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DISTRIBUTED INTELLIGENCE OF MULTI-AGENT SYSTEMS.
PART II: Collective Intelligence of Social Systems

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Abstract. Part II of the multi-part survey is devoted to the features and empirical characteristics of distributed intelligence (DI) as the capability of a collective agent (social system) to perceive, process, and use new information in order to achieve its goals. The implementations of DI in human social systems are considered: the crowd wisdom of unstructured communities and the collective intelligence of small groups, organizational systems (OSs), and big systems (states, peoples, and civilizations in historical time). Unlike the swarm intelligence of social insects and animals, collective intelligence in human communities is built up of individuals capable of deep information processing and creative activity. The tight links between the DI of human organizational and social systems and individual human intelligence are emphasized. The increasing contribution of AI to modern collective intelligence is illustrated by flexible resource management in real time. The factors determining the effectiveness of the DI of a multi-agent system are identified as follows: (a) the cognitive capabilities of individuals, (b) the structure of interactions between them, (c) collective goal-setting, (d) external information recording, compression, and processing, and (e) creation of new "images" of the environment and oneself in it. A modular perception model of external influences by an intellectual agent is discussed.

Keywords: multi-agent social systems, collective intelligence, groups, organizational systems, big systems, modular model of perception.

INTRODUCTION

The analysis of human intellectual activity and its mathematical and technical modeling are the main directions of cognitive psychology, information technology, cybernetics, robotics, and other sciences [1]. These disciplines have established key features common to *human intelligence* (HI), the elements of animal intellectual activity, and the computer implementations of *artificial intelligence* (AI) and *distributed intelligence* (DI) of multi-agent systems. The key features are as follows: the autonomy of agents, including collective ones; information perception (the "reflection" of external influences), processing, and generalization (compression); learnability; the use of information to achieve the agent's objective goals in a variable environment; and several more specific features. DI research includes the study and modeling of cooperative effects in various multi-agent systems: biological, social, economic, and organizational, as well as in groups of autonomous technical devices.

Collective information processing and its use by social systems consisting of people include the dynamics of pedestrian and automobile traffic flows [2, 3], economic activity, stock market, the interaction of organizational systems (OSs), the combat operations of military units, participation of political parties in election campaigns, and many other processes [4–6]. This multi-part survey considers the main kinds of DI known to date in systems of interconnected agents, including biological, technical, social, and organizational ones. Part I, see [7], was devoted to the simple forms of DI without conscious information processing by individuals: the swarm intelligence of social insects [8], flocks of birds and fish schools [9], and groups of interconnected robots [4, 10]. Also, the features of information perception and processing in "proto-intelligent" automatic control schemes [11, 12], the most widespread implementations of AI were briefly described (artificial neural networks [13], logical intelligence [14], and some swarm intelligence simulation methods for *nature-inspired algorithms* [15]). According to part I of the survey, the DI of multi-agent systems is not reducible to standard schemes of individual or collective decision-making or hierarchical or network control: it represents an independent and insufficiently studied aspect of cooperative dynamics [16] that involves elements of the chaotic behavior of agents.

Part II of the survey considers some forms of the collective intelligence of multi-agent social systems (MASSs) consisting of people. Unlike the simpler kinds of DI, the elements of such systems are capable of deep information processing and creative activity, which is reflected by the formal description of their dynamics by game-theoretic methods. Another feature of these systems is the close interaction of individual and collective forms of intelligence, with the gradually growing role of AI in recent decades. Based on the material presented below, we introduce a general classification for all known forms of intelligence and outline a way to its meaningful mathematical description.

1. COLLECTIVE INTELLIGENCE IN HUMAN COMMUNITIES

1.1. The Wisdom of Crowds

The swarm intelligence of biological individuals can be associated with *crowd wisdom* in human society or the DI of an unstructured community of people. Sociologists and philosophers of the 19th–20th centuries (G. Le Bon [17], H. Ortega y Gasset [18], and others) emphasized the reduction of individual consciousness in the human mass as well as the primitiveness and manipulability of the crowd's collective behavior. In the 21st century, following F. Galton's sociological study[1] [19] (a century ahead of modernity), the interest of researchers was attracted by the ability of human MASSs to process external information and the mechanisms of cooperative actions based on it.

In contrast to the conscious elaboration of opinions in groups (see below), crowd wisdom manifests itself as a spontaneous process possibly not affecting the cognitive functions of individuals. For example, a large crowd bypasses an obstacle that is invisible to most of the people moving in it (Fig. 1). The motion mechanism in a crowd of pedestrians—following neighbors and avoiding collisions—is unified for communities of biological individuals and formations of drones [10]. The manifestations of DI in pedestrian traffic flows were considered in several works [2, 4, 16]; they were systematically analyzed by D. Helbing et al. [21]. Sociological studies and mathematical modeling are synthesized in the field of *crowd (mob) control* with numerous tasks to ensure the safety of mass events [22, 23] and suppress their manipulation [24].

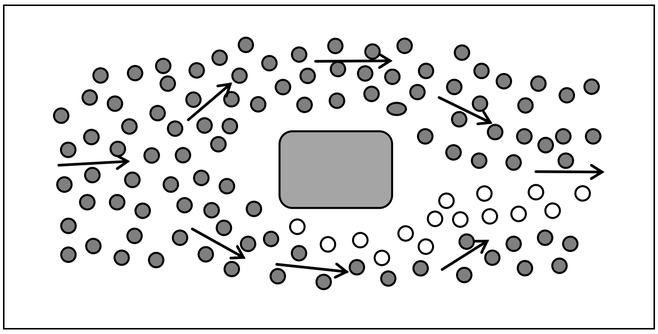

Fig. 1. An obstacle bypassed by a moving crowd.

The analogies with swarm intelligence in unstructured human communities extend to economic relations, including the Invisible Hand of the market and the stock market, where the desire of agents to maximize profits is transformed into a collective evaluation of assets. The highly politicized assertion about the market as the best mechanism for processing economic information is widely presented in textbooks on neoclassical economics [25]. Manifestations of the wisdom of the stock market were discussed in the book [6], although (in our point of view) without any convincing examples. The postulated efficiency of the DI of bidders is utilized by *political markets*, where predictions of political events can be purchased and sold up to their occurrence similar to futures contracts; by assumption, the price of a forecast reflects its quality [26]. Since reliable pricing models in the stock market are still unknown, "market forecasts," becoming widespread in recent years, can be used to manipulate public opinion.

Nowadays, information exchange on the Internet is often treated as an implementation of distributed intel-

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[1] The paper [19], quoted at length in the book [6], discussed the distribution of bull's slaughter weight estimates provided by 787 respondents at an agricultural exhibition in Plymouth. An incentive for accurate estimates (which required specialized knowledge) was given out of the money from selling respondent cards at 6 pennies apiece. The median of all estimates (1207 pounds) exceeded the actual slaughter weight of the animal (1198 pounds) by only 0.8%. The large-scale Internet surveys recently conducted by researchers from Stanford University (2000 participants, 1000 questions in 50 different knowledge domains, and about 500 000 answers) [20] confirmed the higher efficiency of "average" intelligence compared to the majority of individual respondents and the strong effect of collective opinion (when it was reported to participants at the intermediate stage of the experiment), which *worsened* the final result.

ligence [27]. The operation of search engines is a simple and convincing example of crowdsourcing that engages the DI of network users in the analysis of large datasets. In response to a user query, a search engine ranks the hyperlinks to websites in descending order of the number of previous references to these sites. This scheme, first implemented by the Google PageRank search engine in 1998, is currently used by all major search tools on the Internet. The direct network analog of marking ant paths with pheromone is very effective: the list of found sources, which may contain hundreds and thousands of entries, is usually headed by the most relevant and interesting ones. Processes in network structures are often discussed in terms of swarm intelligence [28].

1.2. Opinion Dynamics in Groups

At first glance, the problems of optimal operation of MASSs in human society, where communication and conscious actions play an important role, are far from the dynamics of a school of fish or the life of a bee hive. Nevertheless, they also manifest the general operating principles of DI: connections between system elements, information transfer, and an objective goal pursued by the system that does not necessarily coincide with the goals of its elements. (Some examples are staff reduction in an organization, military operations, etc.) The objects calculated in mathematical models of such systems—plans, opinions, and strategies—are the products of both individual and collective mental activity, e.g., street traffic planning considering the weather forecast. General methods for solving problems of this level of complexity have not been fully developed so far; only the most common partial schemes and heuristics can be discussed here.

