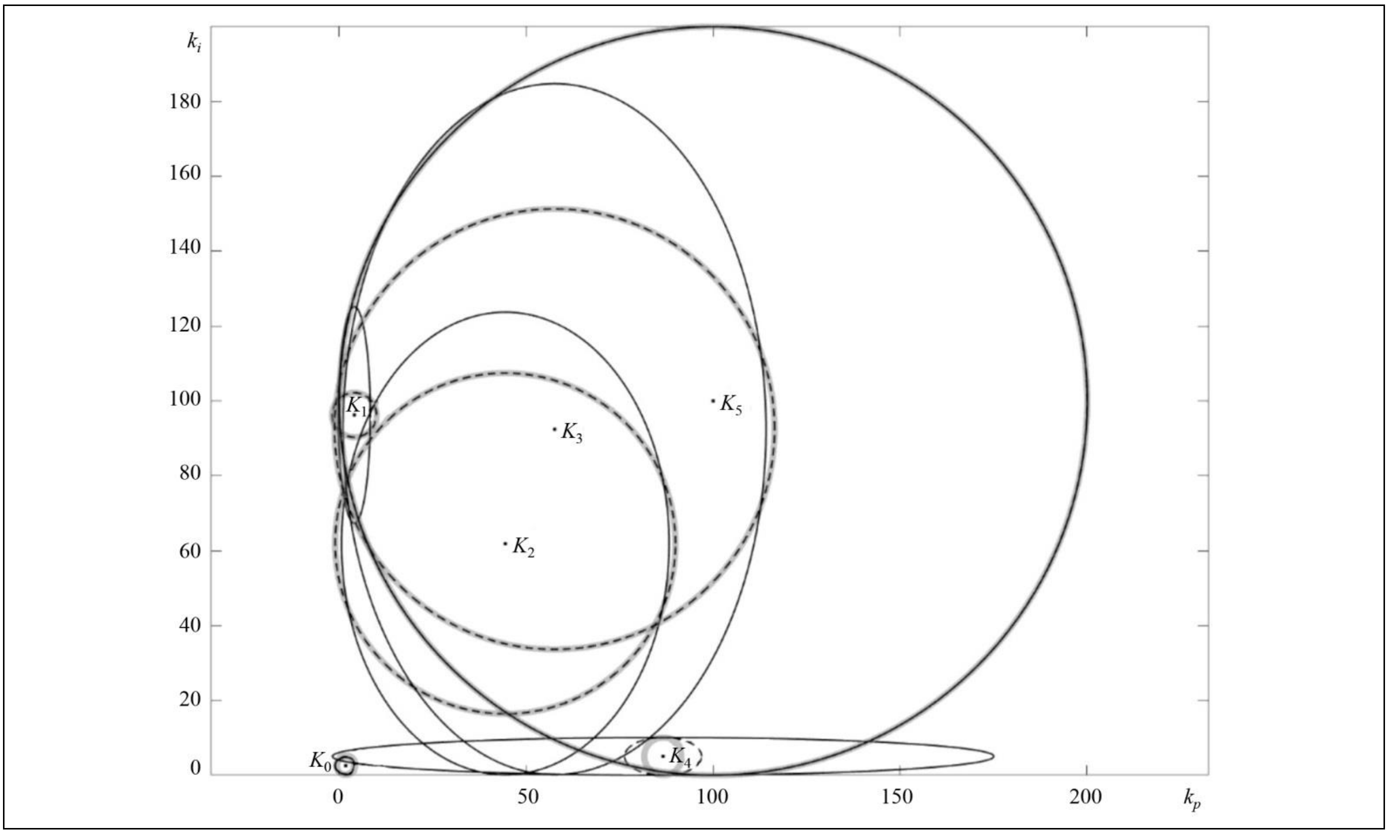

Fig. 4. The regions of stabilizing PI controllers (Example 2).

