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A BLOCK APPROACH TO CSTR CONTROL UNDER UNCERTAINTY, STATE-SPACE AND CONTROL CONSTRAINTS[1]

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Abstract. This paper designs a control law to maintain the temperature in the jacket of a continuous stirred tank reactor (CSTR). The standard mathematical model describing the reactor operation is extended by introducing the actuator's dynamics. The state-space and control constraints are considered by a nonlinear change of the variables of the plant's initial model using linear sat functions. In the transformed system, these constraints are considered by feedback control law design. The block approach allows linearizing the feedback control law by sequentially solving the first-order design subproblems. Under incomplete information on the state vector and the effect of exogenous disturbances, an observer of the state vector and disturbances is constructed to estimate the unknown signals with a given accuracy. The effectiveness of the proposed approach is illustrated by simulating the CSTR–DC motor system in MATLAB.

Keywords: CSTR, tracking problem, block approach, observer of state vector and disturbances, state-space and control constraints.

INTRODUCTION

The continuous stirred tank reactor (CSTR) is widespread in the chemical industry. The CSTR dynamics are usually described by two nonlinear first-order differential equations [1, 2], a reference model for applying and testing new control algorithms.

Nowadays, improving CSTR control is a topical problem that attracts the attention of many control theorists and practitioners [3–8].

Much research in this field involves the sliding mode approach [9–16]: it ensures robust properties of closed loop systems and invariance to exogenous disturbances acting in control channels. Note that within this approach, control laws are often represented by the plant's variables that physically cannot be discontinuous functions, e.g., the flow rate of the coolant in the CSTR jacket. Hence, the practical importance of the sliding mode approach in the automation of various technological processes is significantly reduced.

State observers based on sliding modes and systems with deep feedback [17–21] are widely used to obtain information about the state vector and disturbances. Note that under disturbances, all these vectors can be estimated only within such an approach.

The problem of considering physical constraints on the state vector and control is underinvestigated in control theory. For example, only control constraints were taken into account in [22–24].

This paper proposes a complex solution for CSTR control that develops the original control law design method [25] for mechanical systems with constraints. It is methodologically based on a block approach to control [26], which decomposes high-dimensional problems into independently solvable subproblems of lower dimensions when designing feedback control laws and state observers. Treating the state variables as fictitious control actions, first of all, we satisfy the matching conditions for the disturbances in each subproblem. (In other words, the disturbance belongs to the control space.) In addition, using sat functions in local feedback law design, we ensure the bounded components of the state vector and controls.

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This paper is organized as follows. Section 1 briefly describes the operation principle of a CSTR and its mathematical model and states the problem. In Section 2, we construct an observer of the state vector and disturbances with discontinuous and continuous corrections with large gains. In Section 3, feedback design algorithms are developed by combining local feedback laws with continuous sat functions and discontinuous control of the armature voltage of a DC motor. Section 4 illustrates the effectiveness of the proposed algorithms by simulation modeling in MATLAB.

1. PLANT'S MATHEMATICAL MODEL. PROBLEM STATEMENT

A CSTR is a key component of equipment needed to complete chemical reactions in many chemical and biochemical industries. A complex chemical reaction occurs in a CSTR, e.g., converting a hazardous chemical waste (reagent) into an acceptable chemical product. The schematic diagram of the reactor is shown in Fig. 1.

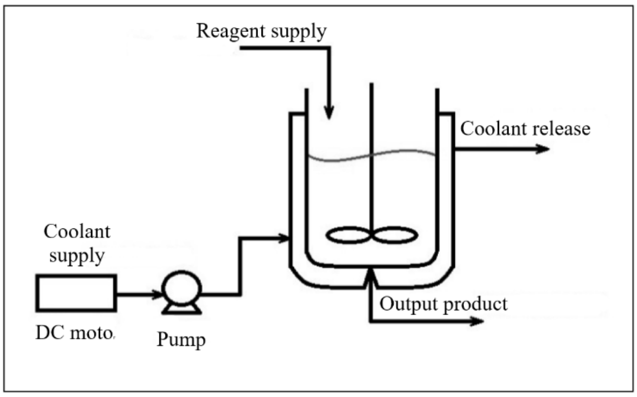

Fig. 1. The schematic diagram of a CSTR.

An irreversible first-order exothermic reaction $A \to B + \Delta H$ occurs in the tank, where A is a reagent, B is a product, and ΔH is the thermal effect of the chemical reaction (enthalpy).

The CSTR volume is equal to V. A reagent having a concentration C_{Af}, a temperature T_f, and a density ρ is supplied to the reactor input at a flow rate q.

The temperature in the reactor is maintained at a certain level through setting a coolant temperature in the jacket, T_c, by controlling the flow rate of the coolant. The reactor's output product is characterized by a temperature T, a concentration C_A, and a flow rate q under the invariable substance volume in the reactor.

Assuming the invariable substance volume in the reactor, ideal mixing, and the invariable substance density in the reactor, the laws of conservation of mass and energy yield the following dynamic model of CSTR [16]:

$$\dot{C}_A(t) = \frac{q}{V}(C_{Af}(t) - C_A(t)) - k_0 C_A(t) e^{-E/RT(t)},$$

$$\dot{T}(t) = \frac{q}{V}(T_f(t) - T(t)) + \frac{(-\Delta H)}{\rho C_p} k_0 C_A(t) e^{-E/RT(t)} + \quad (1)$$

$$\frac{UA}{\rho V C_p}(T_c - T(t)).$$

The parameters of the model (1) are described in Table 1.

Table 1

The parameters of the CSTR model: values and units of measurement

Parameter	Value	Unit of measurement
C_A	The concentration of product *B*	kmol/m^3
T	Temperature in the reactor and product temperature	K
q	Reagent flow rate	m^3/min
V	Reactor volume	m^3
C_{af}	Reagent concentration	kmol/m^3
k_0	First-order reaction rate constant	min^{-1}
E	Activation energy	J/mol
R	Universal gas constant	J/(mol·K)
T_f	Reagent temperature	K
ΔH	Reaction enthalpy	cal/kmol
ρ	Reagent density	g/m^3
T_c	Coolant temperature	K
C_p	Reagent specific heat	cal/(K·g)
U	Heat-transfer coefficient	W/(m^2·K)
A	Heat delivery surface	m^2

The goal of CSTR control is to tune the coolant temperature T_c by changing the coolant flow rate so that the temperature T in the reactor corresponds to the desired values.