The simplest "molecular" form of MASSs in human communities is a *small group* where all individuals (agents) are aware of all other members of the group but differ in the strength and direction of interactions (*influences*): a philatelist club, an enterprise's board of directors, an army platoon, etc. The mutual influence in a small group, which determines its dynamics, is reflected by the arcs of a weighted digraph, whose vertices correspond to the agents [29]. This approach formalizes the contributions of individuals to the collective dynamics of the group, even if their actions are variable and imprecisely known. A complete graph describes an unstructured group where each agent interacts equally with the others.

Opinion exchange and the development of a common position are the main content of the activity of expert councils; in one form or another, they are present in the work of most governing bodies and authorities. Modeling various aspects of this process includes the dynamics of reaching a unified opinion (consensus) and assessing the degree of influence of group members on the resulting decision. At the same time, the well-known ability of a group to propose a new non-standard solution to a problem or to find an adequate response to unforeseen changes in the situation is extremely difficult to formalize and is usually not reproduced in models.

DeGroot's model [30] is a mathematical foundation for reaching consensus. In this model, the mutual influence of agents is reflected by a stochastic *influence matrix* $W = ||w_{ij}||$ with nonnegative elements $\left(\sum_{j=1}^{n} w_{ij} = 1\right)$. The non-diagonal elements of the matrix W correspond to the mutual influence of different agents ($i \to j$ when $w_{ij} > 0$) whereas the diagonal elements to self-influence (the stability of the agent's position). The opinion vector of n agents, $\mathbf{x}(t) = (x_1^{(t)}, x_2^{(t)},..., x_n^{(t)})$, evolve over discrete time t in accordance with the iterative procedure

$$\mathbf{x}(t+1) = W\mathbf{x}((t)). \tag{1}$$

If the mutual influence graph of agents has a tree subgraph in which all vertices are reachable from a single root vertex (i.e., there is an agent directly or indirectly influencing the opinions of all other agents), then the opinion vector in (1) converges to a consensus $\mathbf{x}^*$ [31]:

$$W\mathbf{x}^*(\infty) = \mathbf{x}^*(\infty).$$

The consensus vector is calculated as the eigenvector of the matrix W corresponding to the eigenvalues $\lambda = 1$.

DeGroot's model can be applied to predict the contribution of real persons to the elaboration of decisions of a group (the top management of a competing firm, the board of the defense ministry of a probable enemy, etc.). For this purpose, it is necessary to determine the elements of the matrix W by expertise [29]. In *sparse* influence matrices, many elements (including those without available expert assessments) are assumed to be 0, which additionally complicates the calculations.

DeGroot's model has been developed in thousands of publications over the last decades; for example, see [31]. A simplified consensus search procedure is based on the heuristic *bounded confidence* algorithm, which reproduces the convergence of the parties' positions during the discussion [32]. In this algorithm, agents' opinions are described by a continuous parameter $x_i \in [0, 1]$. Consider a pair of agents in neighbor vertices of the graph reflecting the structure of interactions in the group; their positions (x_i, x_j) converge to one another if the difference between them does not exceed a given *confidence threshold*:

$$\Delta x_{ij} = \left| x_i - x_j \right| < \delta.$$

In this case, at each subsequent modeling step with $x_i > x_j$,

$$x_i(t+1) = x_i(t) - \mu\, \Delta\, x_{ij},$$

$$x_j(t+1) = x_j(t) + \mu\, \Delta\, x_{ij},$$

where $\mu \in [0, 1]$ is the convergence parameter. If $|x_i - x_j| > \delta$, the agents' opinions remain invariable. This can polarize the opinions, dividing the group into fractions (Fig. 2).

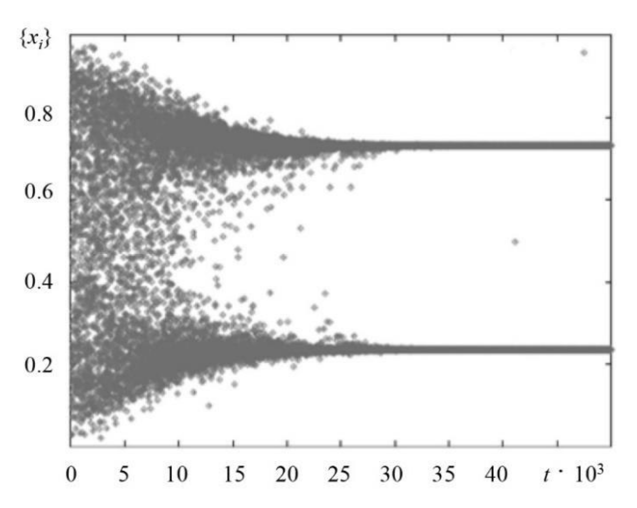

Fig. 2. An unstructured group of agents divided by the continuous opinion parameter $x \in [0, 1]$ into the fractions $x = 0.25$ and $x = 0.75$ under the agreement threshold $\delta = 0.2$ in the bounded confidence model [32].

The bounded confidence algorithm with coefficients reflecting different weights of opinions (e.g., those of a minister and his or her deputies) determines the agents' contributions to the common position, but it also critically depends on the adequacy of expert assessments and the model parameters. A variety of empirical mutual influence schemes have been proposed and used to date; in particular, they are used to analyze opinion formation and propagation in social networks (see the publications [33, 34], part III of the survey [22], and the overviewing chapter [31] in the book [12]). Practical methods for reaching consensus are discussed in the scientific and educational literature [35, 36], including the works explicitly aimed at manipulating public opinion on the issues of "anthropogenic" global warming [37].

Mutual influence schemes for reaching consensus in a group with an initially given set of opinions allow for no fundamentally new solution: they do not reproduce a key property of DI, more often called *collective intelligence* for groups and other relatively regulated human systems. At the same time, computerized decision support systems (DSSs) for collectively elaborating unknown solutions of a problem, including *brainstorming* [38, 39], employ practical group-based recipes for creating new information similar to inventive heuristics [40]: "loosening" the modifiable model, encouraging random associations, and actively expressing any ideas that become common knowledge (see the blackboard of the bee colony algorithm discussed in part I of the survey [7, 41]). We emphasize that successful nonstandard problem-solving by a group is not reduced to the insight of one discussion participant: the new idea must be approved (and usually corrected) by other participants under the guidance of moderators. Thus, brainstorming corresponds to collective creativity, a process that has not been mathematically formalized so far.

Stochastic disturbances of the "states of mind" of individuals participating in a brainstorming session (or, e.g., in a professional scientific seminar) inevitably cause erroneous assumptions and other "noisy" information. But exactly this process facilitates the emergence of insight among the participants. Its results are consolidated by the collective using a mechanism similar to the effect of temporary leaders in a moving flock of birds (see Fig. 6 in part I of the survey [7]): a promising assumption is discussed, criticized, and modified by other discussion participants. Another feature of brainstorming is the enlarged "library of knowledge" due to the different areas of expertise of the participants[2] (similar to the expanded field of view of a school of fish [7]).

The verification of emerging ideas in the general discussion and their consolidation in the individual consciousness of each participant correspond to the creation of new information by the emergent collective intelligence of the group. Another implementation of this approach is *e-expertise*, which also yields nonstandard solutions [42]. The obvious parallels of this process with the manifestations of swarm intelligence in simpler biological communities (see part I of the survey [7]) reflect a single mechanism for implementing group DI that exceeds the individual capabilities of group members.

Despite the fundamental difference between the cognitive abilities of humans and swarm animals, analogies of "creative" intellectual activity of groups with swarm intelligence have been noted many times in the literature [43]. Nevertheless, in its formal models, devising new things is usually postulated as an

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[2] We underline again that the terms "*knowledge*" and "*competence*" are used here in their "naïve" and intuitive clear everyday meaning.

empirically observed effect. The methodologies of creative complex activity were overviewed in the book [44] together with mathematical models for developing, mastering, and using new technologies.

