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AN OPTIMAL ALLOCATION ALGORITHM FOR REENTRANT RESOURCES ON NETWORK GRAPHS#

O. A. Kosorukov* and D. V. Lemtyuzhnikova**

*Moscow State University, Moscow, Russia;
The Presidential Academy (RANEPA), Moscow, Russia
**Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia;
Moscow Aviation Institute (National Research University), Moscow, Russia

*✉ kosorukovoa@mail.ru, **✉ darabbt@gmail.com

Abstract. This paper considers the problem of allocating reentrant resources when performing a set of interdependent works that are represented by a network graph. By assumption, the work completion time linearly depends on the resource amount used. We justify a solution algorithm in the case of a set of works with a predetermined sequence of events in the network graph. Also, we propose an algorithm for reducing the general problem to an auxiliary one with ordered event times and an algorithm for constructing an optimal solution of the original problem. The convergence of this algorithm is ensured by finite iterations at each stage. The overall computational complexity of the algorithm can be estimated as $O(n^2)$, where n denotes the number of vertices in the original network graph. It seems promising to apply this algorithm for planning the sets of interdependent works using reentrant resources.

Keywords: network graph, unordered events, event merging, event splitting, pathfinding.

INTRODUCTION

A considerable amount of literature and scientific publications have been devoted to network scheduling problems. In particular, several statements of optimization problems on network graphs were given in the monograph [1]; however, unlike the dynamic resource allocation problems considered below, they are static. The problems described in [1] involve separable resources, which are often called inventory or material and technical resources. This paper deals with another type of resources, i.e., those that can be reused in different works at different times, e.g., personnel or equipment.

Such resources will be called reentrant; in the literature, they are often called non-stock or power-type resources. Let us briefly review the results for problems of this type.

According to the book [2], there are no exact algorithms for finding an optimal resource allocation in the general case, but there exist several heuristic algorithms. The general case was understood as follows: a set of works given by an arbitrary network graph; the availability of several resources in known amounts; each work is performed by one type of resources; works are performed with a fixed intensity (i.e., the intensity remains the same from the beginning to the end). The authors of the book presented results for independent works with concave intensity functions. As was noted, when a single work is performed by several types of resources, but with a fixed distribution of resource use shares (a particular structure for each work), the results will remain valid. Moreover, if the intensity functions are not concave (the general case), then there always exists an optimal solution containing at most n intervals of constancy on which the intensities do not change. The result for independent works with convex intensity functions was provided. In the case of fixed intensities and independent works, the cited authors proposed an optimization algorithm, which is an analog of the simplex method for a particular class of problems.

The paper [3] considered the case of projects with dependent works (an arbitrary network graph) that are performed with a priori fixed given intensities. The weighted average completion time of all project works was adopted as a schedule efficiency criterion.

In the publication [4], a standard transportation problem was studied and a criterion was derived under which the set of matrix plans of the problem contains a matrix whose elements do not exceed a fixed value. Also, identities were constructed to define the general properties of all matrix plans with their element constraints. The criterion and identities allow finding the minimax matrix (the one with the minimal largest element) and the minimax value as well. Necessary and sufficient conditions were established under which the minimax matrix is unique.

The work [5] discussed the issues of estimating the parameters of network project management models. The peculiarities of building network models were described under different network planning conditions in applications. The impact of model aggregation on the accuracy of schedules was assessed.

Dynamic resource management problems with a close statement were addressed in [6]. Based on physical analogies and laws, the author demonstrated that the optimal use of resources in a hydrostatic model is achieved at the minimum of an objective functional expressing potential energy.

The paper [7] considered problems of scheduling theory with non-stock resources and their solution algorithms based on branch-and-bound methods.

In addition, we mention several publications by foreign researchers. The work [8] considered the *resource-and-material-flow-constrained project scheduling problem* (RCMPSP). Some resource allocation algorithms on network graphs were compared in [9]. The authors [10] surveyed the results on dynamic resource allocation on network graphs, including a detailed bibliography on the subject.

