ADAPTIVE CONTROL OF A SCALAR PLANT IN THE INPUT–OUTPUT FORM BASED ON THE IDENTIFICATION–APPROXIMATION APPROACH

S. P. Kruglov

Irkutsk State Transport University, Irkutsk, Russia

✉ kruglov_s_p@mail.ru

Abstract. This paper considers a scalar plant with current parametric uncertainty in which only the input and output are measured. For such plants, an adaptive control design approach based on simplified adaptability conditions is presented. The approach refers to indirect self-tuning control using the current parametric identification algorithm and an implicit reference model. The tuned model structure in the identification algorithm is selected as simple as possible, corresponding to the main motion of the controlled plant and an elementary dynamic link or links. As a result, the current estimates in this model approximate the plant's motion, which is confirmed by the convergence criterion of the identification residual. Also, it is required to satisfy definite requirements for the current parameter estimates. The estimates, even if imprecise, are used to construct a control law ensuring given properties of the closed-loop control system. This postulate is interpreted as a refinement of the well-known certainty equivalence principle except for the asymptotically accurate parameter estimation requirement to achieve adaptive properties of a self-tuning control system in output-feedback control problems. The main relationships are given for an example when the plant's dominant dynamics are close to an oscillatory process without an additional time delay. The identification algorithm is applied in the form of a recurrent least-squares method with a forgetting factor and some modifications. Two illustrative examples of adaptive control system design are provided: control of the angular motion of an overhead crane and counteraction to the vibrations of an elastic three-mass drive. The approach under consideration is called the identification–approximation one. The possibilities and ways of its further improvement are outlined.

Keywords: adaptive control with self-tuning, current parametric uncertainty, current parametric identification algorithm, certainty equivalence principle, convergence of parameter estimates.

INTRODUCTION

Adaptive control systems have been developed by the scientific community for over 70 years, beginning in the 1950s. Outstanding results have been obtained in the substantiation of theoretical postulates and applications of adaptive control methods. These include self-tuning adaptive control systems, control systems with a reference model, predictive control with self-tuning, sliding mode control, neurocontrollers, control with fuzzy logic, etc. However, many researchers admit that adaptive control methods under current uncertainty (without a predetermined program or parameter setting, even if automatic) are modestly used in practice [1–5].

At the same time, practical tasks require the opposite, due to the wide development of automation tools in modern applications of various fields of technology and the need for further development. As we believe, one reason is the complexity and, sometimes, the practical unattainability of the formulated adaptability conditions of a closed-loop control system. For example, consider self-tuning systems, i.e., those based on the identification approach (systems with a tunable model); the stumbling block here is the so-called *certainty equivalence principle*, which requires asymptotically accurate estimates of unknown parameters from the identifier (e.g., see [6]). In practical conditions, this is extremely difficult to achieve due to a mismatch between the structure of the model being tuned and the plant, uncontrolled disturbances, and noise. In addition, a persistently exciting input signal is required, etc.

Much research into adaptive control systems is devoted to systems based on the MIT rule and stability theory: the use of Lyapunov functions, Popov's hyperstability criterion, and other methods; for example, see [7, 8]. However, such approaches often yield tuning algorithms with constant coefficients that are not obvious to select for particular problems and the discrete implementation, the algorithms have weak identifying properties, and so on. No doubt, multiple modifications related to increasing the robustness of adaptive control, namely, introduction of estimate-feedback loops in the adaptation algorithm, assignment of Lyapunov barrier functions, etc. (e.g., see [5]) contribute to practical implementability but do not completely settle this issue. The latter conclusion follows from the considerations above.

Moreover, many works of the approach with indirect control (when first the parameters of a plant are estimated by an algorithm derived from a Lyapunov function, and then a control law is designed based on them) also require the persistent excitation of the regressor and asymptotically accurate estimates [9, *p. 296*, 10–12].

This paper is an attempt to simplify the adaptive control system design scheme under current parametric uncertainty using self-tuning in the form of a parametric identification algorithm and simplified adaptability conditions. In particular, it refines the certainty equivalence principle by reducing the requirements for the identifier (eliminating the requirements for asymptotically accurate parameter estimates, persistent excitation, etc.) in the problems of output-feedback control. This is demonstrated on an example of a scalar plant in which only the input and output are measured. This study is a logical continuation of the earlier publications [13, 14] and others.

1. PROBLEM STATEMENT

Consider a mathematical model of a scalar plant in the input–output form:

$$\begin{cases} \dot{v} = f_1(v, u, t), \quad v(t_0) = v_0 \\ y = f_2(v, t), \end{cases} \tag{1}$$

where v is the finite-dimension state vector with an initial value v_0; u is a scalar control action (the input); y is the scalar output; t denotes the current time with an initial instant t_0. In engineering applications, the structures of the dependencies $f_1(v, u, t)$ and $f_2(v, t)$ are usually known whereas their parameters at the current time are unknown. These structures and general data about the plant can often be used to obtain a priori information about the qualitative relationship of some parameters and estimate their approximate values.

Assume that in principle, the plant (1) allows achieving a given control objective, which particularly means its controllability. In addition, it has a low-frequency operating range with a known upper limit (denoted by Ω), which matches most engineering applications. Further analysis will be restricted to the class of minimum-phase (input-stable) plants.

Let the dynamics of the plant (1) be close to those of some elementary link from the theory of automatic control or the simplest set of such links, in the range of used controls on a limited time interval and in the operating frequency range. This paper does not consider delayed dynamics (i.e., the presence of a transport delay link) since, in this case, the approach discussed below requires additional solutions.

As an example, the presentation below concerns the case when the input–output relationship of the plant (1) is close to oscillatory dynamics (an oscillatory link). The criterion for no time delays is a small phase delay of the plant's output under low frequencies of the input. This behavior is characteristic of some mechanical systems. For this example, the plant (1) can be approximated by a quasi-stationary link of the second order:

$$\left[p^2 + a_1(t)p + a_0(t)\right]y(t) \approx \left[b_1(t)p + b_0(t)\right]u(t), \tag{2}$$

where $p = d/dt$ indicates the differentiation operator; $a_1(t), a_0(t), b_1(t),$ and $b_0(t)$ are unknown time-varying parameters; the domains of $b_1(t)$ and $b_0(t)$ are approximately known in the sense specified below; due to the minimum phase condition of the plant (1), we have $\operatorname{sign}(b_1) = \operatorname{sign}(b_0)$.

The problem is to design a control law for this plant so that the closed-loop control system dynamics will be close to given dynamics assigned by an implicit reference model.

2. CONTROL ALGORITHM

To solve the problem, we introduce the operator $d(p) = d_1 p + 1$, where $d_1 > 0$ and $d_1^{-1} \geq \Omega$. Dividing equation (2) by the polynomial $d(p)$ yields

$$\tilde{z}_0(t) \approx \theta_1(t)z_1(t) + \theta_2(t)z_2(t) + \theta_3(t)z_3(t) + z_4(t), \tag{3}$$

where $\tilde{z}_0(t), z_1(t), z_2(t), z_3(t)$, and $z_4(t)$ are new variables of time, the first three being given by $\tilde{z}_0(t) = \frac{p^2}{d(p)}y(t)$, $z_1(t) = \frac{p}{d(p)}y(t)$, and $z_2(t)$

$=\frac{1}{d(p)}y(t)$; $\theta_1(t)=-a_1(t)$ and $\theta_2(t)=-a_0(t)$; finally, the variable $z_4(t)$ completes equality (3) to the correct form and consists of the sum of parts proportional to the rates of change of the parameters of equation (2). Due to the quasi-stationarity assumption, this variable has a small contribution in the main dynamic modes compared to the rest of the terms on the right-hand side of equation (3). We will study two cases of the terms $\theta_3(t)$ and $z_3(t)$, arising from a priori information about the parameters $b_1(t)$ and $b_0(t)$:

• Case 1. The contribution of the term $b_1(t)p$ is significantly small compared to $b_0(t)$; then it is possible to suppose that $\theta_3(t)=b_0(t)$ and, consequently, $z_3(t)=\left(\mu_1(t)\frac{p}{d(p)}+\frac{1}{d(p)}\right)u(t)$, where $\mu_1(t)=\frac{b_1(t)}{b_0(t)}$; let $0<\mu_1(t)\le 0.1$.

