A MATHEMATICAL MODEL OF ADAPTIVE TRAFFIC CONTROL IN MOBILE NETWORKS WITH VARIABLE SIGNAL QUALITY

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Abstract. This paper considers the mathematical modeling problem of traffic transmission in mobile networks under high user mobility and spatially heterogeneous coverage, including signal degradation ("dead") zones. Traffic aggregation at the channel level is applied to increase the reliability and stability of data transmission. A Markov model of a communication channel is proposed to study the effectiveness of aggregation algorithms and adapt them to network operating parameters and user velocity. The model is based on a periodic affine motion of a mobile device between base stations (BSs) uniformly distributed along a straight line. Within this model, the concepts of stable coverage zones and transition zones are introduced in terms of the distances to the nearest and next BSs. The channel state is described by a Markov chain with states corresponding to signal quality sampling: stable connection, degraded connection, and disconnection. Transitions between states are governed by a continuous-time Markov process with constant rates, and the parameters of this process are determined from empirical network data. An extension of the model to incorporate the time dependence of channel states is also considered, leading to a semi-Markov framework. For both cases (Markov and semi-Markov chains), explicit expressions are derived for the stationary probabilities of states, and system stability conditions are formulated to ensure bounded traffic queues. In addition, an adaptive control model of the channel throughput is proposed; this model optimizes transmission parameters depending on the current channel state, request queue length, and user velocity. The effectiveness of the approach is demonstrated by numerical simulations: the network has stable performance across a wide range of mobility levels and coverage parameters. The model can be applied to the reliability analysis and optimization of network protocols in highly mobile environments, including high-speed railway transport, vehicle networks, and mobile platforms.

Keywords: mobile networks, Markov model, semi-Markov process, system stability, Laplace transform.

INTRODUCTION

Modern communication systems providing stable data transmission from mobile objects (high-speed trains, unmanned vehicles, drones, and cars) actively use the infrastructure of public cellular networks. However, the quality of communication is significantly reduced outside of densely built-up areas due to the sparsity of base stations (BSs), incomplete coverage, and radio channel instability. This effect is manifested by signal attenuation, disconnections, and significant delays in data transmission.

At the same time, *the quality of service* (QoS) requirements of users in a highly mobile environment are not lower than the demands of users in stable coverage conditions, namely, the stable transmission of heavy multimedia traffic, video streaming support, and real-time data transmission from mobile platforms. Such scenarios are particularly relevant, e.g., in video surveillance, telemetry, or automated control of mobile objects.

Traffic aggregation is an effective way to solve these problems [1, 2]. The existing traffic aggregation techniques, e.g., Link Aggregation Control Protocol (LACP) and Multilink Point-to-Point Protocol (Multilink PPP), are used in LTE and LTE Advanced (LTE-A) architectures. This paper is oriented toward common scenarios in LTE and LTE-A, where aggregation is implemented at the Packet Data Convergence Protocol (PDCP) level via Carrier Aggregation (CA) mechanisms. The model can also be adapted for 5G scenarios, but the features of the New Radio (NR) architecture (New Radio) are not considered in detail. Note that these methods demonstrate high efficiency in networks with stable or moderate coverage but become significantly worse under abrupt changes in signal quality and high user velocity, which requires frequent switching between BSs [2].

The analysis of typical network operation scenarios under high mobility of users allows identifying several fundamental problems:

• Incomplete network coverage. There are significant dead zones outside urban areas, where the signal level is insufficient for a stable connection; as a result, the overall network throughput is reduced, and packet losses are increased.

• Signal dynamics. The quality of the communication channel essentially depends on the path and velocity of the user. The Doppler shift, multipath propagation, and heterogeneous coverage lead to channel instability and frequent disconnections.

• Nonoptimal behavior when losing communication. In case of a disconnection or signal attenuation, multiple packet retransmission attempts are initiated, making both the channel and network infrastructure overloaded, reducing overall transmission efficiency, and increasing power consumption.

The existing methods for mobile traffic aggregation and control are not effective enough to cope with the above problems. They suffer from the following main drawbacks:

• No time correlation. Most models assume independent transitions between network states, thereby ignoring equipment inertia and the physical regularities of signal propagation.

• No adaptation to network topology and user velocity. Several approaches suppose a stationary or quasi-stationary environment, therefore being inapplicable in the case of high velocities and dynamically changing (variable) channel quality.

• High switching latency. Switching algorithms between BSs can lead to long pauses in traffic transmission, especially in unstable signal conditions.

An obvious solution is to improve the existing aggregation methods. However, appropriate mathematical models should be developed to optimize network performance and find the best solution. As a rule, the existing models are either simplified or neglect key parameters of real systems, such as the changing spatial position of servers and users [3, 4], time correlations in the change of network state, dynamic load redistribution, and adaptive data buffering; or they consider the spatial geometry of network elements but not the complex dynamics of queues [5–7]; or they analyze only the connectivity problems of network components using graph theory [8] or percolation theory [9–12], adding time-varying node states [13] and adapting classical methods of statistical physics to wireless network analysis problems [14, 15].

The objective of this paper is to develop a new traffic control model for mobile networks with stochastic variability in communication channel throughputs, the time correlation of network states, and adaptive load balancing to improve connection stability and reduce delays. We present a stochastic aggregate traffic transmission model in which the channel characteristics are described by a finite Markov chain obtained via signal level sampling [16]. The Markov model is justified due to the exponential nature of the statistics of transitions between states in real measurements (e.g., in the analysis of switching scenarios between BSs in LTE) [17].

In contrast to the existing models assuming either a stationary channel state or the independent transitions between states, we propose a stochastic model incorporating time correlation and spatial coverage structure with the user’s motion between BSs along an affine path. The main contribution of this paper consists in:

– the construction of Markov and semi-Markov models of a communication channel based on signal level sampling,

– the analytical derivation of stationary state distributions and queue stability conditions,

– the development of an adaptive traffic control model that optimizes channel throughput considering the current channel state, user velocity, and queue length.

This paper is organized as follows. In Section 1, we build a channel model, present a mathematical model of adaptive traffic control with the main equations, describe a Markov process of transitions between states, and analyze stability conditions of the system. The formulations and proofs of the main results are given in Section 2, including explicit analytical formulas for the stationary distribution and a stability condition of the model. Section 3 provides numerical simulation results for the model in different network operation scenarios; also, the model is compared with the existing traffic aggregation methods. In the Conclusions, we summarize the key findings of the study, assess the effectiveness of the model, and outline directions for further research.

This study is focused on the LTE and LTE-A architectures, where traffic aggregation is performed at the PDCP level via Carrier Aggregation (CA) mechanisms. The model considered is applicable to high-speed travel scenarios (e.g., railways) that are typical of these technologies. Although more sophisticated aggregation schemes (including Dual Connectivity and Service Data Adaptation Protocol (SDAP)) are used in 5G NR, they go beyond the scope of this paper and may be the subject of a later generalization. The main attention is paid to the formalized description of the stochastic structure of a radio channel and its adaptive throughput control in the LTE/LTE-A framework.

