DOI: http://doi.org/10.25728/cs.2024.1.5

CONTROL MODELS IN POWER HIERARCHIES

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Abstract. This paper is devoted to the modeling of control in power hierarchies. Publications in this research area are briefly overviewed. The design principles of such models and the underlying assumptions are described. They are mathematically formalized using difference normal-form games with the information rules of Germeier games. An analytical study is carried out for a system of two-level power hierarchies as a particular case. The general problems of investigating power hierarchies are posed. One-, two-, and *n*-polar power hierarchies are defined, and their emergence conditions are analyzed. Illustrative examples are provided. An alternative resource competition model is considered. The system of power hierarchies is simulated for different cases, and the simulation results are compared. Conclusions are drawn, and some lines of further research are indicated.

Keywords: power hierarchy, simulation, control methods, difference games, coordination of interests, unipolar system, bipolar system, n -polar system, pole comparability, resource competition, influence agents, basic agents, power group, QRS-set.

INTRODUCTION

The mathematical modeling of power distribution dynamics in hierarchical structures was conceptualized in detail by A.P. Mikhailov; see [1–8]. The ideology of this approach was described in the seminal paper [1]. The model is based on balance relations. The main variable of the model is the amount of power $p(x,t)$ as a function of time and the agent's position in the hierarchical structure. For this function, a parabolic partial differential equation is written with some boundary conditions. The hierarchical structure in the basic version is a linear chain, which is quite easily generalized to the case of several agents at each control level. The right-hand side of the dynamics equation is defined by order flows in the hierarchical structure and the society's response function to the actions of authorities. By assumption, the function $p(x,t)$ is bounded from above and below by the functions of the maximum and minimum amount of power, both regulated legally. The substantive hypotheses underlying the model and determining the scope of its applicability were described in detail. The model was initially designed for discrete time, and then the transition to continuous time was carried out.

The model is intended to address several substantive issues, including existence conditions for stationary power distributions and their stability, analysis and forecasting of various-type power crises, the impact of civil society activities on power distribution, etc. Strong simplifications (e.g., the use of linear functions) yield explicit answers to some of these issues; in more general cases, numerical analysis is carried out [1].

Mikhailov summarized the results of the first-stage research in his monograph [2]. In subsequent works, various generalizations and supplements were presented, e.g., the case of two power centers (Principals) [4], the struggle between authorities and opposition [5], etc. In particular, the basic model was used to propose the models of corruption in power hierarchies [3, 6–8] and anti-corruption drive.

An original substantive concept of power was developed by M.L. Khazin [9, 10]. Power was defined as the competitive struggle of small organized groups; the main motives, principles, and modes of behavior of "people of power", as well as the types of their relations, were identified; finally, striking historical examples were given. However, Khazin's concept involves no mathematical models.

In addition, let us mention the publications [11–13] devoted to the modeling of hierarchies.

Below, we model power hierarchies in mathematical terms using the sustainable management of active systems [14]. It continues the theory of active systems and the theory of control in organizations [15, 16]. In particular, the following branches of this theory are used.

• Social and private interests coordination engines (SPICE models). Within these models, each agent divides a personal resource (time, funds, etc.) between the production of some public good and private interests. Accordingly, the agent's payoff consists of the utility from participating in public good consumption and implementing private interests [17].

• Hierarchical control is implemented by the methods of compulsion and impulsion. Under compulsion, an upper level of control forces lower-level agents to perform some actions desirable for it (administrative-legislative impact); under impulsion, it motivates (stimulates) agents to perform such actions (economic impact). In mathematical modeling, compulsion means restricting the range of agent's admissible actions whereas impulsion means affecting its payoff function, usually with agent's control feedback [14].

• The main approach to solving complex dynamic control problems is simulation based on the method of qualitatively representative scenarios (the QRS method). The essence of this method is that the dynamics of a controlled system can be forecasted with sufficient precision using a very small number of control scenarios (a QRS-set). The representativeness of this set is checked through internal and external stability conditions. A QRS-set is internally stable if the Principal's payoffs differ significantly for any two control scenarios in it. External stability means that for any scenario outside a QRS-set, there exists a scenario from this set such that the Principal's payoffs will differ insignificantly [18].

This paper aims to design and study mathematical models of power hierarchies based on the theory of sustainable management of active systems using Mikhailov's and especially Khazin's concepts. To succeed, we solve the following problems:

– propose design principles for the model of three-level power hierarchies with an appropriate mathematical description by a difference normal-form game; consider an alternative modeling approach;

– carry out an analytical study for a particular case of the system of two-level power hierarchies;

– pose the general problems of investigating power hierarchies;

– simulate the system of power hierarchies for various cases and compare the simulation results.

This paper is organized as follows. Section 1 describes the model of three-level power hierarchies of the "Principal–agents" type. In Section 2, we analytically study the system of two-level power hierarchies under simplifying assumptions. Section 3 formulates some problems of investigating power hierarchies and solution approaches. In Section 4, we present the results of numerical calculations and analyze them. Section 5 is devoted to an alternative resource competition model. The outcomes of this paper are summarized in the Conclusions.

1. PRINCIPLES TO MODEL THE SYSTEM OF THREE-LEVEL POWER HIERARCHIES

The object of analysis in the model is a power hierarchy, i.e., a group of people with personal subordination relations that has been united to capture certain resources [10, *vol. 1*, *p. 35*]. Relations within a power group are built on a strictly hierarchical principle: all its members are subordinate to the head of the group and compete with each other to the extent not contradicting the orders of the head.

The problem of each power group, personified by its head, is to maximize its power resource. This notion is difficult to define. In a first approximation, the resource can be supposed financial, although the matter concerns any resource that helps increase power (administrative, political, human, social, informational, nominal, etc.). Briefly, it can be expressed by the chain formula: resource → power → greater resource.

The power group participants have personal loyalty and act as a whole. There are two types of organizing power groups: feudal (monarchical) and tribal (oligarchic). In what follows, the difference between them is neglected.

A power group strives for filling gradually, bottom-top, public positions with its supporters in the system of organizations (private and public) that controls the resources of a country or group of countries, [10, vol. 1, p. 220]. It increases the group's resources and strengthens its relative position.