• Case 2 holds if case 1 is false: $\theta_3(t)=b_1(t)$ and $z_3(t)=\left(\frac{p}{d(p)}+\mu_2(t)\frac{1}{d(p)}\right)u(t)$, where $\mu_2(t)=\frac{b_0(t)}{b_1(t)}$; let $0<\mu_2(t)<10$.

The signal $\tilde{z}_0(t)$ is technically nonimplementable, so we replace it with the approximate variable $z_0(t)=\frac{1}{(d_2p+1)}\tilde{z}_0(t)$, where $0<d_2\le d_1$. Then the plant (3) is modeled by

$$z_0(t)\approx\boldsymbol{\theta}(t)^{\mathrm{T}}\mathbf{z}(t)+z_4(t), \tag{4}$$

where $z_0(t)$ is the plant's response; $\boldsymbol{\theta}(t)=[\theta_1(t),\ \theta_2(t), \theta_3(t)]^{\mathrm{T}}$ is the vector of unknown parameters; $\mathbf{z}(t)=[z_1(t),\ z_2(t),\ z_3(t)]^{\mathrm{T}}$ is the vector of regressors; and the superscript "T" means transpose.

We assign an implicit reference model in the form of the second-order link

$$\begin{gathered}\ddot{y}_{\mathrm{m}}=a_1^m\dot{y}_m+a_2^m y_m+b^m u_{\mathrm{giv}},\\ y_m(t_0)=y(t_0),\ \dot{y}_m(t_0)=\dot{y}(t_0),\end{gathered} \tag{5}$$

where $u_{\mathrm{giv}}=u_{\mathrm{giv}}(t)$ and $y_m=y_m(t)$ are the input and output of the reference model, respectively; a_1^m, a_0^m, and b^m are constant parameters assigned by $a_1^m=-2\xi_m\Omega_m$, $a_0^m=-\Omega_m^2$, and $b^m=k_m\Omega_m^2$, where ξ_m is the relative attenuation coefficient, Ω_m is the natural frequency of oscillations of the reference model, and k_m is the gain of the reference model.

Dividing equation (5) by the polynomial $d(p)$ yields

$$\begin{gathered}z_0^m(t)=a_1^m z_1^m(t)+a_2^m z_2^m(t)+b^m z_3^m(t),\\ y_m(t_0)=y(t_0),\ \dot{y}_m(t_0)=\dot{y}(t_0),\end{gathered} \tag{6}$$

where $z_0^m(t)=\frac{p^2}{d(p)}y_m(t)$, $z_1^m(t)=\frac{p}{d(p)}y_m(t)$, $z_2^m(t)=\frac{1}{d(p)}y_m(t)$, and $z_3^m(t)=\frac{1}{d(p)}u_{\mathrm{giv}}(t)$.

In accordance with the assigned reference model, we introduce the desired plant's response (4):

$$\begin{gathered}\tilde{z}_0^{\mathrm{des}}(t)\triangleq z_0^m\left(z_1(t), z_2(t), z_3^m(t)\right)\\ =a_1^m z_1(t)+a_2^m z_2(t)+b^m z_3^m(t).\end{gathered} \tag{7}$$

Indeed, $\tilde{z}_0(t)\xrightarrow{\approx}\tilde{z}_0^{\mathrm{des}}(t)$ as $z_0(t)\to\tilde{z}_0^{\mathrm{des}}(t)$, and the expression (7) directly implies the behavior of the plant (3) (and hence, that of (4)) tends to the dynamics (6). Therefore, $\ddot{y}(t)\xrightarrow{\approx}a_1^m\dot{y}(t)+a_2^m y(t)+b^m u_{\mathrm{giv}}(t)$ and, consequently, the output (1) tends to the desired one of the reference model (5).

To find the corresponding control action, it is necessary to equate the right-hand sides of the relations (4) and (7). As a result, we obtain a control law, called accurate, based on complete a priori information about the parameters (4) and the variable $z_4(t)$:

$$\begin{gathered}u_1^*(t)=\frac{d_1p+1}{\mu_1(t)p+1}\Big[\left(a_1^m-\theta_1(t)\right)z_1(t)\\ +\left(a_2^m-\theta_2(t)\right)z_2(t)+b^m z_3^m(t)-z_4(t)\Big]\Big/b_0(t),\\ u_2^*(t)=\frac{d_1p+1}{p+\mu_2(t)}\Big[\left(a_1^m-\theta_1(t)\right)z_1(t)\\ +\left(a_2^m-\theta_2(t)\right)z_2(t)+b^m z_3^m(t)-z_4(t)\Big]\Big/b_1(t),\end{gathered} \tag{8}$$

where u_1^* and u_2^* denote the accurate control laws in the first and second cases, respectively.

Since the parameters of the plant (4) are unknown, we will determine them using a current parametric identification algorithm. The variable $z_4(t)$ will be neglected due to its small value and the approximation properties of the identification algorithm. Under the variable parameters of the plant, the recurrent least-squares method with the forgetting factor is the most effective and frequently used [3]. Its discrete implementation in continuous time is described by

$$\begin{cases} \widehat{\boldsymbol{\theta}}(t_i) = \widehat{\boldsymbol{\theta}}(t_{i-1}) + \mathbf{K}(t_i)\varepsilon(t_i) \\ \varepsilon(t_i) = z_0(t_i) - \widehat{\boldsymbol{\theta}}(t_{i-1})^{\mathrm{T}}\mathbf{z}(t_i) \\ \mathbf{K}(t_i) = \mathbf{P}(t_{i-1})\mathbf{z}(t_i)\left(\beta + \mathbf{z}(t_i)^{\mathrm{T}}\mathbf{P}(t_{i-1})\mathbf{z}(t_i)\right)^{-1} \\ \mathbf{P}(t_i) = \left(\mathbf{E} - \mathbf{K}(t_i)\mathbf{z}(t_i)^{\mathrm{T}}\right)\mathbf{P}(t_{i-1})/\beta \\ \mathbf{P}(t_0) = \vartheta\mathbf{E},\ \beta < 1,\ \beta \to 1, \end{cases} \quad (9)$$

where the cap symbol means the estimate of the corresponding parameter; $t_i \in \Delta t\left[(i-1), i\right)$ is the current discrete time with a sampling step Δt, $i = 1, 2, 3, \ldots$; $\varepsilon(t_i)$ is the identification residual; $\mathbf{K}(t_i)$ and $\mathbf{P}(t_i)$ are a vector and a square matrix, respectively, of dimensions corresponding to the vectors $\boldsymbol{\theta}$ and $\mathbf{z}$; ϑ is a large positive number; $\mathbf{E}$ is an identity matrix of appropriate dimensions; β is the forgetting factor that exponentially excludes past information from the algorithm to track the variable parameters of the plant with a past information forgetting time constant equal to $\Delta t/(1-\beta)$ [15].