1. BUILDING THE MATHEMATICAL MODEL

We will consider a mobile network as a set of BSs and mobile users traveling along routes with different signal coverage levels. In this case, a mobile user will be understood as a single aggregator or server that collects and forwards traffic from multiple devices aboard a high-speed train or car. In practice, such a scheme is widespread in modern 4G/5G networks (an example is multi-relay systems, where a train or bus has a single access point to a cellular network) [1]. This allows aggregating the traffic of multiple devices into a single channel and simplifying the process of switching between BSs while individual devices within the coach connect to this "server" [1, 18, 19]. In the context of LTE- and LTE-A architectures, the aggregator is realized as a user terminal or a multi-relay platform that unites onboard traffic of a vehicle and interacts with multiple component carriers (CA).

Assume that the network coverage is not dense (the distances between BSs are considerable enough for coverage breaks) and user velocity is sufficiently high (several BSs should be involved sequentially to transmit the required traffic).

1.1. Model Assumptions

The network topology is naturally modeled as a directed graph $G=(V, E)$ [3, 5, 8], where V stands for a set of network nodes (e.g., BSs) and $E \subseteq V \times V$ is a set of its oriented edges (possible data transmission routes between nodes).

We make several assumptions regarding the model, which simplify the analysis significantly without limiting the generality of the approach and the main qualitative conclusions.

• **The graph of BSs is linear:** they are located along a straight line at points

$$x_j = j\,\Delta x,\ \ j \in \mathbb{Z}_+,$$

where Δx is a fixed distance between neighbor BSs. In other words, the BSs are uniformly distributed at the nodes of the one-dimensional grid $\mathbb{Z}_+$. This assumption reflects the typical coverage on main lines (highways or railways).

Remark. Note that the linear uniform arrangement of BSs along a straight line is an idealized assumption, characteristic primarily of main lines (highways, railways, etc.). In real systems, there may be variations in the mutual location of BSs, curved route segments, inhomogeneous relief zones (tunnels, bridges, etc.). Nevertheless, the linear model provides a convenient basic framework for analyzing the key regularities, especially when moving along relatively straight route segments. This simplification does not affect the overall logic of the methods and conclusions but can be refined if necessary.

• **The aggregator's motion is uniform:** the aggregator moves along the above straight line with a constant velocity:

$$v(t) = v_0,\ v_0 > 0.$$

This assumption is justified to model standard scenarios (e.g., high-speed trains or cars on highways) where velocity changes are relatively small and the aggregator is moving steadily.

Remark. We suppose a constant aggregator velocity $v(t) = v_0$ (or the normalized unit velocity) due to stationary high-speed transportation conditions on long main lines. At the same time, real scenarios may include segments of acceleration (braking) as well as velocity fluctuations. If such effects are significant, the model can be extended. This study is focused on the basic scenario with a constant (or nearly constant) velocity, which ensures periodic traffic and simplifies further analytical considerations without loss of generality.

• **The impact of BSs is local:** the quality of communication at each point of the aggregator's path is determined only by the two nearest BSs, namely, the current one (behind the aggregator) and the next one (in front of the aggregator). This simplification is justified due to the rate of signal attenuation with increasing distance and allows neglecting the impact of far BSs without significant loss of accuracy.

Thus, under the uniform arrangement of BSs and uniform motion of the aggregator, the latter's movements can be modeled as a periodic motion of a point along the half-interval [0, 1); when the aggregator reaches the position 1, the current and next BSs change, and the aggregator starts moving again from the point 0. This interpretation significantly simplifies the mathematical analysis and numerical simulation of network dynamics.

Under these assumptions, the aggregator's state at any time instant can be fully parameterized by its position on the affine half-interval [0, 1); the model can be investigated only on the period of traversing this distance (further called the transit time).

Thus, given the uniform arrangement of BSs and uniform motion of the aggregator, the latter's motion can be formally described as follows.

Let $T = \frac{\Delta x}{v_0}$ denote the aggregator's transit time between two neighbor BSs.

Then the aggregator's position at any time instant t is modeled by a periodic affine parameter $s(t) \in [0,1)$ given by the formula

$$s(t) = \frac{t \bmod T}{T},$$

where $s(t)=0$ corresponds to the aggregator's position exactly at the current BS point and $s(t)=1$ to reaching the next BS.

When the aggregator reaches the position $s(t)=1$, it instantly starts moving on a new segment where the next BS becomes the current one, and the new neighbor BS becomes the next one.

Thus, the aggregator's motion through the network is a periodic process represented by this model as a repetitive motion of a point along the half-interval [0, 1):

$$x(t) = s(t).$$

An important special case is the unit velocity $v_0 = 1$ of the aggregator and the unit distance $\Delta x = 1$ between BSs, which simplifies the model and subsequent analysis as much as possible.

In this case, the transit time between neighbor BSs becomes $T = 1$, and the aggregator's position $x(t)$ on the half-interval [0, 1) is given by the simple periodic dependence

$$x(t) = t \bmod 1.$$

Thus, the aggregator moves along the half-interval [0, 1) with constant unit velocity and, having reached the point 1, instantly moves to the next half-interval, starting again from the position 0. Graphically, this function represents a periodic sawtooth dependence with unit period.

Thus, the aggregator's position on the half-interval [0, 1] completely determines its distance to two neighbor BSs.

In view of the model periodicity, the properties of this system can be analyzed by considering its behavior only on one fixed period $t \in [0, 1]$. The results for the other time intervals will be similar due to periodicity.

1.2. Dynamics of Distances to BSs and Signal Quality Given the Network Topology

At a position $x(t) \in [0, 1]$, the distances between the aggregator and the current (behind it) and next (in front of it) BSs are given by

$$d_{\text{current}}(t) = x(t),\ d_{\text{next}}(t) = 1 - x(t),\ \ 0 \le x(t) \le 0.5;$$

$$d_{\text{current}}(t) = 1 - x(t),\ d_{\text{next}}(t) = x(t),\ 0.5 < x(t) \le 1.$$

These formulas imply that:

- At the beginning of the period ($x(t)=0$), the aggregator is exactly at the current BS (the distance to it equals 0), and the distance to the next station is maximal (equals 1).
- In the middle of the period ($x(t)=0.5$), the aggregator is equidistant (0.5) to the two neighbor BSs.
- At the end of the period ($x(t)=1$), the aggregator reaches the next BS and performs an instantaneous transition to a new segment, where the next BS becomes the current one and the aggregator's position again becomes 0.

Thus, in one period, the aggregator first moves away from the current BS (when passing from 0 to 0.5) and then approaches the next station (when passing from 0.5 to 1). As a result, the distance to the nearest BS in one period first monotonically increases from 0 to 0.5 and then monotonically decreases back to 0.

In addition, assume that each BS has a finite range R of a stable signal received by the aggregator (the state S_1). Beyond the range R, there exists a small transition zone of a width δ, where the signal is degraded (the state S_2).

Thus, there are three coverage zones for each BS:

– a stable signal zone (the state S_1), described by $0 \le d \le R$,

– a degraded signal (transition) zone (the state S_2), described by $R < d \le R + \delta$, and

– a dead zone (the state S_3), described by $d > R + \delta$, where d is the current distance to the BS:

$$d = d(t) = \min\{d_{\text{current}}(t), d_{\text{next}}(t)\}.$$

By assumption, the following natural condition holds:

$$2(R + \delta) < 1.$$

In other words, the total diameter of the coverage zones of one BS (the stable signal zone plus the transition zone on both sides) is significantly smaller than the distance between neighbor BSs. This ensures the presence of dead zones between the coverage zones of neighbor BSs, where the signal completely vanishes.

