MODELS OF FATIGUE AND REST IN LEARNING. PART I: Extension of the General Iterative Learning Model

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Abstract. In this paper, classical iterative learning models are extended by including the factors of fatigue and rest. The existing approaches to model fatigue and rest from various fields—education, production, sports, and medicine—are analyzed, and the need to include these factors in the models is justified accordingly. Mathematical models are proposed to describe learning level dynamics during rest periods, considering the reduced efficiency of acquiring skills due to fatigue. Ten models of growing complexity are studied: from simple models without any fatigue effects to complex ones with the probabilities of skill acquisition and forgetting depending on time and rest periods. As is shown, the breaks of optimal duration allow improving the terminal learning level. In the model with fatigue, rest, and no forgetting, the optimal time to start rest is independent of the probability of skill acquisition as a function of time and lies at the middle of the experience acquisition interval. The models proposed are intended for predicting performance and optimizing training programs, production processes, and training cycles. This study emphasizes the need to consider biological and cognitive constraints when designing adaptive learning systems.

Keywords: experience, iterative learning, learning curve, mathematical modeling, fatigue, rest, skill acquisition, forgetting.

INTRODUCTION

Learning, the process and result of acquiring individual experience [1], underlies the adaptation of living and nonliving systems to changing conditions. In the context of mathematical modeling, learning is understood as a process during which a (biological, technical, or abstract-logical) system optimizes its actions to achieve a given goal. Of particular interest is *iterative learning* (IL), a type of learning based on the system's repetition of actions (trials and errors) to achieve a fixed goal under constant external conditions [2]. This process is the foundation for developing skills in humans, conditioned reflexes in animals, and adaptation algorithms in robotics and artificial intelligence.

Mathematical models of IL describe a sequence of learning levels (the so-called *learning curves*, LCs) [3] through systems of equations, graphs, or algorithms to reveal universal regularities. For example, an LC is the probability of mastering an activity component depending on time or the number of repetitions (iterations).

Based on experimental data, for most systems from humans to neural networks, these curves have a slowly asymptotic character: the rate of performance improvement decreases with time, and the curve tends to some limit [2]. Such behavior of LCs is often approximated by exponential functions, which confirms the "saturation" of the learning process.

Despite the universality of slowly asymptotic regularities, classical learning models neglect such factors as fatigue and rest [3]. To understand these limitations and justify model extensions, it is necessary to analyze the existing approaches to model learning in different contexts, from cognitive processes in education to motor skills in sports and manufacturing.

1. APPROACHES TO MODEL LEARNING AND REST: A REVIEW

Learning plays a key role in almost any human activity, from education to industrial manufacturing and from sports to robotic surgery: understanding the regularities of learning allows optimizing processes and reducing risks. (An *activity* is a human's interaction with the surrounding world in which the former represents a *subject* with a purposeful influence on an *object* [4].) As demonstrated by current research, the modeling of learning requires considering not only individual characteristics but also contextual specifics, whether it is cognitive load in education or physical fatigue in sports or (and) manufacturing.

In **education**, learning is closely related to the management of cognitive resources. The effect of cognitive fatigue on standardized test scores was studied and a clear relationship between the time of day and school performance was revealed [5]. According to the analysis of two million tests of Danish schoolchildren presented therein, the scores deteriorate by 0.9% of the standard deviation every hour after the start of school. This decline is due to the accumulation of fatigue, which impairs concentration and reduces the ability to solve complex tasks. However, the introduction of 20–30 min breaks compensates for this effect and even improves the results by 1.7%, especially for low-performing schoolchildren. For those with learning difficulties, the authors recommended small breaks of 5–10 mins, which decrease cognitive overload and improve learning by 12% [5]. These data emphasize that the educational process should be designed considering biological rhythms. Interestingly, similar regularities are observed in sports and manufacturing, where short pauses help to maintain productivity [6].