To generate the vector $\mathbf{z}(t_i)$ in the algorithm (9) by the dependencies (3), we need information about the value of $\mu(t_i) \in \{\mu_1(t_i),\ \mu_2(t_i)\}$. Assume that its estimate $\widehat{\mu}(t_i)$ is available. (Some ways to achieve this will be discussed below.)

For the above cases, the discrete implementation of the control law will be based on (8) with the parameter estimates delivered by the algorithm (9) instead of their exact values:

$$\begin{aligned} u_1(t_i) &= \frac{(d_1 + \Delta t)\nu(t_i) - d_1\nu(t_{i-1})}{\widehat{b}_0(t_i)\left(\widehat{\mu}_1(t_i) + \Delta t\right)} \\ &\quad + \frac{\widehat{\mu}_1(t_i)}{\widehat{\mu}_1(t_i) + \Delta t} u_1(t_{i-1}), \\ u_2(t_i) &= \frac{(d_1 + \Delta t)\nu(t_i) - d_1\nu(t_{i-1})}{\widehat{b}_1(t_i)\left(1 + \Delta t\widehat{\mu}_2(t_i)\right)} \\ &\quad + \frac{1}{1 + \Delta t\widehat{\mu}_2(t_i)} u_2(t_{i-1}), \end{aligned} \quad (10)$$

where $\nu(t_i) = \left(a_1^{\mathrm{m}} - \widehat{\theta}_1(t_i)\right)z_1(t_i) + \left(a_2^{\mathrm{m}} - \widehat{\theta}_2(t_i)\right)z_2(t_i) + b^{\mathrm{m}} z_3^{\mathrm{m}}(t_i)$.

To prevent high-frequency components of the tuned parameter estimates with the discrete identification algorithm from entering the control signal, it is recommended to apply a low-pass filter with a cutoff frequency of at least Ω to this signal.

The control laws (10) with the identification algorithm (9) are the solution of the problem. Let us consider its main properties.

The peculiarity of identification algorithms is as follows. In the case of open-loop identification (when the algorithm (9) operates autonomously without forming the control law (10)), provided that (a) the regressors (4) are linearly independent on a sliding interval and full enough to describe (2), (b) the value of the parameter β is chosen in accordance with the rate of change of the parameters, and (c) the sampling step Δt is sufficiently small, the identification residual will tend to zero from the first steps of the algorithm and subsequently stay close to it. In addition, the parameter estimates converge very slowly to their true values. Under linear dependent regressors and/or the presence of noise, the estimates even diverge, which is a well-known fact.

Within the identification approach (to self-tuning systems), one often uses the certainty equivalence principle: the structure of the control law is based on calculating complete a priori information about unknown parameters, and the latter are replaced by their current estimates during the control process, assuming that they approach the true values over time. This allows achieving the control objective, which directly requires a persistently exciting ("rich") input signal. Otherwise, the stability of the closed loop system is not guaranteed [1–6].

At the same time, many researchers note a "strange" property of the closed-loop control system: sometimes, the generated control law exactly achieves the objective with the estimates being far from the true values (e.g., see [2, 3, 16–18], etc.). This phenomenon has not been clearly explained so far.

In the author's opinion, a possible recipe is to consider not the parameter estimates and their convergence to the true values but the identification residual and its convergence to zero [13]. Indeed, the plant (4) can be described via the current estimates in continuous form:

$$z_0(t) \approx \widehat{\boldsymbol{\theta}}(t)^{\mathrm{T}}\mathbf{z}(t) + \varepsilon(t). \quad (11)$$

In this expression, the term $\widehat{\boldsymbol{\theta}}(t)^{\mathrm{T}}\mathbf{z}(t)$ is a tunable model of the plant response built on the current parameter estimates. If $\varepsilon(t) \cong 0$, the parameter estimates "build" this model, approximating the current trajectory of the plant (4) and, hence, that of the output of the original plant (1). Consequently, the above conclusions are also valid for the model of the plant (11) built on the current (even inaccurate) estimates. However, there are additional requirements, which are determined (in the problem under study) by the convergence conditions of the identification residual in the closed loop of the control system [13, 14]. They are much simpler than the requirement of asymptotically exact estimates and are basically reduced to definite parameter estimation quality during control. As a result, the need for a persistently exciting input signal, etc. is eliminated.

These requirements were called simplified and defined as sufficient and, at the same time, necessary for the quality of the estimates delivered by the identifier [13]. For the scalar control problem under consideration, the requirements apply only to the estimates $\hat{b}_1$ and $\hat{b}_0$ [14]:

$$\begin{cases} \operatorname{sign}(\hat{b}_1) = \operatorname{sign}(\hat{b}_0) \\ \operatorname{sign}(\hat{b}_1) = \operatorname{sign}(b_1) \\ \quad |\hat{b}_1| > \lambda_i |b_1|/2, \ \hat{b}_1(t) \to \text{const} \\ \operatorname{sign}(\hat{b}_0) = \operatorname{sign}(b_0) \\ \quad |\hat{b}_0| > \lambda_i |b_0|/2, \hat{b}_0(t) \to \text{const}, \end{cases} \tag{12}$$

where $\lambda(t_i) = \mathbf{z}(t_i)^{\mathrm{T}}\mathbf{P}(t_i)\mathbf{z}(t_i), 0 < \lambda(t_i) < 1$. The first condition in (12) is necessary to ensure the stability of the designed law (10), which follows from the law (8) when replacing the parameters with their estimates; the others are necessary for the stability of the identification residual in the closed loop with the control law (10).

It can be shown that conditions (12) settle the "explosive" behavior problem of the closed loop system described in [19] and their violation causes high-frequency oscillations in the closed-loop control system. The upper bounds for the estimates $\hat{b}_1$ and $\hat{b}_0$ exist in the form of the criterion of the resulting control quality: higher absolute values of these estimates lead to longer transients in the closed loop system. The requirements (12) concern only the parameters at the control action; if the ranges of b_1 and b_0 are known, they become simple enough to be implemented in practice.

Under the convergent identification residual, the above requirements for the parameter estimates of the controlled plant can be interpreted as a refinement of the certainty equivalence principle of self-tuning control systems in tracking problems (when the output of a controlled plant should follow a given trajectory, a reference system, etc.).

Based on the requirements (12), in order to simplify the implementation of the control algorithm, we assign constant estimates $\hat{b}_1$, $\hat{b}_0$, and $\hat{\mu}$ while observing (12). (Therefore, these estimates are excluded from the current identification procedure.) Then in the identification algorithm (9), the plant response and the vectors of estimated parameters and regressors are, respectively,

$$z_0(t) - \hat{\theta}_3 z_3(t), \ \boldsymbol{\theta}(t) = \left[\theta_1(t), \theta_2(t)\right]^{\mathrm{T}},$$

$$\mathbf{z}(t) = \left[z_1(t), z_2(t)\right]^{\mathrm{T}}. \tag{13}$$

The solution proposed may increase the rate of change of the exact resulting values of the parameters $\theta_1(t)$ and $\theta_2(t)$ in formula (2) and, hence, enlarge the spread of $z_4(t)$. A recipe is to reduce the value of the parameter β in the identification algorithm (decrease the forgetting time constant). According to studies, the best solution is to assign the forgetting factor to the variables dependent on the identification residual. For this purpose, we employ the approach from [20] with a slight modification:

$$\beta(t_i) = \operatorname{sat}\left[1 - k_\varepsilon \varepsilon_{\mathrm{lf}}^2(t_i)\right], \tag{14}$$

where $\operatorname{sat}(\cdot)$ is the saturation function applied to limit the value of $\beta(t_i)$, i.e., $\beta_{\min} \le \beta(t_i) \le \beta_{\max}$ with assigned minimum $\beta_{\min}$ and maximum $\beta_{\max}$, $0 < \beta_{\min} < \beta_{\max} \le 1$; $\varepsilon_{\mathrm{lf}}^2(t_i)$ is the squared low-frequency component of the current value of the identification residual, obtained at a low-pass filter with a cutoff frequency not smaller than Ω; finally, k_ε is the coefficient chosen depending on the average spread of the squared low-frequency component of the identification residual.