When the aggregator moves along the half-interval [0, 1), the aggregator sequentially passes:

– the stable signal zone of the current station,

– the transition zone of the current station,

– the dead zone,

– the transition zone of the next station, and

– the stable signal zone of the next station.

In general, the impact of the aggregator's velocity and distance to the nearest BS on the quality of communication is modeled by a special loss function combining linear and nonlinear effects [20]:

$$L\left(d(t),\ v(t)\right) = L_0 + a\,d(t)^{\gamma} + k_1 v(t) + k_2 \log\left(1 + v(t)\right) + k_3 v(t)^{\alpha},$$

where L_0 is the basic loss in the ideal case (zero distance and zero velocity); $a\,d(t)^{\gamma}$ is the signal attenuation component depending on the distance $d(t)$ to the BS (the power signal propagation model, usually with $\gamma \in [2,4]$; $k_1 v(t)$ is the linear component due to the Doppler frequency shift; $k_2 \log\left(1 + v(t)\right)$ is the logarithmic attenuation component caused by multipath fading; $k_3 v(t)^{\alpha}$ is the nonlinear effects arising due to switching between BSs and high velocities ($\alpha > 1$).

Thus, the above model simultaneously considers both key factors, i.e., the aggregator's distance to the BS and velocity, therefore being physically adequate and practically useful for further analysis.

The linear model $L(v) = L_0 + ad(t)^{\gamma} + k_1 v(t)$ is a special case ($k_2 = k_3 = 0$) to simplify the basic analysis [20]; this model will be investigated below.

1.3. The Process of Transitions between Discrete Signal Levels in the General Case

Within the model proposed, the aggregator sequentially passes three zones with different signal quality (stable signal, weak signal, and dead zone). In real communication networks, transitions between channel states (S_1, S_2, S_3) occur not instantaneously but with some delay. The main causes of such effects are [20]:

• constant external conditions: the parameters of the communication channel remain relatively stable for some time, so transitions between states occur gradually rather than abruptly;

• communication equipment inertia: switching between BSs requires additional time for coordination and signal processing;

• physical path: along a certain path, transitions between states are related to the network topology and the regularity of BS locations.

These effects create a time correlation between channel states: the probabilities of transitions depend not only on the current state but also on its dwell time. A strict mathematical description of such situations leads to a more general class of processes called *semi-Markov processes* (SMPs) [21, 22].

Consider a random process $X(t)$ modeling the change of signal level in a mobile network. This process is a semi-Markov model with three states: S_1 (stable connection), S_2 (degraded connection), and S_3 (disconnection). The instants of transitions form an increasing sequence $\{\tau_n\}$ of random variables, with $X(t)$ keeping the current state between transitions [23].

The semi-Markov model of signal levels is described by the following system of integral and differential equations for the probabilities of states $p_i(t) = P\{X(t) = S_i\},\ i = 1,\ 2,\ 3$:

$$\frac{dp_i(t)}{dt} = \sum_{j \neq i} \int_0^t p_j(\tau)\alpha_{ji}(t-\tau)\,d\tau - \sum_{k \neq i} \int_0^t p_i(\tau)\alpha_{ik}(t-\tau)\,d\tau, \tag{1}$$

with the transition rate

$$\alpha_{ij}(t) = \lambda_{ij}(t)\exp\left(-\int_0^t \lambda_i(u)du\right),$$

$$\lambda_i(t) = \sum_{k \neq i} \lambda_{ik}(t),\ i = 1,\ 2,\ 3,$$

Let the initial conditions be fixed: $p_1(0) = 1, p_2(0) = p_3(0) = 0$.

The Laplace transform [24] can be conveniently applied to solve system (1). In this case, we obtain the following system of algebraic equations in the Laplace space:

$$s\tilde{p}_i(s) - p_i(0) = \sum_{j \neq i} \tilde{p}_j(s)\tilde{\alpha}_{ji}(s) - \sum_{k \neq i} \tilde{p}_i(s)\tilde{\alpha}_{ik}(s).$$

Generally, the dwell times can have arbitrary distributions $F_i(t)$: $T_i \sim F_i(t)$. Then the system of equations (1) becomes rather complicated for analytical treatment. In such a situation, one can apply numerical schemes on a time grid $\{t_n\}$, e.g.,

$$p_i(t_{n+1}) = p_i(t_n) + \Delta t\left[\sum_{j \neq i}\sum_{m=0}^{n} p_j(t_m)\alpha_{ji}(t_{n+1} - t_m) - \sum_{k \neq i}\sum_{m=0}^{n} p_i(t_m)\alpha_{ik}(t_{n+1} - t_m)\right].$$

This direct numerical approach is effective for more complex distributions (gamma, Weibull, etc.).

In practice, the gamma (Erlang), Weibull, and uniform distributions of dwell times are often used. Below, we present the expressions for the transition rates $\alpha_{ij}(t)=\lambda_{ij}(t)\exp\left(-\int_0^t \lambda_i(u)du\right)$ and their Laplace transforms $\tilde{\alpha}_{ij}(s)=\mathcal{L}\{\alpha_{ij}(t)\}$ under these distributions of dwell times:

- the gamma distribution (the Erlang distribution):

$$\lambda_i(t)=\frac{\lambda_i^k t^{k-1} e^{-\lambda_i t}}{(k-1)!-\sum_{m=0}^{k-1}\frac{(\lambda_i t)^m}{m!}},$$

$$\alpha_{ij}(t)=p_{ij}\lambda_i(t),\ \tilde{\alpha}_{ij}(s)=p_{ij}\frac{\lambda_i^k}{(s+\lambda_i)^k};$$

- the Weibull distribution:

$$\lambda_i(t)=\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1},\ \alpha_{ij}(t)=p_{ij}\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}\exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}\right],$$

$$\tilde{\alpha}_{ij}(s)=p_{ij}\frac{\beta}{\alpha^{\beta}}\int_0^{\infty} t^{\beta-1}\exp\left[-\left(\frac{t}{\alpha}\right)^{\beta}-st\right]dt;$$

- the uniform distribution on the segment $[a,b]$:

$$\lambda_i(t)=\begin{cases}0, & t<a,\\ \dfrac{1}{b-t}, & a\le t<b,\\ \infty, & t=b,\end{cases}$$

$$\alpha_{ij}(t)=\begin{cases}0, & t<a,\\ \dfrac{p_{ij}}{b-a}, & a\le t\le b,\\ 0, & t>b,\end{cases}$$

$$\tilde{\alpha}_{ij}(s)=p_{ij}\frac{e^{-as}-e^{-bs}}{(b-a)s}.$$

By substituting the above parameters into the system of equations (1), we can derive explicit expressions for the probabilistic characteristics of the model with different distributions of dwell times and the most important quality metrics of the system.

However, the analytical study of such models generally leads to cumbersome expressions complicating their interpretation and application. Therefore, we will consider the Markov case with the exponential distribution of dwell times:

$$\mathbb{P}(T_i>t)=e^{-\lambda_i t}.$$

This case corresponds to the absence of memory and considerably simplifies the analysis. Here, time correlations and channel inertia are assumed to be averaged and approximated by constant transition rates of states.

This simplification allows obtaining strictly analytical and easily interpretable results (see Section 2) as the main analytical implications of the model.