The coolant is supplied to the reactor by a pump with a DC motor. According to [27], the operation of this motor is described by the system of equations

$$\dot{x}_3(t) = a_{21}(g x_4(t) - m_L(t)),$$

$$\dot{x}_4(t) = a_{32}(u_2(t) - g x_3(t) - a_{31} x_4(t)), \quad (2)$$

where $x_3(t)$ is the motor shaft rotation frequency; $x_4(t)$ is the armature current; $u_2(t)$ is the armature voltage; $g = \text{const}$ is a magnetic flux; $m_L(t)$ is the load moment; $a_{ij} = \text{const} > 0$ are the motor's design parameters.

The problem is to track a given coolant temperature in the jacket, $T_d(t)$, by the output variable $T(t)$:

$$e_2(t) = T(t) - T_d(t) \to 0.$$

Assume that the pump load is $m_L = mx_3^2(t)$, $m = \text{const}$, and the temperature in the jacket is proportional to the coolant flow rate: $T_c(t) - T(t) = x_3(T_{c0} - T(t))$, where $T_{c0} = \text{const} > 0$ denotes the coolant temperature at the jacket input. Under these assumptions, we write the plant's model (1), (2) in new variables:

$$\begin{aligned} \dot{x}_1(t) &= -ax_1(t) - f_1(x_2)x_1(t) + \xi_1(t), \\ \dot{e}_2(t) &= -ae_2(t) + bf_1(x_2)x_1(t) + \\ &\quad \beta x_3(T_{c0} - T_d(t) - e_2) + \xi_2(t), \\ \dot{x}_3(t) &= a_{21}(gx_4(t) - mx_3^2(t)), \\ \dot{x}_4(t) &= a_{32}(u_2(t) - gx_3(t) - a_{31}x_4(t)), \end{aligned} \tag{3}$$

где $x_1(t) = C_A(t)$, $x_2(t) = T(t)$, $e_2(t) = x_2(t) - T_d(t)$, $\xi_1(t) = aC_{Af}(t)$, $\xi_2(t) = a(T_f - T_d) - \dot{T}_d$, $f_1(x_2) = k_0 e^{-\gamma/x_2(t)}$, $a = \dfrac{q}{V}$, $\gamma = \dfrac{E}{R}$, $b = \dfrac{\Delta H}{C_p\rho}$, and $\beta = \dfrac{UA}{\rho V C_p}$.

For the control of (3), let the output variable $y(t) = e_2(t)$ and the armature current x_4 be measured.

The disturbance $\xi_1(t)$ depends on the input reagent concentration $C_{Af}(t)$ and is difficult to measure in real-time. On the contrary, the input reagent temperature $T_f(t)$ is rather easy to measure. In the sequel, assume that the disturbance $\xi_1(t)$ is unmeasured, whereas the signal $\xi_2(t)$ is measured.

In view of the technological process features, the system variables should satisfy the constraints

$$\begin{aligned} &x_i \in [0, X_i],\ i = 2, 3;\ |x_4| \le X_4; \\ &|u_2| \le U,\ X_i, U = \text{const} > 0. \end{aligned} \tag{4}$$

The problem of considering state-space and control constraints is currently underinvestigated in control theory. This paper proposes to take them into account in the plant's mathematical model by introducing a change of variables—a linear sat function. For details, see Section 3.

Control should maintain given values for the output product's temperature and concentration. In this case, the voltage u_2 at the DC motor armature is the control action in the temperature loop, and the reagent flow rate q is the control action in the concentration loop. In what follows, we will design the temperature control loop in the reactor, assuming the value $a = \dfrac{q}{V}$ to be known. The next section deals with estimating the state vector and disturbances in the system (3).

2. DESIGNING AN OBSERVER OF THE STATE VECTOR AND DISTURBANCES

Consider the problem of estimating the state vector and disturbances in the system (3) from the measured output variables e_2 and x_4 under the assumption that the signals q and ξ_2 are measured.

The observability of the system (3) in the output variables e_2 and x_4 is established based on the following considerations:

- The DC motor model (the last two equations in (3)) does not depend on the variables of the reactor model (the first two equations in (3)), and its observability in the variable x_4 is obvious due to the block observability form (BOF) [28]. The variable x_3 is estimated by an observer designed using the DC motor model.
- Since the variables e_2, x_3, and ξ_2 are measured (see below), the reactor model structurally coincides with the block observability form considering the disturbances [21]. Hence, it is also observable [21].

Note that the desired observer will be constructed designed in the sequence indicated above. Now we formulate some theoretical results on systems with deep feedback, which will be used in the further presentation.

Lemma. *Consider a first-order system of the form*

$$\dot{\varepsilon}(t) = u + \xi(t), \tag{5}$$

where $\varepsilon(t), u(t), \xi(t) \in R$ *denote the state variable, control, and disturbance, respectively, such that* $|\xi(t)| \le E = \text{const}$ *and* $|\dot{\xi}(t)| \le \bar{E} = \text{const}, \forall t \ge 0$.

Then there exists a control law $u(t) = -\alpha\varepsilon(t)$, $\alpha = \text{const} > 0$, *such that the relations:*

1) $|\varepsilon(t)| \le \Delta_0$,

2) $|\dot{\varepsilon}(t)| < \bar{\Delta}_0$,

3) $|\alpha\varepsilon - \xi| < \bar{\Delta}_0$

hold for any given constants $\Delta_0, \bar{\Delta}_0, \alpha, \bar{\alpha} = \text{const} > 0$ *in a finite time* $t_0 > 0$.

P r o o f

1) The convergence of the state variable ε of the system (5) to a given neighborhood of zero, $|\varepsilon| \le \Delta_0$, is ensured by choosing a candidate Lyapunov function $V = 0.5\varepsilon^2$. We define the gain as $\alpha_0 = \dfrac{E}{\Delta_0}$.