The strategic goal of each power group is to integrate into the dominant group: bring its suzerain into the vassals of the first person, push aside the other suzerains, and take control over the main resources; each competing group fights for this [10, *vol. 1*, *p. 223*]. Strategic interaction has the same structure at a lower level of each power group, where there is a struggle between its participants.

Without loss of generality, the dynamics of a system of power hierarchies can be formally analyzed by considering three levels of hierarchy. Now we describe this hierarchical system in detail.

• A power hierarchy will be represented by a tree digraph whose arcs reflect the subordination of its members (agents). An arc means that the final vertex is subordinate to the initial vertex. The root vertex of the tree (the first level of the hierarchy) will be called the Principal. The second level is formed by influence agents subordinate to the Principal. The third level consists of basic agents.[1]

• Each nondegenerate subtree with an influence agent as the root forms a power group within this hierarchy. Hierarchy as a whole is an extreme case of a power group led by the Principal.

• There are several three-level power hierarchies competing for the resource created by the joint efforts of all agents. For simplicity, this resource will be considered financial (and measured in monetary terms).[2] At each discrete-time instant, the share of the resource controlled by any power group, including the hierarchy (its amount of power), is proportional to the total efforts (time cost) of the lower-level agents of this group. Note that the proportional distribution mechanism is a method of economic control (impulsion). The distribution of resource control among power groups does not change its quantity, which is important for determining resource dynamics.

• Each agent (including the Principal) divides personal time between efforts to increase the share of the power resource of its group and competition. Accordingly, the agent's payoff consists of the utility from increasing the jointly created resource and the utility from its win over the competitors (SPICE models). This framework corresponds to the relations of competition–cooperation (coopetition).

• Basic agents compete with other basic agents subordinate to the same influence agent (within the power group). As their control action they use time to increase the power resource of their group.

• Influence agents compete with other influence agents in their power hierarchy. Their control action is to monitor the activities of their basic agents (a lower bound on their efforts to create the resource).

• Principals compete with the Principals of other power hierarchies. Their control action is also to monitor the activities of their influence agents. Thus, the Principal and influence agents implement administrative control (compulsion).

[1]In [9, 10], the terms "suzerain" and "vassal" were used. This paper follows the terminology of the theory of active systems.

[2]Recall that the resource is everything increasing power [9, 10].

Thus, the system of three-level power hierarchies has the following model:

$$J_i = \sum_{t=1}^{T} \delta^t \left[\left(A - \sum_{p=1}^{N} v_p^t \right) v_i^t + R_i^t \right] \to \max , \qquad (1)$$

$$0 \le q_{ij}^t \le 1; \qquad (2)$$

$$J_{ij} = \sum_{t=1}^{T} \delta^t \left[\left(A_i - \sum_{r=1}^{n_i} v_{ir}^t \right) v_{ij}^t + R_{ij}^t \right] \to \max , \qquad (3)$$

$$q_{ij}^t \le q_{ijk}^t \le 1; \qquad (4)$$

$$J_{ijk} = \sum_{t=1}^{T} \delta^t \left[\left(A_{ij} - \sum_{s=1}^{m_{ij}} v_{ijs}^t \right) v_{ijk}^t + R_{ijk}^t \right] \to \max \qquad (5)$$

$$q_{ijk}^t \le u_{ijk}^t \le 1; \qquad (6)$$

$$R^t = \left(1 + \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right) R^{t-1}, \ R^0 = R_0 ; \qquad (7)$$

$$R_i^t = \begin{cases} R^t \left(\sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right) / \left(\sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right), & \sum_{i=1}^{N} \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0, & \text{otherwise}; \end{cases} \qquad (8)$$

$$R_{ij}^t = \begin{cases} R_i^t \left(\sum_{k=1}^{m_{ij}} u_{ijk}^t \right) / \left(\sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t \right), & \sum_{j=1}^{n_i} \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0, & \text{otherwise}; \end{cases} \qquad (9)$$

$$R_{ijk}^t = \begin{cases} R_{ij}^t u_{ijk}^t / \left(\sum_{k=1}^{m_{ij}} u_{ijk}^t \right), & \sum_{k=1}^{m_{ij}} u_{ijk}^t > 0 \\ 0, & \text{otherwise}; \end{cases} \qquad (10)$$

$$i = 1,..., N; \ j = 1,..., n_i ; k = 1,..., m_{ij} ; t = 1,..., T.$$

Here, the number i is associated with a Principal (a power hierarchy); the number j is associated with an influence agent (a power group) of a given hierarchy; the numbers $k, p, r,$ and s are associated with a basic agent of a given group; t denotes the time instant; N is the total number of power hierarchies; n_i is the total number of power groups in a given hierarchy; m_{ij} is the total number of basic agents in a given group of a given hierarchy; J_i, J_{ij}, and J_{ijk} are the payoff of a given Principal, a given influence, and a given basic agent, respectively; R^t, R_i^t, R_{ij}^t, and R_{ijk}^t are the total resource quantity, the resource quantity of a given power hierarchy, the resource quantity of a given power group, and the resource quantity of a given basic agent, respectively; q_{ij}^t is the control action of a given Principal; $\sum_{j=1}^{n_i} q_{ij}^t$ is the share of Principal's efforts i to control its hierarchy; $v_i^t = 1 - \sum_{j=1}^{n_i} q_{ij}^t$ is the share of Principal's efforts to compete with other Principals; q_{ijk}^t is the control action of a given influence agent; $\sum_{k=1}^{m_{ij}} q_{ijk}^t$ is the share of the influence agent's efforts to control its group; $v_{ij}^t = 1 - \sum_{k=1}^{m_{ij}} q_{ijk}^t$ is the share of its efforts to compete with other influence agents of its hierarchy; u_{ijk}^t is the share of the basic agent's efforts to increase the resource quantity; $v_{ijk}^t = 1 - u_{ijk}^t$ is the share of its efforts to compete with other basic agents of its group; A, A_i, and A_{ij} are the competition parameters; $\delta \in (0, 1)$ is the discount rate; R_0 is the initial value of the total resource; finally, T is the planning horizon.

