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A MATHEMATICAL FORMULATION OF CONTROL PROBLEMS ON COGNITIVE MODELS[1]

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Abstract. This paper considers cognitive modeling methods under different types of control. Relevant publications are briefly surveyed. The cognitive model is formally described as a simulation model based on a directed graph (signed or weighted digraph). Mathematical formulations of the optimal, conflict, and hierarchical control problems are proposed for cognitive models in the case of pulse processes and in the general case as well. The methodology is applied to the predator-prey model and the aggregative model of a national economy. The methodological assumptions are detailed for another control problem on a cognitive map (the optimal management of a university). In this model, a university determines the number of commercial places and the price of commercial education. The model is identified on real data for the three largest universities of the Rostov region (the Russian Federation). Some conclusions and recommendations are formulated based on model analysis.

Keywords: control problems, discrete dynamic models, cognitive modeling, control in social and economic systems.

INTRODUCTION

The cognitive modeling of complex systems is an extensive research area that has been actively developing for several decades. In a broad sense, cognitive modeling is understood as the use of various artificial intelligence (AI) models, e.g., neural networks [1]. A particular decision methodology for weakly structured systems based on cognitive maps was proposed by R. Axelrod [2]. A cognitive map is a (signed or weighted) digraph with the vertices corresponding to system components and the arcs to the relations between them. The paper [3] was among the first publications on the subject; the approach was described in detail in the monograph [4]. Fuzzy cognitive map models were surveyed in [5–7].

The methodology of cognitive modeling as the simulation modeling of complex systems based on cognitive maps was developed by N.A. Abramova, Z.K. Avdeeva, et al. (Trapeznikov Institute of Control Sciences RAS) and G.V. Gorelova et al. (Southern Federal University) [8–15].

The paper [8] provided a detailed analysis of the verification problem of cognitive models and an illustrative example of a particular model from this point of view. Some prospects for the development of this research area were outlined in [9, 13].

S.V. Kovriga, E.K. Kornoushenko, and V.I. Maximov (Trapeznikov Institute of Control Systems RAS) considered cognitive modeling as a structure-and-goal analysis tool with application to the development problems of Russian regions as well as stated and solved control problems; see [16–19].

The experience of cognitive modeling of regional socio-economic processes was presented in [20]. The influence of a regional higher education system on the innovative development of the region was studied in [21]. Modern approaches to cognitive modeling were described in [22–24].

Therefore, the works [8–19] pointed to the managerial aspect of cognitive modeling. As we believe, this aspect should be crucial due to the activity of complex

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socio-economic and other weakly structured systems [25–30]. It is reasonable to distinguish dynamic models of optimal control with a single subject [31, 32], conflict control with several competing subjects [33], and hierarchical control with an ordered set of control subjects [34]. The author's approach to solving complex dynamic control problems using simulation modeling was described in [35]. The existence of a cognitive map in the form of a digraph relates cognitive modeling with network control models [36, 37]. Here, the paper [38] by D.A. Novikov is of great interest: he analyzed the feasibility of combining cognitive and game-theoretic approaches, classified cognitive games, and gave an example of a linear impulse cognitive game.

This paper has the following contributions:

- The formal description of the cognitive model as a simulation model based on a weighted digraph (cognitive map) is refined.
- Mathematical formulations of optimal, conflict, and hierarchical control problems are developed for cognitive models.
- The methodological assumptions are detailed for another control problem on a cognitive model: the optimal management of a university.

The remainder of this paper is organized as follows. Section 1 describes the basic cognitive model. In Section 2, we consider control problems of different types. Section 3 is devoted to a particular control problem on a cognitive map (the optimal management of a university). Some conclusions and recommendations are formulated in the Conclusions based on model analysis.

1. THE BASIC COGNITIVE MODEL

The basic cognitive model involves a digraph (cognitive map) in which each vertex and each arc are assigned some real value (as a function of time) and some constant weight (real number), respectively. In particular, the most common ones are signed digraphs, in which the arc weights take values ± 1. Digraph vertices represent the elements of a complex system under study, and digraph arcs represent the connections between them. Each element has some quantitative characteristic that can change over time, whereas each connection has a constant quantitative characteristic.

The basic cognitive model serves to describe and forecast the dynamics of vertex values, which are determined by their initial values and the structure of their connections with arc weights. Forecasting uses several rules reflecting different hypotheses about the dynamics of vertex values.

1.1. The Set of Vertices and Their Values

The vertices of the basic cognitive model represent elements of the system under study. Depending on the nature of the system, these can be employees or divisions of an organization, firms or corporations, biological populations, social groups, countries or their regions, etc. The list of vertices (the cognitive model variables) includes only the system elements with a principal role in the goals of study. This list reflects a compromise between the desire to consider as many system indicators as possible and the real possibilities of study. Formally, the list of vertices is a finite set $V=\{u_1,..., u_n\}$, where n denotes the number of vertices.