Then the condition $\dot{V} = \varepsilon\dot{\varepsilon} = \varepsilon(-\alpha_0\varepsilon + \xi) \le |\varepsilon|(-\alpha_0|\varepsilon| + E) < 0 \Rightarrow -\alpha_0|\varepsilon| + E < 0$ holds out of the domain

$|\varepsilon| \le \frac{E}{\alpha_0} = \Delta_0$. In the case $|\varepsilon(0)| \le \Delta_0$, the variable does not leave the given neighborhood $|\varepsilon(t)| \le \Delta_0,\ \forall t > 0$; in the case $|\varepsilon(0)| > \Delta_0$, it can approach this neighborhood from outside without limit. Hence, choosing $\alpha > \alpha_0$ ensures the convergence of the state variable to the given neighborhood $|\varepsilon(t)| \le \Delta_0$ in a finite time. Really, since $\Delta = \frac{E}{\alpha} < \Delta_0 = \frac{E}{\alpha_0}$, the variable will stay in the new neighborhood $|\varepsilon(t)| \le \Delta,\ \forall t > 0$, if $|\varepsilon(0)| \le \Delta$, or approach the new neighborhood without limit if $|\varepsilon(0)| > \Delta$, reaching the neighborhood $\Delta_0 \le \frac{E}{\alpha_0}$ in a finite time. The solution of (5) satisfies the bound $|\varepsilon(t)| \le |\varepsilon(0)e^{-\alpha t}| + \left|e^{-\alpha t}\int_0^t e^{\alpha\tau}\xi(\tau)\,d\tau\right| \le |\varepsilon(0)|e^{-\alpha t} + \frac{E}{\alpha}(1-e^{-\alpha t})$, and the time t_0 of reaching the domain $|\varepsilon| \le \Delta_0,\ \forall t \ge t_0$, can be estimated as: $(|\varepsilon(0)|-\Delta)e^{-\alpha t} + \Delta = \Delta_0 \Rightarrow t_0 = \frac{1}{\alpha}\ln\left(\frac{|\varepsilon(0)|-\Delta}{\Delta_0-\Delta}\right),\ |\varepsilon(0)| > \Delta_0$.

2) A similar result applies to the derivative of the state variable in the system $\ddot{\varepsilon} = -\alpha\dot{\varepsilon} + \dot{\xi}$ obtained by differentiating both sides of equation (5). Let $|\dot{\varepsilon}| \le \bar{\Delta}_0 = \frac{\bar{E}}{\bar{\alpha}_0}$ be a given neighborhood of zero. We choose $\alpha > \bar{\alpha}_0$ and denote $\bar{\Delta} = \frac{\bar{E}}{\alpha} < \bar{\Delta}_0 = \frac{\bar{E}}{\bar{\alpha}_0}$. Then the relation $|\dot{\varepsilon}| \le \bar{\Delta}_0,\ t \ge \bar{t}_0$, where $\bar{t}_0 = \frac{1}{\bar{\Delta}_0}\ln\left(\frac{|\dot{\varepsilon}(0)|-\bar{\Delta}}{\bar{\Delta}_0-\bar{\Delta}}\right)$ and $|\dot{\varepsilon}(0)| > \bar{\Delta}_0$, will hold in a finite time.

3) Due to equality (5) and item 2), we have $|\alpha\varepsilon(t)-\xi(t)| \le \bar{\Delta}_0,\ \forall t \ge \bar{t}_0$. Hence, the disturbance can be estimated with a given accuracy: $\alpha\varepsilon(t) = \xi(t) + \delta(t)$, $\delta(t) \le \bar{\Delta}_0,\ \forall t \ge \bar{t}_0$. Note that for constant disturbances, the system $\ddot{\varepsilon} = -\alpha\dot{\varepsilon} + \dot{\xi},\ \dot{\xi} = 0$, has the solution $\ddot{\varepsilon} = \dot{\varepsilon}(0)e^{-\alpha t}$. Therefore, the disturbance estimate is asymptotically convergent: $\alpha\varepsilon \to \xi,\ t \to \infty$.

Choosing the parameter α based on the condition $\alpha \ge \max\{E/\Delta_0, \bar{E}/\bar{\Delta}_0\}$ ensures the desired convergence of the variable $\varepsilon(t)$ and its derivative $\dot{\varepsilon}(t)$ to the given domains: 1) $|\varepsilon(t)| \le \Delta_0$, 2) $|\dot{\varepsilon}(t)| \le \bar{\Delta}_0$, 3) $|\alpha\varepsilon(t)-\xi(t)| \le \bar{\Delta}_0$, $\forall t \ge \max\{t_0, \bar{t}_0\}$. The proof of this lemma is complete. ♦

For estimating the state vector of the system (3), we design an observer of the form

$$\begin{aligned}
\dot{z}_1 &= -[a + f_1(x_2)]z_1 + \nu_1,\\
\dot{z}_2 &= -az_2 + \beta z_3(T_{c0} - T_d - e_2) + \xi_2 + \nu_2,\\
\dot{z}_3 &= a_{21}gx_4 + \nu_3,\\
\dot{z}_4 &= a_{32}(u_2 - a_{31}z_4) + \nu_4,
\end{aligned} \tag{6}$$

where ν_i are the observer corrections determined below.

