

A STRATEGIC MANAGEMENT MODEL FOR RESTRUCTURING THE TECHNOLOGICAL CORE OF AN ECONOMY

V. B. Gusev

Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia

✉ gusvbr@ipu.ru

Abstract. This paper considers the multi-sector model of the technological core of an economy, mathematical methods for its analysis, and procedures for calculating an indicative plan to restructure the core. The productivity of this core is proposed as a formalized criterion (indicator) for the effectiveness of structural innovations. The following optimization problem is stated: find a balanced state maximizing productivity by planned changes in the output and price indices. An equivalent transformation method is developed for the model considering the achieved values of the indicators. Several propositions concerning the properties of equilibrium and balanced states are proved. As a result, a multistage procedure is constructed to calculate the trajectory bringing the economic system closer to a balanced state. The multi-sector model is analyzed to compare the uncontrolled and controlled modes of development. The uncontrolled mode simulates the state of a market economy: no centralized management of the economy, sustainability, and relatively low GDP growth rates. The controlled mode involves the strategic planning methodology. As shown below, due to indicative strategic planning, the productivity of Russia's economy can significantly increase even at the first plan implementation stages. The proposed indicative planning methodology is mathematically justified. Numerical examples of its implementation on real statistical data are given. According to the paper's results, centralized planning institutions should be established for developing the technological infrastructure of Russia's economy. Such institutions are of current importance due to the international economic and political situation.

Keywords: the technological core of an economy, controlled development mode, productivity optimum, equilibrium state, balanced state, equivalent transformation, indicative strategic planning, plan of restructuring.

INTRODUCTION

In the modern world, similar technological processes used by countries lead to ambiguous results: the per capita GDP of one country may differ significantly from that of another country. To some extent, this difference is due to the structural peculiarities of their economies. As noted in [1], there is an increasing awareness that the economy's structure causes the main limitations of economic growth in Russia; the matter is an ineffective structure of production, an unproductive structure of incomes, an outdated structure of exports, and an irrational regional structure of the distribution of productive forces.

Russia's economy faces different crisis phenomena. At the microlevel, they include unfavorable business conditions in the industrial sphere, a bias toward trade and services, a short lifespan of small enterprises, and a significant share of bankruptcies. At the macrolevel, they include a low GDP growth rate, the economy's critical dependence on oil and gas exports, unstable exchange rates, a small share of the manufacturing sector and the high-tech sector, the insufficient growth of its basic assets, the economy's dependence on external sanctions, and ineffective management mechanisms. At the external level, these are the special military operation in Ukraine and international economic sanctions.

Improving economic management mechanisms is the aim of the following official documents: Federal Law No. 172-FZ “On Strategic Planning in the Russian Federation” dated June 28, 2014, Presidential Decree No. 474 “On National Development Goals of the Russian Federation for the period up to 2030” dated July 21, 2020, and Presidential Decree No. 633 “On Approving the Basics of State Policy in the Sphere of Strategic Planning in the Russian Federation” dated November 8, 2021. They envisage “the introduction of modern methods of forecasting, modeling, indicative planning, balance accounting, and information technology.”

The documents also note that:

- “The main tools of the strategic planning system are indicative planning, which forms a set of agreed indicators characterizing the state and goals of socio-economic development and national security, and balance accounting with the development of measures to achieve the set goals and their resource supply.”
- “The scientific and methodological support of strategic planning is carried out by a specialized scientific center with the participation of scientific organizations and the federal state budgetary institution ‘The Russian Academy of Sciences.’”

As a target criterion, the strategic planning methodology uses a performance indicator of the economy’s technological core, further called productivity. This methodology includes methods for calculating the dynamics of several output and value parameters of economic development to increase productivity under the existing technological constraints.

1. INDICATORS AND PLANS FOR SUSTAINABLE DEVELOPMENT

In this paper, we analyze the effective use of the existing technological potential of an economic system and determine ways of its development. A possible approach to accelerate economic growth is finding a preferred structure of economic activities and ways to implement this structure. According to numerical calculations, such a possibility is justified [2].

We consider management models and methods aimed at the sustainable self-sufficient development of the technological core of an economy. For this purpose, we employ the closed input-output model of the Leontief type [3]. This model describes the dependence of costs on the output.

Crisis phenomena and external economic sanctions require effective measures to parry them, particularly by transforming the internal mechanisms of economic activity. *The technological core* of an economic system is a set of economic activities available for obser-

vation and measurement and sufficient to adequately represent the system state. According to calculations [2], due to the existing potential of the technological core, the economy’s productivity can be significantly increased compared to the current level. When realized, this potential can overcompensate for the possible volume of losses from external sanctions.

The technological core model allows considering two methods of organizing reproduction, corresponding to the controlled and uncontrolled modes. In the limit, both methods lead to equilibrium states, when the structure of prices and outputs is stabilized with the same growth shares for all types of products and services. In the uncontrolled mode, each sector or activity disposes only of the funds constituting a share of its own value added. As a result, an equilibrium state with the lowest productivity value is implemented. In the controlled reproduction mode, an equilibrium state with the highest productivity value is implemented. In this mode, a stage-by-stage (indicative [4]) plan of structural changes in the output and prices is formed for each sector or activity. The funds for implementing this plan can be redistributed between the sectors.

The reproduction model [2, 5] of a multiproduct system allows defining an economic productivity indicator (the output reproduction index) as a function of structural proportions of the output and prices of the products and services of different sectors. This index reflects the ratio of the output and costs. Maximizing it, we calculate a balanced structure of the outputs and prices corresponding to an equilibrium reproduction mode and a stage-by-stage plan to achieve this goal.

Different models of an economy have different productivity. Hence, the problem is to choose the model with the highest productivity value. The planning problem has the following peculiarity: the maximum (potential) productivity of the technological core of the economic system can be achieved in different ways: by changing the output structure of the sectors, by changing the price structure, or by changing the output and price structures jointly.