1.3. DI of Organizational Systems

The transition from the swarm intelligence of unstructured communities to the collective intelligence of groups, where mutual influence is ordered[3], is accompanied by increasing the accuracy and depth of proposals. Even more ordered *organizational systems* (OSs) are comparable to individual human intelligence in the efficiency of routine information processing; they are often treated as a "bureaucratic machine" under some personal leadership. Similar to the explanation of new discussion results by the individual insight of one participant, this representation is not quite true. For example, the amount of information in an annual report of the enterprise staff goes far beyond the knowledge domain and interests of each coauthor, is addressed to several different groups of experts (technicians, financiers, lawyers, etc.), and is only corrected by the OS top management (hereinafter called the Principal). The information is then collectively processed by the superior organization and leads to organizational measures, e.g., changes in the annual funding. Thus, the DIs of various OSs establish a direct dialog, where the individual intelligence of their elements plays a subordinate role.

Unlike a group, where influence graphs can be arbitrary, organizational systems have a rigid, usually hierarchical architecture (Fig. 3) and are controlled by a single Principal (an individual or a group, e.g., a board of directors). Control actions (red solid arrows in Fig. 3) propagate in the system top-to-bottom to the lower levels of the hierarchy (finally, to direct executors); operational information is transmitted in the opposite direction (blue dashed arrows). By the presence of hierarchy, an OS resembles an inverted artificial neural network (ANN). This similarity is reinforced by the possibility of increasing the connectivity of each layer's node using the indirect influence of the system elements on any departments through the information transmitted to the Principal.

While ANN nodes only amplify or weaken signals of the previous layer, individuals in an OS are capable of deep information processing and creative intellectual activity. In addition to the system structure, their operation in the system is formalized by job descriptions, analogs of activation functions ("filters") in ANNs. Note that collective decisions (in the form of Principal's orders after receiving operational information) are usually made in a non-routine and variable environment. The activity of an OS, well known from everyday reality, certainly includes the reflection of the current situation (reports), information compression (integrated indices), goal-setting, learnability and adaptability, as well as the generation of new knowledge (schemes to fulfill plans, upgrading, response to emergencies, etc.), i.e., all the features of intelligence mentioned above. Information processing and its use in OS dynamics testify to DI in this type of multi-agent systems, but no generally accepted theory has been developed in the modern literature so far.

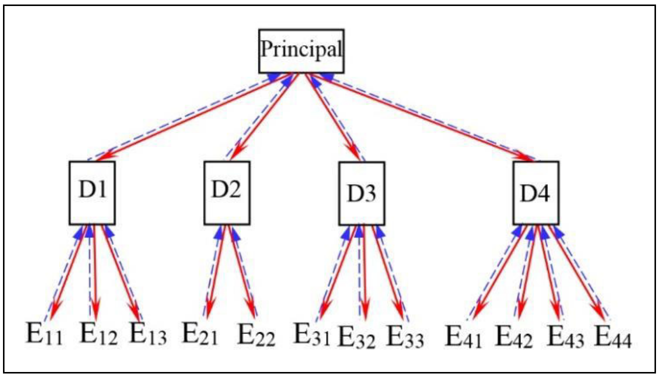

Fig. 3. A structural diagram of an organizational system: D—departments and E—executors.

Usually, the operation of OSs is formally described in terms of *game theory*. As a rule, such descriptions consider the interaction of absolutely rational [45] agents, in which the payoffs of each agent generally depend on the actions or strategies of all agents. (One exception is the models of bounded rationality [46].) The main challenge of game theory is to predict the result of agents' interaction as an equilibrium of their game: a vector of their actions (strategies) that is stable in some sense. Thus, agents in game theory are intelligent by definition (on the one hand) and are kept by game rules (the requirements of rational behavior) within much more rigid bounds than in real multi-agent systems (on the other hand).

In particular, a fruitful direction of this field is *the theory of active systems* (TAS), a branch of control theory for organizational systems whose dynamics are affected by the intelligent behavior (*activity*) of system participants [47]. Several control methods and mechanisms have been developed within TAS for systems containing active elements with their own goal functions. (Pursuing their own interests, such elements can degrade system operation, e.g., by distorting the information transmitted.) Non-manipulable (strategy-proof) mechanisms have been designed for control, resource allocation, motivation, and other tasks in

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[3] Computer support aids of a brainstorming session or, e.g., a scientific seminar in an online format structure the group of participants and organize the discussion process.

models of active systems. In particular, *the fair play principle* (the principle of incentive compatibility) allows maximizing the Principal's goal function on the set of plans corresponding to the maxima of the goal functions of active agents [47, 48]. Such branches of game theory as contract theory and *Mechanism Design* (MD) deal with similar problems; see a comparative review of the results of TAC and MD in [49].

Hierarchical [50] and reflexive games [51] are also used to describe OSs. In a normal form game Γ_0, all agents choose their strategies once, simultaneously, and independently of each other. In a *hierarchical game* Γ_i, contrasting with the game Γ_0, a Principal chooses a strategy x_1 first (the game Γ_1) or informs the second (subordinate) player(s) of the response to the strategy chosen by the latter (the game Γ_3), and so on in the ascending chain. That is, $x_j = f(x_i(x_j(x_i')...))$, where x_2 denotes the strategy (move) of the second player, and the moves of players x_j', x_j'', x_j''' ($j = 1, 2$) at different planning levels in the function f may differ.[4] Multilevel structure is also inherent in *reflexive games* [51, 52], where agents have partial awareness of the strategy sets of their opponents and the latter's awareness of the agent's own strategies. (In classical games, the strategies of all players are common knowledge.) In the graph representation of a reflexive game, the vertices $\{x_i\}$ expressing the agents' strategies are supplemented by the vertices $\{x_{ij}^{(n)}\}$ of phantom agents, where x_{ij} is the strategy of agent j in the belief of agent i and n gives the level (rank) of reflexion. Within this approach, the comparative intelligence levels of agents are expressed by the number of reflexion ranks available to them. Adding vertices to the MASS structure obviously complicates the game description and, consequently, equilibrium search based on it.

In OSs of a more complex structure, considering interactions between same-level agents leads to nested games with the competition of executors and independent Principals (in this case, described by games Γ_0, see Fig. 4). Such games correspond to active systems with coalitions of agents and MASS control by competing Principals, with numerous applications in information confrontation and crowd (mob) control [24, 33]. Game-theoretic methods in OS control were discussed in detail in the book [53].

Multicriteria *complex assessment* mechanisms have been designed for the objective monitoring of the results of active systems, which is required for planning and control actions [54]. This information concentration method in hierarchical OSs is similar in its function to the training procedures of convolutional ANNs. Analogies of information processing in neural networks and in hierarchical organizational systems were considered in [55].

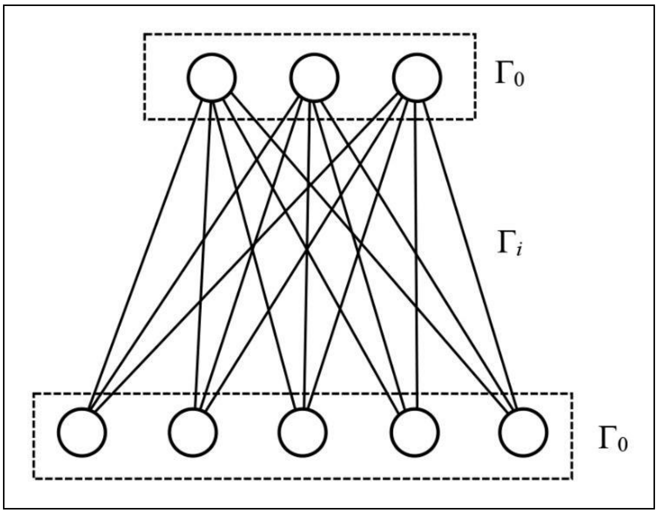

Fig. 4. The nested structure of the game $\Gamma_0(\Gamma_i(\Gamma_0))$ [52].

The formalism of game theory allows analyzing and predicting the actions of intelligent agents, as well as forecasting their outcomes (equilibria), but the set of possible actions must be fixed in the problem statement. Models based on the game-theoretic description of an OS, like formal opinion dynamics algorithms in a group, do not cover emergent events, i.e., fundamental changes in strategies (act of bankruptcy, development of an anti-crisis program, etc.). New knowledge generation by a system of interconnected individuals, empirically well-known, strikingly differs from consensus models as a choice of the best individual program of action or a weighted average of several predetermined programs.