Each vertex $u_i \in V$ is assigned a real value x_i (a function of discrete time), i.e., $x_i:\{0, 1,..., T\} \to \mathbb{R}$. Thus, $x_i(t)$ is the value of the vertex u_i at a time instant t. Of course, the scalarity hypothesis of the value x_i is strongly restrictive because, in reality, each element of the system has several indicators. However, it considerably simplifies the study, still yielding quite interpretable models. The vector $x(t)=(x_1(t),..., x_n(t))$ fully characterizes the system state at each time instant t.

Finally, it is important to determine the initial values of all vertices, $x_0=(x_{10},..., x_{n0})$ (the initial state of the system). This is done based on the available literature, consulting, expertise, etc.

1.2. The Set of Arcs and Their Weights

The arcs of the cognitive model reflect cause-and-effect relations between elements of the system under study. If an arc is positive, increasing the value of the input vertex leads to increasing the value of the output vertex, i.e., the connection is direct. If an arc is negative, increasing the value of the input vertex leads to decreasing the value of the output vertex, i.e., the connection is inverse. The weight of an arc shows the strength of the corresponding connection (the increase/decrease coefficient).

Note that the total number of possible connections between n vertices equals C_n^2 (the number of combinations). The value C_n^2 grows fast with the parameter n, so only the most significant connections should be considered when constructing a cognitive map.

Sometimes the sign of an arc (and even more so its weight) is difficult to determine unambiguously. For example, the plus sign shows a proportional dependence of the price of a bus ticket on the trip length.

However, it is also possible to use the minus sign, which encourages long trips by public transport instead of private cars. In this case, both scenarios should be considered to compare their effect on the system.

Generally speaking, the identification problem is crucial for cognitive models. Researchers often distinguish between structural identification (selection of the sets of vertices and arcs) and numerical identification (selection of the initial values of vertices and the initial weights of arcs). Unfortunately, the identification problem (especially the structural one) is extremely difficult to formalize, which causes inevitable errors in the expertise-based solution [8].

1.3. Value Change Rules for Vertices

It is reasonable to treat cognitive modeling as simulation modeling by cognitive maps. The basic simulation model has the following form:

$$x_j(t+1) = x_j(t) + f(x(t)), \quad x_j(0) = x_{j0}, \ \ j = 1, \ldots, n. \tag{1}$$

Formula (1) describes the value conservation law for the vertex u_j as the balance relation. A particular cognitive model is specified by the function f. The so-called impulse process (step-function) [4]

$$x_j(t+1) = x_j(t) + \sum_{i=1}^{n} a_{ij} p_i(t), \; x_j(0) = x_{j0}, \; j = 1, \ldots, n, \tag{2}$$

is the most widespread rule; here, the difference

$$p_i(t+1) = x_i(t+1) - x_i(t) \tag{3}$$

denotes the impulse in the vertex u_i at the time instant t. Due to formula (3), the rule (2) can be written as

$$p_j(t+1) = \sum_{i=1}^{n} a_{ij} p_i(t), \; x_j(0) = x_{j0}, \; j = 1, \ldots, n. \tag{4}$$

In the vector form,

$$p(t+1) = A^{\mathrm{T}} p(t), \ \ t = 0, 1, \ldots, \tag{5}$$

where $p(t) = (p_1(t), \ldots, p_n(t))^{\mathrm{T}}$. Then it is easy to show by induction that

$$p(t) = A^{\mathrm{T}} p(0), \ \ t = 0, 1, \ldots \tag{6}$$

To emphasize the role of a cognitive map (V, A) defining the system structure, we represent the rule (2) as

$$x_j(t+1) = x_j(t) + \sum_{i \in I(j)} a_{ij} p_i(t), \quad x_j(0) = x_{j0}, \ \ j = 1, \ldots, n, \tag{7}$$

where $I(j)$ denotes the set of all vertices with outgoing arcs to vertex j. The expressions (2) and (7) are equivalent since $a_{ij} = 0$ in the case of no incoming arc (u_i, u_j).

Thus, knowing the weight matrix A and the initial impulse vector $p(0)$, we can forecast the values of all impulses $p(t)$ for any time instant t; knowing the initial value vector x_0, we can calculate the values of all vertices for any time instant t using formula (3), i.e., completely solve the forecasting problem [4].

Thus, the basic cognitive model is described by

$$\langle V, A, x_0, f \rangle \tag{8}$$

with the following notations: $V = \{u_1, \ldots, u_n\}$ is a finite set of vertices; $A = \| a_{ij} \|, \ i = 1, \ldots, n, \ j = 1, \ldots, n,$ is an adjacency matrix (if $a_{ij} \neq 0,$ we have arc (u_i, u_j) with the weight a_{ij}); $x_0 = (x_{10}, \ldots, x_{n0})$ is the vector of initial vertex values. Here, the function f specifies the value change rule, which has the general form (1). When considering an impulse process, we assume that the initial impulse vector $p_0 = (p_{10}, \ldots, p_{n0})$ is given.

2. CONTROL PROBLEMS

The basic cognitive model assumes that the values of vertices change only due to the natural dynamics (1), e.g., those of the impulse process (2). In this case, the change of values of all vertices (the dynamics of the system state) on the entire forecasting period is completely determined by the weight matrix and the initial distribution of impulses and values. If the initial values of impulses can be set arbitrarily, they perform the control function. However, real systems often undergo some external impact. If this impact is purposeful, the control itself and its optimization problem arise in the model. With a certain degree of conditionality, depending on the set of control subjects and its structure, we will distinguish among three types of control: optimal, conflict, and hierarchical.