The efforts to control hierarchies are lower bounds: by assumption, the agents are more prone to competition than to regular activities to increase the power resource of their group. Therefore, the heads must restrict the selfish aspirations of their subordinates, which incurs control costs.

For $i = 1, ..., N$, the relations (1)–(10) determine a normal-form difference game of *N* persons. Let us present the information rules of this game for open-loop strategies without control feedback (the Germeier game Γ_{1t}).

1. The Principals $i = 1, ..., N$ of all power hierarchies choose open-loop strategies $\{q_{ij}^t\}_{t=1}^{T}{}_{j=1}^{n_i}$ simultaneously and independently of each other and then report them to their influence agents.

2. Being aware of the Principal's control action, the influence agents choose open-loop strategies $\{q_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$, simultaneously and independently of all other influence agents (in their power group and the rest of the groups), as a Nash equilibrium in the game of influence agents (3), (4), (9) and then report them to their basic agents.

3. Being aware of the equilibrium control actions of their influence agent, the basic agents of a given power group choose their open-loop strategies $\{u_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$ simultaneously and independently of all other basic agents (in their power group and the rest of the groups). The optimal response of the basic agents to the set $\{q_{ijk}^t\}_{t=1}^{T}{}_{k=1}^{m_{ij}}$ is a Nash equilibrium in the game of basic agents (5), (6), (10) for a given number *j*.

4. Each Principal chooses a control action $\{q_{ij}^t\}_{t=1}^{T}{}_{j=1}^{n_i}$ by solving the optimal control problem (1), (2), (7), (8) on the set of Nash equilibria in the game of influence agents (3), (4), (9).

5. The resulting set $\{q_{ij}^t, q_{ijk}^t, u_{ijk}^t\}_{t=1}^{T}{}_{j=1}^{n_i}{}_{k=1}^{m_{ij}}$ is the solution of the Germeier game Γ_{1t} (1)–(10) for a fixed number *i*, and the set of these solutions for all $i = 1, ..., N$ is the solution of the general difference game.

The information rules of this game are similarly defined for open-loop strategies with control feedback (the Germeier game Γ_{2t}), closed-loop strategies without control feedback (the Germeier game Γ_{1x}), and closed-loop strategies with control feedback (the Germeier game Γ_{2x}).

Therefore, we arrive at the following notion.

Definition 1. *A system of power hierarchies* is a set

$$S_0 = \left\langle N, R_0, \{H_i\}_{i=1}^N \right\rangle,$$

where N is the total number of power hierarchies in this system; R_0 is an initial resource amount; $H_i = (V_i, A_i)$ is the digraph of power hierarchy *i*; $V_i = \{L_{i0}, L_{i11}, ..., L_{ik_in_{k_i}}\}$ is the vertex set of hierarchy *i* (its members); A_i is the arc set of hierarchy *i* (defining the subordination relations between its members).♦

Then model (1)–(10) describes the conflict-controlled dynamics of a system S_0 on a planning horizon T.

2. ANALYSIS OF AN ELEMENTARY SYSTEM OF TWO-LEVEL POWER HIERARCHIES

For the analytical study, we make several simplifying assumptions: $N = n_1 = n_2 = 2$, $A_i = A$, $q_{ij}^t = q_{ij}$, and $u_{ij}^t = u_{ij}$. Let us denote $\bar{u}_i = u_{i1} + u_{i2}$, $i = 1, 2$.

The resulting elementary system of two-level power hierarchies is shown in the figure below.

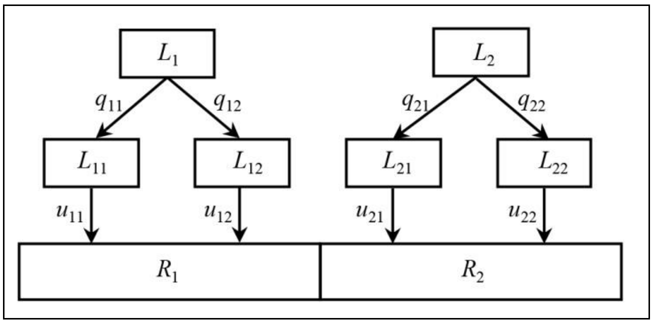

Fig. An elementary system of two-level power hierarchies.

Model (1)–(10) takes the following form:

$$J_i = \sum_{t=1}^{T} \delta^t [(A - v_1 - v_2)v_i + R_i^t] \to \max, \qquad (11)$$

$$0 \le q_{i1} \le 1, 0 \le q_{i2} \le 1;$$

$$J_{ij} = \sum_{t=1}^{T} \delta^t [(A - v_{i1} - v_{i2})v_{ij} + R_{ij}^t] \to \max, \qquad (12)$$

$$q_{i1} \le u_{i1} \le 1, q_{i2} \le u_{i2} \le 1;$$

$$R^t = (1 + \bar{u}_1 + \bar{u}_2)R^{t-1}, R^0 = R_0; \qquad (13)$$

$$R_i^t = \begin{cases} (u_{i1} + u_{i2})R^t / (\bar{u}_1 + \bar{u}_2), \ \bar{u}_1 + \bar{u}_2 > 0 \\ 0, \text{otherwise}; \end{cases} \qquad (14)$$

$$R_{ij}^t = \begin{cases} u_{ij} R_i^t / (u_{i1} + u_{i2}), \ u_{i1} + u_{i2} > 0 \\ 0, \text{otherwise}; \end{cases} \qquad (15)$$

$$i, j = 1, 2; \ t = 1, ..., T.$$

From the expression (13) we obtain

$$R^t = R_0 (1 + \bar{u}_1 + \bar{u}_2)^t, t = 1, ..., T. \qquad (16)$$

Substituting this formula into (14) and then into (15) yields

$$R_{ij}^t = u_{ij} R_0 (1 + \bar{u}_1 + \bar{u}_2)^t / (\bar{u}_1 + \bar{u}_2).$$

Consequently, problem (12) with $v_{ij} = 1 - u_{ij}$ reduces to

$$J_{ij} = \sum_{t=1}^{T} \delta^t \Big[(A - 2 + \bar{u}_i)(1 - u_{ij}) + u_{ij} R_0 (1 + \bar{u}_1 + \bar{u}_2)^t / (\bar{u}_1 + \bar{u}_2) \Big] \to \max,$$

$$q_{i1} \le u_{i1} \le 1, q_{i2} \le u_{i2} \le 1.$$

Obviously, due to the presence of the exponential function, the maximum is achieved at $\bar{u}_1 = \bar{u}_2 = 1$. Thus, $u_{ij} = 1$ and $q_{ij} = 0$, $i, j = 1, 2$, which forms the solution of the game.