However, the case of joint changes in the output and prices is of practical interest: these parameters are interconnected through market mechanisms and vary jointly: an increase in the output leads to a relative decrease in the price and vice versa.

2. OUTPUT REPRODUCTION CONDITIONS

Assume that the direct costs Z_{ij} of sector j for the products or services of type i and the outputs V_j of products or services of type j are given. These data are used to calculate the specific cost coefficients

$$a_{ij} = Z_{ij} / V_j. \quad (1)$$



The input-output model can be described by the relation

$$V_i(t) = \gamma_i \sum_{j=1}^n a_{ij} V_j(t),$$

where γ_i denotes the reproduction index of sector i .

Let the total costs of products or services be equal to their output in the previous stage. Then the system reproduction index γ is given by the minimum reproduction index over all types of products or services: $\gamma = \min_i \gamma_i$.

An optimization problem for the output structure V_i has the following form: find the maximum reproduction index

$$\max_{\gamma, V} \gamma \tag{2}$$

subject to the technological output constraint

$$V_i(t) \geq \gamma \sum_{j=1}^n a_{ij} V_j(t)$$

with the specific cost coefficients a_{ij} . If the matrix $\mathbf{A} = [a_{ij}]$ is nonsingular, the solution of this problem will coincide with the eigenvector of the matrix \mathbf{A} . Indeed, the number of inequalities in the constraint equals the dimension of the output vector. Therefore, the solution of the bilinear programming problem is reached on the equality

$$V_i(t) = \gamma \sum_{j=1}^n a_{ij} V_j(t), \tag{3}$$

coinciding with an eigenvector of the matrix \mathbf{A} .

The outputs satisfying condition (3) in natural indicators are *equilibrium outputs*.

The constraint on the output dynamics can be written as

$$\theta V_i(t-1) \geq V_i(t) \geq \mu V_i(t-1), \quad i=1, \dots, n, \tag{4}$$

where $0 < \mu \leq 1 < \theta$.

The outputs satisfying conditions (3) and (4) are called *balanced*.

If the direct cost coefficient a is given by

$$\min_{a, V} a, \\ a V_i(t) = \sum_{j=1}^n a_{ij} V_j(t),$$

then in the equilibrium mode, we have

$$a = 1/\gamma.$$

The equilibrium outputs corresponding to the eigenvector \mathbf{x} of the matrix \mathbf{A} with the maximum eigenvalue are not optimal for problem (2)–(4).

3. VALUE REPRODUCTION CONDITIONS

By assumption, the production technology in problem (2) remains invariable during one stage. In other words, the natural specific cost coefficients are supposed constant. The solution of this problem does not depend on the prices of products or services but is difficult to implement in practice: for large production systems, measurements are often performed in value terms.

Let the specific cost coefficients be determined based on value indicators rather than the natural ones (formula (1)). In other words,

$$a_{ij}^c = Z_{ij}^c / V_j^c = Z_{ij} P_i / (V_j P_j) = a_{ij} P_i / P_j,$$

where V_j^c denotes the output in value terms and P_j is the price of products of sector j . Then the output structure balancing problem for the production cycle takes the form

$$\max_{\gamma, V^c} \gamma, \tag{5}$$

$$V_i^c(t) \geq \gamma \sum_{j=1}^n a_{ij}^c V_j^c(t). \tag{6}$$

The outputs in value terms satisfying conditions (5) and (6) under fixed prices are *equilibrium*. For such outputs, the sectors have the same reproduction indices.

Assume that both natural volumes and prices can be changed. Problems (5), (6) (on the one part) and (2), (3) (on the other) are not equivalent: in the former, the criterion and the output constraint have a new content. Moreover, the values of the specific cost indicators can initially be determined only at the previous stage ($t - 1$). But they will change with changing the prices. Therefore, to refine these indicators, we should find possible price variations.

Consider the price structure balancing problem for the production cycle. The volumes of products or services are estimated in natural terms. The total value V_i^c of final product or service i is the sum of the values of all intermediate products or services in the production cycle given the price P_j for intermediate product or service j at this stage multiplied by the value increase coefficient for the cycle period (profitability) r_i :

$$V_i^c(t) = r_i \sum_j Z_{ji}(t) P_j. \tag{7}$$

Using this value relation and formula (1), we obtain

$$V_i^c(t) = V_i(t) P_i = r_i \sum_{j=1}^n a_{ji} V_j(t) P_j.$$

Let the reproduction coefficient be determined by the minimum profitability over all products or services. In this case, the price structure optimization problem is written as

$$\max_{r, \mathbf{p}} r, \quad (8)$$

$$P_i(t) \geq r \sum_{j=1}^n a_{ji} P_j(t). \quad (9)$$

The prices satisfying conditions (8), (9) under fixed outputs are *equilibrium prices*. For these prices, all sectors have the same profitability. The additional constraint has the form

$$\theta P_i(t-1) \geq P_i(t) \geq \mu P_i(t-1), \quad i = 1, \dots, n.$$

Its left-hand side restricts possible price reductions and the rate of inflation to the level θ by the types of products or services. Such prices, also satisfying condition (9), are called *balanced*.

4. THE JOINT BALANCING OF OUTPUT AND PRICE REPRODUCTION

Proposition 1. *The maximum values of the criteria y and r for problems (2), (3) and (8), (9) coincide. The solution vectors \mathbf{V} and \mathbf{P} are ambiguous: they are determined within a scalar multiplier.*

This proposition is proved in the Appendix.

To calculate the eigenvector and eigenvalue of the positive Schur matrix \mathbf{A} , we employ the iterative procedure

$$\begin{aligned} \mathbf{x}^0 &= (1, 1, \dots, 1), \\ \mathbf{x}^{k+1} &= \mathbf{A}\mathbf{x}^k / \|\mathbf{x}^k\|, \quad k = 0, 1, 2, \dots \end{aligned} \quad (10)$$

The eigenvalue of the matrix \mathbf{A} , calculated as the limit of $\|\mathbf{x}^k\|$, is the largest one among all its eigenvalues [6]. The process (10) simulates the uncontrolled reproduction mode for the technological core of the economy when interpreting \mathbf{x}^k as the vector of output or price indices at stage k .