2.1. Optimal Control

In this problem statement, there is one control subject influencing the dynamic system (1) with an optimality criterion

$$J = \sum_{t=1}^{T} \delta^t g(x(t), u(t)) + \delta^T G(x(T)) \to \max \tag{9}$$

under control constraints

$$u(t) \in U(t), \ \ t = 1, \ldots, T. \tag{10}$$

Here $\delta \in (0, 1]$ is the discount factor, and $g(\cdot)$ and $G(\cdot)$ are the instantaneous and terminal goal functions. (If $T = \infty$, the term $G(x(T))$ disappears.) The control variable u can be an open-loop $u(t)$ or closed-loop $u(t, x(t))$ strategy. Also, we can introduce additional constraints of the form

$$x(t) \in X^*, t = 1,..., T, \quad (11)$$

or

$$x(T) \in X^*, \quad (12)$$

known as the viability (homeostasis) conditions in the theory of sustainable management of active systems [30]. The strong form (11) (the weak form (12)) means that the state variable of the controlled dynamic system is within a given domain X^* at any time instant (at the terminal time instant, respectively). These conditions can be treated as the goal of control.

With the control impact, the dynamics equation (1) becomes

$$x_j(t+1) = x_j(t) + f(x(t), u(t)),$$
$$x_j(0) = x_{j0}, \ j = 1,..., n, \quad (13)$$

yielding the optimal control model (9), (10), (13) with the state-space constraints (11) or (12).

The control variable (i.e., the function $f(x, u)$ in (13)) can be incorporated into the cognitive model in different ways. Let the vertex set V of the cognitive map be augmented by a control variable v and the arc set A be augmented by arcs (v, u_i) with some weights b_j. If $b_j \neq 0$, then the vertex u_j is controlled.

Then the controlled impulse process takes the form

$$x_j(t+1) = x_j(t) + b_j h_j(u_j(t)) +$$
$$\sum_{i=1}^{n} a_{ij} p_i(t), \ x_j(0) = x_{j0}, \ j = 1,..., n, \quad (14)$$

where the control function $h_j(u_j)$ is, e.g., $h_j(u_j) = u_j^{p_j}$ with $p_j > 0$.

2.2. Conflict Control

In this problem statement, there are several control subjects influencing, simultaneously and independently, the dynamic system (1) with optimality criteria

$$J_k = \sum_{t=1}^{T} \delta^t g_k(x(t), u(t)) +$$
$$\delta^T G_k(x(T)) \to \max, \ k = 1,..., m, \quad (15)$$

under control constraints (10), $u(t) = (u_1(t),..., u_m(t))$, where m is the number of control subjects. By assumption, Nash equilibrium [33] is the solution of the differential game (10), (13), and (15) with the state-space constraints (11) or (12).

In this case, the vertex set V of the cognitive map is augmented by control vertices $v_1,..., v_m$, whereas the arc set A is augmented by arcs (v_k, u_i) with weights b_{kj}, $k = 1,..., m$, $j = 1,..., n$. If $b_{kj} \neq 0$, then the vertex u_j is controlled by the vertex v_k.

The conflict-controlled impulse process takes the form

$$x_j(t+1) = x_j(t) + \sum_{k=1}^{m} b_{kj} h_{kj}(u_{kj}(t)) + \sum_{i=1}^{n} a_{ij} p_i(t),$$
$$x_j(0) = x_{j0}, \ j = 1,..., n. \quad (16)$$

According to [38], cognitive games are classified by several features. In the proposed approach, we fix the following features: nonlinear games, common knowledge, no uncertainty, discrete time, the dependence payoff functions on the actions of all players and the trajectory (closed-loop strategies), a finite horizon, individual constraints, the choice of decisions at each time instant, simultaneous (in the next subsection, sequential) decision making, and no coalitions.

2.3. Hierarchical Control

In this case, the set of control subjects has a hierarchical structure and includes several influence agents and one coordinating center (the Principal). The Principal makes the first move by selecting a control impact

$$u_0(t) \in U_0(t), \ t = 1,..., T, \quad (17)$$

and reporting it to all influence agents. The Principal's optimality criterion has the form

$$J_0 = \sum_{t=1}^{T} \delta^t g_0(x(t), u(t)) + \delta^T G_0(x(T)) \to \max. \quad (18)$$

Knowing the value u_0, the influence agents choose, simultaneously and independently, their control impacts

$$u_k(t) \in U_k(t), \ t = 1,..., T. \quad (19)$$

The influence agents are guided by their optimality criteria (15), $u(t) = (u_0(t), u_1(t),..., u_m(t))$, where m denotes the number of influence agents. Let the optimal response of influence agents to the Principal's control be one of the Nash equilibria in the agents' game. By assumption, Stackelberg equilibrium [33] is the solution of the hierarchical game (13), (15), (17)–(19) with the state-space constraints (11) or (12).

