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MODELING OF THE TARGET'S INTERCEPTION DELAY IN AN ADT GAME WITH ONE OR TWO DEFENDERS

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Abstract. This paper considers the Attacker–Defender–Target (ADT) problem with one or two defenders in a 2D statement. By assumption, the target and defenders move in a straight line with a constant velocity whereas the attacker moves along a catch-up trajectory with an unbounded radius of curvature. Compared to the target's velocity, the defenders move slower whereas the attacker faster. The essence of using defenders is that the attacker first intercepts them and only then switches to pursuing the primary target. As a result, the time of intercepting the primary target increases, and the target may become unattainable for the attacker due to a limited fuel capacity. The angles and times of launching the defenders are optimized, including the case where both defenders are launched on the same side of the target. Different models of the homing system of an autonomous attacking vehicle are studied: moving to the center of mass of all pursued objects and moving to the nearest target by distance or by angular range. Numerical simulations are carried out, showing the importance of choosing the angle of launch of the defenders and the reasonability of using the second defender. Also, scenarios are obtained in which using defenders makes the primary target unattainable for the attacker.

Keywords: pursuit, homing system, defenders, autonomous vehicle, optimization, numerical simulation, interception, ADT.

INTRODUCTION

Group counteraction against autonomous vehicles performing various tasks is becoming more and more topical at the current development level of intelligent algorithms and technologies. Recently, there have appeared publications considering the so-called Missile–Target–Defender (MTD) or Attacker–Defender–Target (ADT) games; for example, see [1–6]. In such problems, a coalition consisting of a target and a defender (defenders) plays against an attacker (a player attacking the target). The defender's task is to intercept the attacker and prevent it from meeting the target. The target executes an evasive maneuver. The defender can be either a mobile striking vehicle (an anti-missile) or a false target (decoy) that distracts the attacker [2].

For each agent in ADT games, control methods were surveyed in [3]. Control actions can be determined based on neural networks [3, 4] and classical approaches. As was noted in [3], neural network approaches can be applied only in relatively simple statements; the authors attempted to develop this methodology using differential game theory.

The paper [5] presented a geometrical solution of the problem with a defender more maneuverable than the target: a control action was determined under which the defender will be in the path of the attacker's motion toward the target.

In [6–8], ADT games were studied using differential game theory in the case of a defender moving faster than the target.

Most of the research works were devoted to the analysis of the game with one target, one defender, and one attacker. However, the results established in [8, 9] concern two attackers at once. In the publication [9], a Riccati equation was solved for this purpose, and nonlinear numerical simulations were carried out. In the paper [10], the case of several pursuers was considered, and the scenario with at most five pursuers тawas simulated numerically. The authors [11] investigated a differential game with several attackers, several defenders, and a stationary target.

This paper deals with an ADT game in a 2D statement. In contrast to the research works mentioned above (a single defender or a stationary target), we analyze the possibility of winning for two defenders moving slower than the mobile target. In addition, the target and defenders have simplified rectilinear motion dynamics with a constant velocity and the attacker's minimum radius of curvature is unbounded.

Other applications-relevant features of the problem statement are as follows: first, the attacker has incomplete information about the game; second, the homing law is known to all players.

The cases of one and two defenders are investigated separately. Compared to the target's velocity, the defenders move slower whereas the attacker faster. By assumption, the attacker has a finite fuel capacity and the pursuit is for a limited time. The defenders are decoys and act in coalition with the primary target. The defenders' task is to divert the attacker's attention to themselves, allowing the target to escape from pursuit.

The attacker, target, and defender are represented by autonomous vehicles. The target and defenders move uniformly and rectilinearly: the target's trajectory is specified whereas those of the defenders are the subject of research.

By assumption, the autonomous attacker also moves with a constant velocity but is equipped with a homing system (HS) and follows a more complex trajectory. At first, the attacker moves to the center of mass of all visible objects, executing a joint pursuit; then, it pursues the objects separately, one after another, until hitting the primary target or reaching the fuel limit. The order is chosen based on the distance or angular range to the objects and may change during the pursuit.

When moving, the attacker uses *Proportional Navigation* (PN), a fairly effective guidance law used, in one form or another, in most autonomous homing vehicles [12–14]. According to PN, the attacker's velocity vector should rotate at a rate proportional to that of the line of sight and in the same direction.

By assumption, at some distance from a defender, the attacker recognizes it as not the primary target and switches to pursuing the next object.

1. PROBLEM STATEMENT

1.1. The Description of the Attack–Evasion Scenario

This paper considers the case of a single attacker attacking a target equipped with one or two defenders (decoys). The attacker, target, and defenders are material points. The case of one defender is considered in Section 2; the case of two defenders, in Section 3. The attack starts at the time instant $t = 0$. Let $d(t)$ denote the distance between the attacker and the primary target at a time instant *t*. At the initial time instant, the distance between the attacker and the target is given by $d(0) = \text{const}$. The target knows of the start of the attack. The attack time is limited by a given value $\tau_{\max}$: afterwards, the attacker has no extra fuel for further pursuit.

The defense tactic is that the target instantaneously launches a defender to distract the attacker's HS, thereby allowing the target to evade. The case in which the target has a second defender is considered separately: it is launched either simultaneously with the first defender (at the initial time instant) or at another (optimal) time instant. Also, we investigate the situation where both defenders are launched on the same side of the target.

1.2. The Motion Model of Autonomous Vehicles

In an interception problem, the velocity ratio of the corresponding objects is important. The attacker's velocity v_A is higher than the target's velocity v_T and the velocities v_D of defenders (decoys), and the target's velocity exceeds that of defenders.

Since the attacker moves faster than the target and defenders, it will catch the target in a finite time. Calculations are carried out until the time instant τ when the attacker catches the primary target. If the result of numerical simulations is $\tau > \tau_{\max}$, the target is considered to have successfully evaded the attack due to the limited fuel capacity of the attacker.