Proposition 2. *Let \mathbf{A} be a nonnegative matrix with a , $|a| < 1$, as the maximum eigenvalue by magnitude. The eigenvector \mathbf{x} of this matrix satisfies the equation*

$$\mathbf{A}\mathbf{x} = a\mathbf{x}.$$

The eigenvector \mathbf{x} with the maximum eigenvalue is found using the iterative procedure

$$\mathbf{x}^{k+1} = \mathbf{A}\mathbf{x}^k / \|\mathbf{x}^k\|,$$

where k denotes the iteration number. The stopping condition has the form $\|\mathbf{x}^{k+1} - \mathbf{x}^k\| < \varepsilon$, where ε is a given accuracy of calculations (e.g., $\varepsilon = 0.001$). The eigenvalue a of the matrix \mathbf{A} is

$$a = \lim_{k \rightarrow \infty} \|\mathbf{x}^k\|.$$

This proposition is proved in the Appendix.

The outputs and prices yielded by this algorithm are equilibrium.

To obtain the output vector maximizing the productivity estimate π for the technological core, we apply the gradient descent method.

We introduce the following notations: \mathbf{I} and \mathbf{E} are a unit vector and an identity matrix of appropriate dimensions, respectively; h is the step value;

$$a(\mathbf{x}) = \|\mathbf{A}\mathbf{x}\| / \|\mathbf{x}\|.$$

On the corresponding rectangular domain, the gradient descent-based procedure to find

$$\min_{\mu \leq x_i \leq \theta, i=1, \dots, n} a(\mathbf{x})$$

has the form

$$\mathbf{x}^{k+1} = \mathbf{x}^k + h(a(\mathbf{x}^k)\mathbf{E} - \mathbf{A}^T)\mathbf{I} / \|\mathbf{x}^k\|,$$

where k is the iteration number.

On the other hand, the resulting optimal solution may be unbalanced.

Proposition 3. *Let*

$$a(\mathbf{x}) = \|\mathbf{A}\mathbf{x}\| / \|\mathbf{x}\|.$$

It is possible to choose a step h so that the gradient projection-based procedure

$$x_i^{k+1} = \min(\theta, \max(\mu, z_i^k)), \quad i = 1, n,$$

where

$$z_i^k = a(\mathbf{y}^k) \sum_j a_{ij} y_j^k, \quad i = 1, n,$$

$$\mathbf{y}^k = \mathbf{x}^k + h(a(\mathbf{x}^k)\mathbf{E} - \mathbf{A}^T)\mathbf{I} / \|\mathbf{x}^k\|,$$

will weakly converge to the conditional optimum

$$\min_{\mu \leq x_i \leq \theta} a(\mathbf{x}).$$

This proposition is proved in the Appendix.

The process of optimizing the output structure generates a sequence of increasing estimates of the technological core productivity, accompanied by a sequence of positive output increments for some sectors. At the same time, for some sectors, positive output increments do not lead higher productivity estimates. The rate of convergence depends on the step h : initially, the rate grows as the step increases; then, it starts decreasing.

The resulting solution is balanced but not equilibrium.

Note that the specific cost coefficients are measured at the previous stage of the production cycle. Hence, the problem of balancing the price proportions in the nonequilibrium mode can be written as



$$\left. \begin{aligned} \max_{r, P} r \\ p_i(t) \geq r \sum_{j=1}^n a_{ji}^c p_j(t) \\ \theta \geq p_i(t) \geq \mu, i = 1, \dots, n \end{aligned} \right\} \quad (11)$$

where $p_i(t) = P_i(t) / P_i(t-1)$ is the price index of product i .

In practice, this approach requires stage-by-stage implementation. For each stage, an additional constraint should be introduced on the deviation from the existing structure. This constraint should agree with the admissible rate of socio-economic processes.

The initial information on the structural balancing problem is formed by analyzing economic statistics. In several situations (e.g., regional or sectoral planning, scenario forecasting), there are no standard methods for collecting and processing data. In the case of such difficulties, statistical data can be combined with expert assessments of the cost structure per unit of output.

Let the solution of the balancing problem (11) be used to tune the specific cost coefficients. In this case, the repeated solution will give price indices in the range $[\mu, \theta]$; moreover, the closer the solution is to the equilibrium state, the smaller increments the growth coefficient r will have. When recalculating the specific cost coefficients, the sectors with small marginal prices may receive the corresponding increments. In addition, the updated direct cost coefficients can be used in the output balancing problem (2), (3).

5. THE INDICATIVE PLAN-FORECAST OF ECONOMIC DEVELOPMENT IN OUTPUT INDICATORS

As a rule, it is impossible to implement a change in the output structure *per saltum*, making all outputs equilibrium. To determine the most rational plan for sectoral development, consider the problem statement

$$\left. \begin{aligned} \max_{\gamma, V} \gamma, \\ V_i(t) \geq \gamma \sum_{j=1}^n a_{ij} V_j(t), \\ \theta V_i(t-1) \geq V_i(t) \geq \mu V_i(t-1), i = 1, \dots, n. \end{aligned} \right\}$$

It includes a technological constraint on the outputs and an output growth condition at the rate θ per one planning stage.

We introduce the following notations: \mathbf{D} is a diagonal matrix with the diagonal elements V_1, V_2, \dots, V_n ; \mathbf{C} is a diagonal matrix with the diagonal elements $1/V_1, 1/V_2, \dots, 1/V_n$.

Varying outputs lead to changes in the specific cost estimates a_{ij} . To fix the results V_i of the previous stage, we recalculate the direct cost coefficients:

$$\bar{a}_{ij} = a_{ij} \cdot V_i / V_j \text{ or } \bar{\mathbf{A}} = \mathbf{D}\mathbf{A}\mathbf{C}.$$

The following result was established in the paper [7]; see Proposition 1 therein.