Using formulas (3) and (6), we rewrite the system in the residues $\varepsilon_i = x_i - z_i$, $i = 1, 3, 4$, and $\varepsilon_2 = e_2 - z_2$:

$$\begin{aligned}
\dot{\varepsilon}_1 &= -(a + f_1(x_2))\varepsilon_1 + \xi_1(t) - \nu_1,\\
\dot{\varepsilon}_2 &= -a\varepsilon_2 + \beta(T_{c0} - T_d - e_2)\varepsilon_3 + bf_1(x_2)x_1(t) - \nu_2,\\
\dot{\varepsilon}_3 &= -a_{21}mx_3^2 - \nu_3,\\
\dot{\varepsilon}_4 &= -a_{32}a_{31}\varepsilon_4 - a_{32}gx_3 - \nu_4.
\end{aligned} \tag{7}$$

Within the cascade approach [28], a design procedure for the observer (7) of the state vector and disturbances includes the following steps:

1. We choose an appropriate correction for the last subsystem of the system (7), i.e., the discontinuous function $\nu_4 = l_4\mathrm{sign}(\varepsilon_4)$, where $l_4 = \mathrm{const} > |a_{32}gx_3|$, to ensure the occurrence of a sliding mode on the line $\varepsilon_4 = 0$. The average (equivalent) value of the discontinuous signal is $\nu_{4eq} = -a_{32}gx_3$. In practice, the equivalent value of the discontinuous control can be obtained using the first-order filter $\mu\dot{\tau} = -\tau + \nu_4$, where $\mu > 0$ and $\nu_{4eq} \approx \tau$ [28].

2. Using the equivalent value and $x_3 = -\nu_{4eq}/(a_{32}g)$, we construct the correction $\nu_3 = -a_{21}m \times (\nu_{4eq}/(a_{32}g))^2 + l_3[-\nu_{4eq}/(a_{32}g) - z_3]$ for the third subsystem of the system (7). As a result, the third subsystem of (7) takes the form $\dot{\varepsilon}_3 = -l_3\varepsilon_3$, and the variable ε_3 asymptotically vanishes with an appropriately assigned coefficient $l_3 > 0$: $\varepsilon_3 \to 0 \Rightarrow z_3 \to x_3$.

3. Under the assumption $\varepsilon_3 \to 0 \Rightarrow z_3 \to x_3$, the second equation of the system (7) takes the form

$$\dot{\varepsilon}_2 = -a\varepsilon_2 + bf_1(x_2)x_1 - \nu_2.$$

We choose the correction $\nu_2 = l_2\mathrm{sign}(\varepsilon_2)$, $l_2 = \mathrm{const} > bf_1(x_2)x_1$, to ensure the occurrence of a sliding mode on the plane $\varepsilon_2 = 0$. The average value of the discontinuous signal is $\nu_{2eq} = bf_1(x_2)x_1$.

4. In the last step, we choose $\nu_1 = (-f_1(x_2) + l_1)\left(\frac{\nu_{2eq}}{bf_1(x_2)} - z_1\right) = (-f_1(x_2) + l_1)\varepsilon_1$.

Then the first equation of (7) takes the form $\dot{\varepsilon}_1 = -(a + l_1)\varepsilon_1 + \xi_1$. Under the assumptions $|\xi_1(t)| \le E_1$ and $|\dot{\xi}_1(t)| \le \bar{E}$, the relations $|\varepsilon_1| \le \dfrac{E_1}{a + l_1} = \Delta_1$ and $|\dot{\varepsilon}_1| \le \dfrac{\bar{E}_1}{a + l_1} = \bar{\Delta}_1$ hold by the lemma. Thus, assigning an appropriate coefficient $l_1 > 0$, we ensure the desired stabilization accuracy $|\varepsilon_1| \le \Delta_1$ in a finite time, thereby estimating the variable $x_1(t) = z_1(t) + \delta_1(t)$ and the disturbance $\xi_1(t) = (a + l_1)\varepsilon_1(t) + \bar{\delta}_1(t)$ with the desired accuracy: $|\delta_1(t)| \le \Delta_1$ and $|\bar{\delta}_1(t)| \le \bar{\Delta}_1$. Note that $\delta_1, \ \bar{\delta}_1 \to 0$ as $l_1 \to \infty$. (The estimation accuracy grows infinitely in the case of large gains.)

Thus, the state vector of the system (3) and the disturbance $\xi_1(t)$ have been estimated.

This design procedure for an observer of the state vector and disturbances involves discontinuous and continuous corrections. Clearly, the design based on sliding modes is easier from a computational viewpoint. However, the average (equivalent) values of corrections are produced by first-order filters [28], which dynamically extends the state space of the original model. The technique for linearizing the residual equations (7) with continuous corrections (Step 4 of the observer design procedure) can be used in all other steps by analogy. In this case, the cascade approach [28] allows decomposing the observer design procedure into one-dimensional subproblems successively solved with a predetermined accuracy of the resulting estimates without extending the state space of the closed loop system.

The next section considers a control law design procedure for the system (7) under complete information (the available estimates of the state vector and disturbances).

3. FEEDBACK LAW DESIGN

Using the block approach, we present the general solution under complete information about the state vector and disturbances provided by the observer (6). In the case of complete information and the state-space and control constraints (4), the feedback law design procedure based on the block approach [21] includes the following steps.

Step 1. We rewrite the second equation of the system (3) as

$$\dot{e}_2 = -ae_2(t) + \beta d(t)x_3 + \bar{\xi}_2(t), \qquad (8)$$

where $\bar{\xi}_2 = bf_1(x_2)x_1 + \xi_2$ and $d(t) = T_{c0} - T_d(t) - e_2(t) < 0$, $|d| \le D = \text{const}$.

In the system (8), the control action is the motor shaft rotation frequency x_3. Under pump loading, this frequency is positive. We introduce the change of variable

$$e_3 = \beta d(t)x_3 + M_3\text{sat}^+(s_3), \qquad (9)$$

where $s_3 = k_2e_2 + \bar{\xi}_2$. Stabilizing the variable $e_3 = \beta d(t)x_3 + M_3\text{sat}^+(s_3) = 0$, we consider the constraint (4) on the variable $x_3 \in [0, X_3]$ by choosing $0 < M_3 < \beta DX_3$.

Definition. Let $M = \text{const} > 0$ and $b = \text{const}$. Then $M\text{sat}(s) = \min(M, |s|)\ \text{sign}(s)$ and $M\text{sat}^+(s) = M\text{sat}(s)0.5[1 + \text{sign}(s)]$.