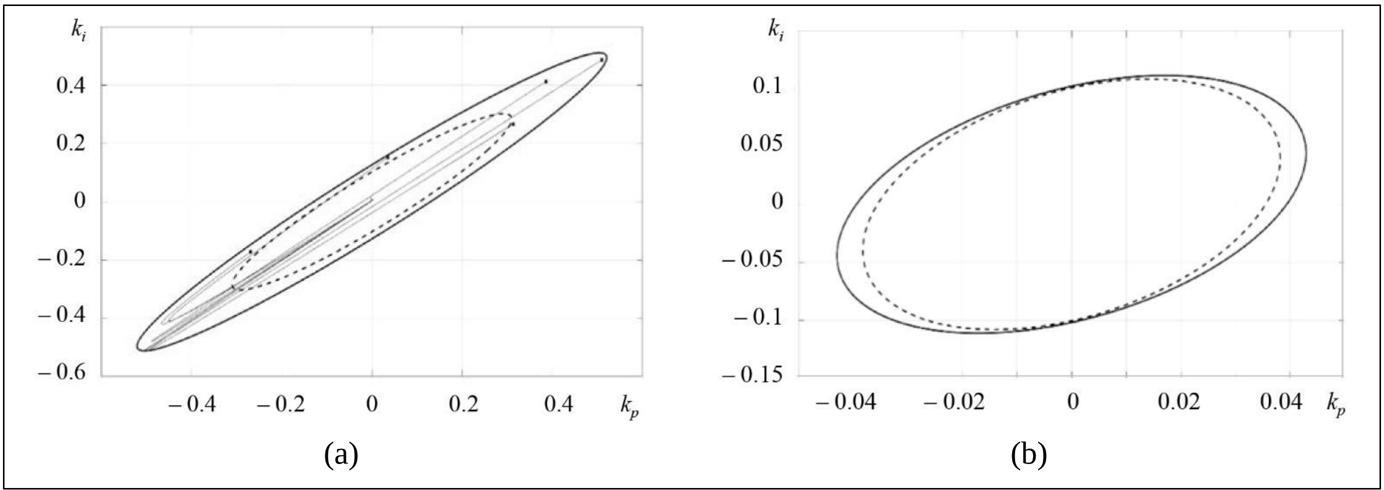

Fig. 5. The non-fragile bounding ellipses (Example 2) for PI controllers with the gains (a) K_0 and (b) K_5.

Example 3. Consider the fourth-order control system

$$\dot{x} = \begin{pmatrix} -15 & -70 & -120 & -64 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} u,$$

$$y = \begin{pmatrix} 0 & 0 & 0 & 64 \end{pmatrix} x,$$

with the state vector x, the observed output y, and the control input u. Let this system be closed by PI controllers with the gains

$$K_0 = \begin{pmatrix} -10 \\ 1 \end{pmatrix} \text{ and } K_1 = \begin{pmatrix} -13 \\ 1.5 \end{pmatrix}.$$

The corresponding elliptical regions of admissible perturbations, as well as the circles with non-fragility radii

$$r_0 = 0.2874, \quad r_1 = 0.0994,$$

are presented in Fig. 6.

Consider the perturbed PI controllers with the gains

$$K_2 = \begin{pmatrix} -9 \\ 1 \end{pmatrix} \quad и \quad K_3 = \begin{pmatrix} -7.5 \\ 0.85 \end{pmatrix},$$

located within the above regions (Fig. 6). The eigenvalues of the closed-loop system matrices with these controllers are

$$\lambda_{K_2} = \begin{pmatrix} -2.0624 \pm j7.5523 \\ -0.1042 \pm j0.7846 \\ -1.6668 \end{pmatrix}, \quad \lambda_{K_3} = \begin{pmatrix} -2.7747 \pm j7.1039 \\ -0.1352 \pm j0.7338 \\ -1.6801 \end{pmatrix}.$$

Accordingly, they are stabilizing.

Figure 7 shows the resulting inner approximation of the set of stabilizing controllers. ♦

CONCLUSIONS

In this paper, we have illustrated, through a series of examples, an LMI-based approach to estimating the fragility of a given stabilizing PI controller. The approach builds guaranteed estimates for admissible perturbations of the controller's gains. The results of numerical experiments and their simple visualization have demonstrated the effectiveness of this approach.