3. GENERAL PROBLEM STATEMENTS AND SOLUTION APPROACHES

Let us formulate the general problems of investigating power hierarchies based on the proposed models. We begin with the following notion.

Definition 2. A system of power hierarchies at a time instant t is said to be:

– *unipolar* if $\exists\, i \in \{1, ..., N\}$ such that $R_i^t \ge 0.75 R^t$;

– *bipolar* if $\exists\, i, j \in \{1, ..., N\}$ such that $R_i^t + R_j^t \ge 0.75 R^t \wedge |R_i^t - R_j^t| \le 0.15 R^t$;

– *multipolar* otherwise. ♦

Of course, unipolarity is defined subjectively: a coefficient of 0.75 is taken based on "qualified majority" considerations; a "pole comparability" coefficient of 0.15 is also arbitrary. Anyway, the emergence of systems of power hierarchies with different numbers of poles and the corresponding conditions are of great interest.

Note that for model (11)–(15), the unipolarity condition (*i*-polarity) has the form

$$\bar{u}_i \ge 3\bar{u}_j; \qquad (17)$$

the bipolarity condition is reduced to the system of inequalities

$$\begin{cases} (1 - \alpha)\bar{u}_i - (1 + \alpha)\bar{u}_j \le 0, & (18) \\ (1 - \alpha)\bar{u}_j - (1 + \alpha)\bar{u}_i \le 0. & (19) \end{cases}$$

Here, α is a small parameter, e.g., $0 < \alpha \le 0.15$. If both inequalities (18), (19) hold, the system of power hierarchies is bipolar; otherwise, it is unipolar with the pole *i* satisfying condition (17). Therefore, two simple results are true as follows.

Proposition 1. *If* $\bar{u}_i = \bar{u}_j$, *then the system of two power hierarchies is bipolar.*

Proposition 2. *If* $\bar{u}_i = 0$ *and* $\bar{u}_j > 0$, *then the system of two power hierarchies is j-polar.*

Thus, for dominance, the Principal's efforts should increase the resource.

Second, it is important to investigate the comparative efficiency of compulsion and impulsion methods, open and closed-loop strategies, as well as the information rules of the Germeier games Γ_1 and Γ_2 from the Principal's point of view considering the interests of agents.

Third, model (1)–(10) can be supplemented with sustainable development requirements for the power hierarchy, to be implemented by the Principal. For example, the amount of power in a hierarchy at any time instant be greater or equal to a given threshold.

Note that analytical study capabilities are limited for model (1)–(10) even under strong simplifying assumptions. Therefore, computer simulation is the main approach to investigating the models of power hierarchy, particularly using the method of qualitatively representative scenarios [18]. Here, the key role is played by planning computational experiments with models.

4. SIMULATION RESULTS

Consider the two-level model

$$J_i = \sum_{t=1}^{T} \delta^t \left[\left(A - \sum_{p=1}^{N} v_p^t \right) v_i^t + R_i^t \right] \to \max ;$$

$$0 \le q_{ij}^t \le 1;$$

$$J_{ij} = \sum_{t=1}^{T} \delta^t \left[\left(A_i - \sum_{r=1}^{n_i} v_{ir}^t \right) v_{ij}^t + R_{ij}^t \right] \to \max ;$$

$$q_{ij}^t \le u_{ij}^t \le 1;$$

$$R^t = \left(1 + \sum_{i=1}^{N} \sum_{j=1}^{n_i} u_{ij}^t \right) R^{t-1}, \; R^0 = R_0;$$

$$R_i^t = \begin{cases} R^t \left(\sum_{j=1}^{n_i} u_{ij}^t \right) / \left(\sum_{i=1}^{N} \sum_{j=1}^{n_i} u_{ij}^t \right), \; \sum_{i=1}^{N} \sum_{j=1}^{n_i} u_{ij}^t > 0 \\ 0, \text{otherwise}; \end{cases}$$

$$R_{ij}^t = \begin{cases} R_i^t u_{ij}^t / \left(\sum_{j=1}^{n_i} u_{ij}^t \right), \sum_{j=1}^{n_i} u_{ij}^t > 0 \\ 0, \text{otherwise}; \end{cases}$$

$$i = 1, \ldots, N, \; j = 1, \ldots, n_i; \; t = 1, \ldots, T .$$

Note that this model is symmetric with respect to the basic agents of each influence agent. Hence, all basic agents of the same influence agent behave the same way: $u_{ij}^t = u_i^t$ and $q_{ij}^t = q_i^t$. The model's symmetry with respect to the influence agents is violated by the different values of n_i and A_i. Therefore, we can eliminate the number *j* from the model but not the number *i*. In view of $u_{ij}^t = u_i^t$ and $q_{ij}^t = q_i^t$, it is possible to determine $v_{ij}^t = 1 - u_i^t$ and $v_i^t = 1 - q_i^t$. As a result, the model becomes simpler:

$$J_i = \sum_{t=1}^{T} \delta^t \left[\left(A - N + \sum_{p=1}^{N} q_p^t \right) \left(1 - q_i^t \right) + R_i^t \right] \to \max, \quad (20)$$

$$0 \le q_i^t \le 1; \quad (21)$$

$$J_{ij} = \sum_{t=1}^{T} \delta^t \left[\left(A_i - n_i \left(1 - u_i^t \right) \right) \left(1 - u_i^t \right) + R_{ij}^t \right] \to \max, \quad (22)$$

$$q_i^t \le u_i^t \le 1; \quad (23)$$

$$R^t = \left(1 + \sum_{i=1}^{N} n_i u_i^t \right) R^{t-1}, \; R^0 = R_0; \quad (24)$$

$$R_i^t = \begin{cases} R^t n_i u_i^t / \left(\sum_{i=1}^{N} n_i u_i^t \right), \sum_{i=1}^{N} n_i u_i^t > 0 \\ 0, \text{otherwise}; \end{cases} \quad (25)$$

$$R_{ij}^t = \begin{cases} R_i^t / n_i, u_i^t > 0 \\ 0, \text{otherwise}; \end{cases} \quad (26)$$

$$i = 1, \ldots, N, \; j = 1, \ldots, n_i; \; t = 1, \ldots, T.$$

In particular, all basic agents of one influence agent receive the same resource quantity.