In this case, the vertex set V of the cognitive map is augmented by control vertices $v_0, v_1,..., v_m$, whereas the arc set A is augmented by arcs (v_k, u_i) with some weights b_{kj}, $k = 0, 1,..., m$, $j = 1,..., n$. If $b_{kj} \neq 0$, then the vertex u_j is controlled by the vertex v_k.

The hierarchically controlled impulse process takes the form

$$x_j(t+1) = x_j(t) + \sum_{k=0}^{m} b_{kj} h_{kj}(u_{kj}(t)) + \sum_{i=1}^{n} a_{ij} p_i(t),$$
$$x_j(0) = x_{j0}, \ j = 1,..., n. \quad (20)$$

Thus, in almost all application-relevant cases, the basic cognitive model (8) is supplemented with the problems of optimal, conflict, or hierarchical control. Then the goal of study is to forecast the dynamics of the controlled system under different influence scenarios and optimize control in some sense.

Briefly, a cognitive model can be defined as a simulation model of a complex system whose structure is specified by a (signed or weighted) digraph and determines the dynamics of the controlled system state under various purposeful control impacts and external factors.

For impulse processes with any control type, simulation scenarios include two components, namely, the initial impulse distribution $p_0 = (p_{10},..., p_{n0})$ and the control trajectory, which has the following form:

- $\{u_j(t),\ j = 1,..., n,\ t = 0, 1,..., T-1\}$ for optimal control,
- $\{u_{kj}(t),\ k = 1,..., m,\ j = 1,..., n,\ t = 0, 1,..., T-1\}$ for conflict control,
- $\{u_{kj}(t),\ k = 0, 1,..., m,\ j = 1,..., n,\ t = 0, 1,..., T-1\}$ for hierarchical control.

For simulations, we apply the method of qualitatively representative scenarios [35].

Example 1. The predator–prey model.

The cognitive map of this model is shown in Fig. 1.

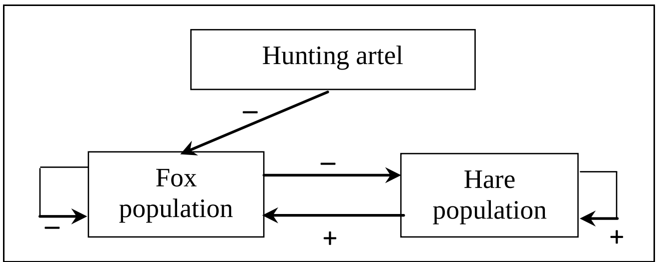

Fig. 1. The cognitive map of the predator–prey model.

The model based on this cognitive map is

$$x_F(t+1) = [1 - s(t)]x_F(t) - a_F x_F(t) + b_F x_F(t) x_H(t),\ x_F(0) = x_{F0}; \quad (21)$$

$$x_H(t+1) = x_F(t) + a_H x_H(t) - b_H x_F(t) x_H(t),\ x_H(0) = x_{H0}. \quad (22)$$

Here $x_F(t)$ and $x_H(t)$ denote the numbers of foxes and hare (the predator and prey, respectively) in year *t*; $a_F > 0$ and $a_H > 0$ are the reproduction rates of the fox and hare populations, respectively; $b_F > 0$ and $b_H > 0$ are the trophic interaction rates of the fox and hare populations, respectively; x_{F0} and x_{H0} are the initial numbers of foxes and hare, respectively; finally, $s(t)$ is the shooting rate of foxes in year *t*.

For the fox population, the optimal exploitation problem has the form

$$J = \sum_{t=1}^{T} [cs(t)x_F(t) - ds^2(t)] \to \max,\ 0 \le s(t) \le 1, \quad (23)$$

$$x_F(T) \ge x_F^*,\ x_H(T) \ge x_H^*, \quad (24)$$

where $c > 0$ is the specific hunting utility, $d > 0$ is the hunting cost coefficient, and x_F^* and x_H^* are the critical numbers of foxes and hare, respectively.

The optimal exploitation problem (23) can be easily generalized to the case of competing hunting artels. In fact, the sustainable development condition (24) is external to hunting, and an environmental body should be introduced to influence hunters. This approach leads to a hierarchical control problem. ♦

Example 2. The Ramsey–Solow aggregate model of a national economy.

The cognitive map of this model is shown in Fig. 2.

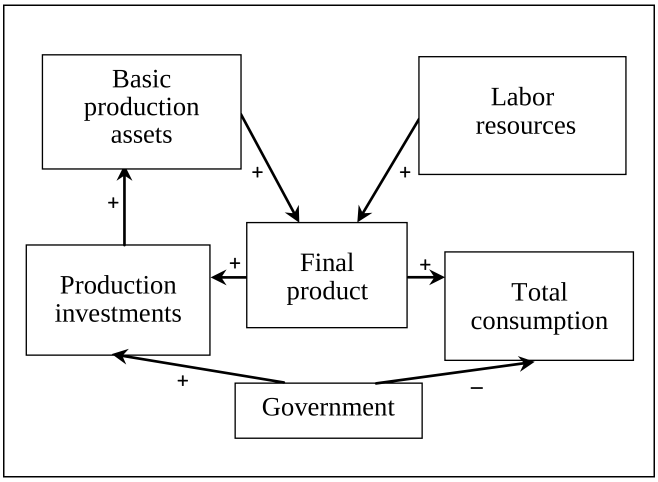

Fig. 2. The cognitive map of the aggregate national economy model.