The attacker intercepts the selected target according to the proportional navigation law: $\Delta\theta = k\Delta\lambda$, where θ is the angle of rotation of the attacker's velocity vector, λ is the target's angle of sight, and *k* is a proportionality coefficient. In this paper, $k = 1$.

Figure 1 shows the initial configuration. At the attack start instant $t = 0$, the origin of the frame of co-

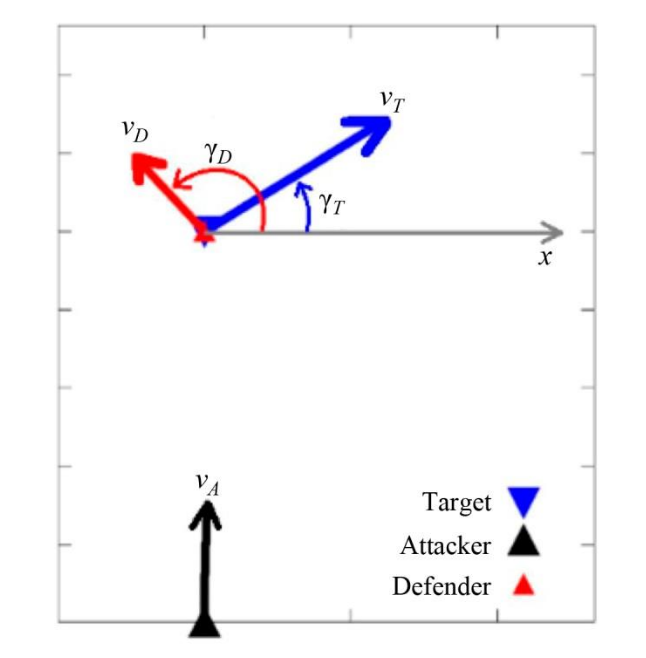


Fig. 1. The initial configuration: the positions and courses of the attacker, target, and defender at the attack start instant.

ordinates coincides with the target's position. The abscissa axis is directed perpendicular to the vector connecting the attacker and the target at the initial time instant. The angles of motion of the target and the defenders (γ_T and γ_{D1}, γ_{D2}, respectively) are counted from this axis in the positive direction.

In the Cartesian frame of coordinates, the motion of the target is described by

$$\begin{cases} \dot{x}_T = v_T \cos(\gamma_T) \\ \dot{y}_T = v_T \sin(\gamma_T), \end{cases}$$

where $(x_T(t),\ y_T(t)) = \mathbf{r}_T(t)$ are its coordinates at the time instant t and $x_T(0) = y_T(0) = 0$.

The motion of the defenders is described by analogy:

$$\begin{cases} \dot{x}_{Di} = v_D \cos(\gamma_{Di}) \\ \dot{y}_{Di} = v_D \sin(\gamma_{Di}), \end{cases}$$

where $i = 1$ or 2, and $(x_{Di}(t),\ y_{Di}(t)) = \mathbf{r}_{Di}(t)$ are the coordinates of the ith defender at the time instant t. Note that $x_{D1}(0) = y_{D1}(0) = 0$ (the first defender is launched immediately at the attack start instant); in the case of two defenders, the coordinates of the second one D_2 at its launch instant τ_D coincide with the target's coordinates, i.e., $x_{D2}(\tau_D) = x_T(\tau_D)$, $y_{D2}(\tau_D) = y_T(\tau_D)$.

The target and defenders move in a straight line; in one scenario, the angles γ_T and γ_{Di} are constant.

We denote by $\mathbf{r}_A(t)$ the attacker's position at the time instant t. When moving along the catch-up trajectory, the attacker's control law can be written as

$$\dot{\mathbf{r}}_A(t) = v_A(t)\frac{\hat{\mathbf{r}}(t) - \mathbf{r}_A(t)}{|\hat{\mathbf{r}}(t) - \mathbf{r}_A(t)|}. \tag{1}$$

Here, $\hat{\mathbf{r}}(t)$ is the position of the target selected for pursuit.

Assume that the attacker first executes a joint pursuit in which $\hat{\mathbf{r}}(t)$ is the center of mass of the primary target T and defenders: $\hat{\mathbf{r}}(t) = \frac{1}{2}(\mathbf{r}_T(t) + \mathbf{r}_{D1}(t))$ in the case of one defender and $\hat{\mathbf{r}}(t) = \frac{1}{3}(\mathbf{r}_T(t) + \mathbf{r}_{D1}(t)$ $+\mathbf{r}_{D2}(t))$ in the case of two defenders. When the distance between the attacker and the center of mass equals the distance between the target and one of the defenders, the attacker switches from the group pursuit to the alternate pursuit.

The goal of the coalition (the primary target and the defenders) is to avoid the equality $\mathbf{r}_T(\tilde{t}) = \mathbf{r}_A(\tilde{t})$ at all time instants $\tilde{t} \in [0, \tau_{\max}]$. (This equality means the interception of the target by the attacker.)

Numerical simulations (see Sections 2 and 3) were carried out for the dimensionless values

$$\begin{cases} v_D = 0.3 \\ v_T = 0.6 \qquad d(0) = 10, \qquad \tau_{\max} = 30 \\ v_A = 0.9, \end{cases}$$

(the velocities of different players, the attacker's initial distance to the target, and the attack time limit, respectively).

In this paper, we consider different fixed values of γ_T. It is required to find the angle of launching one defender, γ_{D1}, or the angles of launching two defenders, γ_{D1} and γ_{D2}, and the launch instant τ_D of the second defender that maximize the interception time of the primary target by the attacker: $\tau \to \max$. The optimization parameters will be separately specified in each subsection of Section 2.