2. THE MAIN RESULTS

2.1. An Optimal Control Mechanism

To adaptively adjust the traffic transmission parameters, we formally introduce a function $u(t)$ depending on the current state of the network:

$$u(t)=\begin{cases}\mu_{\text{norm}} & \text{if } X(t)=S_1\\ \mu_{\text{slow}}\, g(\tau_2,\theta) & \text{if } X(t)=S_2\\ 0 & \text{if } X(t)=S_3,\end{cases}$$

where $g(\tau_2,\theta)$ is an adaptive function depending on the dwell time in the state S_2 and the parameters θ to be optimized.

To optimize the system's operation, we define an objective function of the form

$$J(u)=\int_0^T\left[\omega_1 L_{\text{pack}}(u(t))+\omega_2 D(u(t))\right]dt, \qquad (2)$$

where $L_{\text{pack}}(u(t))$ is a function reflecting packet losses depending on the chosen control strategy $u(t)$; $D(u(t))$ is a function reflecting data transmission delays; ω_1 and ω_2 are significance coefficients of the corresponding criteria; T is a time interval under consideration (the control horizon).

An optimal strategy $u^*(t)$ is achieved by minimizing the objective function (2). For optimization, one can employ, e.g., gradient descent or dynamic programming methods [25, 26]. With gradient descent applied to the parameters θ, the correction is performed using the well-known algorithm [25, 27]

$$\theta_{n+1}=\theta_n-\eta\nabla_{\theta}J(u),$$

where η is the algorithm parameter and $\nabla_{\theta}J(u)$ denotes the gradient of the objective function with respect to the parameters θ. Such approaches are widely used in the calibration of communication systems and flow control in networks.

In addition to controlling the transmission rate, the function $u(t)$ can also affect the transition rates between network states. For example, one can modify these rates to reduce the probability of disconnection or accelerate recovery:

$$\lambda_{ij}\left(v, u(t)\right) = \lambda_{ij}^{0}\, h\left(u(t)\right) e^{-k_v v(t)}, \qquad (3)$$

where λ_{ij}^{0} are the baseline transition rates; the function $h\left(u(t)\right)$ describes the corrective control impact, and $e^{-k_v v(t)}$ reflects the impact of the aggregator's velocity $v(t)$ (if necessary). Such corrections model physical and protocol mechanisms where, e.g., more aggressive resource utilization (higher transmission power) increases the value of $h\left(u(t)\right)$, thereby reducing the probability of passing to a dead zone.

In this study, we use the exponential control function (3). It reflects well the intuitively implied exponential deterioration of the channel parameters when staying long in the degraded connection state [28]. The choice of an exponential function is justified by its analytical simplicity and application in known models of channels with fading, as well as by the possibility of calibrating the parameter α within an optimization procedure [29].

However, other types of functions (logistic, linear, etc.) are possible, depending on physical or protocol constraints. Such a choice of the function $g(\tau_2, \theta)$ can be justified by available data or when approximating mechanisms implemented, e.g., in the adaptive modulation of modern communication protocols (LTE and 5G [30]).

In this paper, the objective function (2) is considered in a general form. For numerical simulation, it can be specified as follows:

$$L_{\text{pack}} = \frac{1}{T}\int_{0}^{T} \mathbb{I}_{\text{loss}}(t)\,dt,$$

where $L_{\text{pack}}\left(u(t)\right)$ is the share of lost packets; $\mathbb{I}_{\text{loss}}(t) = 1$ if a packet at a time instant t was discarded and $\mathbb{I}_{\text{loss}}(t) = 0$ otherwise.

The average packet transmission delay $D\left(u(t)\right)$ can be estimated, e.g., as

$$D\left(u(t)\right) = \frac{1}{N}\sum_{i=1}^{N} W_i,$$

where W_i is the total dwell time of the i th packet in the system.

Thus, introducing the control $u(t)$ and objective functions yields a comprehensive approach to optimization in mobile networks [31]: one can adapt the transmission rate (or other channel parameters) and control the probability of disconnections and recoveries by changing the transition rates λ_{ij}. In practice, a suitable control function $u(t)$ and an appropriate optimization method are chosen depending on quality requirements (reduction of delays and losses) and available hardware (protocol) solutions [32].

Such control can be realized via an additional module (board) with program control, e.g., using neural networks to predict the channel state and adapt the parameters dynamically. In this way, it is possible to respond, in due time, to connection degradations and adapt, in real time, the transmission rate and the probability of disconnections in advance.

Depending on particular service requirements (e.g., delay constraints, loss probability thresholds, or transmission power limits), the objective function (2) can be refined by adding other criteria or penalties for exceeding quality standards. Thus, in particular engineering problems, the form of $L_{\text{pack}}(\cdot)$ and $D(\cdot)$ can be chosen by considering the practical significance of partial network quality metrics (packet losses, downtime, etc.). In this paper, the function $J(u)$ remains in a general form; in the section devoted to partial assumptions and simplifications, we will demonstrate the choice of the objective function in special cases.

2.2. Basic Properties of the System

Consider now the basic properties of the system. For the simplicity and transparency of further analysis, let the transition rates between channel states be constant within each zone and depend only on the current state of the aggregator. This means that the transitions between signal quality levels can be described using fixed parameters reflecting the physical characteristics of the medium and the motion dynamics.

Thus, the model has a finite number of constant transition rates, each characterizing a typical transition regime between the coverage zones of a BS. Under the assumed exponential distribution of dwell times in each state, the model turns into a continuous Markov process with a finite number of states given by the system of Kolmogorov equations. Therefore, from system

(1) we proceed to the consideration of the infinitesimal transition matrix

$$Q=\begin{pmatrix} -\lambda_{12} & \lambda_{12} & 0 \\ \lambda_{21} & -(\lambda_{21}+\lambda_{23}) & \lambda_{23} \\ 0 & \lambda_{32} & -\lambda_{32} \end{pmatrix},$$

and the vector of state probabilities $p(t)=(p_1(t), p_2(t), p_3(t))$ satisfies the system of Kolmogorov equations

$$\frac{dp(t)}{dt}=p(t)Q.$$

Here, the coefficients λ_{ij} have the following physical interpretation:

- λ_{12} is the transition rate from the state S_1 (stable connection) to the state S_2 (degraded connection), which corresponds to passing from the stable signal zone $d \le R$ to the transition zone $R < d \le R+\delta$.
- λ_{21} is the signal recovery rate from the state S_2 to S_1 during the reverse transition to the stable signal zone.
- λ_{23} is the signal loss rate (transition from the state S_2 to the state S_3 (disconnection)), which corresponds to reaching the dead zone.
- λ_{32} is the signal recovery rate when returning from the state S_3 to S_2 (i.e., when passing from the dead zone to the transition zone).

Remark. The network structure is not explicitly included in this model. However, it can be indirectly considered by parameterizing the matrix Q based on the adjacency matrix of the coverage graph or other structural characteristics of the network. Such an approach allows generalizing the model to the case of spatially heterogeneous networks or real telecommunication infrastructures.