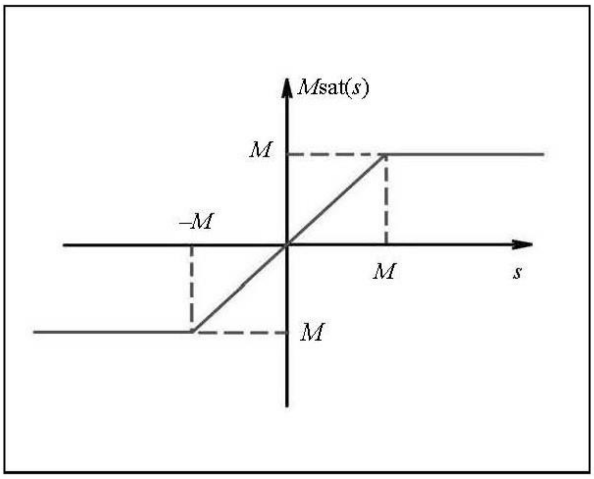

Fig. 2. The graph of *M*sat(*t*).

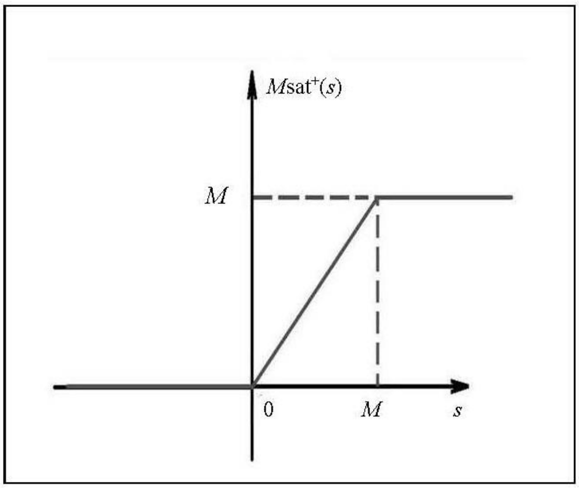

Fig. 3. The graph of *M*sat$^+$(*t*).

Substituting the sum (9) into formula (8), we transform the second equation to

$$\dot{e}_2 = -ae_2 + e_3 - M_3\text{sat}^+(s_3) + \bar{\xi}_2. \qquad (10)$$

If the variable s_3 falls into the linear zone $0 < s_3 < M_3$, equation (10) reduces to

$$\dot{e}_2 = -(a + k_2)e_2 + e_3,$$

where the parameter $k_2 > 0$ determines the convergence of e_2 to a given neighborhood of zero. According to the lemma, this neighborhood is described by $|e_2| \le \Delta_3 / (a + k_2)$ under the condition $|e_3| \le \Delta_3 = \text{const}$.

Falling into the linear zone depends on the amplitude M_3 due to the relation $s_3\dot{s}_3 < 0$ holding in the nonlinear zone $s_3 \notin (0, M_3)$; see Fig. 3. We write the equation for the variable s_3: $\dot{s}_3 = -(as_3 - \bar{\xi}_2) + \dot{\bar{\xi}}_2 + k_2[-M_3\text{sat}^+(s_3) + e_3 + \bar{\xi}_2]$.

The parameters M_3 and k_2 of the function $M_3\text{sat}^+(s_3)$ are assigned from the following considerations.

• For $s_3 \in [M_3, \infty]$ (which implies $-M_3\text{sat}^+(s_3) = -M_3$), the inequality $\dot{s}_3 = -a(s_3 - \bar{\xi}_2) + \dot{\bar{\xi}}_2 + k_2[-M_3 + e_3 + \bar{\xi}_2] < 0$ holds. Hence, the amplitude M_3 should satisfy the constraint $M_3 > \frac{1}{k_2}[a\bar{\xi}_2 + \dot{\bar{\xi}}_2] + e_3 + \bar{\xi}_2$.

• For $s_3 \in (-\infty, 0)$ (which implies $M_3\text{sat}^+(s_3) = 0$ and $M_3\text{sat}^+(s_3) = 0$), the inequality $\dot{s}_3 = -a(s_3 - \bar{\xi}_2) + \dot{\bar{\xi}}_2 + k_2[e_3 + \bar{\xi}_2] > 0$ holds. Hence, under the conditions $e_3 + \bar{\xi}_2 > 0$ and $|e_3| \le \Delta_3 = \text{const}$, the coefficient k_2 should satisfy the constraint $k_2 > \frac{|a\bar{\xi}_2 + \dot{\bar{\xi}}_2|}{e_3 + \bar{\xi}_2}$. Note that due to $bf_1(x_2)x_1 + a(T_f(t) - T_d) > 0$, the requirement $\bar{\xi}_2 + e_3 = bf_1(x_2)x_1 + a(T_f(t) - T_d) - \dot{T}_d + e_3 > 0$ restricts the desired stabilization accuracy $|e_3| \le \Delta_3$ (see the next step) and the rate of change of the given neighborhood: $\Delta_3 < bf_1(x_2)x_1 + a(T_f(t) - T_d(t))$ and $|\dot{T}_d(t)| < bf_1(x_2)x_1 + a(T_f(t) - T_d(t)) - \Delta_3$. In the physical sense, the latter inequality limits the rate of change of the given neighborhood to an acceptable level when the tracking problem becomes solvable.

Step 2. We ensure that the variable e_3 from equation (10) falls into the neighborhood of zero: $|e_3| \le \Delta_3$. According to (9), the dynamics of the variable e_3 are described by

$$\dot{e}_3 = \beta d(t)a_{21}gx_4 + \xi_3. \qquad (11)$$

Treating the variable x_4 in the system (11) as a fictitious control action, we make it equal to

$$e_4 = \beta d(t)a_{21}gx_4 + M_4\text{sat}(s_4), \qquad (12)$$

where $M_4\text{sat}(s_4) = \min(M_4, |s_4|)\text{sign}(s_4)$ (Fig. 2) and $s_4 = k_3e_3 + \xi_3$. Stabilizing the variable $e_4 \to 0$, we consider the constraint $|x_4| \le X_4$ by choosing an amplitude $M_4 < \beta Da_{21}gX_4$.

Equation (11) with the local feedback law (12) takes the form

$$\dot{e}_3 = e_4 - M_4\text{sat}(s_4) + \xi_3.$$

In the linear zone ($|s_4| < M_4 \Rightarrow M_4\text{sat}(s_4) = s_4$), it reduces to

$$\dot{e}_3 = -k_3e_3 + e_4.$$

The amplitude M_4 under which the variable s_4 falls into the linear zone is found using Lyapunov's second method.