The model based on this map has the form

$$K(t+1) = (1-\mu)K(t) + I(t),\ K(0) = K_0; \quad (25)$$

$$L(t+1) = (1+\eta)L(t),\ L(0) = L_0; \quad (26)$$

$$Y(t+1) = AK^{\alpha}(t+1)L^{1-\alpha}(t+1),\ 0 \le \alpha \le 1; \quad (27)$$

$$I(t+1) = s(t+1)Y(t+1),\quad C(t+1) = (1 - s(t+1))Y(t+1); \quad (28)$$

$$0 \le s(t+1) \le 1,\ t = 0, 1,..., T-1. \quad (29)$$

Here $K(t)$ is the amount of basic production assets; $L(t)$ is the amount of labor resources; $Y(t)$ is the final product of the economy; $I(t)$ is the amount of production investments; $C(t)$ is the consumption level; $s(t)$ is the share of investments in the final product (all these parameters, in year *t*); $\mu > 0$ is the amortization rate of production assets; $\eta > 0$ is the reproduction rate of labor resources; finally, K_0 and L_0 are the initial values of the variables K and L, respectively.

The optimal control problem has the form

$$J = \sum_{t=1}^{T} c(t) \to \max,\ 0 \le s(t) \le 1, \quad (30)$$

$$K(T) \ge K^*,\ L(T) \ge L^*, \quad (31)$$

where $c(t) = C(t)/L(t)$ is the specific consumption (per one employee) and K^* and L^* are the target values of the indicators. In this problem, the sustainable development condition (31) can be treated as the government's goal. ♦

3. THE COGNITIVE OPTIMAL MANAGEMENT MODEL OF A UNIVERSITY

As a detailed example, we consider the cognitive optimal management model of a separate university.

The university enrolls in *M* specialties. Students can study in the university on state-funded (budgetary) or commercial places. Budgetary places in the university are allocated by the government: their number does not directly depend on the university management. The number of commercial places for particular specialties can be set by the university independently. Also, the university determines the price of commercial education for each specialty and bears the costs of educating a given number of students (on commercial and budgetary places). Some commercial places provided by a university can remain unclaimed by applicants. We assume that the demand for commercial places in a specialty is directly proportional to the future wage of a graduate and inversely proportional to the price of commercial education. Also, the more graduates of a specialty are employed, the higher attractiveness it will have for applicants. For a chosen planning horizon *T*, we obtain an optimization model of the form:

$$J = \sum_{t=1}^{T}\sum_{j=1}^{M}\left(a_j^C x_j^C(t) - c_j\left(x_j(t)\right)^2\right) \to \max, \quad (32)$$

$$x_j^C(t) \ge 0, a_j^C(t) \ge 0; \quad (33)$$

$$x_j(t+1) = x_j^B(t+1) + \min\{x_j^C(t+1),\ (\gamma_j - a_j^C(t)/4)^{\alpha_j} y_j(t)\},\ x_j(0) = x_{j0}; \quad (34)$$

$$y_j(t+1) = (1-\kappa_j)x_j(t),\ y_j(0) = y_{j0},\ j = 1,\ldots, M,\ t = 1,\ldots, T-1. \quad (35)$$

Here, M is the number of specialties; $x_j^B(t)$ is the number of budgetary places for the *j*-th specialty in year *t* (the exogenous variable); $x_j^C(t)$ is the number of commercial places for the *j*-th specialty in year *t* (the first control variable); $x_j(t)$ is the total number of places for the *j*th specialty in year *t*; $a_{ij}^C(t)$ is the price of commercial education for the *j*-th specialty in year *t* (the second control variable); γ_j is the influence coefficient of potential employment on applying to the *j*-th specialty; κ_i is the share of unemployed graduates with the *j*th specialty; c_j is the education cost coefficient for the *j*th specialty depending on the total number of students in year *t*; $y_j(t)$ is the number of employed graduates with the *j*-th specialty in year *t*; finally, α_j is the elasticity of demand for commercial places in the *j*th specialty. The model is reflected in a cognitive map (Fig. 3).

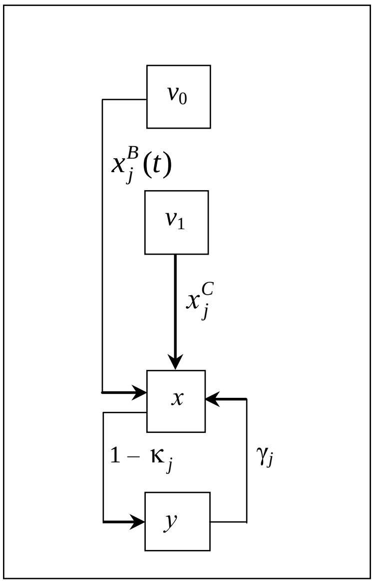

Fig. 3. The cognitive map of the university management model.