For a numerical study, we take model (20)–(26) with the following parameters: $R_0 = 100$ (the initial resource quantity), $\delta = 0.8$ (the discount rate), $T = 5$ years (the planning horizon), $A = A_i = 50$, $N = 2$ (the number of influence agents), $n_1 = 2$ (the number of basic agents for the first influence agent), and $n_2 = 3$ (the number of basic agents for the second influence agent).

Various scenarios were calculated under the condition that all basic agents of the same influence agent apply the same effort. A Stackelberg equilibrium was found. First, we fixed all strategies of the influence agents, e.g., $q_{ij}^t = 0$, $i = 1, 2$, $j = 1, \ldots, n_i$, $t = 1, \ldots, 5$. Different strategies of the basic agents of the first influence agent at the time instant $t = 0$ are presented in Table 1.

Table 1

The strategies and payoffs of basic agents

Strategy u_{1j}^0, j = 1, 2	Payoff J_{1j}, j = 1, 2
0	129.085
0.1	173.389
0.2	177.661
0.3	181.901
0.4	186.109
0.5	190.285
0.6	194.429
0.7	198.541
0.8	202.621
0.9	206.669
1	210.685

Clearly, the strategy $u_{1j}^0 = 1$ gives the greatest payoff to the basic agents of the first influence agent. In other words, at the first time instant, all basic agents of the first influence agent benefit from applying all efforts to increase the resource only. Further experiments confirmed this property for the basic agents of all influence agents at all time instants; moreover, the behavior of the basic agent of one influence agent has no impact on the behavior of the basic agents of another influence agent.

Now, we determine the optimal values of q_{ij}^t. Note that these quantities affect the payoff function of the influence agents but not the payoff function of the basic agent. This influence agent's strategy can only limit the strategy of the basic agents from below. However, this is unnecessary: the basic agent chooses the maximum possible control action. Therefore, due to the impact of the strategy q_{ij}^t on the payoff function of the influence agents, its optimal value is $q_{ij}^t = 0$: the influence agents should not control the basic ones.

Thus, the optimal strategies of system participants in this model are as follows: $q_{ij}^t = 0$ and $u_{ij}^t = 1$. The influence agents have the payoffs $J_1 = 128\,822$ and $J_1 = 193165$. Since the model is symmetric, the basic agents apply equal efforts, and the resource distribution mechanism is proportional, each basic agent has the payoff $J_{ij} = 64\,346.3$.

Table 2 shows the resource dynamics in this model.

Table 2

Resource dynamics

Resource quantity	t =0	t =1	t =2	t =3	t =4	t =5
Total	100	600	3600	21 600	129 600	777 600
The first influence agent	–	240	1440	8640	51 840	311 040
The second influence agent	–	360	2160	12 960	77 760	466 560
Basic agents	–	120	720	4320	25 920	155 520

5. AN ALTERNATIVE RESOURCE COMPETITION MODEL

In this section, we consider a resource competition model of power hierarchies in the following form:

$$J_i = \sum_{t=1}^{T} \delta^t R_i^t \to \max,\ 0 \le u_i^t \le 1; \tag{27}$$

$$R^t = R^{t-1} + F\left(\sum_{i=1}^{N} u_i^t\right),\ R^0 = R_0; \tag{28}$$

$$R_i^t = \begin{cases} \dfrac{1-u_i^t}{\sum_{j=1}^{N}(1-u_j^t)} R^t,\ \sum_{j=1}^{N}(1-u_j^t) > 0 \\ 0, \text{ otherwise,} \end{cases} \tag{29}$$

$$i = 1,\dots,N,\ t = 1,\dots,T.$$

Here, J_i is the payoff of power hierarchy i on a planning horizon T; $R^t = R_1^t + \dots + R_N^t$ is the total resource quantity (amount of power) at a time instant t; R_i^t is the resource quantity of power hierarchy i at a time instant t; N is the total number of power hierarchies; $\delta \in (0,1)$ is the discount rate; u_i^t and $(1-u_i^t)$ are the shares of efforts applied by hierarchy i to increase the resource and gain control over it, respectively, at a time instant t; F is a resource increase function due to the efforts of agents. As above, formula (28) describes the resource dynamics whereas formula (29) the distribution of control over it.

Example 1. Consider the case of two power hierarchies with time-independent strategies and a power resource increase function. Then

$$R^t = R^{t-1} + \sqrt{u_1 + u_2},\ R^0 = R_0;$$

$$R_i^t = \begin{cases} \dfrac{1-u_i}{2-u_1-u_2} R^t,\ u_1 + u_2 \ne 2 \\ 0, \text{ otherwise,} \end{cases} \tag{30}$$

$$i = 1, 2,\ t = 1,\dots,T.$$

For model (27), (30), we choose the characteristic control scenarios $u_i \in \{0; 1/2; 1\}$, $i = 1, 2$. Table 3 shows the corresponding resource quantities whereas Table 4 the payoffs of the hierarchies. In each cell of Table 4, the upper left and lower right corners contain the payoffs of the first and second power groups, respectively.