The learning process of a second language was modeled in [7]; as shown, progress in language skills is not a smooth growth but a series of sharp transitions between steady states. For example, an optimal ratio of new to familiar material (approximately 30% to 70%) triggers a noticeable progress in mastering, whereas an imbalance makes the learning process "stuck" on a "plateau." This phenomenon is well known to educators: facing overly complex tasks, students often lose motivation, and routine exercises without novel elements also inhibit development [8]. Such conclusions agree with corpus studies, where the distribution of errors in the speech of language learners corresponds to power laws [9, 10].

In **professional activities**, especially in high-tech industries, learning is often associated with the problem of fatigue. The cognitive effects of fatigue in employees of engineering companies were studied in [11]; according to the results, after 8-h work, the accuracy of information retrieval in memory drops by 6%, and the time of switching between tasks increases by 120 ms. Moreover, 22% of employees unconsciously switch to less efficient strategies for solving their tasks, thereby saving cognitive resources. These findings are especially relevant for spheres with critical consequences, from air traffic control to medical diagnosis. According to the recommendations in [11], adaptive work schedules should be adopted by alternating periods of intensive workload with rest phases; in addition, simulators should be used for practicing skills under fatigue simulation conditions.

Mastering complex medical technologies such as robotic surgery is of particular interest. Learning curves for nurses working in operating rooms were analyzed in [12]; based on the results, achieving mastery requires an average of 8–11 procedures. However, individual differences are huge: some nurses needed only three operations while others up to 11. As it turned out, the most challenging steps are planning the placement of spinal screws and controlling the robotic arm, where errors are often associated with cognitive overload. To reduce training time, the authors of [12] proposed simulation training on 3D models of the spine, where key skills can be practiced without risk to patients. This approach reduces the level of stress and fatigue by 35%, directly affecting the speed of mastering the technology.

In **manufacturing**, the modeling of learning has become the basis for process optimization. The classical Wright's power curve [13], describing the decrease in task completion time with experience, is still the starting point. However, many tasks combining cognitive and motor components require more sophisticated approaches. The model presented in [14] divides task completion time into two components: cognitive (planning and decision-making) and motor (physical actions). For example, when assembling a mechanical device, the initial stages require active thinking and their time decreases faster, while motor skills improve gradually. Compared to traditional approaches, this model is 23% more accurate in predicting performance in the early stages of learning, which is crucial for production costing and planning.

The best practices for the modeling of learning in manufacturing were systematized in the meta-analysis [15] covering 115 datasets. For tasks involving time or cost reduction, the S-shaped and three-parameter hyperbolic models were found to be the most effective. They consider the "plateau" phase when further improvement becomes minimal. In the context of productivity growth, the leader is the three-parameter exponential model, which well describes processes with the saturation effect. The integration of these models into production management systems reduces inventory planning errors by 18%, preventing both excess inventory and downtime.

In **sports and rehabilitation**, learning is inseparable from fatigue management. According to the study [16], fatigue increases the risk of injury by 40% due to a decrease in proprioception (the ability to orient the body in space). For example, in soccer, 67% of injuries occur in the last 20 minutes of a match when concentration drops and muscles lose elasticity. Objective methods (heart rate variability analysis) are combined with subjective questionnaires to monitor the condition of athletes. Heart rate variability analysis can predict readiness to load: a 15% decrease in heart rate variability correlates with an increased risk of injury. This data is used to personalize training by reducing intensity at the first signs of overfatigue.

The problem of rehabilitation after injuries or strokes deserves special attention. In the model proposed in [17], fatigue is considered as a balance between two components: the objective decrease in performance and the subjective perception of effort. For example, stroke patients often experience muscle weakness even with minimal load. Traditional training aimed at increasing strength can aggravate the condition when neglecting individual fatigue thresholds. The authors of [17] suggested using sensory feedback (e.g., real-time visualization of muscle activity) to help patients learn to distribute effort while avoiding overstrain.