1. PROBLEM STATEMENT

Consider a certain set of interdependent works l_j ($j = 1,..., m$). Let each work have two numerical characteristics, namely, $Q_j > 0$ and $\tau_j > 0$ ($j = 1,..., m$), where Q_j denotes the volume of work l_j and τ_j is the completion time of work l_j. Assume that the sequence of works is restricted by the structure of a directed graph $\{M, N\}$, where $M = \{z_i\}$, $i = 0,..., n$, is the set of vertices (events) and $N = \{l_j\}$, $j = 1,..., m$, is the set of arcs (works). Problems of this type are investigated within the theory of network planning and scheduling. Methods for building such network models and analyzing their characteristics are known; for example, see [1]. Following this theory, for each event z_i, we define a new variable, i.e., the event time t_i ($i = 1,..., n$). The main constraints in network planning are as follows. First, the event corresponding to a given vertex cannot occur before the completion of all incoming works. Second, any work outgoing from some vertex cannot be started before the occurrence of the event corresponding to this vertex.

By the definition of a network graph, we assume the following conditions for the graph $\{M, N\}$ [1]:

(a) There exists a unique vertex z_0 with no works of the form (z_i, z_0), where z_0 is the beginning of the set of works. (In the case of several such vertices, they can be merged by assigning the time t_0^* (the beginning of the set of works) to them.)

b) There exists a unique vertex z_0 with no arcs of the form (z_n, z_i). (Otherwise, we combine these notions by letting their event times be t_n^*, where t_n^* is the completion time of the set of works.)

c) The graph has no closed paths.

d) For any vertex z_i ($i = 1,..., n - 1$), there exists a path from z_0 to z_n passing through z_i. For any arc l_j ($j = 1,..., m$), there exists a path containing this arc.

These conditions define the concept of a network graph.

Suppose that the resource amount c_j allocated for work l_j does not change during its completion time. This time, denoted by τ_j, depends on the resource amount c_j as follows:

$$\tau_j = \theta_j^2 - \theta_j^1 = \frac{Q_j}{c_j}, \; j = 1,\ldots, m.$$

Let work l_j correspond to network arc (z_{j1}, z_{j2}). Then its start and end times and the times of the start and end events are related by

$$t_{j1} \le \theta_j^1 < \theta_j^2 \le t_{j2}, \;\; j = 1,..., m,$$

where θ_j^1 and θ_j^2 are the start and end times of work l_j, respectively.

Due to these relations, the event z_{j2} cannot occur before the completion of all incoming works of vertex z_{j2}. In addition, the start time of all works, t_0^*, and the completion time of all works, t_n^*, are specified.

It is required to determine the event times t_i ($i = 1,..., n - 1$), the start and end times of all works (θ_j^1, θ_j^2) ($j = 1,..., m$), and resource allocation functions $c_j(t)$ that minimize the maximum resource amount used to perform the given set of works.

This problem can be mathematically stated as follows:

$$\min_{\bar{t},\bar{\theta}} \max_{t\in\left[t_0^*, t_n^*\right]} u(t), \tag{1}$$

$$\bar{t} = (t_1,..., t_{n-1}), \bar{\theta} = (\theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2);$$

$$u(t)=\sum_{j=1}^{m}c_j(t),\ t\in[t_0^*,\ t_n^*],$$

$$c_j(t)=\begin{cases}\dfrac{Q_j}{\theta_j^2-\theta_j^1}, & t\in[\theta_j^1,\ \theta_j^2]\\ 0, & t\notin[\theta_j^1,\ \theta_j^2],\end{cases}$$

$$t_{j1}\le\theta_j^1<\theta_j^2<t_{j2},\ \ j=1,\dots,\ m,$$

$$t_0=t_0^*,\ t_n=t_n^*,$$

where j_1 and j_2 are the start and end vertices of arc j.

Proposition 1. *If there exists an admissible solution* $(\bar t,\ \bar\theta)$ *of problem* (1) *such that*

$$u(t)\equiv c=\frac{\sum_{j=1}^{m}Q_j}{t_n^*-t_0^*},\ \ t\in[t_0^*,t_n^*],\tag{2}$$

then the vector $(\bar t,\ \bar\theta)$ *is its optimal solution*.