As is known, when using small values of the forgetting factor, the norm of the matrix $\mathbf{P}(t_i)$ of the algorithm (9) may strongly increase, which reduces the stability of the delivered estimates and the generated control. The same effect occurs when there is a linear dependence in the vector of regressors, e.g., on a degenerate motion. To exclude this, we use a modification of the algorithm (9) described in [21]. It consists in regularizing the matrix $\mathbf{P}(t_i)^{-1}$, i.e., limiting the norm of $\mathbf{P}(t_i)$ by checking whether its diagonal elements exceed an assigned number $q_{\max}$ and performing its subsequent correction by the dependence (immediately after computing this matrix in the original algorithm):

$$\mathbf{P}_{\mathrm{cor}}(t_i) = \Lambda(t_i)\mathbf{P}(t_i)\Lambda(t_i),$$

where $\mathbf{P}_{\text{cor}}(t_i)$ is the corrected value of the matrix $\mathbf{P}(t_i)$; $\Lambda(t_i)$ is a diagonal matrix of appropriate dimensions with unit elements on the diagonal, except the elements (with conventionally denoted numbers *j*) corresponding to the inequality $\rho_j > \rho_{\max}$; ρ_j is the diagonal element of the original matrix $\mathbf{P}(t_i)$ with the number j; $\rho_{\max}$ is the limit assigned for the diagonal elements; the value of the *j*th element of the matrix $\Lambda(t_i)$ equals $\sqrt{\rho_{\max}/\rho_j}$.

An additional way to improve the quality of control (in some cases, unnecessary) in condition (13) is to write the control laws (10) considering the low-frequency component of the identification residual as part of the variable $\nu(t_i)$:

$$\nu(t_i) = \left(a_1^m - \widehat{\theta}_1(t_i)\right) z_1(t_i) + \left(a_2^m - \widehat{\theta}_2(t_i)\right) z_2(t_i) + b^m z_3^m(t_i) - \tilde{\varepsilon}_{\text{lf}}(t_i), \quad (15)$$

where $\tilde{\varepsilon}_{\text{lf}}(t_i)$ is the low-frequency component of the current value of the identification residual (different from the one in (14)), obtained at a low-pass filter with a cutoff frequency smaller than Ω. This solution improves the adequacy of the original plant's model with the desired estimates $\widehat{\theta}_1(t_i)$ and $\widehat{\theta}_2(t_i)$ under the accepted conditions.

We propose to call this adaptive control design approach for the plant (1) based on simplified adaptability conditions the identification method of adaptive control with approximation or identification–approximation control.

3. APPLICATION TO OVERHEAD CRANE CONTROL

Consider the control of an overhead crane to move a cargo in the horizontal plane along one axis. The control action is the force applied to the crane trolley (Fig. 1).

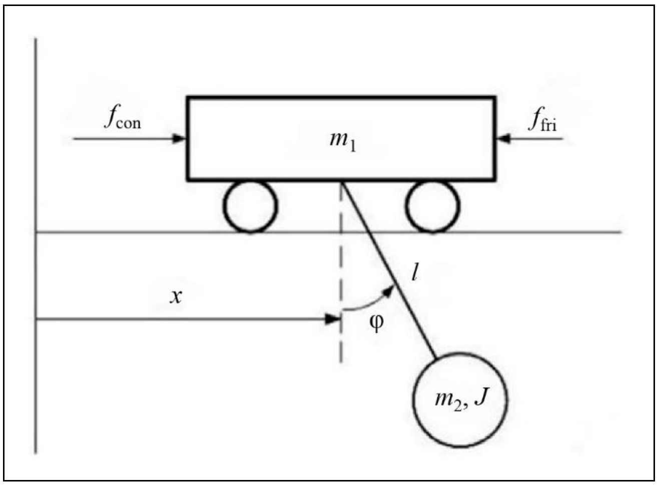

Fig. 1. The kinematic diagram of the overhead crane with a cargo along the horizontal axis.

When neglecting the cable mass, angular motion friction, wind disturbances, and dry friction, the cargo dynamics are described by the system of nonlinear differential equations [22]

$$\begin{cases} (m_1 + m_2)\ddot{x} + (m_2 l \cos\varphi)\ddot{\varphi} \\ \quad = m_2 l \dot{\varphi}^2 \sin\varphi + f_{\text{con}} - f_{\text{fri}} \\ (m_2 l \cos\varphi)\ddot{x} + \left(m_2 l^2 + J\right)\ddot{\varphi} = -m_2 g l \sin\varphi, \end{cases} \quad (16)$$

where *x* is the linear movement of the crane trolley; φ is the deviation angle of the cargo suspension from the vertical axis; f_{con} is the control force applied to the trolley; $f_{\text{fri}} = k_x \dot{x}$ is the friction force counteracting trolley movements; k_x is the viscous friction coefficient; m_1 and m_2 are the masses of the crane trolley and transported cargo, respectively; l is the suspension length; J is the cargo's central axial moment of inertia; finally, g is the free fall acceleration.

Consider the control problem with the signal f_{con} as the controlled plant's input and the velocity of the deviation angle $(\omega = \dot{\varphi})$ as the output. Elementary physics implies an oscillatory relationship between the input and output of the plant (16). Moreover, this input–output combination is used since, according to the preliminary studies, they have a minimum phase delay at low frequencies. (This issue has been discussed above.)

Let the crane have the parameters $m_1 = 450\ \text{kg}$, $m_2 = 1000\ \text{kg}$, $J = 1000\ \text{kg}\cdot\text{m}^2$ (which corresponds to the cargo's radius of inertia equal to 1 m), $l = 7\ \text{m}$, and $k_x = 0.3\ \text{N}\cdot\text{s/m}$. Due to the smallness of the angle φ and its velocity, the transfer function from the input f_{con} to the output ω can be obtained by linearizing equations (16):

$$\frac{\omega(t)}{f_{\text{con}}(t)} = \frac{\tilde{b}_2 p^2}{p^3 + \tilde{a}_2 p^2 + \tilde{a}_1 p + \tilde{a}_0} \approx \frac{-2.98 \cdot 10^{-4} p^2}{p^3 + 6.38 \cdot 10^{-4} p^2 + 4.24 p + 8.77 \cdot 10^{-4}}, \quad (17)$$

where $\tilde{b}_2 = -m_2 l/\gamma$, $\tilde{a}_2 = \left(J + m_2 l^2\right) k_x/\gamma$, $\tilde{a}_1 = (m_1 + m_2) m_2\, \text{lg}/\gamma$, $\tilde{a}_0 = m_2 l k_x g/\gamma$, and $\gamma = (m_1 + m_2) J + m_1 m_2 l^2$. From this point onwards, all transfer functions are written through the differentiation operator; by assumption, the initial values of the input and output of the corresponding link and their derivatives are zero.

The natural frequency of oscillation of this mechanical system is 2.1 rad/s with the variations of the crane parameters $\Omega \approx 5\,\mathrm{s}^{-1}$ specified below. Let the output variable ω be directly measured.