Table 3

The dynamics and control of the resource in characteristic control scenarios

	$u_2=0$	$u_2=1/2$	$u_2=1$
$u_1=0$	$R^t=R_0$ $R_i^t=R_0/2, i=1,2$	$R^t=R_0+t\sqrt{2}/2$ $R_1^t=2R^t/3=2R_0/3+t\sqrt{2}/3$ $R_2^t=R^t/3=R_0/3+t\sqrt{2}/6$	$R^t=R_0+t$ $R_1^t=0$ $R_2^t=R^t=R_0+t$
$u_1=1/2$	$R^t=R_0+t\sqrt{2}/2$ $R_1^t=R^t/3=R_0/3+t\sqrt{2}/6$ $R_2^t=2R^t/3=2R_0/3+t\sqrt{2}/3$	$R^t=R_0+t$ $R_i^t=(R_0+t)/2, i=1,2$	$R^t=R_0+t\sqrt{6}/2$ $R_1^t=R^t=R_0+t\sqrt{6}/2$ $R_2^t=0$
$u_1=1$	$R^t=R_0+t$ $R_1^t=R^t=R_0+t$ $R_2^t=0$	$R^t=R_0+t\sqrt{6}/2$ $R_1^t=0$ $R_2^t=R^t=R_0+t\sqrt{6}/2$	$R^t=R_0+t\sqrt{2}$ $R_i^t=0, i=1,2$

Table 4

The payoffs of power hierarchies in characteristic control scenarios

	$u_2=0$	$u_2=1/2$	$u_2=1$
$u_1=0$	$\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}$ $\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}$	$\frac{2\delta R_0(1-\delta^T)}{3(1-\delta)}+\frac{\sqrt{2}}{3}\sum_{t=1}^{T}\delta^t t$ $\frac{\delta R_0(1-\delta^T)}{3(1-\delta)}+\frac{\sqrt{2}}{6}\sum_{t=1}^{T}\delta^t t$	$\frac{\delta R_0(1-\delta^T)}{1-\delta}+\sum_{t=1}^{T}\delta^t t$ 0
$u_1=1/2$	$\frac{\delta R_0\left(1-\delta^T\right)}{3\left(1-\delta\right)}+\frac{\sqrt{2}}{6}\sum_{t=1}^{T}\delta^t t$ $\frac{2\delta R_0(1-\delta^T)}{3(1-\delta)}+\frac{\sqrt{2}}{3}\sum_{t=1}^{T}\delta^t t$	$\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}+\sum_{t=1}^{T}\delta^t t$ $\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}+\sum_{t=1}^{T}\delta^t t$	$\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}+\frac{\sqrt{2}}{2}\sum_{t=1}^{T}\delta^t t$ 0
$u_1=1$	0 $\frac{\delta R_0(1-\delta^T)}{1-\delta}+\sum_{t=1}^{T}\delta^t t$	0 $\frac{\delta R_0(1-\delta^T)}{2(1-\delta)}+\frac{\sqrt{2}}{2}\sum_{t=1}^{T}\delta^t t$	0 0

We recall a definition from [18]. Let $\Omega=S_1\times\ldots\times S_N\times X_1\times\ldots\times X_N$, where $S_i=(s_i\geq 0;\ \sum_{i=1}^{n}s_i\leq S)$ and $X_i=\left(x_i\geq 0\right), i=1,2,\ldots,N,$ are the sets of all admissible control actions of agents and the Principal, respectively. A set

$$QRS=S^{QRS}\times X^{QRS}$$

$$=S_1^{QRS}\times S_2^{QRS}\times\ldots\times S_N^{QRS}\times X_1^{QRS}\times X_2^{QRS}\times\ldots\times X_N^{QRS}$$

$$=\left\{(s,x)=\left(s_1,\ldots,s_N;x_1,\ldots,x_N\right);\right.$$

$$\left.s_i\in S_i^{QRS}\subset S_i;\ x_i\in X_i^{QRS}\subset X_i\right\}$$

is called a *QRS*-set of a hierarchical game with precision Δ if:

– For any two elements $(s,x)^{(i)}$, $(s,x)^{(j)}\in QRS$: $\left|J_0^{(i)}-J_0^{(j)}\right|>\Delta$ (internal stability);

– For any element $(s,x)^{(l)}\notin QRS$, there exists an element $(s,x)^{(j)}\in QRS$ such that $\left|J_0^{(l)}-J_0^{(j)}\right|\leq\Delta$ (external stability).

Thus, any scenarios from a *QRS*-set significantly differ in terms of the players' payoffs; for any "external" scenario, one can select a scenario from this *QRS*-set so that the difference in payoffs will be insignificant. In other words, consideration of a small number of scenarios from a *QRS*-set is necessary and sufficient for a qualitative analysis of system dynamics.

In our case, for a given admissible payoff error Δ, the set of control scenarios will be internally stable if

$$\left|\frac{\delta R_0\left(1-\delta^T\right)}{6(1-\delta)}-\frac{\sqrt{2}}{6}\sum_{t=1}^{T}\delta^t t\right|>\Delta,$$

$$\frac{\delta R_0\left(1-\delta^T\right)}{2(1-\delta)}+\sum_{t=1}^{T}\delta^t t>\Delta,$$

$$\left|\frac{\delta R_0\left(1-\delta^T\right)}{6(1-\delta)}-\frac{3-\sqrt{2}}{3}\sum_{t=1}^{T}\delta^t t\right|>\Delta.$$

Note that the larger the initial total resource quantity is, the more likely the internal stability conditions will be satisfied for the *QRS*-set.

Due to the problem's symmetry, we construct a *QRS*-set based on the payoffs of the first hierarchy for $R_0 = 100$, $\delta = 0.8$, and $T = 5$. First, we consider a potential *QRS*-set from the nine scenarios above, i.e., the strategy set $S_0 = \{0;\ 0.5;\ 1\}$ of one hierarchy. Let the admissible payoff error be $\Delta = 30$ (see Table 5).

Table 5

The payoffs of the first power hierarchy for the strategy set S_0

	$u_2 = 0$	$u_2 = 0.5$	$u_2 = 1$
$u_1 = 0$	134.464	182.535	275.821
$u_1 = 0.5$	91.2673	137.91	277.37
$u_1 = 1$	0	0	0

Scenarios that satisfy the internal stability condition are highlighted in blue. Are they externally stable? Let us expand the strategy set of each hierarchy to five elements: $S_1 = \{0; 0.25; 0.5; 0.75; 1\}$ (Table 6).