P r o o f. If a solution of problem (1) is admissible, we have the relations

$$\int_{t_0^*}^{t_n^*}u(t)\,dt=\sum_{j=1}^{m}\int_{t_0^*}^{t_n^*}c_j(t)\,dt=\sum_{j=1}^{m}\int_{\theta_j^1}^{\theta_j^2}\frac{Q_j}{\theta_j^2-\theta_j^1}\,dt=\sum_{j=1}^{m}Q_j.$$

Assume the existence of an admissible solution $(\bar t',\ \bar\theta')$ with the dynamic resource use function $u_1(t)$ such that $\max\limits_{t\in[t_0^*,t_n^*]}u_1(t)<c.$

Then

$$\int_{t_0^*}^{t_n^*}u_1(t)dt\le\max_{t\in[t_0^*,t_n^*]}u_1(t)(t_n^*-t_0^*)<c(t_n^*-t_0^*)=\sum_{j=1}^{m}Q_j.$$

Hence, contrary to the assumption made, the vector $(\bar t',\ \bar\theta')$ is not an admissible solution. This contradiction proves the validity of Proposition 1.

2. THE PROBLEM WITH ORDERED EVENT TIMES

We begin with problem (1) under the condition that the event times are strictly ordered, i.e., the events occur in the same order in all admissible solutions of problem (1). Let us order the events by their times: $\bar t=(t_1,\dots,t_{n-1}),\ \ t_0^*<t_1<\dots<t_{n-1}<t_n^*.$

In this case, problem (1) can be written as

$$\min_{\bar t,\bar\theta}\max_{t\in[t_0^*,t_n^*]}u(t),\tag{3}$$

$$\bar t=(t_1,\dots,t_{n-1}),\ \bar\theta=(\theta_1^1,\ \theta_1^2,\dots,\theta_m^1,\ \theta_m^2),$$

$$u(t)=\sum_{\substack{j:\ j1\le k\\ j2\ge k+1}}c_j(t),\ \ t\in[t_k,t_{k+1}],\ \ \ k=0,\dots,n-1;$$

$$c_j(t)=\begin{cases}\dfrac{Q_j}{\theta_j^2-\theta_j^1}, & t\in[\theta_j^1,\ \theta_j^2]\\ 0, & t\notin[\theta_j^1,\ \theta_j^2],\end{cases}$$

$$t_{j1}\le\theta_j^1<\theta_j^2\le t_{j2},\ \ j=1,\dots,\ m,$$

$$t_0^*<t_1<t_2<\dots<t_{n-1}<t_n^*,$$

$$t_0=t_0^*,\ \ t_n=t_n^*.$$

Theorem 1. *There exists an optimal solution of problem* (3) *that satisfies relation* (2).

P r o o f. It suffices to establish the existence of an admissible solution $(\bar t,\ \bar\theta)$ for which the conditions of Theorem 1 are valid. Its optimality will follow from Proposition 1. Let

$$\theta_j^1=t_{j1},\ \theta_j^2=t_{j1+1}.\tag{4}$$

We verify the constraints (3) in this case. Indeed, since $\theta_j^1=t_{j1}<t_{j1+1}=\theta_j^2\le t_{j2}$, we have

$$c_j(t)=\begin{cases}\dfrac{Q_j}{\theta_j^2-\theta_j^1}=\dfrac{Q_j}{t_{j1+1}-t_{j1}}, & t\in[t_{j1},\ t_{j1+1}]\\ 0, & t\notin[t_{j1},\ t_{j1+1}].\end{cases}$$

Therefore, for k = 0,..., n–1 and $t\in[t_k,t_{k+1}]$,

$$u(t)=\sum_{\substack{j:\,j_1\le k,\\ j_2\ge k+1}}c_j(t)=\sum_{\substack{j:\,j_1=k,\\ j_2\ge k+1}}c_j(t)$$

$$=\sum_{\substack{j:\,j_1=k,\\ j_2\ge k+1}}\frac{Q_j}{t_{k+1}-t_k}=\frac{1}{t_{k+1}-t_k}\sum_{\substack{j:\,j_1=k,\\ j_2\ge k+1}}Q_j.$$

Further, letting $u(t)\equiv c,\ t\in[t_0^*,t_n^*],$ gives the system of equations

$$\sum_{\substack{j:\,j_1=k,\\ j_2\ge k+1}}Q_j=c(t_{k+1}-t_k),\ \ k=0,\dots,n-1.\tag{5}$$