We write the derivative of the function s_4 as

$$\dot{s}_4 = k_3[-M_4\text{sat}(s_4) + \varphi_4(.)] + \dot{\xi}_3,$$

where $\dot{\xi}_3 = 2\beta da_{21}mx_3\dot{x}_3 + \frac{d^2}{dt^2}[M_3\text{sat}^+(s_3)]$ and $\varphi_4 = e_4 + \beta da_{21}mx_3^2 + \frac{d}{dt}[M_3\text{sat}^+(s_3)]$.

We choose a candidate Lyapunov function of the form $V = 0.5s_4^2$. Then the requirement $\dot{V} = s_4\dot{s}_4 < 0$ outside the linear zone ($|s_4| \ge M_4$) yields the derivative $\dot{V} = s_4\{k_3[-M_4\text{sign}(s_4) + \varphi_4(.)] + \dot{\xi}_3\} < 0$. Hence, the amplitude should be assigned from the condition $M_4 > \Phi_4 + \bar{E}_3 / k_3$, where $|\dot{\xi}_3| \le \bar{E}_3 = \text{const}$ and $|\varphi_4| \le \Phi_4 = \text{const}$.

Step 3. The last step is to stabilize the variable (12) described by

$$\dot{e}_4 = \beta da_{21}ga_{32}u_2 + \xi_4, \qquad (13)$$

where $\xi_4 = \beta\dot{d}(t)a_{21}gx_4 - \beta d(t)a_{21}ga_{32}(gx_3 + a_{31}x_4) + \frac{d}{dt}[M_4\text{sat}(s_4)]$.

We choose the discontinuous control

$$u_2 = M_2\text{sign}(e_4). \qquad (14)$$

In the system (13), a sliding mode occurs on the plane $e_4 = 0$ in a finite time under the existence condition $M_2 > \left|\frac{\xi_4}{\beta d a_{21} g a_{32}}\right|$.

For clarity, we write the dynamic equation of the system (5) in this sliding mode:

$$\dot{x}_1 = -a x_1 - f_1(T_d) x_1 + \xi_1(t),$$
$$\dot{e}_2 = -(a + k_2) e_2 + e_3,\ \dot{e}_3 = -k_3 e_3,\ e_4 = 0. \quad (15)$$

In the system (15), the variables e_3 and e_2 asymptotically vanish: $e_4 = 0 \Rightarrow e_3 \to 0 \Rightarrow e_2 \to 0$. At the same time, the first subsystem is an equation of zero dynamics. Due to $f_1(T_d) > 0$, this equation is stable.

Thus, the discontinuous control (14) ensures a sliding motion on the plane $e_4 = 0$ in a finite time, described by the stable system (15) of linear differential equations.

Note that if the subsystem (13) is stabilized using the continuous feedback law

$$\beta d a_{21} g a_{32} u_2 + \xi_4 = -k_4 e_4 \Rightarrow u_2 = -\frac{\xi_4 + k_4 e_4}{\beta d a_{21} g a_{32}},$$

then the last equation of the system (15) will take the form $\dot{e}_4 = -k_4 e_4$. Hence, the closed loop system will be stable. Among additional requirements for implementing this continuous control, we mention information about the variable ξ_4 and a pulse-width modulation device to control the DC motor voltage inverter. For implementing the discontinuous control (14), we need only an upper bound on this variable: $|\xi_4| \le E_4 = \text{const}$.

The first subsystem of (15), treated as an equation of zero dynamics, has a bounded solution: $|x_1| \le \frac{E_1}{a + f_1(T_d)}$ for $|\xi_1(t)| = a C_{Af} \le E_1 = \text{const}$; see the lemma. Particularly for $\xi_1 = E_1 = \text{const}$ and $x_2 = T_d = \text{const}$, the lemma implies $x_1 \to \frac{q C_{Af}}{q + V f_1(T_d)}$.

(Recall that $a = \frac{q}{V}$.) Choosing the parameter q as the control action in the product concentration loop, we have the following limit relations: $q \to \infty \Rightarrow C_A \to C_{Af}$ and $q \to 0 \Rightarrow C_A \to 0$. Therefore, we can maintain the product concentration within a reasonable range $C_A \in [C_{A1}, C_{A2}]$ by tuning the reagent flow rate into the reactor.

Clearly, increasing (decreasing) the reagent flow rate into the reactor, we decrease (increase, respectively) the product concentration.

Consider three sets of parameters in which the values $C_{Af} = 0.9$ and $q = 0.9$ are fixed, whereas the desired temperature varies: 1) $T_d = 350$, 2) $T_d = 380$, 3) $T_d = 400$. According to the relation $x_1 \to \frac{q C_{Af}}{q + V f_1(T_d)}$, we obtain: 1) $x_1 \to 0.3251$, 2) $x_1 \to 0.3214$, 3) $x_1 \to 0.3192$.

Thus, the product concentration decreases as the temperature in the reactor increases, and this conclusion agrees with the simulation results; see Fig. 10 in Section 4.

4. SIMULATION RESULTS

The effectiveness of the proposed approach was verified by numerical simulations of the CSTR–DC motor system in MATLAB. The parameters for the system (3), observer (6), and control (14) were selected from Table 2.