Proposition 4. *If all $V_i \neq 0, i = 1, \dots, n$, then the transformed matrix with the coefficients $\bar{a}_{ij} = a_{ij} \cdot V_i / V_j$ has the same eigenvalues as the matrix \mathbf{A} , and its eigenvector equals the original one up to the deformation \mathbf{D} .*

The latter transformation does not affect the spectrum of the direct cost (technological) matrix \mathbf{A} , and its eigenvectors are preserved within the deformation \mathbf{D} . Repeating the described procedures (finding the optimal solution and recalculating the direct cost matrix) stage-by-stage, we obtain an indicative multistage plan-forecast of the joint development of different sectors in the technological core of the economy.

The indicative planning procedure involves absolute and relative outputs. At stage 1, we use the absolute output vector \mathbf{V}^0 for passing to the relative outputs \mathbf{v}^1 by transforming the technological matrix:

$$\mathbf{A}^0 = \mathbf{D}^0 \mathbf{A} \mathbf{C}^0, \text{ where } \mathbf{D}^0 = \text{diag}(\mathbf{V}^0) \text{ and } \mathbf{C}^0 = (\mathbf{D}^0)^{-1}.$$

Then we obtain the relative output vector \mathbf{v}^k by solving the problem

$$\left. \begin{aligned} \max_{\mathbf{v}^k} \gamma, \\ \mathbf{v}^k \geq \gamma^k \mathbf{A}^k \mathbf{v}^k, \\ \mu \mathbf{I} \leq \mathbf{v}^k \leq \theta \mathbf{I}, k = 1, 2, \dots \end{aligned} \right\} \quad (12)$$

where $\mathbf{A}^k = \mathbf{D}^{k-1} \mathbf{A}^{k-1} \mathbf{C}^{k-1}$, $\mathbf{D}^i = \text{diag}(\mathbf{v}^i)$, and $\mathbf{C}^i = (\mathbf{D}^i)^{-1}$.

The relative output vector \mathbf{v}^k (the solution of problem (12)) at stage k is called a local equilibrium under the technological output constraint and the relation output growth condition with the rate $\theta, \theta > 1$, per one planning stage. (Recall that \mathbf{I} denotes a unit vector of an appropriate dimension.) If the problems have a solution at stage $k \geq 1$, the indicative outputs in absolute terms will be

$$\mathbf{V}^k = \prod_{j=k}^1 \text{diag}(\mathbf{v}^j) \mathbf{V}^0.$$

The following result was established in the paper [7]; see Proposition 3 therein.

Proposition 5. *The sequence \mathbf{V}^k tends to the eigenvector of the technological matrix \mathbf{A} , and the estimate γ^k tends to the eigenvalue of this matrix.*

Remark. When transforming the technological matrix \mathbf{A} by the deformation $\mathbf{D} = \text{diag}(\mathbf{v}^*)$, after reaching the output vector \mathbf{v}^* , further planning becomes trivial: $\mathbf{v} = c\mathbf{I}$, where $\theta \geq c \geq \mu$. In other words, the output structure will no longer change after reaching the technological equilibrium.

6. THE COMBINED PLAN-FORECAST OF ECONOMIC DEVELOPMENT IN OUTPUT AND PRICE INDICATORS

An isolated indicative change in the prices and outputs is of little practical importance: in reality, these parameters vary simultaneously. Equilibrium outputs are unstable under fixed non-equilibrium prices, like equilibrium prices under fixed outputs. A concerted equilibrium pair of the prices and outputs is stable: in this case, the profitability r and the reproduction indices γ have the same values for all sectors, leading to the sustainable development of the economic system in the long run. However, at the initial stages of eliminating the disproportions, the sectoral indicators change very unevenly.

Consider the process of calculating the indicative dynamics of output and price indicators leading to a joint equilibrium of the output and prices.

The transformation procedures

$$\mathbf{A}^k = \mathbf{D}^{k-1} \mathbf{A}^{k-1} \mathbf{C}^{k-1}$$

for the matrices \mathbf{A} and \mathbf{A}^T and the solution procedures for problems (10), (11) are performed sequentially under a given upper bound $\theta > 1$ for the profitability r and the reproduction index γ . Note that:

$$\mathbf{D}^i = \text{diag}(\mathbf{v}^i) \text{ and } \mathbf{C}^i = (\mathbf{D}^i)^{-1}$$

in the output problem;

$$\mathbf{D}^i = \text{diag}(\mathbf{p}^i) \text{ and } \mathbf{C}^i = (\mathbf{D}^i)^{-1}$$

in the price problem.

At stage k , the intermediate cost deflator for the price index vector \mathbf{p} is calculated as

$$d = \sum_{i,j} a_{ij}^k p_i^k / \sum_{i,j} a_{ij}^k.$$

At stage k , the intermediate cost index for the output index vector \mathbf{v} is calculated as

$$w = \sum_{i,j} a_{ij}^k v_j^k / \sum_{i,j} a_{ij}^k.$$

The productivity π of the economic system (the value added divided by the intermediate costs) has the simple relation

$$\pi = \gamma - 1$$

to the output reproduction coefficient γ ; for details, see the paper [7].

The same relation holds for the profitability r :

$$\pi = r - 1.$$

Due to Proposition 1, both values coincide in the equilibrium mode.

7. INDICATIVE PLANNING OF OUTPUT PROPORTIONS

In calculations, the values V_i are interpreted as proportions of the volume of transport services; under the assumed nondecrease property, the constraints imposed on them have the form

$$V_i(t) \geq 1, i = 1, \dots, n.$$

The equilibrium output proportions may significantly differ from the existing ones. Therefore, we solve a series of optimization problems with θ close to unity ($\theta = 1.2$). In other words, the proportions can grow at most by 20%. Consider the statistical data on Russia's multisector economy [8]: the supply and use tables for 2019. For these data, the curves of indicative dynamics of 61 output indices were obtained under fixed prices. In several stages, the curves lead to a balanced price structure. We present a few curves below for demonstration purposes. (These are the first dozen of non-constant curves in succession.)