This system includes n equations whereas the number of unknowns is (n–1). We show the linear dependence of these equations. Let k = (n–1) and $l_m=(z_{n-1},z_n)$. Then

$$\frac{1}{t_n^*-t_{n-1}}\sum_{\substack{j:\,j_1=n-1,\\ j_2\ge n}}Q_j=\frac{1}{t_n^*-t_{n-1}}Q_m=\frac{\sum_{j=1}^{m}Q_j-c(t_{n-1}-t_0^*)}{t_n^*-t_{n-1}}$$

$$=\frac{1}{t_n^*-t_{n-1}}\left[\sum_{j=1}^{m}Q_j-\frac{\sum_{j=1}^{m}Q_j}{t_n^*-t_0^*}(t_{n-1}-t_0^*)\right]$$

$$=\frac{\sum_{j=1}^{m}Q_j}{t_n^*-t_0^*}\,\frac{t_n^*-t_0^*-t_{n-1}+t_0^*}{t_n^*-t_{n-1}}=\frac{\sum_{j=1}^{m}Q_j}{t_n^*-t_0^*}=c.$$

Obviously, if condition (5) holds for $k = 0,..., (n–2)$, it will be valid for $k = (n–1)$. Hence, this condition for $k = (n–1)$ can be eliminated from the system of equations (5). The remaining equations admit an explicit analytical solution:

$$\bar{t} = (t_1,..., t_{n-1}),\ t_{k+1} = t_k + \frac{\sum\limits_{\substack{j:j_1=k,\\ j_2\ge k+1}} Q_j}{c}, \qquad (6)$$

$$k = 1,..., n-2,\ t_0 = t_0^*.$$

Based on equations (4), we calculate the vector $\bar{\theta} = (\theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2)$. Obviously, the vectors $\bar{t}$ and $\bar{\theta}$ satisfy the problem constraints (3). By the construction procedure, the function $u(t)$ does not change over the entire interval $t \in [t_0^*, t_n^*]$; according to Proposition 1, $(\bar{t}, \bar{\theta})$ is the optimal solution of problem (3). ♦

Remark. Due to the linear dependence of the system of equations, the last one has not been used. However, any other equation can be excluded as well. When discarding the equation with number i, the explicit solution is given by

$$t_{k+1} = \frac{\sum\limits_{\substack{j:j_1=k,\\ j_2\ge k+1}} Q_j}{c} + t_k,\ k = 0,..., i-1,\ t_0 = t_0^*,$$

$$t_k = t_{k+1} - \frac{\sum\limits_{\substack{j:j_1=k,\\ j_2\ge k+1}} Q_j}{c},\ k = n-1,..., i+1,\ t_n = t_n^*.$$

3. THE PROBLEM WITH UNORDERED EVENT TIMES

In its original statement, problem (1) does not imply a given sequence of events. We can prove that the sequence of events z_i and z_j is predetermined if and only if there exists a path between the vertices z_i and z_j. (By the properties of network graphs listed above, this condition is equivalent to the existence of a path between z_0 and z_n that contains the vertices z_i and z_j.)

Theorem 2. *Problem* (1) *always has an optimal solution that satisfies relation* (2).

P r o o f. Suppose that the times of some events z_{i1} and z_{i2} are unordered. We modify problem (1) by merging the events z_{i1} and z_{i2} and assigning the same time t_i' to them: $t_{i1} = t_{i2} = t_i'$.

This modification of the problem corresponds to merging the vertices of the network graph $\{M, N\}$. It can be demonstrated that the modified graph still meets all the requirements for network graphs. Obviously, the transformed network graph satisfies conditions a), b), and d) (see Section 1). To verify condition c), we note that the merging operation in a network graph may cause a closed path if and only if there exists a path between the vertices being merged in the original graph. Due to the unordered times of these events (see the assumption above), such a path is absent; in this case, condition c) holds.

Now, we show that if the vectors $\bar{t'} = (t_1,..., t_i,..., t_{n-1})$, $\bar{\theta} = (\theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2)$ are an admissible solution for the modified network graph, then

$$\begin{aligned} \bar{t} &= (t_1, ..., t_{i1}, ..., t_{i2}, ..., t_{n-1}), \\ \bar{\theta} &= (\theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2), \end{aligned} \qquad (7)$$

where $t_{i1} = t_{i2} = t_i',$ – is an admissible solution for the original network graph. Since the events z_{i1} and z_{i2} are unordered, there is no path between them in the graph. In this case, the constraints on the components of the vectors $\bar{t}$ and $\bar{\theta}$ will coincide in the modified and original problem up to the change of variables.