In some works on **robotic systems and industrial design,** the learning process is described using biomechanical analysis. For example, differential equations of muscle fatigue dynamics were presented in [6]. Recovery from a 5 kg muscle load requires an average of 2.4 mins of rest; this data is used to design workstations on conveyors. With simulators integrating such models, ergonomics can be tested before production starts. In the aircraft industry, this approach reduced cumulative fatigue by 22% by optimizing tool angles and work surface heights.

Thus, modern research into the learning process demonstrates the absence of universal learning models with fatigue effects: each context requires consideration of its specifics. In education, the focus is now shifting to the management of cognitive load and periods; in manufacturing, to the separation of cognitive and motor components; in sports, to the balance between fatigue and recovery. The above approaches indicate an important role of fatigue and rest in experience acquisition dynamics. Hence, it is necessary to extend the existing classical learning models by including these factors. Let us take the model [3] as a basis for further extension.

2. FATIGUE AND REST IN EXPERIENCE ACQUISITION MODELS

Consider the following learning models. Let a learner (further called the agent) gain individual experience, i.e., master some type of activity through successive trials (iterative learning). Assume that the experience in each period is characterized by two possible states: “formed” or “not formed.” In each period $t = 1, 2, ...$, if the experience has not been gained, experience acquisition occurs with a probability $0 \le w(t) < 1$, generally depending on time. In parallel, the experience is forgotten with a probability $0 \le u(t) < 1$, also generally depending on time. The value $q(t)$ of the agent’s individual experience criterion belongs to [0, 1]. The value $q(t)$ is the probability that in period t the agent’s experience will be gained and not forgotten. Both the dependence of the probability of experience acquisition on time (a decreasing function) and the dependence of the probability of forgetting on time (an increasing function) can reflect the effects of fatigue, exhaustion, etc. during education and (or) a productive activity.

Consider several learning models covering such processes as mastering, forgetting, fatigue, and rest, sequentially in ascending order of their complexity. The presence of certain processes in the models is reflected in the table below.

The processes and parameters of experience acquisition models

Models	Mastering	Forgetting	Fatigue	Rest
Model 1	Time-invariant	-	-	-
Model 2	-	Time-invariant	-	-
Model 3	Time-invariant	Time-invariant	-	-
Model 4	Time-varying	-	+	-
Model 5	Time-varying	Time-invariant	+	-
Model 6	Time-invariant	Time-varying	+	-
Model 7	Time-varying	-	+	+
Model 8	Time-varying	Time-invariant, considered only on rest intervals	+	+
Model 9	Time-varying	Time-invariant	+	+
Model 10	Time-invariant	Time-varying	+	+

Model 1: In the basic (simplest) learning model [3], there is no forgetting and the probability of expe-

rience acquisition w does not depend on time. Here, the learning level dynamics are described by

$$q(t+1)=q(t)+(1-q(t))w,\ t=0,1,2,..., \quad (1)$$

with a known initial value $q(0)$.

In continuous time, the difference equation (1) turns into the differential equation

$$\dot{q}=(1-q)w,$$

and its solution has the form

$$q(t)=1-(1-q(0))e^{-wt}. \quad (2)$$

The learning curve (2) is non-decreasing and asymptotically tends to unity. Figure 1 shows the graph of the function (2) with $q(0)=0$.

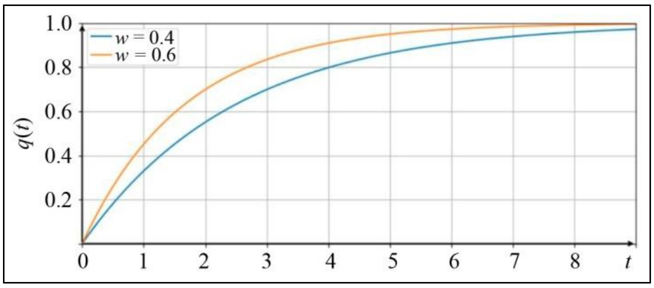

Fig. 1. The learning curve for Model 1: an illustrative example.