Theorem 1 (on the existence of a piecewise smooth control function). *For the communication channel in one of the three states: S_1 (stable connection), S_2 (degraded connection), and S_3 (disconnection), there exists a piecewise smooth control function of the general form $u(t)$ depending only on the current channel state $X(t)$ such that*

$$u(t)=\begin{cases} u_1, & X(t)=S_1, \\ u_2, & X(t)=S_2, \\ u_3, & X(t)=S_3, \end{cases}$$

where u_i is the control parameters in the state S_i $(i=1, 2, 3)$. In addition, the service rates $\mu_i(u(t))$ in each channel state and the transition rates $\lambda_{ij}(u(t))$ between channel states are well-defined and controllable on each state constancy interval.

This control function determines the service rate $\mu_i(u(t))$ in each channel state and can also impact the transition rates $\lambda_{ij}(u(t))$ between the states (e.g., by a controllable acceleration/deceleration of channel recovery/degradation). In other words, when the channel is in the state S_i, the system applies the control action u_i, which sets the current service rate $\mu_i(u_i)$ and can change the transition rates $\lambda_{ij}(u_i)$ to other states.

P r o o f. Rationale behind choosing this form of $u(t)$. Control depending only on the current state $X(t)$ is natural in the context of adaptive communication systems. Such a piecewise function $u(t)$ allows instantaneously responding to changes in channel quality: in the event of passing from stable to degraded connection or disconnection, the control jumps to a new value corresponding to the degraded communication conditions. This provides adaptive service control: e.g., in case of channel degradation (S_2), it is possible to reduce the service rate or activate a more reliable transmission regime; in case of disconnection (S_3), initiate communication restoration procedures (which is equivalent to increasing the transition rate back to the state S_1 or S_2). In the stable connection state S_1, control can return to the maximum throughput regime. With this control function $u(t)$, the system is dynamically adapted depending on the current channel state, improving service stability during channel degradation and reducing downtimes during disconnections. ♦

Theorem 2 (a stability criterion for the queueing system). *Consider a Markov process $X(t)$ with three states $\{S_1, S_2, S_3\}$, where S_1 and S_2 have service rates μ_1 and μ_2, respectively, and no transmission is possible in S_3 ($\mu_3=0$). Let λ_{in} be the rate of an incoming (Poisson) flow of requests. We denote by π_1, π_2, and π_3 the stationary probabilities of channel's dwelling in the states S_1, S_2, and S_3, respectively (if they exist).*

The queueing system with unlimited buffer and variable signal quality is stable if and only if

$$\lambda_{\text{in}} < \pi_1\mu_1 + \pi_2\mu_2. \quad (4)$$

According to condition (4), the average input load must be less than the average channel throughput $\pi_1\mu_1 + \pi_2\mu_2$. If condition (4) is violated, the queue (with unlimited buffer) grows infinitely; see [3, 24].

P r o o f. In the *M/M/1* model, the stability condition $\lambda_{\text{in}} < \mu$ is necessary and sufficient [2]. For the case with the channel state-dependent variable μ, the average service rate is $\sum_{i=1}^{3}\pi_i\mu_i$. Since $\mu_3 = 0$, the state S_3 has zero contribution to the throughput. Hence, the criterion turns into inequality (4). If condition (4) holds, there is a unique stationary distribution of the queue length; otherwise, no stationary distribution exists.

The stationary probabilities π_1, π_2, and π_3 are determined from the Kolmogorov equations (1). The stationary vector $\pi = (\pi_1, \pi_2, \pi_3)$ satisfies the system

$$\begin{cases} -\lambda_{12}\pi_1 + \lambda_{21}\pi_2 = 0 \\ \lambda_{12}\pi_1 - (\lambda_{21} + \lambda_{23})\pi_2 + \lambda_{32}\pi_3 = 0 \\ \pi_1 + \pi_2 + \pi_3 = 1. \end{cases}$$

Resolving this system gives

$$\pi_2 = \frac{\lambda_{12}}{\lambda_{21}}\pi_1,\ \pi_3 = \frac{\lambda_{23}}{\lambda_{32}}\pi_2 = \frac{\lambda_{12}\lambda_{23}}{\lambda_{21}\lambda_{32}}\pi_1.$$

Substituting these expressions into the normalization condition, we obtain

$$\pi_1 + \pi_2 + \pi_3 = \pi_1\left(1 + \frac{\lambda_{12}}{\lambda_{21}} + \frac{\lambda_{12}\lambda_{23}}{\lambda_{21}\lambda_{32}}\right) = 1$$

$$\Rightarrow \pi_1 = \left(1 + \frac{\lambda_{12}}{\lambda_{21}} + \frac{\lambda_{12}\lambda_{23}}{\lambda_{21}\lambda_{32}}\right)^{-1}.$$

Finally, with π_1, π_2, π_3 substituted into inequality (4), the explicit numerical condition takes the form

$$\lambda_{\text{in}} \le \left(\frac{\lambda_{21}\lambda_{32}}{\Delta}\right)\mu_1 + \left(\frac{\lambda_{12}\,\lambda_{32}}{\Delta}\right)\mu_2,$$

where

$$\Delta = \lambda_{21}\,\lambda_{32} + \lambda_{12}\,\lambda_{32} + \lambda_{12}\,\lambda_{23}. \ ♦$$

Corollary (a stability condition for the system with control). *In the stationary regime, the stability condition for the system with optimal control is given by*

$$\lambda_{\text{in}} < \pi_1\left(u^*\right)\mu_{\text{norm}} + \pi_2\left(u^*\right)\mu_{\text{slow}},$$

where the stationary probabilities $\pi_i\left(u^*\right)$ *depend on the optimal strategy* $u^*(t)$.

2.3. Performance Characteristics of the System in the Stationary Mode

The previous sections have been focused on the description of the channel (the states S_1, S_2, S_3) and its transition rates. In a real queueing system, however, the full state at a time instant t is given by the "channel state–queue length" pair.

With the channel evolution estimated in terms of the states $\{S_1, S_2, S_3\}$ and the control function $u(t)$, we can assess the quality of the system considering traffic in real systems. For this purpose, let the Markov model of the channel be combined with the queueing model of the *M/M/1* type (or *M/M/1/N* with a finite buffer), where the service rate depends on the current channel state and, if necessary, on the control function $u(t)$.

This approach is based on the classical results of queueing theory [3, 4], applied to the situation where service parameters (and even channel transitions between states) may vary depending on the control strategy. As shown below, in the stationary regime, the key stochastic performance characteristics—the probability of packet loss, the average number of requests in the system, and throughput—are expressed through the standard Erlang and Little formulas considering the average (effective) speeds given by the channel states.

Consider a queueing system served by a *controllable* Markov channel with the three states $\{S_1, S_2, S_3\}$, where the control action $u(t)$ can affect both the service rate $\mu_i(u)$ in the state S_i and the transition rates $\lambda_{ij}(u)$. Assume that:

- The queue has a finite buffer: at most N packets waiting (hence, a total of $N+1$ packets along with the one being served).
- In the stationary regime, the channel spends a fraction of time $\pi_i\left(u^*\right)$ in the state S_i (under optimal or fixed control u^*).
- If $\mu_3(u) = 0$ (the dead zone), then $\bar{\mu} = \pi_1\left(u^*\right)\mu_1\left(u^*\right) + \pi_2\left(u^*\right)\mu_2\left(u^*\right)$ is the *effective* (average) service rate.
- The rate of the incoming flow of requests is λ_{in}.

Under the above assumptions, we can find the performance characteristics of the system.