Table 2

Model parameters

Group of parameters	Parameter values
CSTR parameters	$q = 0.9, V = 1, \beta = 0.003, \gamma = 80, b = 5,$ $k_0 = 2$, $T_{c0} = 300$, $C_{Af} = 0.9 + 0.005\sin(0.03\pi t)$, $T_f = 395 + 0.01\sin(0.05\pi t)$.
DC motor parameters	$a_{21} = 0.8,\ g = 0.7,\ m = 0.0001,$ $a_{31} = 12.5, a_{32} = 2$
Initial conditions and reference	$x_1(0) = 0.3,\ x_2(0) = 400,\ x_3(0) = 100,$ $x_4(0) = 20$
Simulation scenario	$T_d(0) = 350$ for $t \in [0,\ t_1]$, $T_d(t_1) = 380$ for $t \in (t_1, \infty], t_1 = 75$
Observer parameters	$z_i(0) = 0,\ i = \overline{1,4}$, $l_1 = 100,\ l_2 = 3,\ l_3 = 100,\ l_4 = 500$
Controller parameters	$M_2 = 400, M_3 = 50, M_4 = 170,$ $k_2 = 0.5,\ k_3 = 0.1$
Physical constraints on state variables	$C_A \in [0,1],\ T \in [0, 400],\ x_3 \in [0, 100],$ $|x_4| < 150, u_2 = \pm 400$

The temperature in the reactor jacket, x_2, the armature current x_4, the reagent flow rate q, and its temperature T_f were assumed measurable in the plant. The state observer (6) was constructed to obtain complete information about the state variables of the CSTR–DC motor system and the exogenous disturbances (to estimate the unknown signals with a given accuracy).

The observation errors $\varepsilon_i = x_i - z_i, i = \overline{1,4}$, are presented in Figs. 4–7.

Figure 8 shows the graph of the exogenous disturbance $\xi_1 = 0.9(0.9 + 0.005\sin(0.03\pi t))$ reconstructed using the observer (6). The dashed line corresponds to the real disturbance values and the solid line to the restored ones. Figure 9 shows the graph of temperature variations in the reactor jacket.

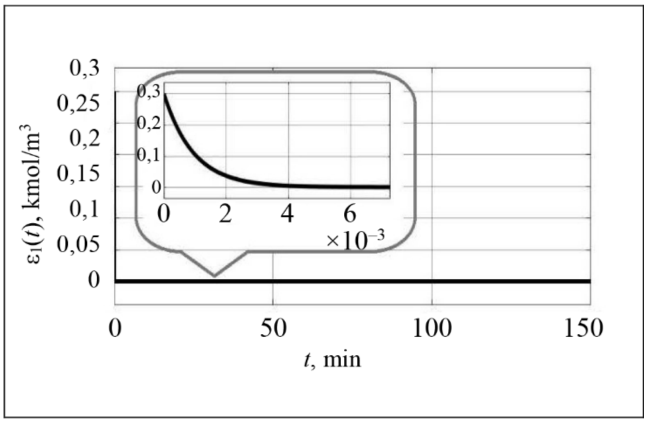

Fig. 4. The observation error ε₁(*t*).

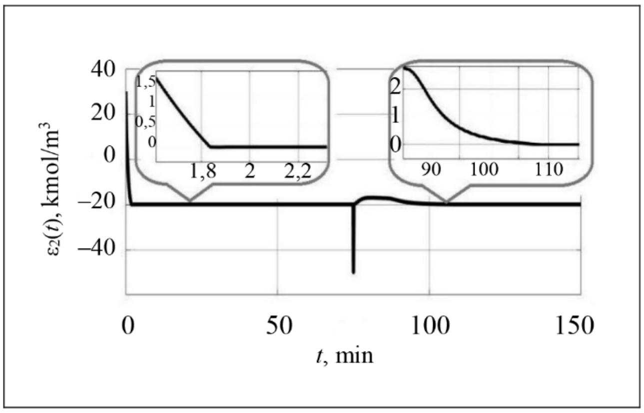

Fig. 5. The observation error $\varepsilon_2(t)$.

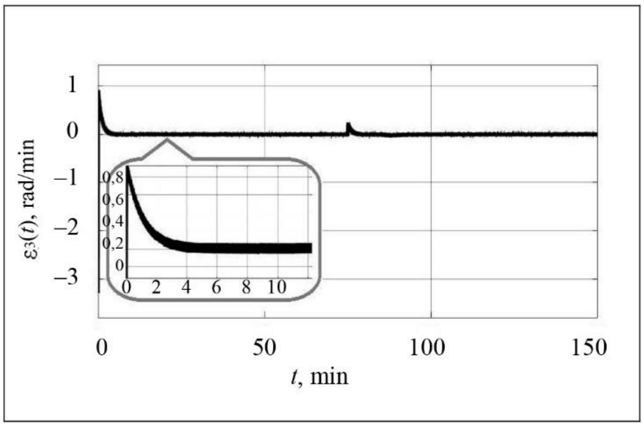

Fig. 6. The observation error $\varepsilon_3(t)$.

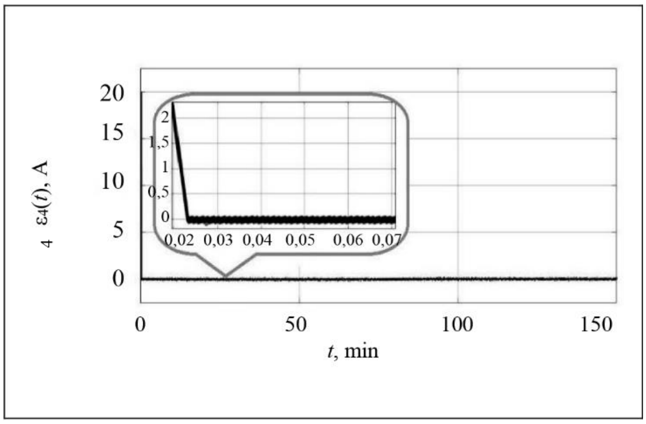

Fig. 7. The observation error $\varepsilon_4(t)$.

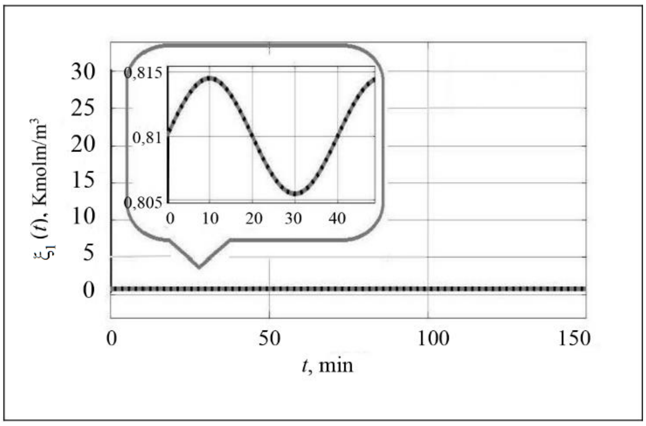

Fig. 8. The exogenous disturbance $\xi_1(t)$.