The demand of applicants for commercial places in the *j*th specialty (see the function (34)) is given by

$$(\gamma_j - a_j^C(t)/4)^{\alpha_j} y_j(t).$$

The expression in parentheses can be explained as follows. By some estimates [39], long-term investments in higher education in Russia have an average payback period of 10 years: the price of commercial education is covered by future wages in a specialty at the rate of at least 10% annually. According to the Tabiturient portal (https://tabiturient.ru/vuzcost/), the average price of higher education in Russia in 2022 is 174 533 rubles per year (about 17 453 rubles per month). As discovered by the analytical center of Synergy University (https://ria.ru/20210914/zarplata-1749940260.html), the average desired wage is about 50–70 thousand rubles per month, which provides a sufficient level of comfortable life. Thus, for an effective return on investment in higher education in Russia, the future wage of a graduate should be at least three times higher than the monthly expenditures on education.

Let us introduce two simplifying assumptions:

- All budgetary places allocated are filled.
- There is no expulsion, i.e., all university applicants become graduates.

Thus, at each time instant, a university is informed by the government about the number of budgetary places allocated for each specialty. This is important for universities since education costs depend on the total number of students. After the university receives information on budgetary places, it decides on the maximum possible enrollment of commercial students and determines the price of commercial education in each specialty.

We investigate the model by computer simulation [40]. The uncontrolled parameters of the model are identified, and then different control scenarios are analyzed. Each scenario consists in specifying:

- the vector of exogenous variables

$$\{x_{ij}^{B}(t)\},\ \ j=1,...,M,\ t=0,...,T-1,$$

- the vector of control variables

$$\{a_j^{C}(t),\ x_j^{C}(t)\},\ \ j=1,...,M,\ t=0,...,T-1.$$

Six enlarged groups of specialties were taken for the study as the most important ones: pedagogy, medicine, economics, engineering, construction, and agriculture. Then the key universities of the Rostov region with the corresponding specialties were selected: Rostov State Medical University (RostSMU), Don State Technical University (DSTU), and Southern Federal University (SFedU).

RostSMU is a specialized regional university with 11 faculties (medicine, pharmacology, and psychology). DSTU is a regional supporting multidisciplinary university of the Rostov region with 24 faculties (engineering, agriculture, and social and humanitarian specialties). SFedU is the largest research and educational center in the South of Russia, which includes 29 structural units (natural sciences, engineering, and social and humanitarian specialties).

Table 1 shows the specialties of these universities.

Table 1

The specialties of universities in the Rostov region

Specialties	RostSMU	DSTU	SFedU
Pedagogy		+	+
Medicine	+		
Economics		+	+
Engineering		+	+
Construction		+	+
Agriculture		+	

Let us describe the parameter identification procedure. The components of the vector

$$\left(\{\gamma_j\}_{j=1}^{M},\{\kappa_j\}_{j=1}^{M},\{c_j\}_{j=1}^{M},\{x_{j0}\}_{j=1}^{M},\{y_{j0}\}_{j=1}^{M},\{\alpha_j\}_{j=1}^{M}\right),$$

which form the uncontrolled parameters of the model, were to be identified. Consider them in detail.

The parameter γ_j is the influence coefficient of potential employment on applying to the *j*th specialty. As this parameter, we took the average wage of the corresponding profession in the Rostov region. Note that its value does not depend on a particular university. The data were provided by the territorial body of the Federal State Statistics Service in the Rostov region (Rostovstat; see https://rostov.gks.ru). For each industry and specialty, the average wages were calculated for several years. The data for 2020 were taken as the parameter γ_j.

Table 2

The influence coefficient of potential employment on applying to specialties

Specialties	Parameter	Value, in roubles
Pedagogy	γ_1	28550
Medicine	γ_2	35849
Economics	γ_3	35000
Engineering	γ_4	53000
Construction	γ_5	47000
Agriculture	γ_6	23726

The parameter x_{j0} is the number of graduates for the *j*-th specialty in the initial year of the planning horizon. The data were taken from public documents (self-evaluation reports, enrollment orders, and enrollment statistics by year) on the official portals of RostSMU, DSTU, and SFedU. The number of graduates and enrolled students for 2020 was considered (Table 3).

The parameter κ_j is the share of unemployed graduates with the *j*th specialty. This parameter was calculated based on public documents on the official portals of the universities. The share of unemployed graduates was determined by subtracting that of employed graduates from 1. For the calculations, this parameter was set equal to 1 for the universities without appropriate specialties. The resulting values are presented in Table 4. The parameter y_{j0} is the number of employed graduates with the *j*th specialty in the initial year of the planning horizon. It was calculated (see Table 5) through the parameters x_{j0} and κ_j by the formula

$$y_{j0}=\left(1-\kappa_j\right)x_{j0}.$$

Table 3

The number of graduates in the initial year

Specialties	Parameter	RostSMU	DSTU	SFedU
Pedagogy	x_{10}	–	40	666
Medicine	x_{20}	165	–	–
Economics	x_{30}	–	319	296
Engineering	x_{40}	–	675	895
Construction	x_{50}	–	586	150
Agriculture	x_{60}	–	138	–