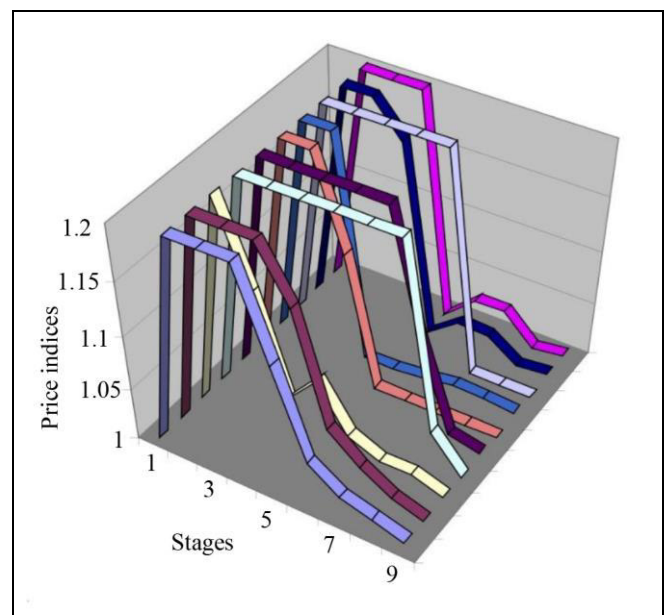


Fig. 1. Indicative dynamics of output indices with an upper bound of 1.2 for different sectors:

- Products of forestry, logging and related services;
- Products of textiles, clothing, and leather;
- Paper and paper products;
- Printing and recording services;
- Basic pharmaceutical products and pharmaceutical preparations;
- Rubber and plastic products;
- Other non-metallic mineral products;
- Electrical equipment;
- Machinery and equipment n.e.c.;
- Repair and installation services of machinery and equipment.

The indicative productivity dynamics are shown below. Figure 2 shows the productivity graph when repeatedly solving problem (7) for Russia's economy data. (The upper bound of the price indices is 1.2, and productivity is measured in percent.)

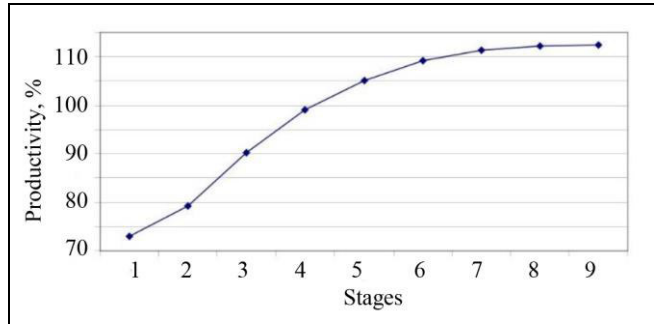


Fig. 2. Productivity under repeated output balancing.

This graph gives an idea of how far the initial state of the economy (the solution at stage 1) is from the equilibrium state (the asymptote for the repeated solution). In addition, the slope of the curve can be used to judge the sustainability of the economy. The closer the economic structure is to the equilibrium state, the higher its tolerance will be to price changes. According to Figs. 1 and 2, the closer the price structure is to the balanced one, the less the prices and productivity index will change in one stage.

Note that the output indices are stabilized at the upper bound of 1.2. This result qualitatively differs from the calculations [7] for 2016, where the output indices were stabilized at a level of 1. The reason lies in the nonunique equilibrium state and changes in the optimization algorithm. In the latter (second) case, the minimum index value was chosen. Due to the non-uniqueness of the optimal solution, both algorithms yield the same optimal value of the objective function. However, the index values of the second algorithm are preferable from the implementation point of view. Moreover, in the first case, the productivity index dynamics reach the target at a lower level (112% vs. 153%). This can be explained as follows: the problem dimension (the number of activities) was 95 (the second case) vs. 61 (the first case), which obviously reduces the choice space of planning.

8. THE JOINT DEVELOPMENT STRATEGY OF THE TECHNOLOGICAL CORE

Several constrained optimization problems with given admissible ranges of the indices are solved to obtain an indicative plan considering the joint change in the outputs and prices. If the stage duration is 1

year, the variation limits of the output indices are chosen depending on the possibility to invest in the stock formation in the current planning year. The variation limits of the price indices are determined by the allowable inflation or deflation requirements. The lower limit of the indices is set equal to 1: according to the plan, the output and prices for the sectors should not decrease.

Consider an example of joint planning of the outputs and prices. Below we present the calculation results for 10 products according to the All-Russian Classification of Products by Economic Activities (OKPD) with index values differing from 1.

For the data [8] on Russia's multisector economy, we applied the method described above to obtain indicative dynamics curves for 60 output indices and 60 price indices. These curves lead to a balanced structure in several stages, which agrees with the highway property of optimization models of economic dynamics [9]. The standard software package for mathematical programming problems [10–13] gives similar results. The plan was calculated under the interval constraint [1, 1.2] for the output indices (Fig. 3) and the price indices (Fig. 4). For demonstration purposes, only the first dozen of non-constant curves in succession is provided for each group of indices.