Model 2 (the forgetting model). Assume there is no experience acquisition but only time-invariant forgetting:

$$q(t+1)=q(t)-uq(t),$$

$$\dot{q}=-qu,$$

$$q(t)=q(0)\mathrm{e}^{-ut}. \quad (3)$$

The graph of the function (3) with $q(0)=1$ is shown in Fig. 2.

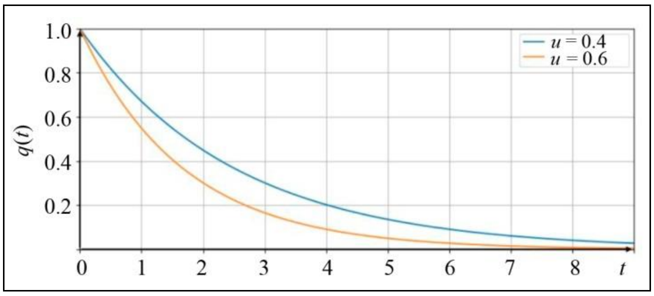

Fig. 2. The learning curve for Model 2: an illustrative example.

Model 3. Within Model 1, consider time-invariant forgetting (equivalently, time-invariant mastering within Model 2):

$$q(t+1)=q(t)+(1-q(t))w-uq(t),$$

$$\dot{q}=(1-q)w-qu, \quad (4)$$

$$q(t)=\frac{w}{w+u}-\left(\frac{w}{w+u}-q(0)\right)\mathrm{e}^{-(w+u)t}. \quad (5)$$

The learning curve (5) with $q(0)<\frac{w}{w+u}$ is non-decreasing and asymptotically tends to $\frac{w}{w+u}$. The graph of the function (5) with $q(0)=0$ and $w=0.6$ is shown in Fig. 3.

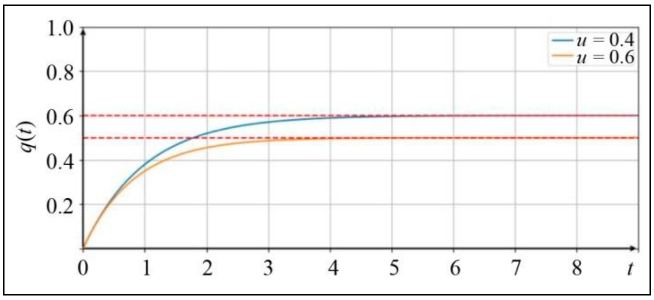

Fig. 3. The learning curve for Model 3: an illustrative example.

Model 4. Within Model 1, let the probability of experience acquisition depend on time and be positive: $w(t)>0$. Assume that $w(\cdot)$ is a continuous non-increasing function, conditionally called the "*fatigue curve*". In a practical interpretation, this function reflects the agent's *fatigue* and/or *exhaustion* in the learning process. Then

$$q(t+1)=q(t)+w(t)(1-q(t)),$$

$$\dot{q}=(1-q)w(t),$$

$$q(t)=1-(1-q(0))e^{-W(t)}, \quad (6)$$

where $W(t)=\int_0^t w(\xi)\,d\xi$. Obviously, the latter function has the following properties.

Lemma 1. *$W(\cdot)$ is a continuous, positive, monotonically increasing, and concave function of time such that $W(0)=0$.*

The learning curve (6) is non-decreasing and asymptotically tends to 1. In the special case $w(t)=w$, Model 3 turns into Model 1.

For illustrations below, we choose $w(t)=w_0\mathrm{e}^{-\alpha t}$ and $W(t)=\frac{w_0}{\alpha}(1-\mathrm{e}^{-\alpha t})$. The graph of the function (6) with $q(0)=0$ and $w_0=0.6$ is shown in Fig. 4.