In some sociological models, moods and morale in real collectives and groups are considered by introducing stylized "humanitarian" variables: the level of trust, mutual assistance, etc. [29]. Indirectly reflecting the presence of DI in OSs, this approach also does not reproduce its operation mechanisms since the results of collective information processing appear to be predetermined.

Analogies with this situation can be found in studies of the causes of altruistic behavior in MASSs of biological (i.e., inherently selfish) individuals [56, 57]. Game-theoretic models confirm the advantages of communities with altruistic agents whose equilibrium share depends on external conditions [58], but they do not reproduce the emergence mechanism of such

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[4] The addition of feedback loops complicating the function f is limited by *Germeier's theorem* [50, 52]: the Principal's maximal guaranteed payoff in games Γ_m with an even (odd) number m does not exceed that in the game Γ_2 (Γ_3, respectively).

agents: altruism has to be postulated [56]. Thus, game theory describes the manifestations of DI in OSs only phenomenologically. At the same time, the emergence of altruism in an "intelligent" MASS is explained at the qualitative level by the objective goal of self-preservation of the system and the variability of agents' behavior as a useful feature fixed by biological evolution or (and) learning.

1.4. Emergent Intelligence in Resource Management Systems

The intermediate position between standard OSs, usually involving from several tens to several thousands of individuals, and the DI of macroscopic systems (see the next subsection) is occupied by *emergent intelligence* (EI), a special form of AI in transportation, logistics, economic, and combined management systems of enterprises at the municipal or industrial level (see [59] and other books in this series, as well as [60]). An intelligence feature of such systems is the ability to allocate limited resources in a changing environment without direct operator intervention. The number of varying parameters in such problems makes rigid centralized control, if it is possible at all, critically dependent on computer power and complicates the correction of random disturbances in real time due to breakdowns, failures, weather conditions, etc.

An alternative to direct centralized control and optimization is mathematical models of flexible systems with information exchange among agents (the suppliers and consumers of resources or services). The *Belief–Desire–Intention* (BDI) model proposed in the 1990s, in which agents are initially intelligent and their interactions are defined by mathematical logic, has encountered difficulties in the formal definition of basic functions of intelligence and software implementation [61]. As it has turned out, a much more promising model is a system of "boundedly intelligent" agents with the algorithmically defined aspiration to maximize an objective function (*virtual money*): using the known values of payoffs and losses, these agents establish and switch connections between the sets of suppliers and consumers in a variable environment. In such models, a real-time *intelligent resource management* system is built based on the dynamic *network of needs and opportunities* [62].

The emergent intelligence of multi-agent resource exchange models exhibits all the main features of DI: variable dynamics of interconnected supplier and consumer agents, model information exchange between them in the form of redistributing supply and demand volumes, agents' striving for maximum individual payoffs, and evolution of the system (represented by a *dispatching* agent) to maximize the general goal function. The representative paper [63] briefly overviewed the state-of-the-art in this area and presented a smart ecosystem model minimizing the difference between demands and available resources under external disturbances.

The metaheuristic of "market" agents (see NIMs in subsection 3.3 of part I of the survey [7]), which seek to maximize the payoff by redistributing themselves among the most profitable "orders," has been effective in a wide range of applications [62]. The number of research works in the field of EI has been growing over the last decade [64], particularly reflecting the popularity of new terminology in the traditional problem of planning and operational resource reallocation (see [1]). The introduction of computer AI into the distributed intelligence of a social system is very evident here.

1.5. DI of "Macroscopic" Systems

Macroscopic systems in physics include sets of interacting particles whose number $N \sim 10^{20}$–10^{24} can be only a few orders of magnitude smaller than Avogadro's number $6.02 \cdot 10^{23}$. Brownian particles containing 10^{12}–10^{15} atomic and molecular subunits already belong to mesoscopic systems. In this sense, all social systems are either micro- or mesoscopic; this explains well the large-scale fluctuations of their parameters. However, in human society, "macroscopic," or *large*, systems are MASSs containing tens of thousands or more individual agents. Such systems—countries, peoples, economic sectors, etc.—usually have a complex and weakly ordered structure and an incompletely studied operation mechanism with a strong effect of random factors.

The phenomenological description of human society consisting of collectives as "generalized persons" and the projections of physical laws onto the social environment were systematically presented, apparently for the first time, by V.M. Bekhterev in his book [65], which was far ahead of its time. The dynamics of large social systems are still predominantly analyzed at the qualitative level in the humanities.

The hypothetical *Global Brain* of computer users united on the Internet [66] can be considered the ultimate representative of macroscopic DI. However, the declared analogies of this dynamic network with neural networks of the brain [67], stated only at the verbal level, look doubtful from the point of view of the connection of functions with the structure (for the Internet, it seems chaotic without a control center) and hardly indicate its cognitive capabilities (see Section 2). The coordination of elements and the birth of new knowledge in macroscopic subsystems of the modern world (transportation, scientific, and commercial ones) were described in Surowiecky's book [6], a popular introduction to the range of DI problems.

From a general point of view, the ability of large systems to process information and act accordingly is beyond doubt, but the DI of such systems has been described only at the qualitative level in the literature. At this level, it is possible to identify its key characteristics consistent with manifestations of other forms of collective intelligence:

- the disordered interactions of collective actors, with competitive (economy, scientific and technical environment, and domestic policy) and antagonistic relations (foreign policy, wars) prevailing;
- no features of personal governance (in particular, weakened ethical standards);
- systems tending to equilibrium;
- biosimilar life cycles.

An unexpected feature of the DI of large human systems is often a low level of information processing and solving specific current tasks compared to individual human norms. Like the reduction of ethics, this is due to the non-personal mechanism of cooperative thinking and a weakly ordered structure of inter-agent interactions. The decreasing role of ethical norms in large-scale collective intelligence, up to their temporary abolition (wars), is caused both by the size of large systems (the relative value of a single agent becomes insignificant against their background) and by the long (centuries-scale) time of their existence. The latter circumstance explains the low efficiency of large intelligence in solving immediate problems: the DI of large systems solves problems on a different time scale.

A large field of *biopolitics* [69] is based on analogies between the behavior of political actors and animals [68]; parallels between the collective actions of animals and humans are also considered in *human ethology* [70]. Another characteristic feature of this kind of DI is the possibility of false and often pathologically mass goal-setting due to the strong dependence of collective dynamics on the state of individuals (e.g., a drunken crowd). As a consequence, the matter concerns not only collective intelligence but also the collective psyche of large systems. The latter term is used in social psychology [71] and underlies *psychohistory* [72]. A combination of collective intelligence and collective psyche at the level of a state and its large social subsystems is usually called an *ideology*.

The confrontation of parties to conflict unites them into a single metasystem, which is also capable of processing and using information. The application of game theory in political science and military sciences is based on the tendency of such systems to equilibrium [73–75]. In particular, the existence of equilibrium configurations even under a strong antagonism of the parties is illustrated by the graph of military losses of the USSR in the Great Patriotic War (Fig. 5). In 1942, the spring offensive of the Red Army in the conditions of the thaw did not achieve its goals and was accompanied by heavy human and equipment losses (with proportional figures for the German army); in the unfavorable periods of spring 1943 and spring 1944, the parties did not conduct intensive combat operations (see the book [76]). Thus, all parties to conflict contribute to the DI of the system of opposing actors. In particular, this fact underlies the game-theoretic interpretation of wars within T. Schelling's *bargaining theory* [73].