The reference model (5) has the following description: $y_\mathrm{m} \triangleq \omega_\mathrm{m}$, where ω_m corresponds to the angular velocity ω; u_giv is a given value of the angular velocity of the suspension (denoted by ω_giv); $\xi_\mathrm{m} = 1$, $\Omega_\mathrm{m} = 3\,\mathrm{s}^{-1}$, and $k_\mathrm{m} = 1$.

Based on the value of Ω, we choose $d_1 = 0.2\,\mathrm{s}$ and $d_2 = 0.1\,\mathrm{s}$. Hence,

$$z_0(t) = \frac{p^2}{(0.2p+1)(0.1p+1)}\omega(t),$$

$$z_1(t) = \left[p/(0.2p+1)\right]\omega(t),$$

$$z_2(t) = \left[1/(0.2p+1)\right]\omega(t),$$

$$z_3^\mathrm{m}(t) = \left[1/(0.2p+1)\right]\omega_\mathrm{giv}(t).$$

Note that formula (2) does not structurally coincide with the expression (17), so on the current trajectory the transfer function (17) can be approximated only using $\boldsymbol{\theta} = \boldsymbol{\theta}(t)$. In other words, model (2), (3) is time-varying (nonstationary).

For control design, we need a priori information about the values of the parameters b_1 and b_0 in equation (2) approximating the plant (16), (17). Due to formula (17), their sign is negative. More accurate information can be obtained in different ways. The first method is to estimate preliminarily all parameters of (2) (calculate their approximate values). The second one is to get only the estimates $\hat{b}_1$ and $\hat{b}_0$ based on identification outside the operating frequency range of the plant, as was suggested in the patent [23].

The third method, more simple, consists in the following. As is known, too small absolute values of $\hat{b}_1$ and $\hat{b}_0$ cause high-frequency unstable motions in the closed-loop control system by conditions (12). Therefore, it is possible to select their values experimentally on the control system model. We apply this method here, taking the second case of model (3), $\hat{b}_1 \equiv -3\cdot 10^{-4}\,(\mathrm{m}\cdot\mathrm{kg})^{-1}$, and $\hat{\mu}_2 \equiv 5\,\mathrm{s}^{-1}$. As a result, $z_3(t) = \left[(p+5)/(0.2p+1)\right] f_\mathrm{con}(t)$.

The algorithm (9), (13), (14) with the parameters $\Delta t = 0.01\,\mathrm{s}$, $\vartheta = \rho_\mathrm{max} = 10$, $\beta_\mathrm{min} = 0.01$, $\beta_\mathrm{max} = 0.99$, and $k_\varepsilon = 10\,\mathrm{s}^2$ was used for identification. (The discrete control law $u_2(t_i)$ (10), (15) was implemented with the same sampling step.) The low-frequency components of the identification residual were $\varepsilon_\mathrm{lf}(t) = \left[1/(0.1p+1)\right]\varepsilon(t)$ and $\tilde{\varepsilon}_\mathrm{lf}(t) = \left[1/(p+1)\right]\varepsilon(t)$. The signal f_con generated by the system was filtered on the link $1/(0.1p+1)$ before supplying to the controlled plant.

Numerical simulations were carried out in Matlab/Simulink/Multibody for the controlled plant's model corresponding to the dependencies (16) with the above parameter values.

Figure 2 shows the simulation results for these parameter values of the control algorithm with the assigned angular velocity reference curve (Fig. 2c). Obviously, the current angular velocity (ω) almost follows the reference one (ω_m), and the angular motion is stable. Of course, in practice, such a system requires an additional control loop on trolley movements; this issue goes beyond the scope of the problem under consideration.

This control law yielded almost the same-quality behavior of the signals ω and φ under other values of the crane parameters in large ranges: load mass from 50 to 10 000 kg, suspension length from 2 to 10 m, and the cargo's radius of inertia from 0.5 to 2 m (proportionally to the cargo mass). Only the control actions changed, i.e., we can speak about adaptive control under the current parametric uncertainty.

In addition, the robust properties of the control algorithm were assessed under real factors affecting the control system. In particular, the output of the plant (16) was measured with centered Gaussian noise, and a delay was introduced into the control action. According to the simulation results, the noise with a standard deviation up to 0.001 rad/s and a delay up to 0.005 s have almost no effect on control quality. The hypothetical case of large amplitudes of the angle φ, up to 30° and more, with nonlinear effects was also investigated. In this case, the closed-loop system demonstrated a lower reference tracking quality, but the angular motion remained stable.

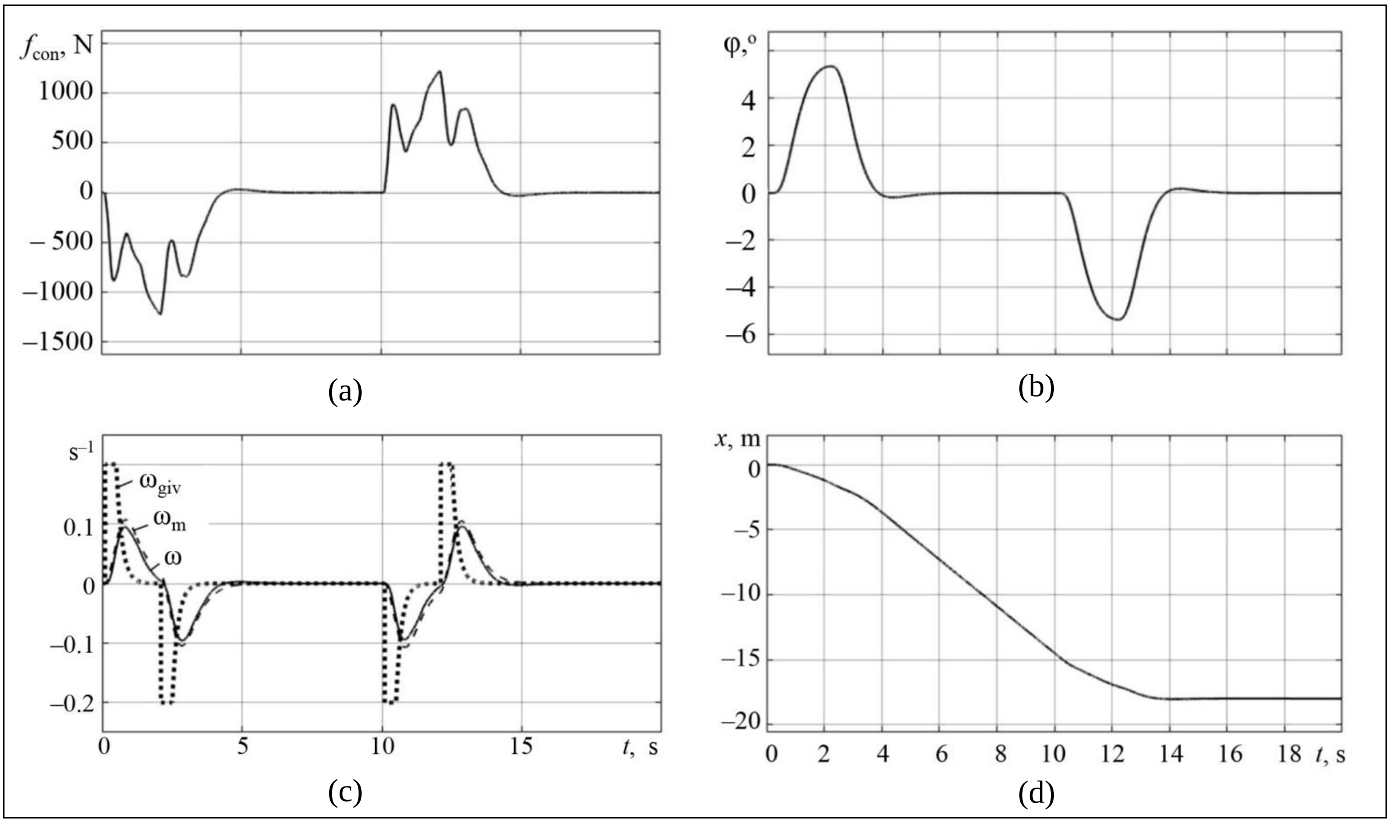

Fig. 2. The closed-loop control system of the overhead crane: (a) control force, (b) the deviation angle of cargo suspension, (c) given angular velocity ω_{giv}, reference angular velocity $\omega_{м}$, and the velocity of deviation angle ω and (d) the distance traveled by the crane trolley.