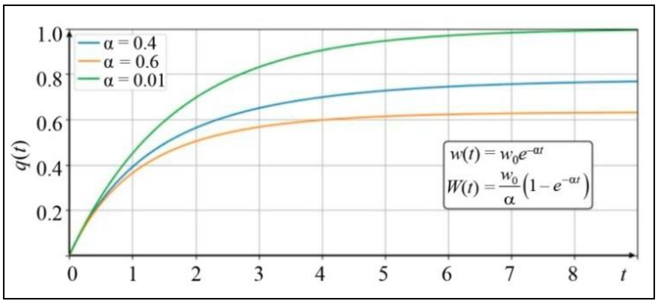

Fig. 4. The learning curve for Model 4: an illustrative example.

Model 5. Within Model 4, consider time-invariant forgetting: $u(t) = u$. Then

$$q(t+1)=q(t)+w(t)(1-q(t))-uq(t),$$

$$\dot{q}=(1-q)w(t)-uq,$$

$$q(t)=\mathrm{e}^{-(W(t)+ut)}\cdot\int_0^t w(\xi)\cdot \mathrm{e}^{(W(\xi)+u\xi)}\mathrm{d}\xi. \qquad (7)$$

The graph of the function (7) with $w_0=0.6$ and $u=0.1$ is shown in Fig. 5. As can be observed, the learning curve in Model 5 has a maximum.

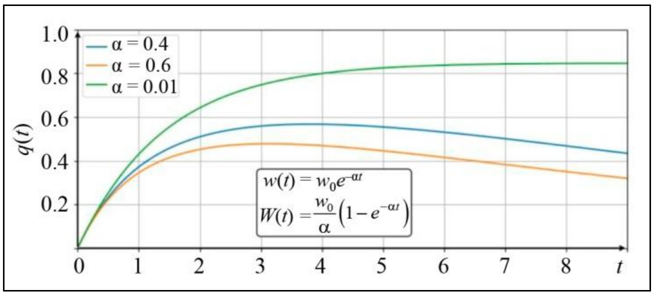

Fig. 5. The learning curve for Model 5: an illustrative example.

Model 6. Within Model 3, let the probability of forgetting be a continuous non-decreasing function $u(\cdot)$ of time. In this case, the solution of equation (4) is the learning curve

$$q(t)=\mathrm{e}^{-U(t)}\left(q(0)+w\int_0^t \mathrm{e}^{U(\xi)}d\xi\right), \qquad (8)$$

where $U(t)=\int_0^t (w+u(\xi))d\xi$. For $U(t) = u$, the expression (8) becomes (5). The graph of the function (8) with $w=0.6$, $q(0)=0$, $u(t)=1-\mathrm{e}^{-\alpha t}$, and $U(t)=(w+1)t-\frac{1}{\alpha}\left(1-\mathrm{e}^{-\alpha t}\right)$ is shown in Fig. 6.

Lemma 2. *$U(\cdot)$ is a continuous, positive, monotonically increasing, and convex function such that $U(0) = 0$.*

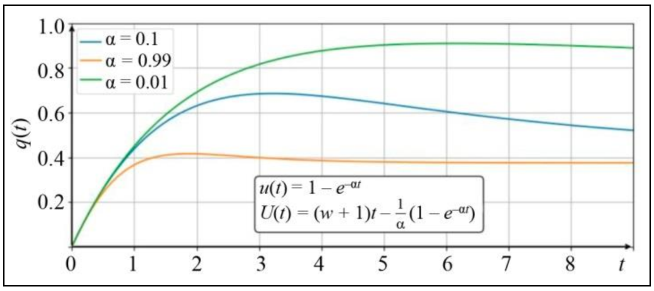

Fig. 6. The learning curve for Model 6: an illustrative example.

The graph of the function (8) with $w_0=0.6$ and $u_0=0.1$ is shown in Fig. 5.

The curve (8) can have a maximum at a point t^* satisfying the condition

$$\left(w+u\left(t^*\right)\right)\int_0^{t^*}\mathrm{e}^{u(\xi)}d\xi=\mathrm{e}^{u\left(t^*\right)}.$$

The value t^* can be interpreted as the maximum reasonable duration of learning.