2. THE CASE OF ONE DEFENDER

Sections 2 and 3 present the results of numerical simulations for different scenarios: an attack from different angles, one or two defenders, and optimization of the launch time of the second defender. A software package was developed in the C language to perform the calculations. Numerical simulations were carried out using a grid in the problem's parameter space, the method of gradient descent was applied, and a parametric study was conducted. The attacker's trajectory was integrated with a controlled relative local error; see [15, 16].

In Figs. 3–7, the trajectories of the attacker $\mathbf{r}_A(t)$, target $\mathbf{r}_T(t)$, and defenders $\mathbf{r}_{Di}(t)$ are represented by the black, blue, and red curves, respectively, in the Cartesian frame of coordinates. The triangles of corresponding colors indicate the positions of the attacker, target, and defender at the initial time instant. The bold dots of corresponding colors on the trajectories are the positions of the attacker, target, and defender at the final time instant of the group pursuit. The green square marks the center of mass of the target and defender at this time instant. Finally, the circles are the positions of the attacker and the first defender at the launch time τ_D of the second defender.

2.1. Choosing the Optimal Launch Angle for the Defender

In the case of a single defender launched at the initial time instant, the optimality criterion has the form

$$\tau \underset{\gamma_{D1}}{\to} \max. \tag{2}$$

Figure 2 demonstrates the importance of choosing the angle of launching the defender in this case. The solid curve corresponds to pursuing first the defender and then the target; the dashed curve corresponds to the scenario in which, after moving to the center of mass, the attacker immediately chooses to pursue the primary target.

In Fig. 2a, the optimal launch angle of the defender is $\gamma_{D1} = 128.7°$, the target's interception time τ increases from 20 to 51.5; in Fig. 2b, $\gamma_{D1} = 140.5°$ and τ increases from 26.6 to 47.7; in Fig. 2c, $\gamma_{D1} = 146°$ and τ increases from 29.4 to 45.4.

Note that without a defender, in all these cases, the attacker successfully intercepts the target within the attack time limit $\tau_{\max} = 30$. If a defender is used, the target has a good chance of evading the attacker during this time.

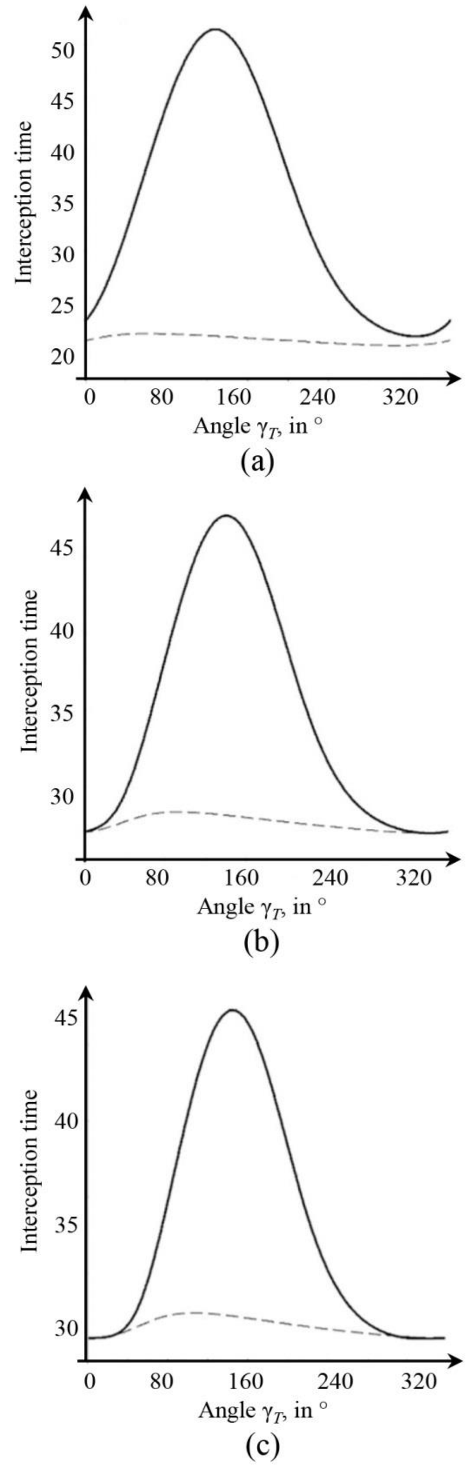


Fig. 2. The interception time as a function of the defender's launch angle γ_{D1} for different angles of the target: (a) $\gamma_T = 0$, (b) $\gamma_T = 30°$, and (c) $\gamma_T = 45°$.

2.2. Trajectories for the Optimal Values of the Defender's Launch Angles

Figure 3 demonstrates the trajectories of the attacker, target, and defender corresponding to the opti-