Table 4

The share of unemployed graduates

Specialties	Parameter	RostSMU	DSTU	SFedU
Pedagogy	κ_1	–	0.45	0.45
Medicine	κ_2	0.16	–	–
Economics	κ_3	–	0.45	0.18
Engineering	κ_4	–	0.45	0.18
Construction	κ_5	–	0.45	0.07
Agriculture	κ_6	–	0.45	–

Table 5

The number of employed graduates in the initial year

Specialties	Parameter	RostSMU	DSTU	SFedU
Pedagogy	y_{10}	–	22	366
Medicine	y_{20}	139	–	–
Economics	y_{30}	–	171	213
Engineering	y_{40}	–	371	644
Construction	y_{50}	–	323	144
Agriculture	y_{60}	–	76	–

The parameter c_j is the education cost coefficient for the *j*th specialty depending on the total number of students in year *t*. It is directly related to the prime cost of tutoring in this specialty. This parameter was assigned through expertise as 80% of the price of commercial education available from public sources: the official portals of SFedU (https://sfedu.ru), DSTU (https://donstu.ru), and RostSMU (http://rostgmu.ru). See Table 6 below.

The values in Table 6 are not the values of the parameter c_j. Assuming quadratic costs, the value c_j is given by

$$c_j = \frac{c}{x_{j0}},$$

where c denotes the prime cost of tutoring.

Table 6

The prime cost of tutoring, in roubles

Specialties	RostSMU	DSTU	SFedU
Pedagogy	–	86 000	88 000
Medicine	125 000	–	–
Economics	–	86 000	107 000
Engineering	–	100 000	104 000
Construction	–	100 000	113 000
Agriculture	–	100 000	–

The resulting values of the parameter c_j are combined in Table 7. For the calculations, this parameter was set equal to almost infinity for the universities without appropriate specialties.

The parameter α_j is the elasticity of demand for commercial places in the *j*th specialty. It characterizes demand variations under changing the future wage or the price of commercial education. The data were taken from https://iq.hse.ru/news/177671083.html (IQ: Research and Education Website, National Research University Higher School of Economics). The cited source indicates the relative variation *Pov* under increasing the demand of applicants for a specialty with a 40% increase in graduate wages.

Therefore, we calculated this parameter by the formula

$$\alpha_j = \log_{1.4}\left(1 + \frac{Pov}{100}\right);$$

see Table 8.

Even at the identification stage, we arrive at the following conclusion: it is unprofitable for applicants to study medicine and agriculture on commercial places. Really, the expression $\gamma_j - a_j^C(t)/4$ (the basis for calculating the demand) is negative even at the prime cost of tutoring. For agriculture, it can be explained by low wages; in the case of medicine, the reason is the high prime cost of tutoring. Engineering and construction attract applicants for commercial places with high future wages. Economics and pedagogy lie at the borderline: the future wages are commensurate with the price of commercial education.

For the prices $a_j^C(t) > 4\gamma_j$, there is no demand for commercial education: see medicine and agriculture as examples. Therefore, the problem for RostSMU has a trivial solution and will not be considered below.

In view of (34), the university need not enroll commercial students above the demand $(\gamma_j - a_j^C(t)/4)^{\alpha_j} y_j(t)$. (Although universities incur no losses from the excessive commercial enrollment.) When increasing the number of students, the costs grow faster than the income (32). Hence, there exists a finite optimal number of commercial students for the university: for this number, the goal function (32) achieves maximum. The university should enroll precisely this number of commercial students.

The optimal control problem for the university is solved in two stages.

- For each specialty, it is required to determine the maximum number of commercial students $\{x_j^C(t)\}$, $j = 1,\dots, M$, $t = 0,\dots, T-1$, profitable for the university considering the goal function (32) and the demand for commercial places (34).
- For each specialty, it is required to select the maximum price of commercial education $\{a_j^C(t)\}$, $j = 1,\dots, M$, $t = 0,\dots, T-1$, that maximizes the function (32). The price is determined as a markup to the prime cost of tutoring.

If the university does not receive any budgetary places, the model takes the following form:

$$J = \sum_{t=1}^{T}\sum_{j=1}^{M}\left(a_j^C x_j(t) - c_j\left(x_j(t)\right)^2\right) \to \max,$$

$$x_j^C(t) \ge 0,\ a_j^C(t) \ge 0;$$

$$x_j(t+1) = \min\{x_j^C(t+1),\ (\gamma_j - a_j^C(t)/4)^{\alpha_j} y_j(t)\},$$

$$x_j(0) = x_{j0};$$

$$y_j(t+1) = (1-\kappa_j)x_j(t),\ y_j(0) = y_{j0},$$

$$j = 1,\dots, M,\ t = 1,\dots, T-1.$$

The calculation results for DSTU and SFedU are shown in Tables 9 and 10, respectively.