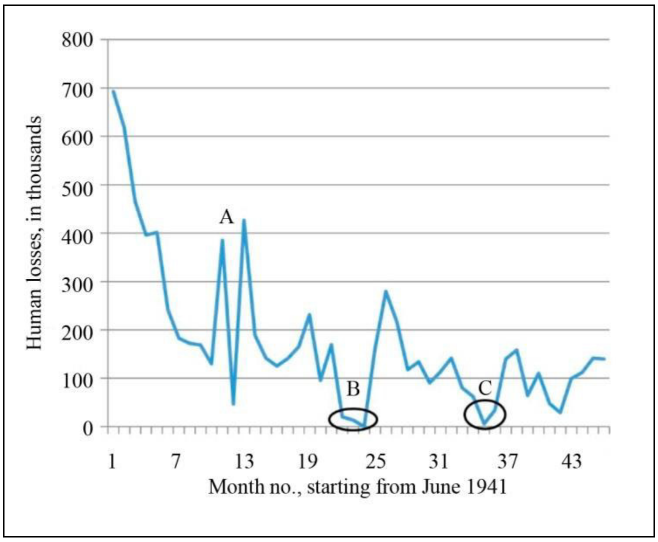

Fig. 5. Monthly military losses of the USSR from June 1941 to May 1945 according to [76]: A—May 1942, B and C—strategic pauses in April–June 1943 and May–June 1944, respectively.

Since the late 20th century, the evolution of mankind as a single system has been discussed in the humanities in the context of the *Universal History* of the Earth, including geophysical, environmental, technical, and social factors [77]. In terms of our survey, this process primarily reflects the dynamics of the DI of world-scale systems (states, peoples, and civilizations [78]) in the historical time scale with its inherent nonuniformity: upsurges, stagnation, and crises. At the universal level, the inseparability of individual human intelligence from the educational and cultural environment is clearly manifested: this fact, trivial for humanitarian knowledge bearers, is still not fully reflected in formal models.

Despite that the DI of the world economic-political system with states as agents is unfortunately not yet able to prevent crises and wars, its activity in peacetime is mainly creative. In this sense, outbreaks of wars and revolutions with the destruction of people, material values, and the least viable social institutions act as analogs of the chaotic movement of agents in the simplest MASS with swarm intelligence. The intermittent improvement of living conditions of the Earth's population, which increased in the 20th century almost fourfold despite wars and epidemics [79], is based on the achievements in technology, economics, medicine, social sphere, and other aspects of implementing DI. At the descriptive level, scientific, technical, and social progress in all known historical periods is obviously the result of developing the collective intelligence of mankind and its large subsystems. This emphasizes the importance of studying DI and ways for its adequate mathematical description.

The representation of society as an "atomized" system of interacting individuals, which emerged in the 19th century, took the emergent manifestations of DI beyond sociological theories, although religious, psychological, and "organicist" concepts of society[5] continued to exist until the first half of the 20th century [80]. In modern literature, the dominating description of an MASS as a set of individual agents with fixed strategy sets does not imply the generation of new features by the system of their dynamic relations. However, the mathematical modeling of historical processes sometimes uses "humanitarian" variables, e.g., the morale of the people (*asabiyyah*, or collective solidarity) in the balance of state resources [81]. This indirectly manifests the objective need to consider the influence of collective intelligence and collective psyche on historical processes.

2. THE FACTORS FORMING DI

All kinds of DI considered in this survey are summarized in Table 1; the efficiency of new information generation is evaluated in the last column on a purely qualitative level. (Question mark indicates the kinds of DI that are not generally accepted.) Even the simplest forms of swarm intelligence, which emerged in different biological species for collective survival, form a flexible response of MASSs to unprogrammed external influences. When treating the adaptive changes of organisms during evolution as the creation of new information, it is necessary to recognize the existence of very effective "evolutionary DI." Despite its use in computational *genetic* algorithms [15], this direction is still poorly developed. The formal description of the DI of the largest-scale social systems—states, nations, and civilizations—is insufficiently studied as well. Nevertheless, global technical and social progress certainly reflects the development of this form of collective intelligence. At the same time, reliable manifestations of the Global Brain, built from users of global computer networks, are not yet known. Given the chaotic structure of networks, the existence of such distributed intelligence is rather a hypothesis.

Let us summarize the brief overview of the known types of distributed intelligence. The modern applications of DI in the field of management, control, and planning are constantly improving, have no clear separation from AI (EI, NIMs), and far excel individual human intelligence by capabilities in some applications when combined with AI. Like the model of a fully rational and omniscient *homo economicus* in neoclassical economic disciplines and game theory, individual human intelligence (HI) is rather an abstraction reflecting the early sociological notions of an "atomized" society [80]. Both of these models implicitly incorporate the influence of collective intelligence (the entire society and the economic aspects of its activities, respectively). The inseparable connection between the HI of individuals and collective intelligence clearly manifests itself at all its levels: learning, personality formation, use of knowledge accumulated by society, creativity, and many others.

Different levels of any carriers of intelligence (people, living organisms, different types of society, technical devices, and systems) are summarized in Fig. 6. The nested features of intelligent agents reflect, among other things, the historical development of their research. At the successive levels of intelligence, its fundamental features (see Section 1 in part I of the survey [7]) can be filled with different content, e.g., from a goal set externally to a robot or ANN, through the disordered but purposeful actions of a swarm, to pursuit of a goal by animals or "intelligent" logistic schemes, and, further, to conscious autonomous goal-setting and its adaptive editing by a creative person.

The outer contour of the diagram in Fig. 6 corresponds to the capabilities of technical devices and automatic control systems, including those not considered intelligent outside of specialized disciplines. The remaining levels are covered by different types of intelligence, including HI or DI combined with AI. Nowadays, the field of artificial intelligence is dominated by third-level systems, which are massive and intensively developed. There are some successes at the fourth level, but the fifth one still remains a dream: the inner shaded area is still the sovereign territory of human intelligence.

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[5] Many of these concepts appeared within racial-anthropological theories and ideas of social Darwinism. Therefore, since the middle of the 20th century, they have been forgotten together with the corresponding approaches to analyzing the DI of large systems.

Table 1

The kinds and features of distributed intelligence

Kinds of DI	Intended purpose	Structure	Noise level	Volume of new knowledge
Swarm intelligence (social insects, fish, birds, etc.)	Collective survival	Weak	High	+
The “social” DI of social animals	Survival and collective operation	Hierarchy	Any	++
The crowd wisdom of human communities (mob, market, electorate)	Optimizing collective operation	Weak	High	+
The “evolutionary DI” of biological species	Adaptation and survival of the species	Weak	High	?
Computer simulation of swarm intelligence: agent-based models, NIMs	Reproducing the dynamics of “natural” DI and optimizing calculations	Fixed	Given	?
The AI of artificial social systems (formations of drones, etc.)	Optimizing collective functioning	Assigned	Any	?
The DI of small groups	Decision-making	Weak	Low	++
Hybrid human-computer systems, brainstorming software tools	Decision-making, searching for new information, and collective creativity	Fixed	Given	++++
The DI of organizational systems	Supporting all operation processes of the society	Hierarchy	Low	+++
The emergent intelligence of large-scale systems (transportation, logistics, urban economy, etc.)	Optimizing economic activity and solving given tasks	Complex hierarchy	Medium	++
DI in economics, policy, and military affairs	Optimizing collective operation and solving given tasks	Hierarchy; complex hierarchy	Any	+++
Large systems at the state level and above (history, culture, and civilizations)	Survival; technical and social progress	Complex, weakly ordered	Variable	++++
Hypothetical Global Brain	Optimizing the operation of mankind as a system	Rather unknown	Variable	?

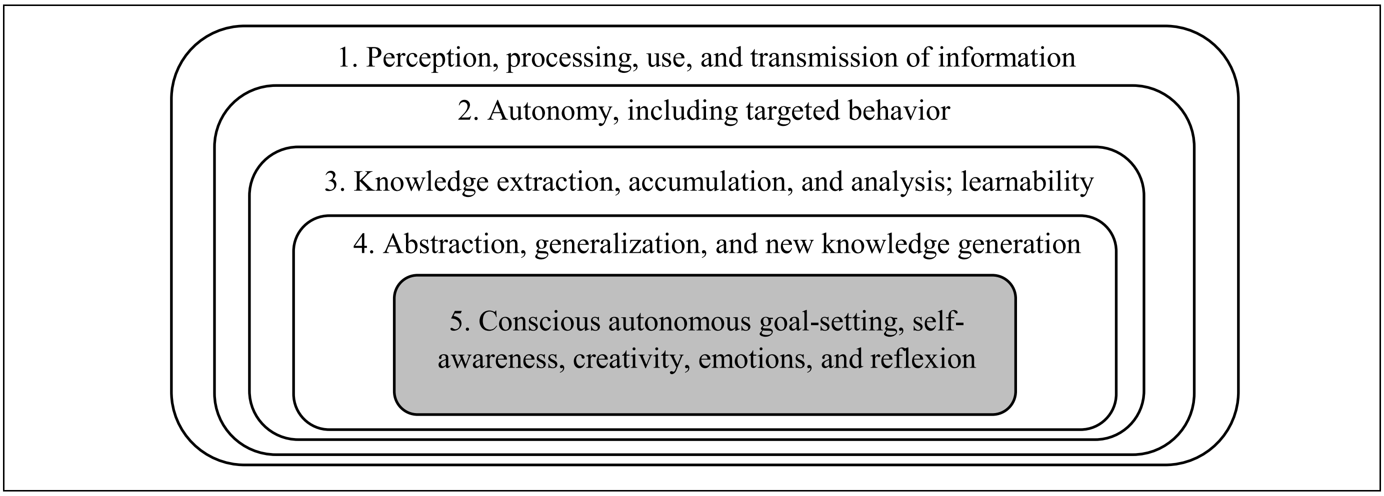

Fig. 6. The general “levels of intelligence” [82, 83].