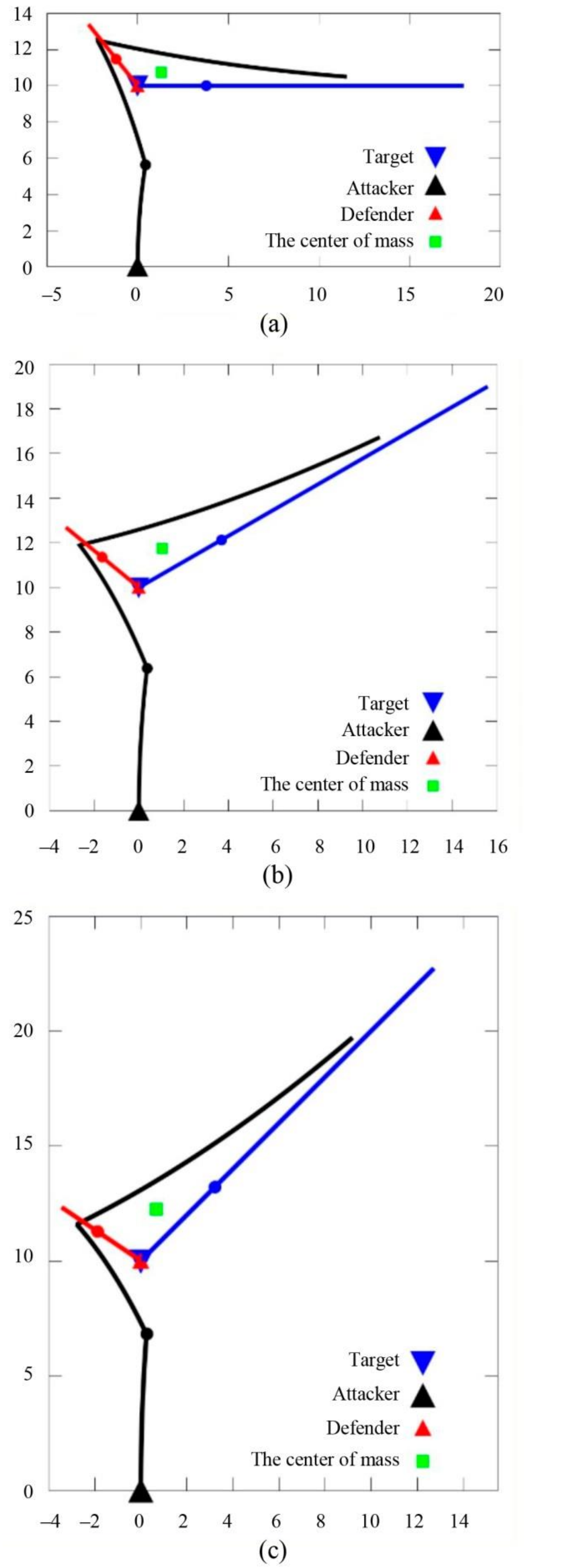


Fig. 3. The trajectories of the attacker (black), target (blue), and defender (red) corresponding to the optimal angle γ_{D1} of the defender's launch (see Fig. 2) when the attacker first pursues the center of mass of the defender and target, next the defender, and then the target: (a) $\gamma_T = 0$, (b) $\gamma_T = 30°$, and (c) $\gamma_T = 45°$.

mal angles γ_{D1}. These trajectories were plotted using the simplified law of the attacker's motion along the catch-up trajectory (1) and the optimality criterion (2).

In Fig. 3a (the case $\gamma_T = 0°$), the group pursuit ends at the time instant $t = 6.29$, when the attacker's distance to the center of mass is equal to the distance between the target and the defender (5.17). The attacker identifies the defender as a decoy at $t = 14.51$; at $t = 30$ (reaching the fuel limit), the attacker's distance to the target is 6.48.

In Fig. 3b (the case $\gamma_T = 30°$), the group pursuit ends at $t = 7.11$, when the attacker's distance to the center of mass is 5.40. The defender is overtaken at $t = 14.12$, the distance to the target at this instant is $d(30) = 5.33$. In Fig. 3c (the case $\gamma_T = 45°$), the group pursuit ends at $t = 7.57$, when the attacker's distance to the center of mass is 5.45. The attacker identifies the defender as a decoy at $t = 13.88$; at $t = 30$, the distance to the target is $d(30) = 4.65$.

Thus, choosing the correct angle γ_{D1} of launching the defender may significantly increase the interception time of the primary target. It is reasonable to use several defenders to increase the probability of failure of the attacker.

3. THE CASE OF TWO DEFENDERS

3.1. The Case $\gamma_T = 0$

Figure 4 presents the trajectories with two defenders in the case $\gamma_T = 0$. The attacker first pursues the center of mass of the target and the defenders and then switches to the alternate pursuit of the nearest objects.

In Fig. 4a, both defenders are launched at the initial time and the problem is solved with the optimality criterion $\tau \underset{\gamma_{D1},\, \gamma_{D2}}{\to} \max$. The first defender is launched at an angle of $\gamma_{D1} = 174°$; in this case, the optimal angle of launching the second defender is $\gamma_{D2} = 98°$, and the target's interception time is $\tau = 56$. When the attacker reaches the fuel limit ($t = 30$), its distance to the target is $d(30) = 7.77$.

In Fig. 4b (the case $\gamma_T = 0$), the variables of the optimization problem are the launch angles of the defenders (γ_{D1} and γ_{D2}) as well as the launch time τ_D of the second defender, i.e.,