Table 7

The education cost coefficient

Specialties	Parameter	RostSMU	DSTU	SFedU
Pedagogy	c_1	–	2150	132
Medicine	c_2	758	–	–
Economics	c_3	–	270	361
Engineering	c_4	–	148	116
Construction	c_5	–	171	753
Agriculture	c_6	–	725	–

Table 8

The elasticity of demand for commercial places

Specialties	*Pov*	α_j
Pedagogy	76	$\alpha_1 = \log_{1.4} 1.76 = 1.68$
Medicine	131	$\alpha_2 = \log_{1.4} 2.31 = 2.48$
Economics	77	$\alpha_3 = \log_{1.4} 1.77 = 1.70$
Engineering	42	$\alpha_4 = \log_{1.4} 1.42 = 1.04$
Construction	51	$\alpha_5 = \log_{1.4} 1.51 = 1.22$
Agriculture	41	$\alpha_6 = \log_{1.4} 1.41 = 1.02$

Table 9

Calculation results for DSTU (no budgetary places)

Specialties*	Academic year					
	First		Second		Third	
	Commercial enrollment	Markup, %	Commercial enrollment	Markup, %	Commercial enrollment	Markup, %
Pedagogy	100	3	38	3	95	3
Engineering	56	30	63	30	5	30
Construction	67	19	65	19	8	19
Economics	130	9	85	9	35	9

* There is no information about agriculture due to no demand.

Table 10

Calculation results for SFedU (no budgetary places)

Specialties*	Academic year					
	First		Second		Third	
	Commercial enrollment	Markup, %	Commercial enrollment	Markup, %	Commercial enrollment	Markup, %
Pedagogy	250	11	250	11	250	11
Engineering	86	18	98	18	9	18
Construction	28	1	27	1	5	1
Economics	250	8	250	8	250	8

* There is no information about agriculture due to no demand.

At SFedU, admission to pedagogy and economics is limited by the university's capacity and does not exceed 250 places. The demand for commercial places for these specialties is above 250. Note that generally, the demand for commercial places decreases over time due to the rational search for budgetary ones. Consequently, universities have to reduce the price of commercial education. The new values are presented in Table 11.

According to the calculation results, commercial enrollment is not profitable for the university: under a high price of education, there is no demand for commercial places; under a low price, the university suffers losses due to the quadratic costs.

Table 11

Additional budgetary places for DSTU

Specialty	$x_j^B(t)$
Pedagogy	1988
Economics	816
Engineering	459
Construction	331
Agriculture	331

Note that this study covers only the first level of higher education (bachelor's degree). In the case of master's and other postgraduate programs, all qualitative conclusions will remain valid, whereas quantitative conclusions will rest on the identification of model parameters. The demand for postgraduate commercial places can be ensured under the following conditions:

- The employer must clearly understand the increased qualifications of a graduate with a master's degree (and realize the potential utility of attracting him or her to a higher wage) compared to an employee with a bachelor's degree. The wage differential should be motivating when deciding to invest in a master's degree.

In other words, the wage jump must satisfy the condition

$$\gamma_j^{\text{mast}} - \gamma_j^{\text{bach}} > \frac{a_j^C(t)}{4}.$$

There should be a greater market demand for employees with higher qualifications (master's degree), who have competencies lacking in bachelor's degree and a greater potential utility for employers. This assertion is confirmed especially during the systemic economic recession (a reduced supply of jobs and an increased level of unemployment).

CONCLUSIONS

This paper has presented a cognitive modeling methodology under different types of control (optimal, conflict, and hierarchical).

A cognitive optimization model of a university has been studied. The analysis allows drawing several conclusions as follows. Within the model, if a university is allocated many budgetary places in a specialty, it has low demand for commercial enrollment. In this case, commercial places in excess of the budgetary ones incur losses. (The model does not allow the university's refusal from the allocated budgetary places.) A large number of commercial places remains economically justified only in case of no budgetary enrollment (SFedU). Allocating a small number of budgetary places has economic advantages: the university provides the maximum of commercial places within the resource potential and gains higher income.

The university and the government are recommended to choose the priority of enrollment (budgetary or commercial). If budgetary places are the priority, the control problem will have another statement, yielding other conclusions. Changes are also possible in the case of considering master's degree programs and university rankings.

It seems very promising to combine cognitive simulation modeling with the mathematical apparatus of network games, which has been intensively developed recently [41].

As we believe, there are no fundamental theoretical limitations on the applicability of this methodology. It is possible to simulate any dynamic active systems of any complexity. However, there may arise technical limitations related to the model dimension and the need to collect and process relevant data. Following standard practice in applied systems analysis, we have to compromise between the desired accuracy and the capabilities of study.

The model adequacy should be assessed primarily through the meaningful analysis of the results based on expert opinions. Of course, it is possible to apply traditional methods of applied statistics (e.g., hypothesis testing). Their usefulness, however, seems limited due to obvious difficulties when satisfying formal requirements for the available data. In addition, when modeling complex socio-economic systems, the conclusions and recommendations are mainly qualitative in nature. They should be validated through expertise.

The opinion of the expert community is even more useful when interpreting the results. We have not yet considered in detail the well-known methods to aggregate and process expert information [42]. This is one subject for further research.

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