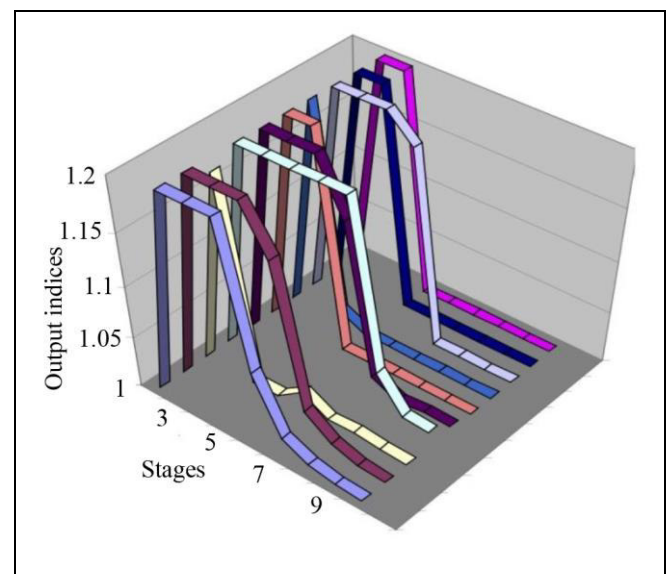


Fig. 3. Dynamics of output indices on range [1, 1.2] for different sectors:

- Products of forestry, logging and related services;
- Products of textiles, clothing, and leather;
- Paper and paper products;
- Printing and recording services;
- Basic pharmaceutical products and pharmaceutical preparations;
- Rubber and plastic products;
- Other non-metallic mineral products;
- Electrical equipment;
- Machinery and equipment n.e.c.;
- Repair and installation services of machinery and equipment.

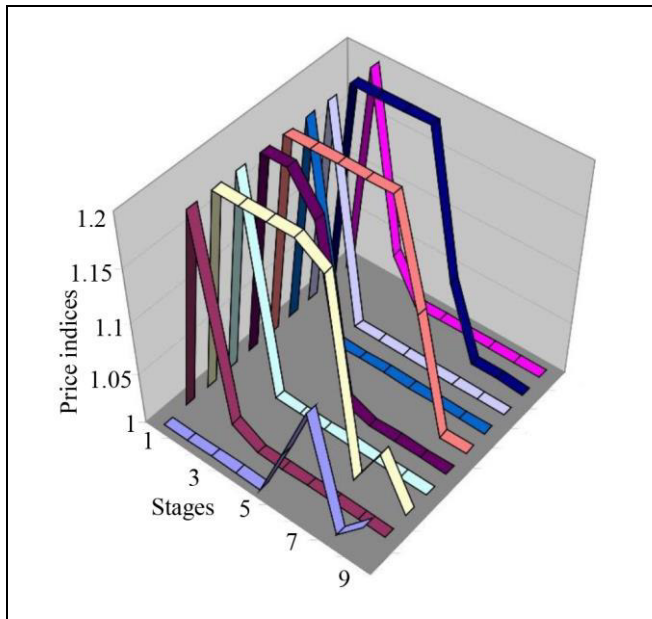


Fig. 4. Dynamics of price indices on range [1, 1.2] for different sectors:

- Products of forestry, logging and related services;
- Products of textiles, clothing, and leather;
- Paper and paper products;
- Printing and recording services;
- Basic pharmaceutical products and pharmaceutical preparations;
- Rubber and plastic products;
- Other non-metallic mineral products;
- Electrical equipment;
- Machinery and equipment n.e.c.;
- Repair and installation services of machinery and equipment.

According to the figures, the prices rose in the sectors with nonincreasing output plans. In other words, market pricing holds considering the allowable inflation.

Figure 5 shows the productivity curve when solving the stage-by-stage planning problem for Russia's economy data. (Productivity was measured in percent.)

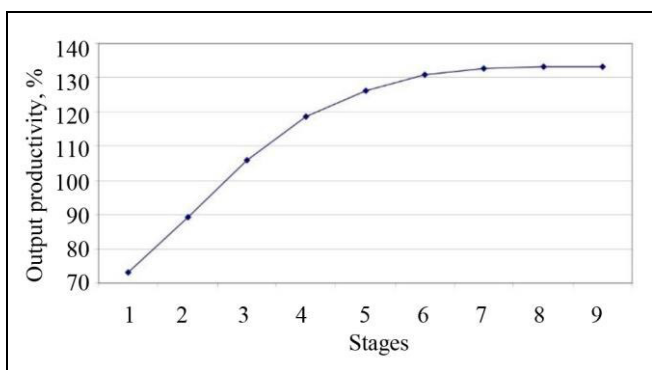


Fig. 5. Productivity under multistage output and price balancing.

The graph gives an idea of how far the initial state of the economy (the solution at the initial step was 90%) from the equilibrium state (the asymptote for the

repeated solution is close to 190%). In addition, the slope of the curve can be used to judge the sustainability of the economy.

This graph gives an idea of how far the initial state of the economy (the solution at the initial stage, 90%) is from the equilibrium state (the asymptote for the repeated solution, close to 190%). In addition, the slope of the curve can be used to judge the sustainability of the economy.

According to Fig. 5, the closer the price structure is to the equilibrium state, the higher tolerance the economy will have to price changes. Also, see Fig. 5, the first few stages of the indicative plan are the most effective. Let us compare these results with the productivity dynamics for the isolated output changes (Fig. 2). Obviously, the marginal level of productivity is higher in the case of joint changes in the prices and output. This can be explained as follows: the marginal level of productivity under output variations only corresponds to the optimum on the smaller-dimension admissible set of the parameter space; a larger value of the optimum is obtained when passing to the additional variation of prices.

Price changes cause inflation. Figure 6 shows the deflator dynamics for the entire list of products and services.

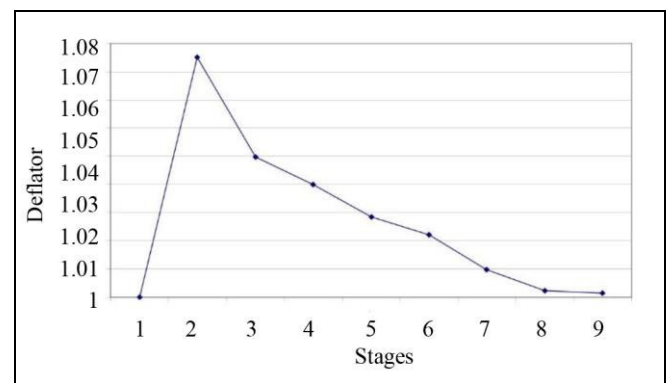


Fig. 6. The deflator under multistage output and price balancing.