In the field of DI, intelligent systems sometimes include even second-level systems demonstrating emergent (superadditive) properties as a result of interactions of multiple agents: the expanded field of view of a school of fish, ant paths, crowd wisdom as the averaging of estimates, etc. The third level here includes multi-agent systems for distributed problem-solving (EI, routine actions of OSs) that increase the efficiency of information processing through agent interaction and, moreover, can extract information and learn. Finally, collective human intelligence, as a highly developed form of DI with relatively regulated interactions and partial perturbations of the structure (many organizational systems, including e-expertise, brainstorming, etc.), directly uses human creativity (the fifth level).

A characteristic feature of distributed intelligence in human society is the purposeful strengthening of all features constituting the core of people's intellectual activity (the central block[6] in Fig. 6). At this level, individual and collective intelligences are inseparable (education, organization of creative activity, culture, and civilization) and should be considered in a single context. For these reasons, the classification and formalization of all known kinds of DI remains highly relevant. In the structured subsystems of society, distributed intelligence is implemented as an emergent quality of the "ensemble" of individual intelligences, also with the use of AI in recent decades. Hence, all types of intelligence should be jointly analyzed on the unified background.

Now we list the factors that most obviously correlate with the presence of DI in social systems (including artificial systems) and with its effectiveness. To a greater or lesser extent, these factors manifest themselves in the collective dynamics of all the MASSs considered above; the effectiveness of DI is determined by their combined action. (At the same time, the presence of a leader, planning, reaching consensus, and some other features of the dynamics of human OSs are not as generic and are reducible to particular combinations of key DI components.) In this sense, hopefully, the list below is inexhaustible and sufficiently comprehensive:

1. the multi-agent nature of the system and the presence of connections between autonomous agents;
2. the density and intensity of inter-agent connections;
3. the individualized cognitive capabilities of agents;
4. system structure and ordered agent dynamics;
5. fluctuations in the structure and dynamics of the system and the impact of the environment.
6. information reflection, compression, and recording.

These features are characteristic of any "intelligent" systems, including individual intelligence (as repeatedly noted in the literature on AI and DI in the 20th century, brain neurons act as interconnected elements), ANNs, and NIMs. A fundamental manifestation of multi-agent system's intelligence is "collective rationality," which does not necessarily involve individual understanding of an objective goal. The joint pursuit of a goal not realized by process participants is characteristic of all sufficiently large and complex systems whose objective needs are beyond the comprehension of individual agents, including mass political and social events.

The presence of individual intelligence (the third factor), despite its obviousness for human MASSs, is not a prerequisite: the swarm DI of insects is composed of individuals with low cognitive abilities. In different combinations with the structure and elements of stochastic dynamics, the individual intelligence of agents can be both reduced (a street crowd) and amplified within given tasks (groups of experts, actions of economic agents, etc.).

The fourth and fifth factors, i.e., the ordered structure and its violations, formally contradict each other, but they are implemented together in many "intelligent" systems considered here. (An example is brainstorming schemes and other forms of collective creative activity [38, 43].) Finally, the means of information compression (as a necessary condition for processing an infinite volume of data) and information recording (the sixth factor) are the dynamic "images" of external influence perceived by MASS elements. The imprints of external influences in different systems can be "blocks" of consciousness [84], areas of the cerebral cortex [85], ant paths (see Fig. 8a in part I of the survey [7]), ethical norms in society [56, 57], complex assessments [54], control mechanisms for OSs [47, 48], and many other entities. All these factors determine the mechanism of information processing and effective use in different-type MASSs, i.e., in the varieties of DI considered here.

The rationality of individual behavior of people increases with reducing choice alternatives (crossing the street on a green light and stock market trading with a fixed stock price) and becomes bounded when choosing among many alternatives (trying to cross the street on a red light and stock market trading with high price volatility). The rationality of the collective dynamics of a system also increases when the set of possible actions of agents is limited and when there is a "library" of standard responses to external influences

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[6] General concepts, such as *education*, *culture*, *science*, *civilization*, and many others, are actually metaphors for different manifestations of the collective intelligence of large systems.

(a *theater fire*[7]). It can be assumed that all diverse manifestations of intelligence, both individual and distributed, conceal a single mechanism of perception of external influences by an "intelligent" agent based on structuring their features.

The model of "intellectual" information perception and processing proposed in [86] associates with an external influence a finite combination of blocks (modules), each reflecting a certain characteristic (*feature*) of an object. The "imprint" of the influence in the agent's perception is represented by the weighted sum

$$\xi_i = \sum_{j=1}^{n} w_j m_j^{(t)},$$

where $\{m_j^{(i)}\}$ are the modules forming the image ξ_i and $\{w_j \in [0, 1]\}$ are their weights. The combination of several features defines the image of an object like a word in hieroglyphic writing. This scheme serves to represent an unlimited number of external influences in a cost-effective way by small ($n < 10$) combinations of modules under a reasonable library size $N \sim 1000$ (reflecting the technical constraints of human memory). In addition, this scheme reproduces the birth of new information as the construction of a new combination of available modules for an object not encountered previously.

The modular structure of the image of external influences naturally extends to the manifestations of distributed intelligence of multi-agent social systems. In the case of OSs, the function of blocks in the modular interpretation of external influences is performed by service instructions and norms. The role of a "block" image is played by the personnel's actions according to instructions; the correction of the image in changed conditions with the replacement of blocks corresponds to the search for the best combination of available actions; the memorization of new information corresponds to the modification of instructions. In "living" MASSs with more primitive agents, there are modules as well that guide collective dynamics and change in a variable environment: bees dancing in a hive, ant paths, reproducible motion modes of individuals in a flock, etc. (see part I of the survey [7]).

Table 2 illustrates the relationship between the "depth" of the system's collective intelligence and the degree of order of its structure and agents' actions. The strictly ordered operation of the "ideal commission" (the left column of this table) is a heuristic that increases its efficiency. At the same time, a completely informal system without any constraints (the right column) can hardly make any common decision regardless of the intellectual level of its participants. Disordered "human" MASSs or their parts with interactions described by a complete graph are unable to critically perceive external influences and usually serve as an object of manipulation (street crowd [24], information bubbles in social networks [33], the main part of the electorate during an election campaign [22], etc.). At the same time, efficient information processing in ANNs and OSs is directly determined by their rigid structure. Some collective decision processes constituting the core of DI in different-scale "human" systems were analyzed in [87]; the publication [88] presented their identification results for online social networks.

Table 2

Operation features of some model MASSs

"The Ideal Commission"	Political rally	"The Collective Imbecile"
General knowledge of the specialty	General intentions	Nothing in common
Targeted selection of participants by qualification criterion	Random selection of participants based on the proximity of sentiment	Free entry
Strong management (chair with a casting vote)	Weak management	No management
Formalized exchange of information and opinions, excluding emotions	Informal exchange of opinions and emotions	Random exchange of emotions
Quantitative comparison of the significance of opinions (voting)	Declarations of opinions (appeals)	No formulated opinions
Subordination of the minority to the majority	Insubordination of the minority to the majority	No majority
Obligation to execute decisions	No obligation to execute decisions	No decisions

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[7] In this classical sociological example, a theater fire threatens to cause panic and stampede despite the high cultural and educational level of the majority of the audience. However, following the orders of their commanders, a company of soldiers in the auditorium is very likely to evacuate everybody without casualties.