4. APPLICATION TO THREE-MASS ELASTIC PLANT CONTROL

Consider the adaptive control of an elastic three-mass plant, a model of many fast-response drives of mechatronic systems. Its kinematic diagram is presented in Fig. 3 [24].

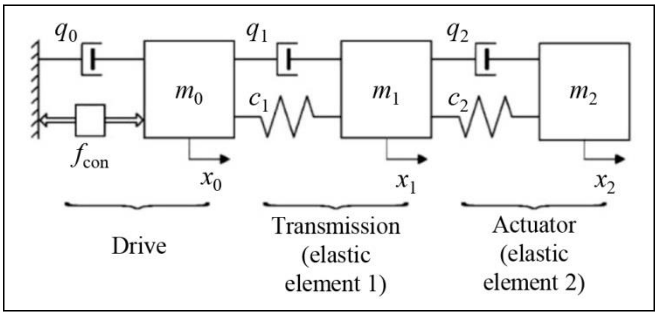

Fig. 3. Kinematic diagram of a three-mass elastic actuator.

This figure has the following notations: $m_0, m_1,$, and m_2 are the reduced masses of drive bodies, mechanical motion transmission, and actuator, respectively; $x_0, x_1,$, and x_2 are the movements of these bodies relative to their initial position; c_1 and c_2 are the reduced stiffness coefficients of the elastic elements; $q_0, q_1,$ and q_2 are the viscous friction coefficients; finally, f_{con} is the control force of the drive. The diagram contains two elastic elements (see Fig. 3) with the natural frequencies $\Omega_1 = \sqrt{c_1/m_1}$ and $\Omega_2 = \sqrt{c_2/m_2}$.

This system is described by the differential equations

$$\begin{cases} m_0\ddot{x}_0 + q_0\dot{x}_0 + q_1(\dot{x}_0 - \dot{x}_1) + c_1(x_0 - x_1) = f_{con} \\ m_1\ddot{x}_1 + q_1(\dot{x}_1 - \dot{x}_0) + c_1(x_1 - x_0) \\ \qquad + q_2(\dot{x}_1 - \dot{x}_2) + c_2(x_1 - x_2) = 0 \\ m_2\ddot{x}_2 + q_2(\dot{x}_2 - \dot{x}_1) + c_2(x_2 - x_1) = 0. \end{cases} \tag{18}$$

Let the first elastic element be at least as fast as the second, i.e., $\Omega_1 \ge \Omega_2$: $m_0 = 10\,\text{kg}$, $m_1 = 1\,\text{kg}$, $m_2 = 0.5\,\text{kg}$, $q_0 = 3\,\text{N}\cdot\text{cm}^{-1}$, $q_1 = q_2 = 0.5\,\text{N}\cdot\text{cm}^{-1}$, $c_1 = 10000\,\text{N}\cdot\text{m}^{-1}$, and $c_2 = 30\,\text{N}\cdot\text{m}^{-1}$. These parameters correspond to $\Omega_1 = 100\,\text{s}^{-1}$ and $\Omega_2 \approx 7.7\,\text{s}^{-1}$. Assume that only the second elastic element is subject to parameter variations, so $\Omega = 100\,\text{s}^{-1}$.

In this system, we choose the variable f_{con} as the input and the variable $x = x_2 - x_0$ as the output, which characterizes the elastic properties of the system. The relationship between these variables is oscillatory, and the phase delay is negligible at low frequencies. The transfer function from the input to the output has the form

$$\frac{x(t)}{f_{\text{con}}(t)} = \frac{-\left(\tilde{b}_3 p^3 + \tilde{b}_2 p^2 + \tilde{b}_1 p\right)}{p^5 + \tilde{a}_4 p^4 + \tilde{a}_3 p^3 + \tilde{a}_2 p^2 + \tilde{a}_1 p + \tilde{a}_0}$$

$$\approx \frac{-\left(0.1p^3 + 0.2p^2 + 1009p\right)}{p^5 + 2.35p^4 + 11091p^3 + 14561p^2 + 693\cdot 10^3 p + 18\cdot 10^4},$$

where $\tilde{b}_3 = m_1 m_2/\gamma$, $\tilde{b}_2 = \left[q_1 m_2 + q_2\left(m_1 + m_2\right)\right]/\gamma$,

$$\tilde{b}_1 = \left(c_1 m_2 + c_2 m_1 + c_2 m_2\right)/\gamma,$$

$$\tilde{a}_4 = \left(q_0 m_1 m_2 + q_1 m_0 m_2 + q_2 m_0 m_1 + q_1 m_1 m_2 + q_2 m_0 m_2\right)/\gamma,$$

$$\tilde{a}_3 = \left[\begin{array}{l} q_0 q_1 m_2 + q_0 q_2\left(m_1 + m_2\right) \\ + q_1 q_2\left(m_0 + m_1 + m_2\right) \\ + c_1 m_2\left(m_0 + m_1\right) + c_2 m_0\left(m_1 + m_2\right) \end{array}\right] \Big/ \gamma,$$

$$\tilde{a}_2 = \left[\begin{array}{l} q_0 q_1 q_2 + q_0\left(c_1 m_2 + c_2 m_1 + c_2 m_2\right) \\ + \left(q_1 c_2 + q_2 c_1\right)\left(m_0 + m_1 + m_2\right) \end{array}\right] \Big/ \gamma,$$

$$\tilde{a}_1 = \left[q_0\left(q_1 c_2 + q_2 c_1\right) + c_1 c_2\left(m_0 + m_1 + m_2\right)\right]/\gamma,$$

$$\tilde{a}_0 = q_0 c_1 c_2/\gamma, \text{ and } \gamma = m_0 m_1 m_2.$$

The reference model (5) is described by $y_{\text{m}} \triangleq x_{\text{m}}$, where x_{m} corresponds to $x = x_2 - x_0$; u_{giv} is a given value of *x*: $x_{\text{giv}} \equiv 0$; $\xi_{\text{m}} = 1$, $\Omega_{\text{m}} = 10\,\text{s}^{-1}$, and $k_{\text{m}} = 1$.

Based on the value of Ω, we choose $d_1 = 0.01\,\text{s}$ and $d_2 = 0.001\,\text{s}$. Hence, $z_0(t) = \frac{p^2}{(0.01p+1)(0.001p+1)} x(t)$, $z_1(t) = \frac{p}{0.01p+1} x(t)$, $z_2(t) = \frac{1}{0.01p+1} x(t)$, and $z_3^{\text{m}}(t) \equiv 0$. We calculate the estimates $\hat{b}_1$ and $\hat{b}_0$ by the same method as in Section 3, taking the first case of model (3), $\hat{b}_0 \equiv -0.4\,\text{kg}^{-1}$, and $\hat{\mu}_1 \equiv d_1$. Therefore, $z_3(t) = f_{\text{con}}(t)$.