Model 7. Within Model 4, let the agent have a rest interval $[\tau, \tau + \Delta]$, where $\tau > 0$ is the time to start a rest of duration $\Delta \geq 0$. Recall that there is no forgetting in the case under consideration: $q(\tau + \Delta) = q(\tau)$. The result of rest (an increase in the probability of experience acquisition) will be reflected by assuming that $\Delta \geq \Delta_0$, i.e., the duration of rest is sufficient for the agent to fully recover:

$$w(t)=w(t-\tau-\Delta),\ t\geq\tau+\Delta.$$

where the minimum necessary duration $\Delta_0 > 0$ is known. Then

$$q(t)=\begin{cases}1-(1-q(0))\mathrm{e}^{-W(t)},\ t\in[0,\ \tau],\\ 1-(1-q(0))\mathrm{e}^{-W(\tau)},\ t\in[\tau,\ \tau+\Delta],\\ 1-\left[(1-q(0))\mathrm{e}^{-W(\tau)}\right]\mathrm{e}^{-W(t-\tau-\Delta)},\ t\geq\tau+\Delta.\end{cases} \qquad (9)$$

For the sake of simplicity, we take the initial level of learning to be $q(0) = 0$. The graph of the function (9) with $w_0=0.6$ and $\alpha=0.6$ is shown in Fig. 7.

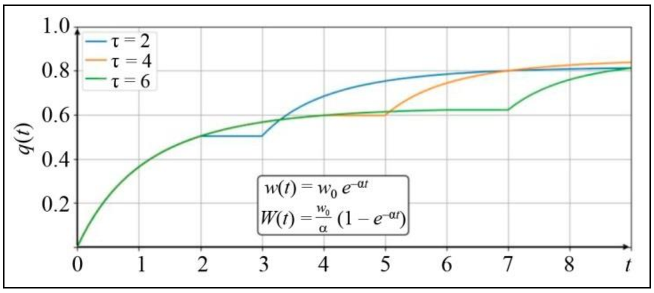

Fig. 7. The learning curve for Model 7: an illustrative example.

Two problems can be posed and solved by choosing the time to start rest and its duration.

The first problem is to maximize the terminal learning level on a planning horizon *T*:

$$1-e^{-W(\tau)-W(T-\tau-\Delta)} \to \max_{\tau\in(0,T-\Delta_0),\,\Delta\in[\Delta_0,T-\tau]}. \quad (10)$$

The second problem is to maximize the integral value of the learning level:

$$\int_0^{\tau}\left(1-e^{-W(\varsigma)}\right)d\varsigma+\int_{\tau+\Delta}^{T}\left(1-e^{-W(\tau)}e^{-W(\xi-\tau-\Delta)}\right)d\xi \to \max_{\tau\in(0,T-\Delta_0),\,\Delta\in[\Delta_0,T-\tau]}. \quad (11)$$

The integral criterion can be interpreted as the amount of agent's correct actions if learning occurs in productive activity [18].

Problem (10) can be written in the form

$$W(\tau)+W(T-\tau-\Delta)\to \max_{\tau\in(0,T-\Delta_0),\,\Delta\in[\Delta_0,T-\tau]}. \quad (12)$$

And problem (11) can be written in the form

$$\int_0^{\tau}e^{-W(\varsigma)}d\varsigma+e^{-W(\tau)}\int_0^{T-\tau-\Delta}e^{-W(\xi)}d\xi \to \min_{\tau\in(0,T-\Delta_0),\,\Delta\in[\Delta_0,T-\tau]}. \quad (13)$$

Due to Lemma 1 and the structure of the criteria (12) and (13), we arrive at the following result.

Proposition 1. *Within Model 7, the optimal rest duration* $\Delta^{(*)}$ *in both problems (maximizing the terminal and integral learning levels) is equal to the minimum necessary one:* $\Delta^* = \Delta_0$.