CONCLUSIONS

According to the material presented in this survey, the manifestations of DI at all its known levels reveal common features determined by collective processing of information not necessarily reflexed by the consciousness of agents. (In the systems of social insects, fish, and birds, as well as in nature-inspired computational metaheuristics, there is nothing about the consciousness of agents; however, the information content of mass collective processes in human society is usually not realized by their participants.) The capabilities of DI are determined by the intensity and structure of interactions between agents, their cognitive abilities, as well as by the balance between the degree of order of the system and random “noise,” which plays an important role in optimizing system dynamics. In several well-known examples (brainstorming, scientific discussion, meetings in a creative environment, etc.), inventing new things is stimulated by the purposeful amplification of “noise,” which blurs stereotypes and increases the probability of insight. Analogies in the manifestations of individual, collective, and artificial intelligences—with the necessary reservation about the absence of a generally accepted definition of this phenomenon—indicate the prospect of analyzing all its known forms from a unified standpoint.

The above analysis of different kinds of DI in biological, social, and artificial multi-agent systems shows the similar forms of its implementation and, moreover, reveals the incompleteness of existing formal models, complicating the unified interpretation of this phenomenon. Given the abundance of publications and directions in the field of cognitive sciences, the aspects of intelligence under consideration still contain no formal algorithms for inventing new things, i.e., creating previously non-existing knowledge beyond its logical inference. While classifying the stages of creative activity [83], modern models face the necessity to create mathematical images of objects that do not exist within the available set of knowledge before executing some creative act; modern mathematics, apparently, does not yet possess a developed apparatus for describing the nonexistent. At the same time, the modular model of perception circumvents this problem by presenting new images as a new combination of the features of external influences already known to the agent.

All aspects of intelligence activity are reproduced to varying degrees in the cooperative dynamics of diverse multi-agent systems, indicating the presence of DI in such systems. The numerous forms of DI in biological and human communities overviewed here demonstrate both the obvious parallels of DI dynamics with the manifestations of individual intelligence and the absence of emergent effects of the birth of new information in its existing theories. Meanwhile, such effects are reproduced by phenomenological multi-agent models, are used in modern natural computing algorithms, and are directly considered in DSSs and other computerized tools for supporting human intellectual activity. The unified formal modeling of both individual and distributed intelligence therefore seems to be a very topical problem.

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FORMING THE GENERATIONS OF NEW TECHNOLOGICAL PRODUCTS AS A SET COVERING PROBLEM

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Abstract. The development of any enterprise implies improving its control mechanisms for the manager to make decisions based on the achievements of science rather than intuitive ideas of his (or her) personal experience. It is necessary to improve the model-building process in order to eliminate the coinciding peaks of resource consumption when working on multiple projects. For this purpose, the concept of a generation of new technological products can be adopted: a new product is formed from separate prototypes (operating models), which can serve to determine some features of the project under development. Naturally, it is unreasonable to include the entire model range in the generation of new technological products: one should select the minimum number of prototypes required. This problem belongs to the class of set covering problems: complete covering (when the selected prototypes must possess the entire set of properties possessed by the model series under development) or partial covering (when the selected prototypes must possess only some of these properties). Exact algorithms and approximate heuristic algorithms are presented to solve both problems.

Keywords: placement problem, complete set covering problem, partial set covering problem, innovation lifecycle, generation of new technological products, prototype, properties matrix.

INTRODUCTION

The process of creating and improving new technological products (hereinafter called products for brevity) is a priority direction of Russia's development. Hence, the creation of generations of new products predetermines certain progress in the field of design and technological solutions, based on which the former are developed [1, 2]. Applying new solutions in the design and technological sphere implies obtaining new products with new functional properties. Thus, the new functional properties of a developed product are formed using new design and technological solutions [3, 4].

The creation of new products is a very cost-intensive process requiring significant material, human, and financial resources. Any fruitful idea underlying development works will be applied in the new generations of products forming a model range. The concept of a prototype is therefore widely used in engineering. A prototype is an operating model of the processes occurring in a new product being created.

Note that the problem of resource provision arises in the course of any project. Its potential solution is based on that the resources can be distributed over time during the implementation period of the project, i.e., they are not required all at once. As a rule, an enterprise is engaged in several projects simultaneously; hence, it is necessary to organize the project implementation process so that different projects have noncoinciding peaks of resource consumption [5–7]. A generation of new products should be formed by selecting a minimum number of prototypes to reduce the amount of resources required.

In this case, the problem of forming a generation of new products can be solved using optimization methods with some optimality criteria [7, 8]. In other words, the following question arises immediately: what should be optimized?

1. PROBLEM STATEMENT

As a rule, any fruitful idea has a continuation expressed in the creation of a whole series of new products with new functional properties. What can be used as a prototype to develop the next generation of innovation? Naturally, the selection is based on the principle of an identical scope of application: a tractor carries out one set of works whereas a tank another.

An excellent illustrative example is the development of aircraft technology. Consider TU-104, a Soviet jet passenger airplane (the third largest one in the world). In the first two years of its operation (1956–1958), TU-104 was the only jet passenger airplane in the world after the discontinuation of the British De Havilland Comet in the summer of 1956 and until the introduction of the American Boeing 707 into commercial operation in October 1958. Officially, the airplane had the following modifications: TU-104, TU-104A, and TU-104B. It was produced until 1960 and operated until 1981. In addition to TU-104, the TU-124 airplane was developed, including three modifications (TU-124A, TU-124B, and TU-124B). TU-124A, the most successful modification, was soon transformed into a new type of airplanes (TU-134), which also had 19 modifications. TU-134 was manufactured until 1989 and is still in service.

The same picture can be observed for Western products. For example, the British De Havilland Comet—the world's first jet passenger airliner—had Comet 1, Comet 1A, Comet 1XB, Comet 2X, and 12 other modifications. The airplane was manufactured until 1964 and was in service until 1997.

A passenger airliner must combine speed and capacity to transport as many passengers as possible to the desired destination as quickly as possible. This is the key factor when forming the generations of new products of this type.

The simplest solution to form the model range would be the selection of all the created products as prototypes. But this solution is somewhat redundant. As a rule, individual products have sufficiently close properties, and it seems unreasonable to choose all of them as prototypes. A reasonable approach is to take some *N* of them. Thus, the first optimization criterion is the minimum number of prototypes selected for developing an innovative product.

Assume that there are *N* products possessing *M* properties. To describe the particular properties of different products, we introduce the properties matrix *A* with the following elements: $a_{ij} = 1$ if product *i* possesses property *j* and $a_{ij} = 0$ otherwise.

Thus, the property matrix *A* consists of zeros and ones. Let the properties matrix be formed using a set of *N* products ordered by the time of creation; since the properties are linked to the product, the products developed in a later period may have new properties but may lose some old ones. As an example, we recall a classical example, vividly manifested during the Chernobyl disaster elimination: the use of semiconductor elements led to the emergence of completely new properties but the anti-radiation stability of the equipment was lost. Therefore, the matrix *A* will have a ribboned structure: the elements $a_{ij} = 1$ will be grouped mainly near the main diagonal, forming a kind of "ribbon."

Thus, economic considerations [9] bring to the problem of selecting the minimum number of prototypes to develop a product with a given set of properties.

We write this problem as an integer linear programming problem with the objective function

$$\sum_{i=1}^{N} x_i \to \min \tag{1}$$

and the constraints

$$\begin{gathered}
\sum_{i=1}^{N} a_{ij} x_i \ge y_j, \quad j = \overline{1, M}, \\
\sum_{j=1}^{M} y_j = m, \qquad (2) \\
x_i \in \{0, 1\}, \quad i = \overline{1, N}, \\
y_j \in \{0, 1\}, \quad j = \overline{1, M},
\end{gathered}$$

with the following notations: x_i is a binary variable equal to 1 if product *i* is selected as a prototype and to 0 otherwise; *m* specifies the number of properties to be satisfied for all the selected prototypes; y_i is a binary variable equal to 1 if product *i* must have this property and to 0 otherwise.