• *The load factor.* The relative load is given by

$$\rho = \frac{\lambda_{\text{in}}}{\bar{\mu}} = \frac{\lambda_{\text{in}}}{\pi_1\left(u^*\right)\mu_1\left(u^*\right) + \pi_2\left(u^*\right)\mu_2\left(u^*\right)}.$$

If $\rho < 1$ for a given value u^*, the system can operate without collapse in an unlimited buffer; if $\rho \geq 1$, the queue grows or large losses occur.

• *The loss probability.* For a finite number N, similar to Erlang's formula, the probability of packet loss in an overloaded queue is given by

$$P_{\text{loss}} = P\{\text{queue} = N\} = \frac{\rho^{N+1}}{\sum_{k=0}^{N+1} \rho^k},$$

where ρ is defined through $\pi_i\left(u^*\right)$ and $\mu_i\left(u^*\right)$. As $N \to \infty$, we have $P_{\text{loss}} \to 0$ if $\rho < 1$ and $P_{\text{loss}} \to 1$ if $\rho > 1$.

• *The average number of packets and delay.* We denote by $L = E[n]$ the expectation of the number of packets in the queue and service. For $\rho \neq 1$, the following formula is valid:

$$L = \frac{\rho}{1-\rho} - \frac{(N+2)\rho^{N+2}(1-\rho)}{1-\rho^{N+2}}.$$

The average number of packets in the queue, L_q, is obtained by subtracting the average busy service position; similarly, for the average dwell time (W) and waiting time $\left(W_q\right)$, we apply Little's law:

$$L = \lambda_{\text{eff}} W, \quad L_q = \lambda_{\text{eff}} W_q,$$

where $\lambda_{\text{eff}} = \lambda_{\text{in}}\left(1 - P_{\text{loss}}\right)$ is the rate of the actual flow of incoming and served packets.

• *The average throughput.* In the stationary regime, the system serves the flow with the rate

$$\lambda_{\text{out}} = \lambda_{\text{eff}} = \lambda_{\text{in}}\left(1 - P_{\text{loss}}\right).$$

If $\rho < 1$, $\lambda_{\text{out}} = \lambda_{\text{in}}$ as $N \to \infty$ (no losses). If $\rho > 1$, the queue is full and $\lambda_{\text{out}} \approx \bar{\mu}$.

2.4. Implications and Remarks

• **Boundary cases (reducibility to classical models).** If the communication channel is almost always in the stable connection (good) state S_1 (i.e., $\pi_1 \approx 1$, $\pi_2 \approx \pi_3 \approx 0$, and $\mu_1 = \mu$), then the entire system reduces to the classical *M/M/1*. In this case, the stability criterion takes the form $\lambda_{\text{in}} < \mu$, and the above formulas for the probability of packet loss, the average number of packets, and the waiting time are simplified to well-known results (Erlang's formula, Little's formulas, etc.). In contrast, if the aggregator stays in the dead zone (the disconnection state S_3, in which π_3 is large and $\mu_3 \approx 0$) for a large share of time, the effective rate $\bar{\mu}$ will drop dramatically and the system will behave like a slow server with a high load ρ.

• **An example of the dead zone's impact on packet losses.** Let $\mu_1 = 1$, $\mu_2 = 0.5$, and $\mu_3 = 0$, and let the fractions of time in the corresponding states be $\pi_1 = 0.8$, $\pi_2 = 0.15$, and $\pi_3 = 0.05$. Then $\bar{\mu} = 0.8 \cdot 1 + 0.15 \cdot 0.5 = 0.875$. For an incoming flow with $\lambda_{\text{in}} = 0.7$, we have $\rho \approx 0.8 < 1$, and the system operates without significant packet losses. However, for $\lambda_{\text{in}} = 1.0$ $(\rho \approx 1.14 > 1)$, the queue starts growing infinitely (an arbitrarily large buffer) or reaches the maximum N (a bounded buffer); an appreciable probability of packet loss arises $(P_{\text{loss}} \approx 0.2)$, and the resulting throughput saturates at $\lambda_{\text{out}} \approx \bar{\mu} = 0.875$.

• **The physical meaning of controlling transitions and service rate.** If one can control the rates $\lambda_{ij}(u(t))$ (accelerating recovery from the state S_3 or decelerating withdrawal from the state S_1) and/or modify $\mu_i(u(t))$ (selecting more aggressive or safer regimes), then it is actually possible to control the share of time π_3 (staying in the disconnection state) and the effective rate $\bar{\mu}$. Thereby, the load ρ, the probability of packet loss, and the average delay can be significantly reduced. However, in practice, increasing μ_i or reducing the dwell time in the state S_3 may require additional resources (power, redundant channels, etc.), so a trade-off between cost and gain in the quality of service should be found.

• **The scope of application and prospects.** The above assumptions (exponential distributions, Markov or semi-Markov transitions) simplify the analysis and provide elegant formulas but can only approximate real channels with correlated traffic and complex signal propagation dynamics in a heterogeneous medium. Nevertheless, the main conclusion (the need to satisfy the stability condition $\lambda_{\text{in}} < \bar{\mu}$) and the optimization principles of the control function $u(t)$ remain valid. The model allows estimating the marginal network performance and adapting the service and transition strategy in modern mobile networks, where important criteria include network throughput, power consumption, connection recovery time, etc..

• **Relation to *M/G/*1 systems with interruptions. A**s a generalization of the system with $n>3$ states [33–35], the three-state channel model with variable service rate and periods of complete unavailability (disconnections) considered in this paper is conceptually close to *M/G/*1 systems with interruptions; for example, see [36, 37]. However, unlike typical M/G/1 models with interruptions, we use an additional control function $u(t)$ to impact the transition rates and the service rate. As a result, the parameters are adapted to the current conditions of the communication channel, which extends the classical approaches and opens up the possibility of adaptive optimization.

• Thus, there is a similarity with *M/G/*1 with interruptions, and the results of this paper (particularly the criterion $\lambda_{\text{in}} < \sum_i \pi_i \mu_i$) repeat the logic of classical models. However, owing to the dynamic control, the physical interpretation of the states (radio channel), and the objective function $J(u)$, the model presented has a more flexible structure, opening new possibilities of adaptation in engineering applications.

3. THE STABILITY AND PERFORMANCE OF THE ADAPTIVE MODEL: NUMERICAL ANALYSIS

In this section, we provide simulation results confirming the above theoretical conclusions. Consider a stochastic channel with the three states $\{S_1, S_2, S_3\}$ and an incoming flow of requests with a rate λ_{in}. The throughput of the system depends on the current channel state and the control function $u(t)$.

The simulation was carried out using the discrete event method on a horizon $T_{\max}$. The key characteristics investigated include system stability, queue length, the probability of packet loss, and the impact of the network parameters and control strategy on the network quality. The experiment parameters, metrics, and visualization of the system behavior in different regimes are given below.

3.1. Model Parameters

The flow of requests is modeled by a Poisson process with the rate λ_{in}. All requests are processed according to the FIFO (*first in, first out*) principle with a finite buffer.

The channel can be in three states:

– S_1 (stable connection), the corresponding throughput is μ_{norm},

– S_2 (degraded connection), the corresponding throughput is μ_{slow} (or $\mu_{\text{slow}} e^{-\alpha\tau_2}$), and

– S_3 (disconnection), the service is unavailable $(\mu = 0)$.