$$\tau \underset{\gamma_{D1},\, \gamma_{D2},\, \tau_D}{\to} \max. \quad (3)$$

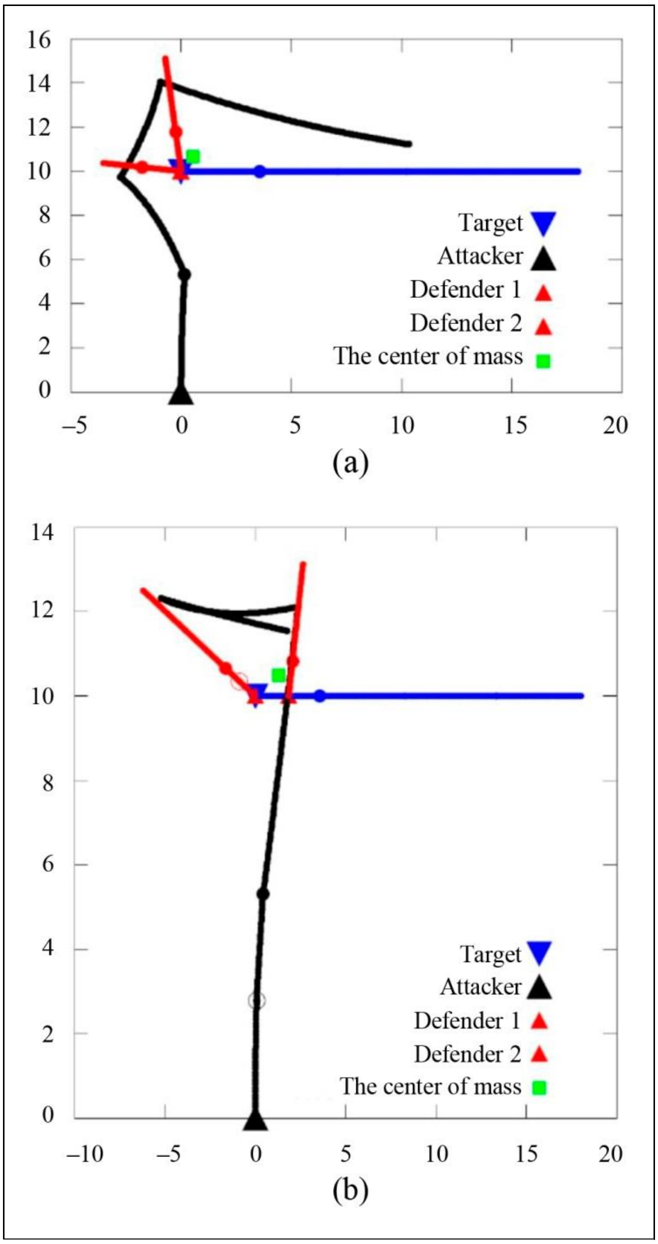


Fig. 4. The trajectories of the attacker, target, and two defenders corresponding to (a) the optimal launch angle γ_{D2} of the second defender and (b) the optimal launch angles of both defenders γ_{D1} and γ_{D2} and the optimal launch time τ_D of the second defender.

In this case, the interception time is 84.6, which exceeds the attack time limit, and the launch angles are $158°$ (the first defender) and $76°$ (the second defender). After the start of the attack, the optimal launch time of the second defender is $\tau_D = 3.1$. In this case, the attacker first tries to intercept the second defender and then switches to the first defender: when the second defender is identified as a decoy, the first defender will be nearer to the attacker than the primary target. Having identified the first defender as a decoy, the attacker will try to intercept the primary target. When the attacker reaches the fuel limit, its distance to the target is $d(30) = 16.32$.

Figures 5–7 show the simulation results during the time $\tau_{\max}$ with two defenders for $\gamma_T = 30$ (the graphs on the left) and $\gamma_T = 45°$ (the graphs on the right). The variables of the optimization problem are the launch angles of the defenders as well as the launch time of the second defender. The angle of motion of the target γ_T is set, and the first defender is launched at the initial time instant.

The defenders are marked in red and vanish in the figures after their identification as a decoy. The interception time was obtained by numerical simulations of the dynamics up to the interception; the graphs do not show the interception itself because the simulations were carried out before the time instant $t = 30$ (when the attacker reaches the fuel limit). If the attack time limit is $\tau_{\max} = 30$, interception in these scenarios with defenders is impossible.

The legends in Figs. 5–7 contain the interception time τ, the launch angles γ_{D1} and γ_{D2} of the defenders, the launch time τ_D of the second defender, and the attacker's distance $d(30)$ to the target at the final time instant (when the attacker reaches the fuel limit).

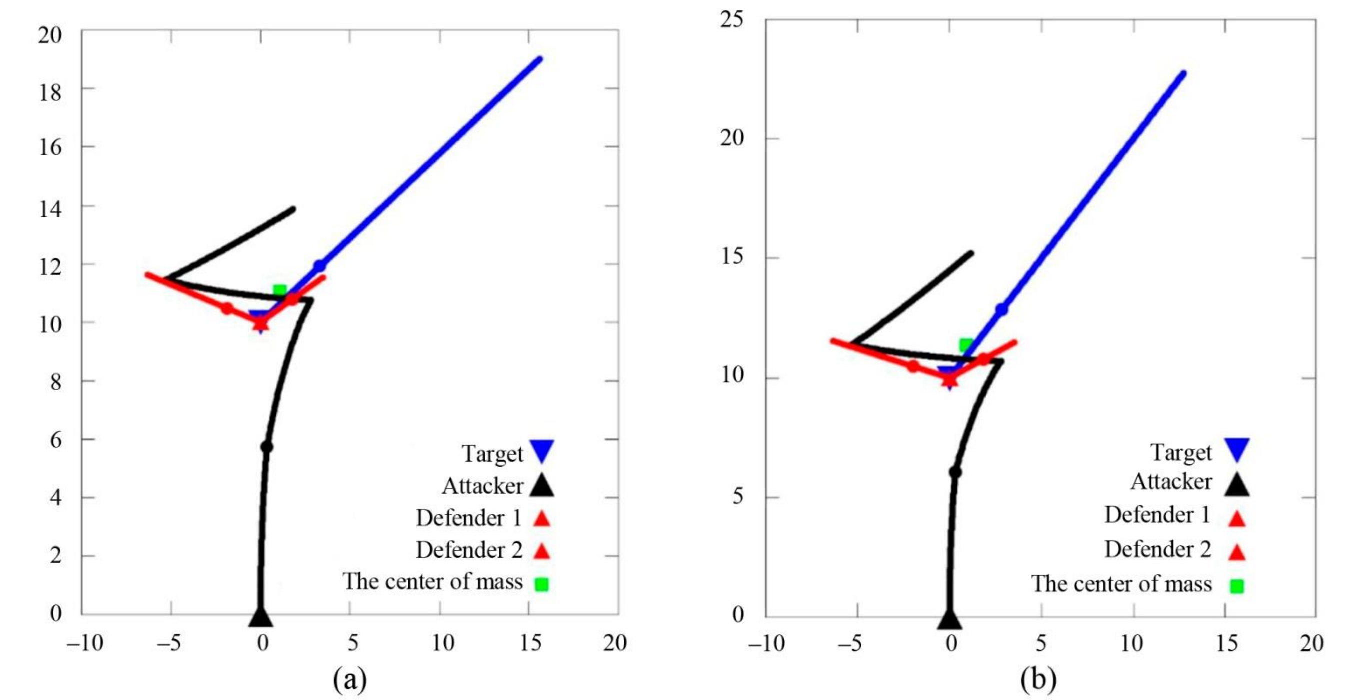
Next, the functional and optimization parameters (3) are discussed in Section 3.2.