The lines of further research include generalizing the above results to the case of PID controllers, which will require solving the corresponding problems in the three-dimensional space. And, of course, a promising line is to pass from the analysis to the design of non-fragile PI/PID controllers; and visualization methods can become an effective tool for selecting optimal settings.

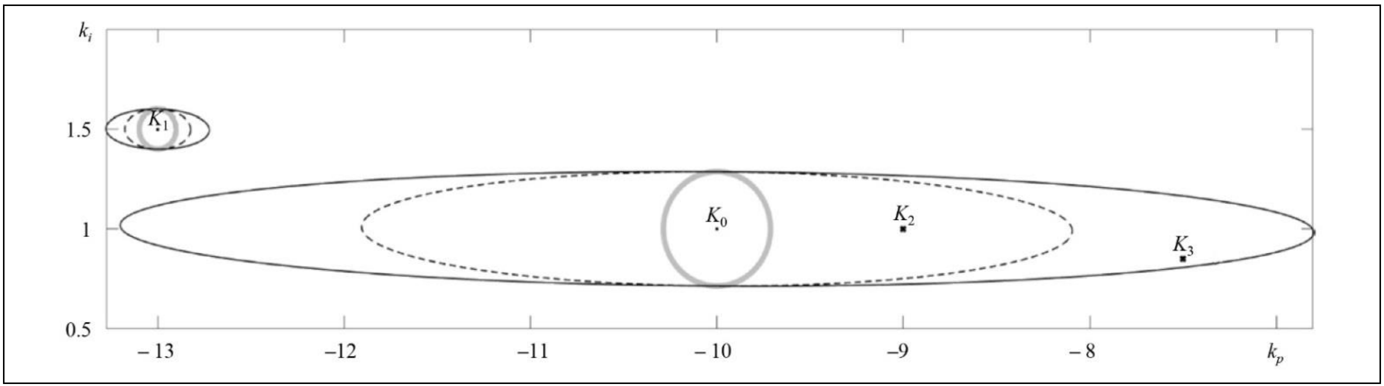

Fig. 6. The regions of stabilizing PI controllers (Example 3).

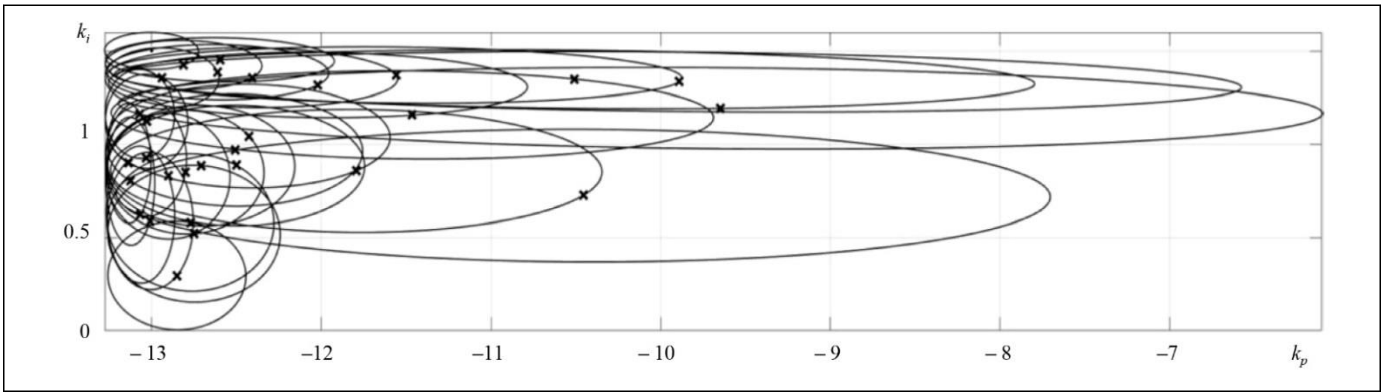

Fig. 7. The inner approximation for the set of stabilizing controllers (Example 3).

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