Based on Proposition 1, problems (12) and (13) are reduced to the following problems of selecting the optimal time to start rest:

$$W(\tau)+W(T-\tau-\Delta_0)\to \max_{\tau\in(0,T-\Delta_0)}. \quad (14)$$

and

$$\int_0^{\tau}e^{-W(\varsigma)}d\varsigma+e^{-W(\tau)}\int_0^{T-\tau-\Delta_0}e^{-W(\xi)}d\xi \to \min_{\tau\in(0,T-\Delta_0)}, \quad (15)$$

respectively.

The first-order optimality condition for the solution $\tau^{(*)}$ of problem (14) is $w(\tau^*) = w(T-\tau^*-\Delta_0)$. By the continuity and monotonicity of the function $w(\cdot)$, we obtain

$$\tau^* = \frac{T-\Delta_0}{2}. \quad (16)$$

The second derivative of (14) at this point is negative, which can be easily verified.

Within Model 7, it is better to take a break near the middle of the experience acquisition interval in terms of maximizing the terminal learning level.

Proposition 2. *Within Model 7, the optimal time to start rest (in terms of maximizing the terminal learning level) is given by* (16) *and does not depend on the fatigue curve.*

The expression (16) yields the optimal time to start rest if a rest of duration Δ_0 must be taken. Meanwhile, is a rest break necessary? To answer, we should compare the value of the criterion (10) at the point τ^* with the maximum value of the learning level $1-e^{-W(T)}$ achieved without rest. Of course, the answer depends both on the properties of the function $w(\cdot)$ and on the values of *T* and Δ_0 and is provided below.

Proposition 3. *A rest break is reasonable in terms of maximizing the terminal learning level if*

$$W\left(\frac{T-\Delta_0}{2}\right)\ge\frac{W(T)}{2}. \quad (17)$$

Condition (17) *can be written as*

$$\int_0^{\frac{T-\Delta_0}{2}}w(\varsigma)d\varsigma \ge \int_{\frac{T-\Delta_0}{2}}^{T}w(\varsigma)d\varsigma. \quad (18)$$

Due to the decreasing integrand, inequality (18) naturally holds for sufficiently small Δ_0 and/or sufficiently large *T*. At the same time, for a fixed Δ_0, it is possible to find a value of *T* turning (18) into equality, i.e., the minimum planning horizon on which a rest of a given duration will still be reasonable. Conversely, for a fixed *T*, we can find a value of Δ_0 turning (18) into equality, i.e., the maximum duration of rest that will still be reasonable on a given planning horizon.

Now we pass to problem (15). The first-order optimality condition for its solution τ^{**} is given by

$$\int_0^{T-\tau^{**}-\Delta_0}e^{-W(\xi)}d\xi=\frac{1-e^{-W\left(T-\tau^{**}-\Delta_0\right)}}{w\left(\tau^{**}\right)}.$$

As is easily verified, the second derivative of (15) takes a negative value at this point.

Model 8. Within Model 7, let forgetting occur only during rest with a constant and time-invariant probability *u*. According to the expression (3) of Model 2, when the rest phase ends, the learning level will decrease to

$$q(t+\Delta_0)=q(\tau)e^{-u\Delta_0}.$$

Then

$$q(t) = \begin{cases} 1 - e^{-W(t)}, \ t \in [0, \tau], \\ \left(1 - e^{-W(\tau)}\right) e^{-u(t-\tau)}, t \in [\tau, \tau + \Delta_0], \\ 1 - \left[1 - \left(1 - e^{-W(\tau)}\right) e^{-u\Delta_0}\right] e^{-W(t-\tau-\Delta_0)}, t \geq \tau + \Delta_0. \end{cases} \quad (19)$$