Let the network graph be modified as described above. If it still contains pairs of events with an unspecified order, we repeat the merging operation for these vertices. Obviously, applying this procedure finitely many times, we will obtain a graph with an ordered set of events. By the results of Section 2, there exists an optimal solution of such a problem with the function satisfying condition (2). This solution has the general form $\bar{\theta} = (\theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2)$ and $\bar{t^*} = (t_1^*, ..., t_q^*)$. Let the vector $\bar{t}$ in the original problem have dimension $(n–1)$. Then $q < (n–1)$. Successively splitting the merged events, we construct a vector $\bar{t}$ containing $(n–1)$ components that are the components of the vector $\bar{t^*}$; this vector will be admissible for problem (1). Next, we calculate the function $u(t)$ using the components of the constructed vectors $(\bar{t}, \bar{\theta})$. As is easily verified, it satisfies condition (2). In this case, by Proposition 1, the vector $(\bar{t}, \bar{\theta})$ is the optimal solution. ♦

Apparently, one could consider other ways of reducing the general problem to the one with ordered event times. For example, it is possible to declare some order of occurrence for the unordered events. However, the merging procedure discussed above is advantageous: it reduces the number of variables.

Theorem 3. *Relation* (2) *is valid for any optimal solution of problem* (1).

P r o o f. Assume on the contrary that condition (2) fails for some optimal solution $(\bar{t}, \bar{\theta})$, i.e., $u(t) \not\equiv c$. Based on Theorem 1, we have $u(t) \le c,\ t \in [t_0^*, t_n^*]$. All components of the vector $(\bar{t}, \bar{\theta})$ belong to the interval $[t_0^*, t_n^*]$; we denote their set by

$$T = \{t_0^*, t_1, ..., t_{n-1}, t_n^*,\ \theta_1^1, \theta_1^2,..., \theta_m^1, \theta_m^2\}.$$

On each interval formed by the neighbor points of this set, the function $u(t)$ is a constant. Hence, there exist points $\tau_1 \in T,\ \tau_2 \in T$ such that $u(t) \equiv \tilde{c} \le c,\ t \in [\tau_1, \tau_2]$. Hence,

$$\int_{t_0^*}^{t_n^*} u(t)dt = \int_{t_0^*}^{\tau_1} u(t)dt + \int_{\tau_1}^{\tau_2} u(t)dt + \int_{\tau_2}^{t_n^*} u(t)dt$$

$$\le c(\tau_1 - t_0^*) + \tilde{c}(\tau_2 - \tau_1)$$

$$+ c(t_n^* - \tau_2) < c(t_n^* - t_0^*) = \sum_{j=1}^{m} Q_j.$$

This strict inequality contradicts Proposition 1, and the proof of Theorem 3 is complete.

4. AN ALGORITHM FOR OBTAINING THE OPTIMAL SOLUTION

Let us summarize the results presented above. For the dynamic resource allocation problem (1) on a network graph, there always exists an optimal solution for which the integral dynamic resource use function is given by (2).

In this paper, we have actually justified the following algorithm for obtaining the optimal solution.

1) *Forming the modified network graph.* Applying a pathfinding method for graphs [11], we find and merge the events with unordered times of their occurrence. This can be implemented, e.g., using the backtracking algorithm [12]. If the graph structure is specified by the adjacency matrix, the computational complexity of this algorithm can be estimated as $O(n^2)$, where n denotes the number of graph vertices.

2) *Solving the modified problem.* The optimal solution elements are calculated based on relations (4) and (6).

3) *Solving the original problem.* By splitting the identified events, we obtain the solution of the original problem in the form (7).

The convergence of this algorithm is ensured by finite iterations at each stage; see the considerations above. The computational procedures in Stages 2 and 3 have a complexity not exceeding $O(n^2)$. In view of the computational complexity of Stage 1, the overall computational complexity of the algorithm can be estimated as $O(n^2)$, where n is the number of vertices in the original graph.