Table 6

The payoffs of the first power hierarchy for the strategy set S_1

	$u_2 = 0$	$u_2 = 0.25$	$u_2 = 0.5$	$u_2 = 0.75$	$u_2 = 1$
$u_1 = 0$	134.464	155.643	182.535	219.918	275.821
$u_1 = 0.25$	116.732	136.901	164.938	206.866	276.634
$u_1 = 0.5$	91.2673	109.959	137.91	184.423	277.37
$u_1 = 0.75$	54.9795	68.9552	92.2115	138.685	278.046
$u_1 = 1$	0	0	0	0	0

According to Table 6, all scenarios with the strategies u_i = 0.25 should not be included in the *QRS*-set, whereas those with u_i = 0.75 should be. To verify the external stability of the set of "blue" scenarios, we expand the strategy set of each hierarchy further to seven elements: $S_2 = \{0;\ 0.25;\ 0.5;\ 0.625;\ 0.75;\ 0.875;\ 1\}$ (Table 7).

According to Table 7, all scenarios with the strategies u_i = 0.625 should not be included in the *QRS*-set, whereas those with u_i = 0.875 should be. To verify the external stability of the set of "blue" scenarios, we expand the strategy set of each hierarchy further to nine elements: $S_3 = \{0;\ 0.25;\ 0.5;\ 0.625;\ 0.75;\ 0.8125;\ 0.875;\ 0.9375;\ 1\}$ (Table 8).

According to Table 8, all scenarios with the strategies u_i = 0.8125 should not be included in the *QRS*-set, whereas those with u_i = 0.9375 should be. To verify the external stability of the set of "blue" scenarios, we expand the strategy set of each hierarchy further to ten elements: $S_4 = \{0;\ 0.25;\ 0.5;\ 0.625;\ 0.75;\ 0.8125;\ 0.875;\ 0.9375;\ 0.96875;\ 1\}$ (Table 9).

Table 7

The payoffs of the first power hierarchy for the strategy set S_2

	$u_2 = 0$	$u_2 = 0.25$	$u_2 = 0.5$	$u_2 = 0.625$	$u_2 = 0.75$	$u_2 = 0.875$	$u_2 = 1$
$u_1 = 0$	134.464	155.643	182.535	199.547	219.918	244.778	275.821
$u_1 = 0.25$	116.732	136.901	164.938	183.584	206.866	236.776	276.634
$u_1 = 0.5$	91.2673	109.959	137.91	157.851	184.423	221.608	277.37
$u_1 = 0.625$	74.8302	91.7919	118.388	138.317	166.206	208.027	277.715
$u_1 = 0.75$	54.9795	68.9552	92.2115	110.804	138.685	185.143	278.046
$u_1 = 0.875$	30.5973	39.4627	55.4021	69.3425	92.5715	139.023	278.366
$u_1 = 1$	0	0	0	0	0	0	0

Table 8

The payoffs of the first power hierarchy for the strategy set S_3

	$u_2 = 0$	$u_2 = 0.25$	$u_2 = 0.5$	$u_2 = 0.625$	$u_2 = 0.75$	$u_2 = 0.8125$	$u_2 = 0.875$	$u_2 = 0.9375$	$u_2 = 1$
$u_1 = 0$	134.464	155.643	182.535	199.547	219.918	231.698	244.778	259.39	275.821
$u_1 = 0.25$	116.732	136.901	164.938	183.584	206.866	220.826	236.776	255.175	276.634
$u_1 = 0.5$	91.2673	109.959	137.91	157.851	184.423	201.327	221.608	246.393	277.37
$u_1 = 0.625$	74.8302	91.7919	118.388	138.317	166.206	184.795	208.027	237.895	277.715
$u_1 = 0.75$	54.9795	68.9552	92.2115	110.804	138.685	158.597	185.143	222.306	278.046
$u_1 = 0.8125$	43.4433	55.2066	75.4976	92.3974	118.947	138.857	166.729	208.535	278.208
$u_1 = 0.875$	30.5973	39.4627	55.4021	69.3425	92.5715	111.153	139.023	185.472	278.366
$u_1 = 0.9375$	16.2119	21.2646	30.7991	39.6491	55.5764	69.5116	92.7359	139.183	278.522
$u_1 = 1$	0	0	0	0	0	0	0	0	0

Table 9

The payoffs of the first power hierarchy for the strategy set S_4

	u_2=0	u_2=0.25	u_2=0.5	u_2=0.625	u_2=0.75	u_2=0.8125	u_2=0.875	u_2=0.9063	u_2=0.9375	u_2=0.96875	u_2=1
u_1=0	134.5	155.6	182.5	199.5	219.9	231.7	244.8	251.9	259.4	267.4	275.8
u_1=0.25	116.7	136.9	164.9	183.6	206.9	220.8	236.8	245.6	255.2	265.5	276.6
u_1=0.5	91.3	110.0	137.9	157.9	184.4	201.3	221.6	233.3	246.4	261.0	277.4
u_1=0.625	74.8	91.8	118.4	138.3	166.2	184.8	208.0	222.0	237.9	256.3	277.7
u_1=0.75	55.0	69.0	92.2	110.8	138.7	158.6	185.1	202.0	222.3	247.1	278.0
u_1=0.8125	43.4	55.2	75.5	92.4	118.9	138.9	166.7	185.3	208.5	238.4	278.2
u_1=0.875	30.6	39.5	55.4	69.3	92.6	111.2	139.0	158.9	185.5	222.6	278.4
u_1=0.90625	23.6	30.7	43.8	55.5	75.8	92.7	119.2	139.1	167.0	208.8	278.4
u_1=0.9375	16.2	21.3	30.8	39.6	55.6	69.5	92.7	111.3	139.2	185.6	278.5
u_1=0.96875	8.4	11.1	16.3	21.4	30.9	39.7	55.7	69.6	92.8	139.3	278.6
u_1=1	0	0	0	0	0	0	0	0	0	0	0

According to Table 9, the *QRS*-set should include only those scenarios containing the strategies of hierarchies from the set $\{1-0.5^n\}_{n=0}^{\infty}$. Obviously, $u_i = 0$ is the dominant strategy of each player. Therefore, the Nash equilibrium is $NE = \{(0, 0)\}$. ♦