For Model 8, we consider the problem of maximizing the terminal learning level. The integral criterion is analyzed similar to Model 7:

$$1 - \left[1 - \left(1 - e^{-W(\tau)}\right) e^{-u\Delta_0}\right] e^{-W(t-\tau-\Delta_0)} \to \max_{\tau \in (0, T-\Delta_0)},$$

which means that

$$\left[1 - \left(1 - e^{-W(\tau)}\right) e^{-u\Delta_0}\right] e^{-W(T-\tau-\Delta_0)} \to \min_{\tau \in (0, T-\Delta_0)}. \quad (20)$$

The first-order optimality condition for problem (20) has the form

$$e^{u\Delta_0} - 1 = \left(\frac{w(\tau^*)}{w(T - \tau^* - \Delta_0)} - 1 \right) e^{-W(\tau^*)}.$$

The graph of the function (19) with $w_0 = 0.6$, $u_0 = 0.6$, and $\alpha = 0.6$ is shown in Fig. 8.

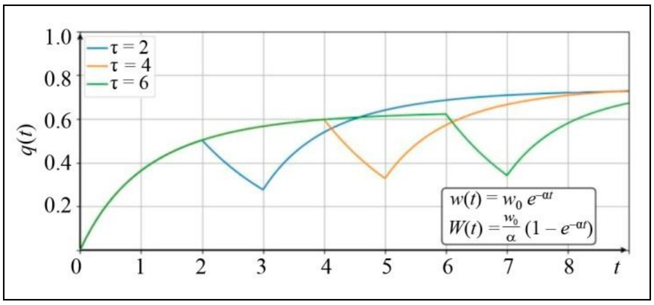

Fig. 8. The learning curve for Model 8: an illustrative example.

Model 9. Within Model 8, consider time-invariant forgetting during rest and learning as well. In this case, the section of $q(t)$ before rest will match formula (7) of Model 5. The corresponding learning curve may no longer be monotonic and, e.g., have a maximum.

Model 10. Within Model 9, let mastering be time-invariant whereas forgetting time-varying. In this case, the section of $q(t)$ before rest will match formula (8) of Model 6. Then the analog of the expression (19) for this model has the form

$$q(t) = \begin{cases} e^{-U(t)} \cdot \left(w \int_0^t e^{U(\xi)} d\xi + q(0) \right), \\ \qquad t \in [0, \tau], \\ e^{\hat{U}(t-\tau)} \cdot \left[e^{-U(\tau)} \cdot \left(w \int_0^\tau e^{U(\xi)} d\xi + q(0) \right) \right], \\ \qquad t \in [\tau, \tau + \Delta_0], \\ e^{-U(t-\tau-\Delta_0)} \cdot \left\{ w \int_0^{t-\tau-\Delta_0} e^{U(\xi)} d\xi \right. \\ \left. + e^{\hat{U}(t-\tau)} \cdot \left[e^{-U(\tau)} \cdot \left(w \int_0^\tau e^{U(\xi)} d\xi + q(0) \right) \right] \right\}, \\ \qquad t \geq \tau + \Delta_0, \end{cases}$$

where generally $\hat{U}(t) \neq U(t)$.

It seems unreasonable to study this model (and even more complex ones) in the analytical form further since the current results are cumbersome and yield no analytical conclusions about the regularities of experience acquisition. Numerical methods can be used instead. However, for this purpose, it is necessary to use particular functions as the probabilities of mastering and forgetting depending on time.

CONCLUSIONS

In this paper, we have extended the experience acquisition models from [3] by including the factors of fatigue and rest. An important result is the identification of optimal time intervals for breaks in terms of maximizing the terminal learning level: for problems described by Model 7, the optimal time to start rest does not depend on the time-varying probability of experience acquisition and is approximately at the middle of the experience acquisition interval. In more complex models with known learning and/or forgetting functions, it is also possible to identify the maximum reasonable learning time and the optimal time to start rest.

Thus, supplementing experience acquisition models with fatigue and rest processes increases their practical applicability, allowing one to predict performance degradation and optimize resources by calculating rest intervals.

Part II of this research will be devoted to datasets with experience acquisition dynamics and examples of the application of the above models to describe the results of well-known experimental studies.

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