3.2. The Cases $\gamma_T = 30°$ and $\gamma_T = 45°$

3.2.1. Attacker's Motion toward the Nearest Object

At the beginning of the trajectory in each case, the attacker moves for some time to the center of mass of all visible objects. When they are at a significant distance from each other, the attacker chooses one of them to pursue. If the attacker detects a more convenient target during motion or identifies the current object as a decoy, it switches to the next target, which is determined by different algorithms in the models under consideration. The results of these algorithms are presented in the graphs below.

When the attacker moves to the nearest object, we obtain the trajectories in Fig. 5a (the interception time of the primary target $\tau = 79.3$) and Fig. 5b (the interception time $\tau = 76.3$). The second defender distracts the attacker considerably far away from the target after the attacker identifies the first defender as a decoy; as a result, the interception time increases significantly compared to the case of a single defender. However, such trajectories are rather unrealistic: at the instant of identifying the first defender, the primary target is in the attacker's field of view whereas the second de-



$\gamma_T = 30°$, $\tau = 79.27$, $\gamma_{D1} = 23.8°$, $\gamma_{D2} = 165.6°$, $\tau_D = 0$, and $d(30) = 14.70$

$\gamma_T = 45°$, $\tau = 76.35$, $\gamma_{D1} = 21.8°$, $\gamma_{D2} = 167.2°$, $\tau_D = 0$, and $d(30) = 13.79$

Fig. 5. The trajectories of the attacker, target, and two defenders corresponding to the attacker's motion toward the nearest object.

fender out of it. In other words, the attacker must continue moving toward the primary target. The attacker's field of view will be considered in subsection 3.2.2.

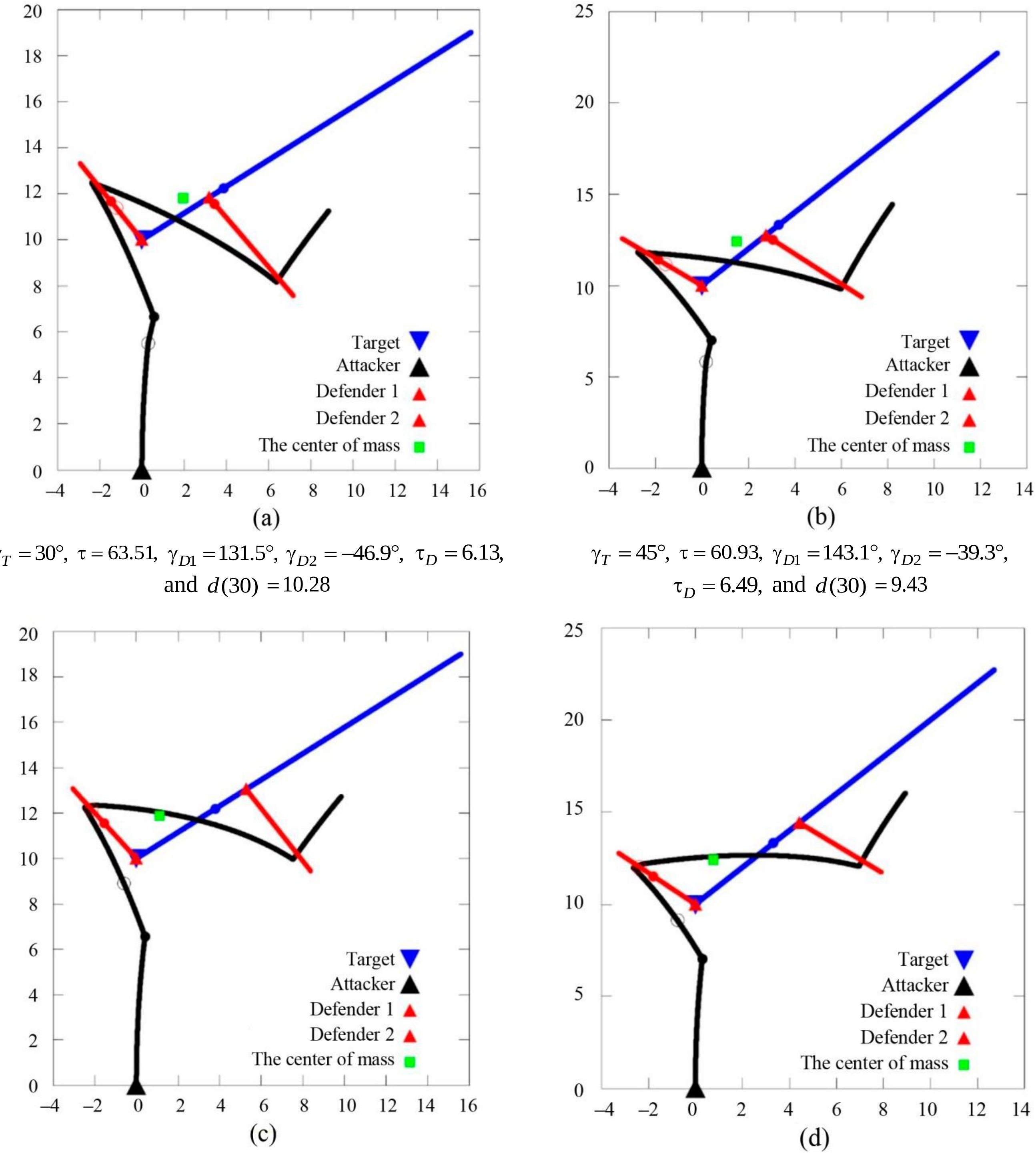
3.2.2. Consideration of the Attacker's Field of View

This subsection describes the following scenario. When choosing the next target after the joint pursuit stage, the attacker determines the nearest target in its field of view, i.e., in the interior of an angle L containing the attacker's velocity vector and being symmetric about it. If there are no targets in the field of view, the attacker will pursue the nearest target to the field of view in terms of the angular distance (not the nearest one in terms of the absolute 2D distance).