The identification algorithm was applied with the same parameters as in Section 3, except $\Delta t = 0.001\,\text{s}$ and $k_\varepsilon = 0.5\,\text{m}^2$; the low-frequency components of the identification residual were set equal to $\varepsilon_{\text{lf}}(t) = \left[1/(0.1p+1)\right]\varepsilon(t)$ and $\tilde{\varepsilon}_{\text{lf}}(t) = \left[1/(0.05p+1)\right]\varepsilon(t)$. The signal f_{con} was filtered on an aperiodic link with a time constant of 0.01s.

Figure 4 shows the initial behavior of the plant (18) under the control signal $f_{\text{con}} = f_{\text{con}}^{\text{d}}$, where $f_{\text{con}}^{\text{d}}$ is a disturbance. Next, Fig. 5 presents the simulation results for the closed-loop control system with $f_{\text{con}} = f_{\text{con}}^{\text{d}} + f_{\text{con}}^{\text{t}}$, where $f_{\text{con}}^{\text{t}}$ is the target control generated by the control system to ensure the given properties of the closed-loop control system.

Numerical simulations were also carried out with other values of the parameters (18) of the second elastic element, which were varied in a random combination: c_2 from 3 to 240 N·s/m and m_2 from 0.1 to 6 kg. (These values correspond to a change in the frequency Ω_2 from 0.7 to 50 s^{-1}.) According to the simulation results, in all cases, the control system provides the

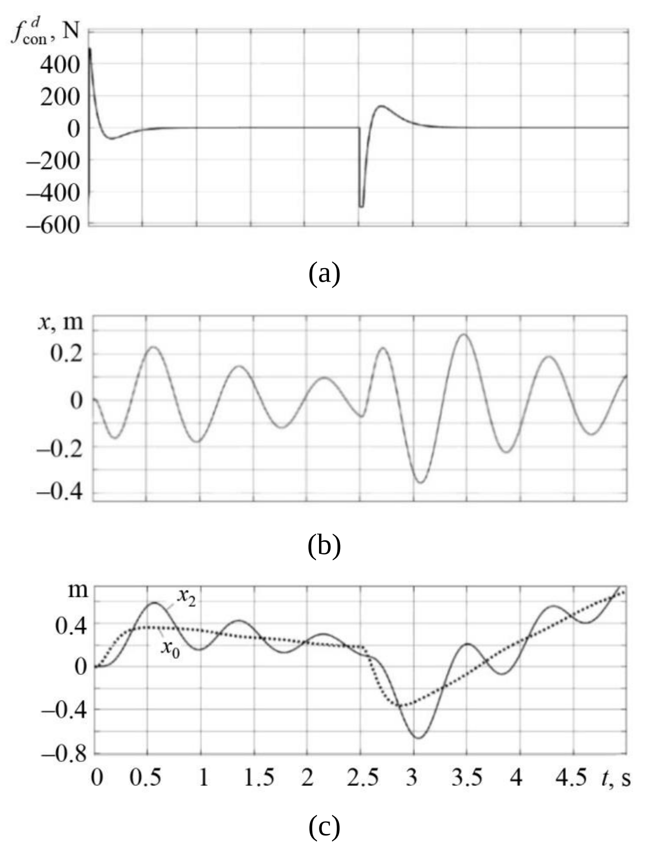

Fig. 4. The behavior of the elastic three-mass drive under the disturbance: (a) the exogenous disturbing force, (b) the relative movement of the drive masses, (c) the movements of bodies with masses m_0 and m_2.

stable behavior of the variable $x = x_2 - x_0$: the error on this variable did not exceed 0.002 m in 1–1.5 s after the disturbing effect. Note that the transients may slightly differ from those illustrated in Fig. 5. The control system retains its control quality even if the measurements of the output variable y include centered Gaussian noise with a standard deviation of no more than 0.1 mm and there is a control delay of no more than 0.003 s.

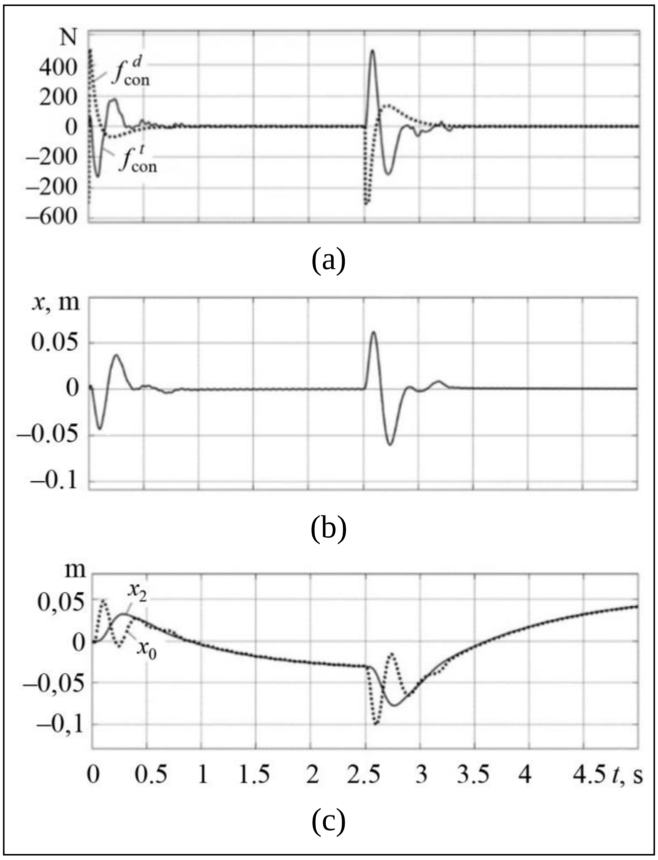

Fig. 5. The closed-loop control system of the elastic three-mass drive: (a) the exogenous disturbing force f^{d}_{con} and the control force f^{t}_{con}, (b) the relative movement of the drive masses, and (c) the movements of bodies with masses m_0 and m_2.

Obviously, the system successfully damps elastic vibrations: by the end of the exogenous disturbance, $x \to 0$ within a large variety of the plant parameters.

CONCLUSIONS

This paper has proposed an approach based on simplified adaptability conditions, or the identification–approximation approach for self-tuning systems, to design adaptive control of a scalar plant in which only the input and output are measured. The main peculiarity of the approach is the use of structural and parametric approximation of plant dynamics. The former is implemented through describing the dynamics by a simple-structure model (a simple link or a set of such links) compared to that of the plant. The latter is reduced to the use of current parameter estimates delivered by the identification algorithm, not necessarily tending in asymptotics to their exact values and being time-varying. Then exact parameter estimates are not required to achieve the properties of the closed-loop control system close to an assigned reference model: it suffices to fulfill conditions (12) and make the identification residual convergent. A control law is constructed based on this model. This result can be interpreted as a refinement of the well-known certainty equivalence principle of self-tuning systems with output-feedback control of the plant. A concomitant positive feature of the latter is the nonnecessity of persistently exciting regressors (the "richness" of the input signal).