The price growth constraint in the initial stages can be toughened to reduce the deflator peak. As a result, the growth of productivity at these stages will decrease.

CONCLUSIONS

This paper has considered analysis methods for control mechanisms of developing systems in crisis situations. Methods for assessing productivity indicators in natural and value terms have been developed within the economic system operation model in the autonomous mode. The procedure for calculating the



indicative dynamics of price and output proportions in the autonomous mode has been described. An illustrative example has been given: calculation results for the structure of Russia's intersectoral balance.

Despite the existing high potential for development, Russia's economy faces crisis phenomena. These factors cause the incomplete realization of the potential of its technological core [5]. The results of this paper demonstrate the possibilities of increasing the efficiency of the economy through the systematic structural modification of the technological core. Proceeding from the Rosstat data, the possible growth of productivity of the economy can be exceeded more than 2 times. Such possibilities should be implemented by elaborating strategic economic development plans. Along with choosing priority directions of development of the technological core, it is necessary to apply adequate forecasting methods for multisectoral dynamics considering the main aspects of economic activity: asset formation, accumulation, final consumption of the state and households, and export-import flows [14, 15]. New-level planning also involves appropriate organizational and institutional mechanisms [16].

Unlike a directive plan, an indicative plan is non-binding. If any sectors deviate from the plan, it will be recalculated to reflect the changed circumstances. The new plan and estimates of under-received benefits are reported to economic agents. Within fixed technological outputs, the stage of controlled (indicative plan-based) restructuring takes a limited time. The additional funding for asset formation and wages can be provided either by the sectors' own value added or by redistributing the funds between different sectors. In the latter case, the restructuring period may be shorter. The strategic plan is considered fulfilled after the restructuring period. However, the emergence of new technologies and refined data, as well as the adjustment of structural constraints on the outputs and prices, may require elaborating a new strategic plan. Therefore, the indicative plan should be corrected on a regular basis.

Adequate assessments of the socio-economic state of national and global systems and the consequences of control impacts are required at the strategic decision level (the state and interstate affairs). The following elements should be assessed:

- the imbalance of the national economy,
- the current and potential state of intersectoral interaction,
- strategic decision-making under instability,
- structural innovation planning.

The results above illustrate the specifics of the proposed methodology with relevant calculation and analysis tools. This class of problems has several peculiarities. Here, high-dimensional models are of real

interest. (The analysis may cover hundreds of economic activities.) Besides, the practical application of technological core models requires effective algorithms for solving mathematical programming problems [10, 12, 17] and linguistic calculation management means, particularly their integration into the working environment [11, 13]. Free access is needed to actual verified data and modern information technology, including an appropriate computing environment and interface devices. The paper [18] described the application of a similar open-access toolkit (Thread Pool Executor of Akka) for the processing of high-dimensional problems.

APPENDIX

P r o o f of Proposition 1.

Indeed, the maximum values of the criteria γ and r are the eigenvalues of the matrices \mathbf{A} and \mathbf{A}^T , respectively. Let \mathbf{V} and \mathbf{P} be the solutions of problems (2) and (8) with constraints (3) and (9), respectively. Since these constraints are homogeneous, the solutions under consideration are determined within the multiplier and, hence, are ambiguous.

The characteristic polynomials for both matrices coincide:

$$\begin{aligned} L(\gamma) &= (1 - \gamma a_{11})(1 - \gamma a_{22}) \dots (1 - \gamma a_{nn}) - \\ & a_{12} a_{21} (1 - \gamma a_{33}) \dots - a_{13} a_{31} (1 - \gamma a_{22}) - \dots, \\ L(r) &= (1 - r a_{11})(1 - r a_{22}) \dots (1 - r a_{nn}) - \\ & a_{12} a_{21} (1 - r a_{33}) \dots - a_{13} a_{31} (1 - r a_{22}) - \dots, \end{aligned}$$

where γ is the eigenvalue of the matrix \mathbf{A} and r is the eigenvalue of the matrix \mathbf{A}^T . ♦

P r o o f of Proposition 2.

Indeed, consider the transformation $\mathbf{y} = \mathbf{Ax} / \|\mathbf{x}\|$. We decompose the vector \mathbf{x} with respect to the eigenvectors \mathbf{s}^i , $i = 1, \dots, n$, of the matrix \mathbf{A} corresponding to the eigenvalues λ_i :

$$\mathbf{x} = \sum_{i=1}^n b_i \mathbf{s}^i.$$

Then

$$\mathbf{Ax} = \sum_{i=1}^n b_i \mathbf{As}^i = \sum_{i=1}^n b_i \lambda_i \mathbf{s}^i$$

and

$$\mathbf{Ax} / \|\mathbf{x}\| = \frac{\sum_{i=1}^n b_i \lambda_i \mathbf{s}^i}{\left\| \sum_{i=1}^n b_i \mathbf{s}^i \right\|}.$$

By assumption, the matrix spectrum satisfies the condition $|\lambda_i| < 1$, $i = 1, \dots, n$. Hence, this transformation is a contraction, and \mathbf{s}^1 is a fixed attraction point of this transformation.

If the initial approximation has nonnegative components, all subsequent iterations will give the same result as the obtained eigenvector. The eigenvectors \mathbf{s}^i correspond

ing to the other eigenvalues λ_i , $i \neq 1$, contain negative components because they are orthogonal to \mathbf{s}^1 . Therefore, due to the nonnegativity of the matrix \mathbf{A} ,

$$|\lambda_i| = \frac{\|\mathbf{A}\mathbf{s}^1\|}{\|\mathbf{s}^1\|} > \frac{\|\mathbf{A}\mathbf{s}^i\|}{\|\mathbf{s}^i\|} = |\lambda_i|, i \neq 1.$$

Thus, deviations from the equilibrium vector \mathbf{s}^1 will decrease the estimate $\|\mathbf{Ax}\|/\|\mathbf{x}\|$ for $\mathbf{x} \neq \mathbf{s}^1$; due to the contraction property of the operator, this estimate will increase at the subsequent iterations. Therefore, the iterative process will converge to the vector \mathbf{s}^1 and the value λ_1 . ♦

Proof of Proposition 3.