The simulation results of this behavioral scenario are presented in Figs. 6 and 7. First, the attacker moves for some time to the center of mass of all visible objects; then, it starts moving to the nearest object in the field of view, which is determined at each observation cycle. If there exists only one object in the field of view, the attacker moves to it, regardless of the distance, without switching to near objects outside this field. As it turns out, even in this more realistic case, the defenders significantly increase the interception time, making the primary target unattainable for the attacker.

The trajectories in Figs. 6a and 6b have a similar nature. After identifying the first defender as a decoy, the attacker switches to the primary target. But when moving toward this object, the attacker almost immediately detects the second defender within the attacker's field of view $L < 30°$ close to its angular boundary. In this case, the second defender is nearer to the attacker than the primary target, so the attacker switches to the former object with a sharp turn. After identifying the second defender as a decoy, the attacker pursues the primary target again. In Fig. 6a (the case $\gamma_T = 30°$), the interception time is $\tau = 63.5$, which exceeds the attack time limit; moreover, this result is significantly higher than the interception time $\tau = 47.7$ with the same value γ_T when using only one defender. In Fig. 6b (the case $\gamma_T = 45°$), the interception time is $\tau = 61$, which exceeds the attack time limit and is significantly higher than the interception time $\tau = 45.4$ with the same value γ_T when using only one defender.

The cases in Figs. 6c and 6d correspond to the attacker's switch toward the nearest object in the field of



(a) $\gamma_T = 30°$, $\tau = 63.51$, $\gamma_{D1} = 131.5°$, $\gamma_{D2} = -46.9°$, $\tau_D = 6.13$, and $d(30) = 10.28$

(b) $\gamma_T = 45°$, $\tau = 60.93$, $\gamma_{D1} = 143.1°$, $\gamma_{D2} = -39.3°$, $\tau_D = 6.49$, and $d(30) = 9.43$

(c) $\gamma_T = 30°$, $\tau = 57.88$, $\gamma_{D1} = 134.9°$, $\gamma_{D2} = 310.7°$, $\tau_D = 10.15$, and $d(30) = 8.52$

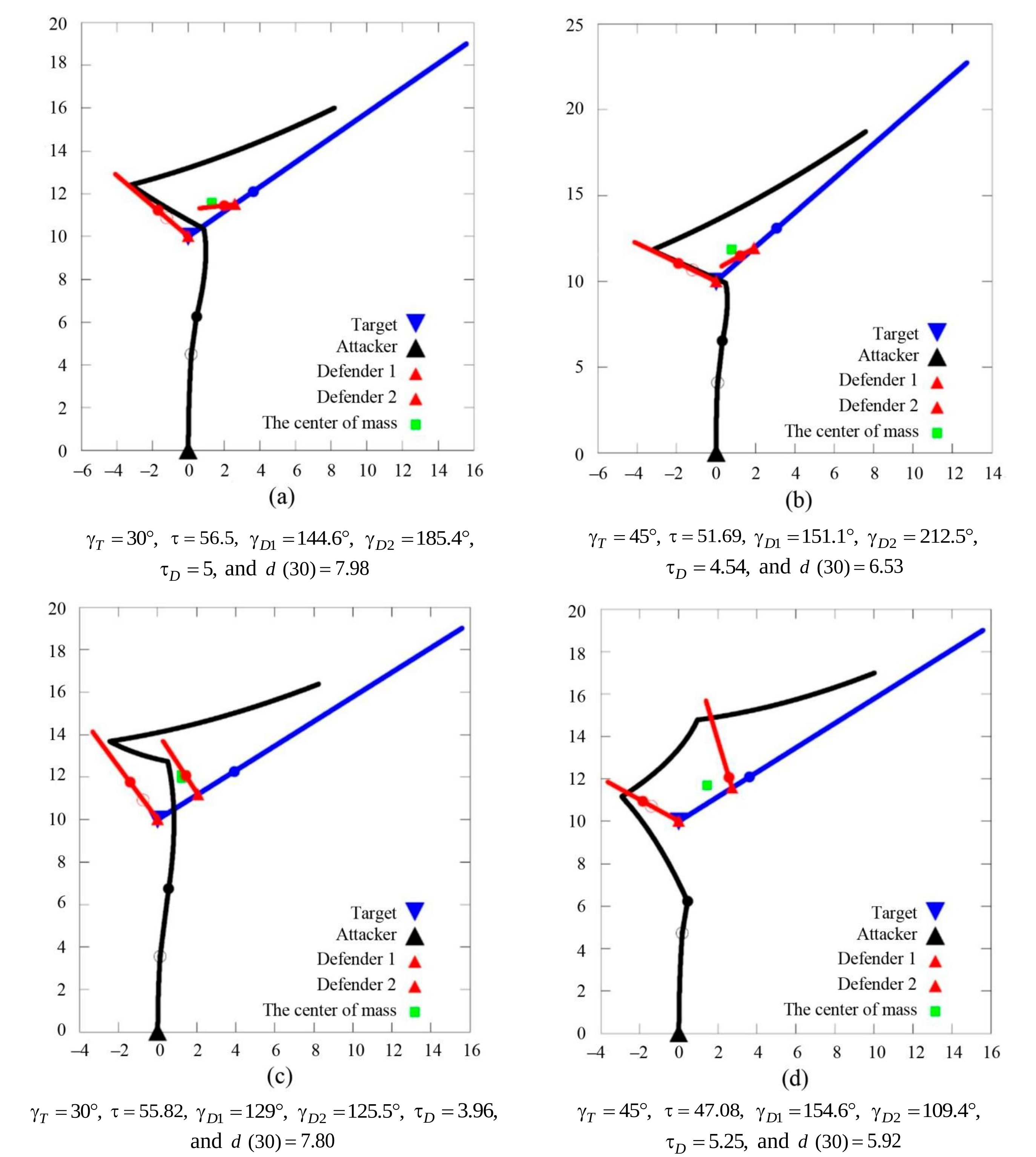
(d) $\gamma_T = 45°$, $\tau = 55.27$, $\gamma_{D1} = 139.6°$, $\gamma_{D2} = 322.4°$, $\tau_D = 10.43$, and $d(30) = 7.69$