This approach allows for different modifications. For example, it is possible to apply Kaczmarz's sequential projection algorithm [3] instead of the identification algorithm (9). The constant parameter estimates at the control action of the approximating model can be replaced by the variables delivered by two-stage identification [23]. Also, to improve the robustness of the current parametric identification procedure and adaptive control in general, one may utilize the approaches described, e.g., in [2, 3], etc. These include: introducing a feedback loop on small parameter estimates into the identification algorithm, "attracting" the estimates to their a priori known values, using the insensitivity zone of the identification residual, limiting its spread, and others. Note also that this paper has addressed only the plants with oscillatory dynamics approximated by an oscillatory link without any transport delay. At the same time, generalizations to other classes of plants are obvious.

Undoubtedly, the above method cannot be attributed to universal adaptive control design procedures. Nevertheless, many applications-relevant problems can be effectively solved by this method: in addition to the illustrative examples of Sections 3 and 4, it has been successfully applied, e.g., in [25] and other publications.

Acknowledgments. *This work was supported by the Russian Science Foundation, project no. 23-29-00654; https://rscf.ru/project/23-29-00654/.*

REFERENCES

1. Tao, G., *Adaptive Control Design and Analysis*, Hoboken, NJ: John Wiley & Sons, 2003.
2. Ioannou, P. and Fidan, B., *Adaptive Control Tutorial*, Philadelphia: Society for Industrial and Applied Mathematics, 2006.

3. Åström, K.J. and Wittenmark, B., *Adaptive Control*, 2nd ed., Mineola, NY: Dover Publications, 2008. URL: bayanbox.ir/view/8821671619590593184/Adaptive-Control-Astrom-2nd-edition.pdf (Accessed October 10, 2024.)
4. Nguyen, N.T., *Model-Reference Adaptive Control*, Advanced Textbooks in Control and Signal Processing, Cham: Springer, 2018. URL: https://link.springer.com/book/10.1007/978-3-319-56393-0 (Accessed October 10, 2024.)
5. Annaswamy, A.M. and Fradkov, A.L., A Historical Perspective of Adaptive Control and Learning, *Annual Reviews in Control*, 2021, vol. 52, pp. 18–41.
6. Bobál, V., Böhm, J., Fessl, J., and Macháček, J., *Digital Self-tuning Controllers: Algorithms, Implementation and Applications*, Berlin: Springer, 2005. URL: https://matlab.fei.tuke.sk/raui_new/subory/prednasky/Digital_Self-tuning_Controllers.pdf (Accessed October 10, 2024.)
7. Fradkov, A.L., Miroshnik, I.V., and Nikiforov, V.O., Nonlinear and Adaptive Control of Complex Systems, Dordrecht: Kluwer Academic Publishers, 1999.
8. Eremin, E.L., Telichenko, D.A., Semichevskaya, N.P., et al., *Upravlenie tekhnicheskimi sistemami v usloviyakh neopredelennosti* (Control of Technical Systems under Uncertainty Conditions), Blagoveshchensk: Amur State University, 2014. (In Russian.)
9. Efimov, D.V., *Robastnoe i adaptivnoe upravlenie nelineinymi kolebaniyami* (Robust and Adaptive Control of Nonlinear Oscillations), St. Petersburg: Nauka, 2005. (In Russian.)
10. Loria, A., Kelly, R., and Teel, A., On Uniform Parametric Convergence in the Adaptive Control of Mechanical Systems, *European Journal of Control*, 2005, no. 11, pp. 90–101.
11. Dobriborsci, D., Kolyubin, S., and Bobtsov, A., Output Adaptive Control for Linear Systems under Parametric Uncertainties with Finite-Time Matching Input Harmonic Disturbance Rejection, *ArXiv abs/2005.07639*, 2020. URL: https://arxiv.org/pdf/2005.07639 (Accessed October 10, 2024.)
12. Glushchenko, A.I. and Lastochkin, K.A., Approximation-Based Approach to Adaptive Control of Linear Time-Varying Systems, *Automation and Remote Control*, 2024, vol. 85, no. 5, pp. 512–531.
13. Kruglov, S.P., Adaptability Issues of Control Systems with the “Identifier + Reference” Scheme, *Trudy 4-oi mezhdunarodnoi konferentsii “Identifikatsiya sistem i zadachi upravleniya”* (Proceedings of the 4th International Conference “System Identification and Control Problems”), Moscow, 2005, pp. 1307–1348. (In Russian.)
14. Kruglov, S.P., Convergence of the Residual Identification Error in the Control System with Parametrical Adaptation, *Information Technology and Mathematical Modeling in the Management of Complex Systems*, 2019, no. 1, pp. 27-40. URL: http://ismm-irgups.ru/toma/12-2019 (Accessed October 10, 2024.) (In Russian.)
15. Ljung, L., *System Identification: Theory for the User*, New York: Prentice Hall, 1987.
16. Sragovich, V.G., *Adaptivnoe upravlenie* (Adaptive Control), Moscow: Nauka, 1981. (In Russian.)
17. Isermann, R., *Digital Control Systems*, Berlin–Heidelberg: Springer-Verlag, 1981.
18. Bukov, V.N., *Adaptivnye prognoziruyushchie sistemy upravleniya poletom* (Adaptive Predictive Flight Control Systems), Moscow: Nauka, 1987. (In Russian.)
19. Anderson, B.D.O., Adaptive Systems, Lack of Persistency of Excitation and Bursting Phenomena, *Automatica*, 1985, vol. 21, no. 3, pp. 247−258.
20. Dumont, G.A. and Huzmezan, M., Concepts, Methods and Techniques in Adaptive Control, *Proceedings of the 2002 American Control Conference*, 2002, vol. 2, pp. 1137–1150.
21. Kruglov, S.P. Modifications of the Recurrent Method of Least Squares with Forgetting Factor for the Functional Stability of the Current Parametrical Estimation of Dynamic Processes, *Information Technology and Mathematical Modeling in the Management of Complex Systems*, 2019, no. 1, pp. 1–12. URL: http://ismm-irgups.ru/toma/12-2019 (Accessed October 10, 2024.) (In Russian.)
22. Chernousko, F.L., Ananievski, I.M., and Reshmin, S.A., *Control of Nonlinear Dynamical Systems: Methods and Applications*, Berlin–Heidelberg: Springer-Verlag, 2008.
23. Bukov, V.N., Kruglov, S.P., Bronnikov, A.M., and Segedin, R.A., Patent RU 2231819 S2, *Byull. Izobret.*, 2004, no. 18. (In Russian.)
24. Kuznetsov, N.K., Active Vibration Control of Actuators of Mechatronic Systems, *Modern Technologies. System Analysis. Modeling*, 2011, no. S-4 (32), pp. 101–110. (In Russian.).
25. Kruglov, S.P. and Kovyrshin, S.V., Identification-Based Speed Control of an Overhead Crane with a Reduced Cargo Transfer Model, *Control Sciences*, 2023, no 4, pp. 25–33.

This paper was recommended for publication by S.A. Krasnova, a member of the Editorial Board.

Received October 13, 2024,
and revised December 5, 2024.
Accepted December 9, 2024.

Author information

Kruglov, Sergey Petrovich. Dr. Sci. (Eng.), Irkutsk State Transport University, Irkutsk, Russia
✉ kruglov_s_p@mail.ru
ORCID iD: https://orcid.org/0000-0001-9241-3352

Cite this paper

Kruglov, S.P., Adaptive Control of a Scalar Plant in the Input–Output Form Based on the Identification-Approximation Approach. *Control Sciences* **6**, 32–42 (2024).

Translated into English by *Alexander Yu. Mazurov*,
Cand. Sci. (Phys.–Math.),
Trapeznikov Institute of Control Sciences,
Russian Academy of Sciences, Moscow, Russia
✉ alexander.mazurov08@gmail.com