All transitions between the states are described either by a fixed matrix of transition rates (the Markov case) or by functions $\alpha_{ij}(t)$ in the semi-Markov model. A request is served if the communication channel has the state S_1 or S_2 and the queue is non-empty. In the state S_3, requests are still received, but their service is suspended.

The main quality metrics of the network are queue length dynamics and the impact of the rate λ_{in} and velocity on stability.

3.2. Simulation Results

The graphs below demonstrate the behavior of the model in two regimes:

– stable (subcritical, $\lambda_{\text{in}} < \pi_1\mu_{\text{norm}} + \pi_2\mu_{\text{slow}}$) and

– unstable (supercritical, $\lambda_{\text{in}} > \pi_1\mu_{\text{norm}} + \pi_2\mu_{\text{slow}}$).

In the subcritical regime (Fig. 1), the queue (the aggregator's buffer level) is stabilized and does not grow infinitely. However, there is a positive drift during overload (Fig. 2).

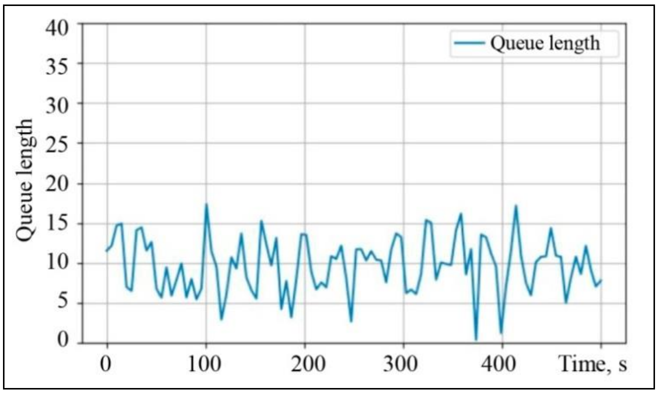

Fig. 1. Queue length dynamics in the stable regime.

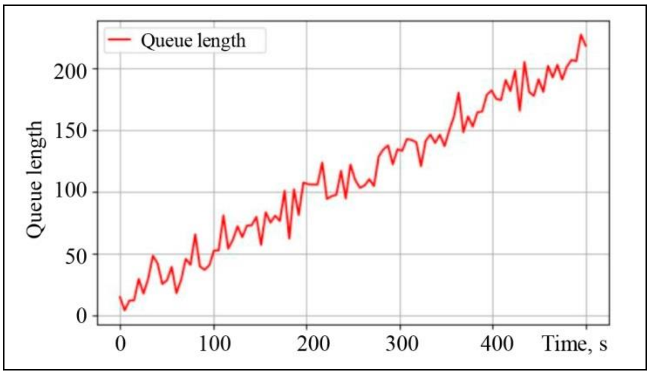

Fig. 2. Queue length dynamics in the unstable regime.

According to Fig. 3, in the subcritical regime, the communication is restored quite quickly even in case of disconnections. Under overload, even short service interruptions cause an avalanche-like growth of the aggregator's buffer.

Figure 4 shows the impact of exponential fading in the state S_2: under a high load, the dwell time in the state S_3 increases, which drastically deteriorates stability.

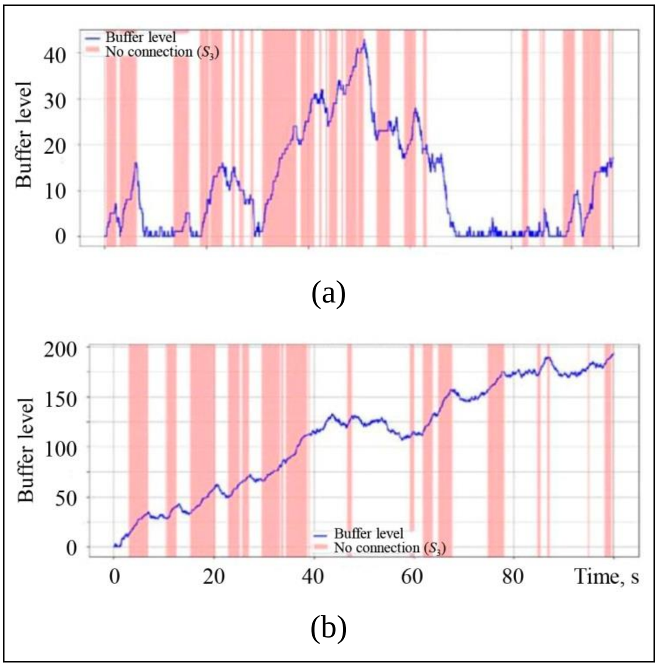

Fig. 3. Queue: (a) in the stable regime $(\lambda = 3.0$ reqs/s, $p = 30$ m/s) and (b) in the unstable regime $(\lambda = 7.0$ reqs/s, $p = 30$ m/s). The red zones indicate disconnection periods.