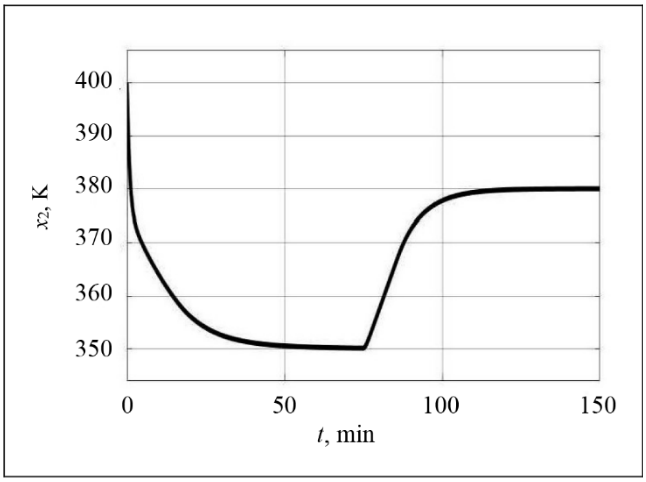

Fig. 9. Temperature in the jacket, $x_2(t)$.

Figures 10–13 show the graphs of the product concentration $x_1(t)$, the motor shaft rotation frequency $x_3(t)$, the armature current $x_4(t)$, and the armature voltage $u_2(t)$, respectively.

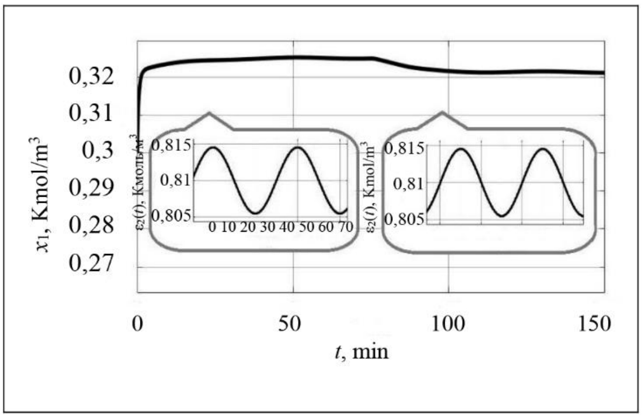

Fig. 10. The product concentration $x_1(t)$.

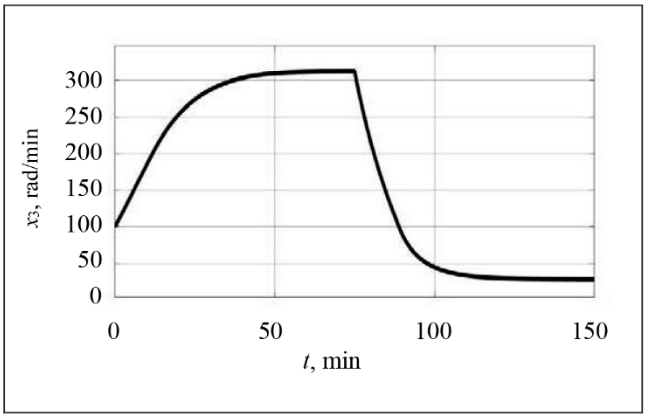

Fig. 11. The motor shaft rotation frequency $x_3(t)$.

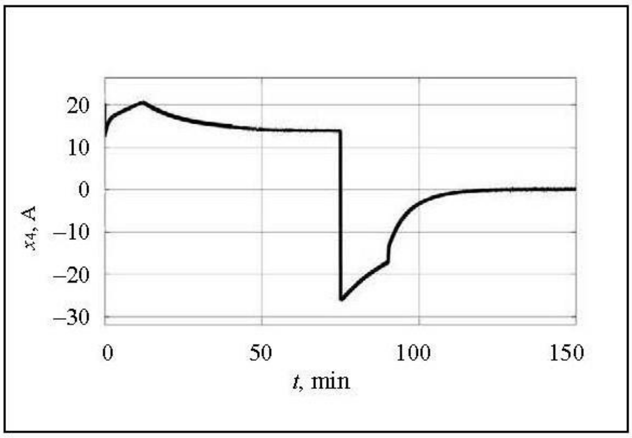

Fig. 12. The armature current $x_4(t)$.

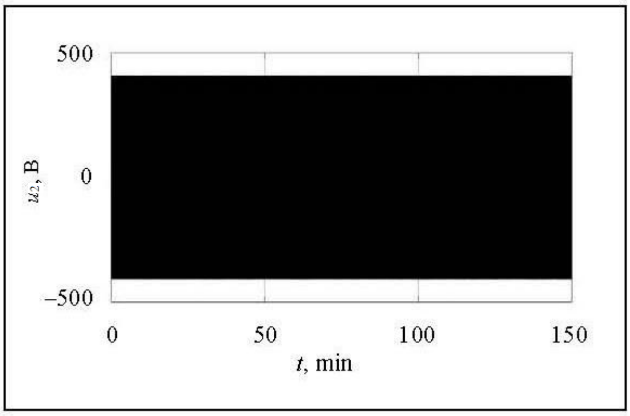

Fig. 13. The armature voltage $u_2(t)$.

CONCLUSIONS

The mathematical model of a continuous stirred tank reactor has been extended by introducing the dynamics of an actuator (DC motor) to apply the theory of sliding modes by (de)activating the switches of the voltage inverter.

An observer with mixed corrections (discontinuous and continuous) has been designed to obtain information about the unmeasured state variables and disturbances.

In the proposed feedback law design procedure, an observer estimates the real signals with a given accuracy both in a real sliding mode and when using deep feedback. A key feature of this work is the feedback law designed within the block approach to consider state-space and control constraints for the CSTR.

The effectiveness of this control algorithm has been validated analytically and illustrated numerically by simulations in MATLAB.

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