Indeed, the gradient of $a(\mathbf{x})$ has the form

$$\nabla_x a(\mathbf{x}^k) = -(a(\mathbf{x}^k)\mathbf{I} - \mathbf{A}^T \mathbf{I}) / \|\mathbf{x}^k\|,$$

\mathbf{z}^k is the projection of the point \mathbf{y}^k on the technological constraint

$$\mathbf{y}^k = a\mathbf{A}\mathbf{y}^k,$$

and \mathbf{x}^{k+1} is the projection of the point \mathbf{z}^k on the constraint

$$\mu \leq x_i \leq \theta, i = 1, \dots, n.$$

Since $\nabla_x a(\mathbf{x}^k) \neq 0$, the projection operators are monotonic, and there exists a step $h > 0$ such that the sequence $a(\mathbf{x}^k)$ will monotonically decrease. Moreover, because the admissible domain is a compact set, the limit point $a(\mathbf{x}^*)$ will be bounded and represent a constrained optimum. ♦

REFERENCES

1. Uzyakov, M.N., Problems of Economic Measurements and Possibilities of Structural Analysis, *Studies on Russian Economic Development*, 2020, vol. 31, no 1, pp. 3–4. (In Russian.)
2. Gusev, V.B., Equilibrium Models of Multi-Resource Self-Developing Systems, *Control Sciences*, 2007, no. 3, pp. 18–24. (In Russian.)
3. Leontief, W.W., *Essays in Economics. Theories, Theorizing, Facts, and Policies*, New York: Oxford University Press, 1966.
4. *Indikativnoe planirovanie i provedenie regional'noi politiki* (Indicative Planning and Regional Policy Implementation), Abdikeev, M.N., et al., Moscow: Finansy i Statistika, 2007. (In Russian.)
5. Gusev, V.B., Models of Autonomous Control in the Developing Systems, *Control Sciences*, 2018, no. 6, pp. 2–17. (In Russian.)
6. Polyak, B.T., Khlebnikov, M.V., and Rapoport, L.B., *Matematicheskaya teoriya avtomaticheskogo upravleniya* (Mathematical Theory of Automatic Control), Moscow: URSS, 2019. (In Russian.)
7. Gusev, V.B., The Technological Core Model of a Large-Scale Economic System: Optimal Characteristics, *Control Sciences*, 2021, no. 6, pp. 25–33.
8. *Supply and Use Tables of the Russian Federation for 2019 (in Current Prices, mil. rub.)*. Published by the Federal State Statistics Service of the Russian Federation, January 26, 2022. <https://rosstat.gov.ru/statistics/accounts>. (In Russian.)
9. Dorfman, R., Samuelson, P.A., and Solow, R.M., *Linear Programming and Economic Analysis*, New York: McGraw-Hill, 1958.
10. Fox, W.P. and Burks, R., Mathematical Programming: Linear, Integer, and Nonlinear Optimization in Military Decision-Making, in *Applications of Operations Research and Management Science for Military Decision Making*, New York: Springer, 2019, pp. 137–191.
11. Mason, A.J., OpenSolver - An Open Source Add-in to Solve Linear and Integer Programmes in Excel, *Operations Research Proceedings 2011*, Klatte, D., Lüthi, H.-J., and Schmedders, K., Eds., Berlin-Heidelberg: Springer, 2012, pp. 401–406. http://dx.doi.org/10.1007/978-3-642-29210-1_64.
12. Bergstra, J., Bardenet, R., Bengio, Y., and Kégl, B., Algorithms for Hyper-parameter Optimization, *Proceedings of the 24th International Conference on Neural Information Processing Systems (NIPS'11)*, Red Hook, NY, USA: Curran Associates Inc., 2011, pp. 2546–2554.
13. Doumic, M., Perthame, B., Ribes, E., et al., Toward an Integrated Workforce Planning Framework Using Structured Equations, *European Journal of Operational Research*, 2017, vol. 262, iss. 1, pp. 217–230.
14. Samuelson, P.A., *Economics*, New York: McGraw-Hill, 1989.
15. Petrov, A.A., Pospelov, I.G., and Shaninin, A.A., *Opyt matematicheskogo modelirovaniya ekonomiki* (An Experience of Mathematical Modeling of an Economy), Moscow, Energoatomizdat, 1996. (In Russian.)
16. Antipov, V.I., *GOSPLAN. Vchera, Segodnya, zavtra* (State Planning: Yesterday, Today, Tomorrow), Moscow: Kontseptual, 2019. (In Russian.)
17. Polyak, B.T., *Introduction to Optimization*, Optimization Software, 1987.
18. Hai, T.N., Tien, V.D., and Csaba, R., Optimizing the Resource Usage of Actor-based Systems, *Journal of Network and Computer Applications*, 2021, vol. 190:103143. DOI: <https://doi.org/10.1016/j.jnca.2021.103143>.

This paper was recommended for publication by V.V. Klochkov, a member of the Editorial Board.

Received August 13, 2022,
and revised November 6, 2022.
Accepted November 23, 2022

Author information

Gusev, Vladislav Borisovich. Cand. Sci. (Phys.–Math.), Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
✉ gusvbr@ipu.ru

Cite this paper

Gusev, V.B., A Strategic Management Model for Restructuring the Technological Core of an Economy, *Control Sciences* **6**, 11–20 (2022). <http://doi.org/10.25728/cs.2022.6.2>

Original Russian Text © Gusev, V.B., 2022, published in *Problemy Upravleniya*, 2022, no. 6, pp. 14–25.

Translated into English by Alexander Yu. Mazurov, Cand. Sci. (Phys.–Math.), Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
✉ alexander.mazurov08@gmail.com