Fig. 6. The trajectories of the attacker, target, and two defenders corresponding to the attacker's motion: (a), (b) to the nearest object in the field of view $L < 30°$; (c), (d) to the nearest object in the field of view $L < 30°$ after identifying the first defender as a decoy (the attacker's course is changed from the target to the other defender only if at some instant after the time $\Delta t = 0.5$ of the first defender's identification, the second defender is nearer to the attacker than the primary target within the field of view $L < 15°$).

view $L < 30°$ after identifying the first defender. The attacker's course is changed from the target to the other defender only if at some instant after the time $\Delta t = 0.5$ of the first defender's identification, the second defender is nearer to the attacker than the primary target within the field of view $L < 15°$. In this scenario, the attacker faster intercepts the primary target and has a smoother turn after identifying the first defender as a decoy due to a significantly later launch of the second defender: the attacker observes the second defender inside the field of view as far from its boundary as possible.

3.2.3. Launching Defenders on the Same Side of the Target

The case in Fig. 7 is based on the assumptions corresponding to Figs. 6a and 6b; in addition, the defenders are launched on the same side of the target. In oth-



(a) $\gamma_T = 30°$, $\tau = 56.5$, $\gamma_{D1} = 144.6°$, $\gamma_{D2} = 185.4°$, $\tau_D = 5$, and $d\,(30) = 7.98$

(b) $\gamma_T = 45°$, $\tau = 51.69$, $\gamma_{D1} = 151.1°$, $\gamma_{D2} = 212.5°$, $\tau_D = 4.54$, and $d\,(30) = 6.53$

(c) $\gamma_T = 30°$, $\tau = 55.82$, $\gamma_{D1} = 129°$, $\gamma_{D2} = 125.5°$, $\tau_D = 3.96$, and $d\,(30) = 7.80$

(d) $\gamma_T = 45°$, $\tau = 47.08$, $\gamma_{D1} = 154.6°$, $\gamma_{D2} = 109.4°$, $\tau_D = 5.25$, and $d\,(30) = 5.92$

Fig. 7. The trajectories of the attacker, target, and two defenders corresponding to the launch of both defenders on the same side of the target.

er words, the velocity vectors of both defenders must lie in the same half-plane relative to the straight trajectory of the primary target.

Two fundamentally different families of trajectories were obtained as follows.

Figures 7a and 7b show the best trajectories by the optimality criterion in which the second defender is launched in the time $\tau_D = 4.5 \div 5$ after the start of the attack toward the initial position of the target. Thereby, the second defender takes the attacker to the first defender from the target that has already left this position.

The trajectories in Figs. 7c and 7d correspond to essentially different locally optimal families in which no defenders are sent to the target's initial position. However, they slightly lose, in terms of the values of the optimality criterion, to the trajectories in Figs. 7a and 7b. In Fig. 7c (the case $\gamma_T = 30°$), the attacker first pursues the second defender, next the first defender, and then the primary target; the interception time is $\tau = 55.8$. In Fig. 7d (the case $\gamma_T = 45°$), on the contrary, the attacker first moves toward the first defender, next toward the second defender, and finally toward the primary target. In this case, the interception time is minimal among all the scenarios considered $(\tau = 47)$, but the attacker still fails to reach the target. On this trajectory, the attacker very reliably switches from the first defender to the second one: at the switching instant, the second defender and the target are at very close angles of view for the attacker, but the defender is much nearer to the attacker than the target.

CONCLUSIONS

In this paper, we have modeled the interception of a target by an attacker in the case of one or two defenders (false targets) with different pursuit rules. It has been assumed that the primary target and defenders move in a straight line with constant velocities. The launch angles of the defenders have been optimized. In addition, the problem of optimizing the launch time of the second defender and the case of launching the defenders on the same side of the target have been considered.

A software package has been developed for the ADT game, and the results of numerical simulations for different models of the attacker's behavior have been presented: motion toward the nearest object, motion toward the nearest object in the attacker's field of view, and motion toward the nearest object in the attacker's field of view with additional conditions imposed on switching from the primary target to the defender.

As has been demonstrated, in all the scenarios investigated, the second defender is reasonable to use: it significantly increases the interception time compared to the case of only one defender. The additional defender also increases the probability of choosing one of the defenders for intermediate pursuit instead of the primary target.

Under the conditions on the starting positions, velocities, and fuel capacity, we have designed launch schemes for the defenders under which the attacker will distract to them, thereby becoming unable to hit the primary target due to reaching the fuel limit.

Promising lines for further research include the consideration of a larger number of defenders as well as more complex dynamics of the primary target and defenders.

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