CONCLUSIONS

This paper has been devoted to the dynamic resource allocation problem when performing a set of interdependent works with the minimum total amount of available resources. The problem has been solved under the linear dependence of the work completion time on the resource amount used. We have justified a solution algorithm in the case of a set of works with a predetermined sequence of events in the network graph. Also, we have proposed an algorithm for reducing the general problem to an auxiliary one with ordered event times and an algorithm for constructing an optimal solution of the original problem. The convergence of this algorithm is ensured by finite iterations at each stage. The overall computational complexity of the algorithm can be estimated as $O(n^2)$, where n denotes the number of vertices in the original network graph. It seems promising to apply this algorithm for planning the sets of interdependent works using reentrant resources.

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REFERENCES

1. Davydov, E.G., *Igry, grafy, resursy* (Games, Graphs, Resources), Moscow: Radio i Svyaz', 1981. (In Russian.)
2. Burkov, V.N., Gorgidze, I.A., and Lovetskii, S.E., *Prikladnye zadachi teorii grafov* (Applied Problems of Graph Theory), Tbilisi: Metsniireba, 1974. (In Russian.)
3. Kosorukov, O.A., Lemtyuzhnikova, D.V., and Mishchenko, A.V., Methods and Models of Project Resource Management under Uncertainty, *J. Comput. Syst. Sci. Int.*, 2023, vol. 62, no. 2, pp. 304–322.
4. Mironov, A.A. and Tsurkov, V.I., Transport-Type Problems with a Minimax Criterion, *Automation and Remote Control*, 1995, no. 12, pp. 109–118.
5. Lyakhov, O.A., Resources in Network Planning of Complex Sets of Works, *Problemy Informatiki*, 2013, no. 1 (18), pp. 27–36. (In Russian.)
6. Razumikhin, B.S., The Problem on Optimal Resources Distribution, *Automation and Remote Control*, 1965, vol. 26, no. 7, pp. 1227–1246. (In Russian.)
7. Mishenko, A., Kosorukov, O., and Sviridova, O., Optimization of Works Management of the Investment Project, Proceedings of the 2nd International & European Conference "Modelling and Simulation of Social-Behavioural Phenomena in Creative Societies" (MSBC-2022), Vilnius, 2022, pp. 201–217.
8. Gehring, M., Volk, R., and Schultmann, F., On the Integration of Diverging Material Flows into Resource-Constrained Project Scheduling, *European Journal of Operational Research*, 2022, vol. 303, no. 3, pp. 1071–1087.
9. Bianco, L., Caramia, M., and Giordani, S., Project Scheduling with Generalized Precedence Relations: A New Method to Analyze Criticalities and Flexibilities, *European Journal of Operational Research*, 2022, vol. 298, no. 2, pp. 451–462.
10. de Lima, V.L., Alves, C., Clautiaux, F., et al., Arc Flow Formulations Based on Dynamic Programming: Theoretical Foundations and Applications, *European Journal of Operational Research*, 2022, vol. 296, no. 1, pp. 3–21.
11. Minieka, E. *Optimization Algorithms for Networks and Graphs*, New York: M. Dekker, 1978.
12. Filippova, A.S., Porechnyi, S.S., and Ramazanova, R.R., *Osnovy kombinatornykh algoritmov* (Foundations of Combinatorial Algorithms), Ufa: Bashkir State Polytechnic University, 2018. (In Russian.)

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Author information

Kosorukov, Oleg Anatol'evich. Dr. Sci. (Eng.), Moscow State University, Moscow, Russia; the Presidential Academy (RANEPA), Moscow, Russia
✉ kosorukovoa@mail.ru
ORCID iD: https://orcid.org/0000-0001-8235-4360

Lemtyuzhnikova, Dar'yana Vladimirovna. Cand. Sci. (Phys.–Math.), Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia; Moscow Aviation Institute (National Research University), Moscow, Russia
✉ darabbt@gmail.com.
ORCID iD: https://orcid.org/0000-0002-5311-5552

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Translated into English by *Alexander Yu. Mazurov*,
Cand. Sci. (Phys.–Math.),
Trapeznikov Institute of Control Sciences,
Russian Academy of Sciences, Moscow, Russia
✉ alexander.mazurov08@gmail.com