Example 2. Letting $F = u_1 + u_2$, we consider the model of Example 1 in continuous time without discounting:

$$J_i = \int_0^T R_i(t)dt \to \max, \ 0 \le u_i \le 1, \ i = 1, 2 ; \quad (31)$$

$$\dot{R} = u_1 + u_2, \ R(0) = R_0 ; \quad (32)$$

$$R_i(t) = \begin{cases} \dfrac{1-u_i}{2-u_1-u_2} R(t), \ u_1 + u_2 \ne 2 \\ 0, \text{ otherwise,} \end{cases} \quad (33)$$

$$i = 1, 2.$$

Formula (32) implies $R(t) = (u_1 + u_2)t + R_0$; due to (33), problem (31) takes the form

$$J_i = \frac{1-u_i}{2-u_1-u_2} \int_0^T [(u_1 + u_2)t + R_0]dt \to \max,$$

$$0 \le u_i \le 1, \ i = 1, 2,$$

or, after trivial transformations,

$$J_i = \frac{1-u_i}{2-u_1-u_2} (T^2(u_1+u_2)/2 + R_0 T) \to \max,$$

$$0 \le u_i \le 1, \ i = 1, 2.$$

The first-order conditions yield the system of equations

$$(1-u_1)(2-u_1-u_2) + (u_2-1)(u_1+u_2-R_0T) = 0 ;$$

$$(1-u_2)(2-u_1-u_2) + (u_1-1)(u_1+u_2-R_0T) = 0 .$$

By symmetry (there are no other solutions), we have

$$2(1-u)(1-u) - (1-u)(2u - R_0T) = 0,$$

$$u = u_1 = u_2 .$$

If $u=1$, then $R_i = J_i = 0,\ i=1,2$. Therefore, the solution of the game (31) is given by

$$u = u_1 = u_2 = \frac{2+R_0T}{4},$$

and

$$J_1 = J_2 = T(2T+R_0T+4R_0)/2.$$

Now examine the number of poles. Note that the conditions from Definition 2 do not depend on the type of function for the quantity R. The i *-polarity* condition has the form

$$\frac{R_i^t}{R^t} = \frac{1-u_i}{2-u_i-u_j} \geq \frac{3}{4},$$

or

$$u_i \leq 3u_j - 2. \quad (34)$$

For the system of two power hierarchies, the first bipolarity condition always holds. Of interest is the second condition

$$|R_i^t - R_j^t| \leq 0.15R^t = \alpha R^t.$$

In this example, it reduces to

$$\left|\frac{u_j - u_i}{2-u_i-u_j}\right| \leq \alpha,$$

which is equivalent to the system of inequalities

$$\begin{cases} (1+\alpha)u_j - (1-\alpha)u_i \leq 2\alpha, & (35) \\ (1+\alpha)u_i - (1-\alpha)u_j \leq 2\alpha. & (36) \end{cases}$$

If both inequalities (35), (36) are true, the system of power hierarchies will be bipolar. Otherwise, it is unipolar with the pole i satisfying condition (34). For example, for $u_1 = u_2 = 0.5$, both conditions (35), (36) hold; therefore, the system is bipolar. If $u_1 = 0$ and $u_2 = 1$, condition (36) breaks but condition (34) remains valid; therefore, the system is unipolar. Thus, in Example 2, dominance is achieved by gaining control over the resource instead of increasing its quantity. ♦

CONCLUSIONS

A.P. Mikhailov's mathematical theory of power hierarchies [1–8] is based on natural but rather abstract assumptions. Being much closer to reality, the concept of power hierarchies proposed by M.L. Khazin [9, 10], however, involves no mathematical models. Despite mathematical formalization difficulties, we have attempted to implement this concept based on the sustainable management of active systems [14].

This paper has outlined the design principles of a mathematical model of power hierarchies and presented the model for the case of three-level hierarchies. The information rules of the corresponding difference game have been described, and the definition of a system of power hierarchies has been given. An elementary system of two-level power hierarchies has been studied analytically. General problems of investigating power hierarchies have been posed, the number of power poles has been analyzed, and computational experiments have been carried out for numerical examples.

In addition, an alternative resource competition model has been proposed and studied, first analytically and then numerically using the method of qualitatively representative scenarios [18].

As we believe, this paper contributes mainly by demonstrating the possibilities of a mathematical formalization of the theory of power [9, 10] (using some ideas from the works [1–8]). The theory of power [9, 10] seems very interesting and is supported by many convincing examples. Nevertheless, its mathematical formalization is a difficult problem, and such attempts have not yet been carried out.

The results of this research are as follows:

– Design principles for a model of three-level power hierarchies have been proposed, and the corresponding mathematical formalization has been provided in terms of a difference normal-form game. Also, an alternative approach to model design has been considered.

– A particular case of a system of two-level power hierarchies has been analytically studied.

– General problems of investigating power hierarchies have been posed.

– The system of power hierarchies has been simulated for different cases, and the simulation results have been compared. We emphasize the conclusions regarding the number of poles, which are extremely topical in the current geopolitical situation. Indeed, before the end of World War II, the world was multipolar; from 1945 to 1991, bipolar (the USA and the USSR with their allies); then, unipolar (only the USA). Nowadays, the world is returning to multipolarity again, which is fundamentally important.

The results of this paper are primarily illustrative. However, it seems that the approach may be fruitful when studying real power hierarchies.

Further research is expected to:

– clarify the main hypotheses underlying the model;

– continue the comparative analysis of various information rules;

– consider the continuous-time modifications of the model;

– establish general conditions for the existence of systems of power hierarchies with a different number of poles.

Acknowledgments. *This work was supported by the Russian Science Foundation, project no. 23-21-00131 (O. I. Gorbaneva).*

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This paper was recommended for publication by RAS Academician D.A. Novikov, a member of the Editorial Board.

Received September 20, 2023,
and revised November 27, 2023.
Accepted November 29, 2023.

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Cite this paper

Gorbaneva, O.I. and Ougolnitsky, G.A., Control Models in Power Hierarchies. *Control Sciences* **1,** 34–45 (2024). http://doi.org/10.25728/cs.2024.1.5

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