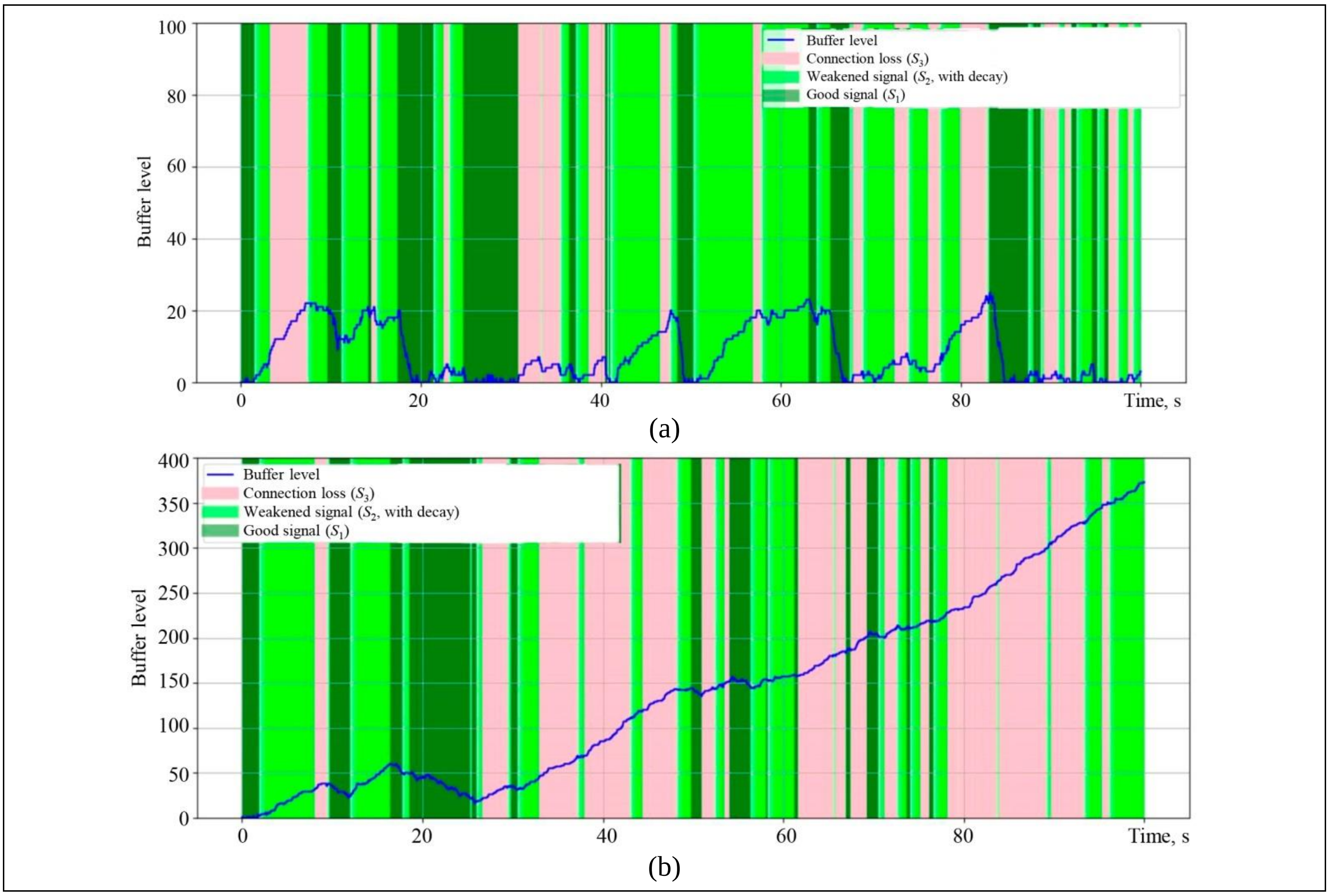

Fig. 4. Decaying throughput in the state S_2 : (a) stable regime $(\lambda = 3.0)$ and (b) unstable regime $(\lambda = 7.0)$.

The advantage of adaptive control is demonstrated in Fig. 5: in the neighborhood of the critical load, this control effectively stabilizes the queue length.

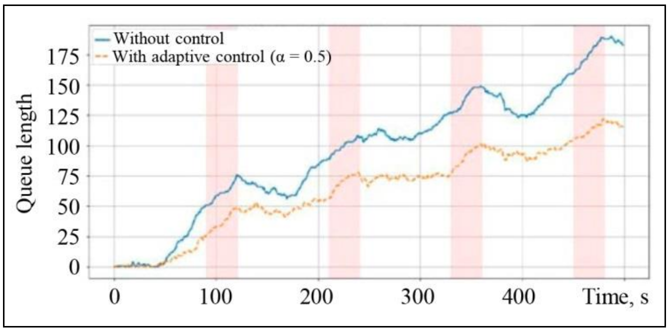

Fig. 5. Queue length dynamics: with adaptive control vs. without control.

Figure 6 shows the stability margin. The adaptive model demonstrates a shift in the overload threshold, allowing the system to operate closer to the critical values.

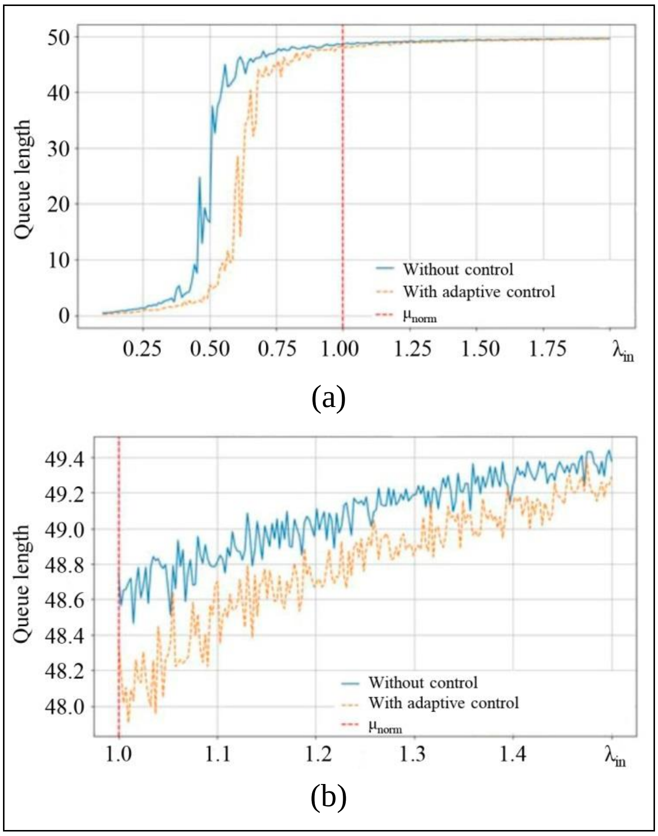

Fig. 6. Phase transition: average queue length as a function of λ_{in}.

The impact of semi-Markov effects is illustrated in Fig. 7: under the gamma distribution and distributions, the system becomes more inertial, which increases its sensitivity to overload.

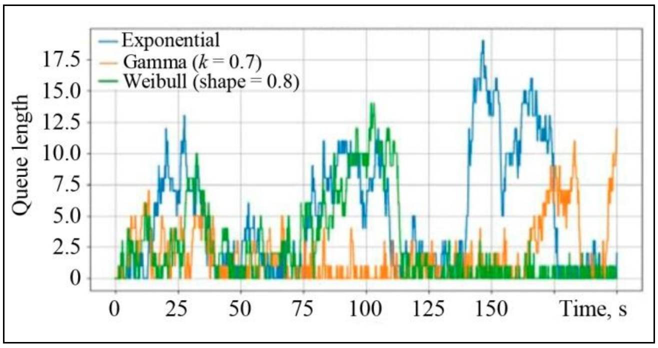

Fig. 7. Queue length dynamics under different distributions of service times.

CONCLUSIONS AND PROSPECTS

The model proposed in this paper describes an adaptive control system under stochastically varying characteristics of the communication channel, which is in one of three states: stable connection, degraded connection, and disconnection. As has been shown, introducing a control function $u(t)$ (an impact on both the service rate and the transition rates between states) allows one to adapt the system behavior dynamically to the current conditions. Unlike classical models with fixed parameters, this approach involves parameters as functions of the strategy to consider the impact of control actions on network performance.

According to the numerical analysis results, the system is stable for $\lambda_{in} < \bar{\mu}$: the queue remains bounded. In the case of overload ($\lambda_{in} > \bar{\mu}$), an avalanche-like growth of queue length occurs. However, adaptive control decelerates this growth and actually shifts the stability margin. The effects of temporal correlation have also been demonstrated: when passing to semi-Markov models, the system becomes inertial, but the general stability criteria are preserved. This feature emphasizes the universality of the model and its applicability in both classical and more complex telecommunication scenarios.

In practice, this model can be implemented in the form of a programmable unit embedded in the aggregation device. Such a module can utilize neural network schemes to estimate the current channel state and control transmission parameters. In addition, the model can be generalized towards Markov control (when solutions depend not only on the current state but also on the dwell time in it). This is naturally realized through systems with memory or logic with the history of states. It is also possible to realize intermediate communication states using multi-valued logic to describe not only abrupt but also smooth degradations of the channel.

Thus, the model combines stochastic dynamics, control, and queues into a single system and can be applied in theoretical stability analysis of such systems and in the development of new stacks of data transmission protocols, especially in the context of new-generation mobile networks. Prospects for further research include multiuser scenarios, the investigation of control strategies on data, and the construction of energy-efficient algorithms for devices with limited computational resources based on intelligent control. As an independent theoretical challenge, we note the study of phase transition; an open question is to eliminate the exponential nature of queue growth and decelerate adaptive control.

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