Problem (1), (2) is to determine the minimum number of prototypes with given properties, a basis for the further development of this innovation.

By its nature, the problem under consideration resembles the placement problem of infrastructure objects in a given domain: the role of such objects is played by the existing products, and the domains are the properties to be satisfied for the selected prototypes [9].

In this case, two formulations of the problem are possible:

1. The number of properties to be satisfied for the selected set of prototypes is equal to the total number

of properties characteristic of the entire set of products, i.e.,

$$m = M.$$

2. The number of properties to be satisfied for the selected prototypes is equal to or less than their total number, i.e.,

$$m \le M.$$

In graph theory, problems of the first type belong to the class of complete set covering ones; possible algorithms for solving them were described in [10]. Problems of the second type belong to the class of partial set covering ones. Both types of problems are NP-hard.

2. AN ALGORITHM FOR SOLVING THE COMPLETE SET COVERING PROBLEM

This problem is formally described by the objective function (1) and the constraints (2) without the second one. In other words, the objective function (1) remains unchanged, and the system of constraints is written as follows:

$$\begin{gathered}\sum_{i=1}^{N} a_{ij} x_i \ge 1, \quad j = \overline{1, M}, \\ x_i \in \{0,\, 1\}, \quad i = \overline{1,\, N}.\end{gathered} \tag{3}$$

Problem (1), (3) belongs to the class of integer linear programming problems, and the simplex method turns out to be inapplicable here.

The system of inequalities in (3) expresses the requirement that each property is satisfied for at least one product.

If only one product possesses some property, such a product will be called unique.

Proposition 1. *Unique products must be included in the set of selected prototypes.*

Proof. Only a unique product has a particular property; no other product possesses that property. Hence, it is necessary to include this product in the set of selected prototypes to cover all properties. ♦

Unique products are easy to determine: it suffices to calculate the sums of all columns of the properties matrix. If the sum of a column equals 1, then the product corresponding to this property is unique and must be included in the solution.

Multiplying all inequalities included in the constraints, i.e.,

$$\prod_{j=1}^{M} \sum_{i=1}^{N} a_{ij} x_i \ge 1, \tag{4}$$

yields a Boolean polynomial of degree M after removing the brackets. In this case, we can formulate the following result.

Proposition 2. *Each term in the Boolean polynomial of degree M of the expanded expression* (4) *represents an admissible solution of the problem satisfying the constraints* (3) *but is generally not the optimal solution.*

Proof. The desired solution must satisfy the system of unstrict inequalities (3). (Each property must be satisfied for at least one product.) If a product with this property is single, the corresponding constraint will hold as equality and the product will be unique. Therefore, the entire system of constraints—the unstrict inequalities (3)—can be replaced by a single constraint when multiplying the cofactors for the set of products with the property in question. There may exist several such products. If none of the products possesses a certain property, the corresponding cofactor will contain only zeros and will be equal to 0; therefore, the entire expression will be equal to 0, which violates the constraints. Indeed, the expression (4) is a product with M cofactors (the number of properties possessed by all products). In turn, each cofactor is a kind of list of products that have this property. If we expand the polynomial (4), each term will have degree M and describe an admissible solution. ♦

The trivial solution is to select the entire set of products as prototypes, i.e., all N products. However, the number of candidates for a new generation of innovations can be generally reduced. This is possible if some product partially satisfies the properties that are inherent in some other products. (In this case, they need not to be selected.) This will decrease the number of prototypes selected for a generation of new products. Note that at the stage of data preparation, the problem dimension can be reduced by using the concept of a unique product, which has properties absent in other products. As a rule, such products are those of the latest development. Naturally, such products must be included in the solution.

Thus, the problem is to select the minimum number of prototypes possessing all the properties corresponding to a given model range. There may exist several such sets. All of them are selected and presented to the decision maker, who determines an acceptable solution.

The minimum value of the objective function will be achieved when all constraints in the form of nonstrict inequalities hold as equalities. In other words, each property is implemented for one product only. But this situation is usually not the case in practice: each property is often characteristic of several products. As a result, the constraints (3) will be implemented in the form of strict inequalities [10–12]. Due to the binary character of the variables x_i, we can replace the system of inequalities with a recurrent system of Boolean equations. To be more precise, the following result is valid.

Proposition 3. *Problem* (1), (3) *is equivalent to the following sequence of Boolean equations*:

$$\prod_{j=1}^{M}\sum_{i=1}^{N} a_{ij}x_i = k,\ k = 1, 2, \ldots, l\ , \tag{5}$$

Here, theoretically, the upper bound for l will be the value $l = N^M$, i.e., the admissible solution in which all N products have all M properties. But this solution is trivial.

Proof. The inequality sign in the constraint (3) means that, in principle, several products may possess the same property and the expression (3) will accordingly hold as a strict inequality. Considering the integer nature of the problem, we can replace inequality (4) with the sequence of equalities (5). Expanding the expression (5) yields a Boolean polynomial of degree M. Each such term describes an admissible solution; therefore, to minimize the number of selected products, it is necessary to choose the polynomial term with the smallest number of cofactors but of the maximum degree. ♦

By sequentially solving equation (5) for different k, we arrive at the desired solution in a finite number of iterations.

To solve the problem, it is necessary to write the expression (5) as a polynomial of degree M. A common approach is to choose the most frequently encountered variable in (4): it will have the maximum possible degree after removing the brackets in the Boolean polynomial [10, 13, 14]. In this case, the following result is true.

Proposition 4. *In the Boolean polynomial* (5), *the terms with the minimum number of cofactors correspond to the optimal solution of problem* (1), (3). *In addition, such a term contains the smallest number of the variables* x_i, *but each of them has the maximum possible degree.*

Proof. Each product may have several properties. Hence, by selecting products with the maximum number of properties, one reduces the number of products to be selected. ♦

Based on the properties of Boolean polynomials described in Propositions 1–4, we can develop an exact algorithm for solving the problem. For this purpose, it is necessary to obtain the expanded expression for the polynomial (4). Moreover, the entire set of terms of the expression (4) is not required in explicit form: it suffices to obtain only a few first terms with the minimum number of cofactors of the maximum degree. The degree of each cofactor of the Boolean polynomial (4) must equal the number of properties to be satisfied for the selected prototypes. According to Proposition 4, each such term will be a solution of the problem under consideration.

Note that for a high-dimensional problem, extracting the first terms with the minimum number of cofactors of the maximum degree from the Boolean polynomial is a rather time-consuming and very difficult operation. It has not been computerized so far. Therefore, we propose a heuristic algorithm for solving problem (1), (3), which will be convenient for computer implementation. This algorithm is based on Propositions 1–4 and operates the properties matrix.

Preliminary step. Create the properties matrix of dimensions $N \times M$ and fill it with zeros and ones according to the following rule: if product i has property j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$. In this matrix, the number of rows will be equal to the number of products and the number of columns to the number of their properties: $N' = N$ and $M' = M$, where N' and M' are auxiliary variables.

Step k. Check whether the properties matrix contains any uncrossed-out rows. If such rows are absent, i.e., $N' = 0$, then the solution is found; terminate the computations. If $N' \neq 0$, then check the existence of a number $1 \le i \le N$ satisfying the relation

$$\sum_{j=1}^{M} a_{ij} = 1, \qquad i = \overline{1, N}.$$

Such products are unique and must be included in the selected set. Cross out row i of the properties matrix and set $N' = N' - 1$; also, cross out the corresponding columns for which $a_{ij} = 1\ \ \forall j\ = \overline{1, M'}$.

If there are no unique products, the original matrix will remain unchanged. Calculate the row and column sums for the matrix (either modified or original).

Find the row with the largest sum. If there are several such rows, take the row with the smallest number. The product with this number must be included in the set of prototypes, and the rows and columns associated with it must be crossed out. Repeat step k.

Thus, we obtain the solution of the problem in a finite number of iterations.

The number of possible iterations will be surely smaller than the number of products: $k^{\max} < N$. Indeed, each product has several properties, and crossing out the columns corresponding to these properties implicitly reduces the number of products. At the next step, some product with a certain number of properties coinciding with those of the already selected one will not be included in the set of prototypes: the sum of the row corresponding to this product will be non-maximum (or even equal to 0).

Example 1. Consider seven products with twenty properties, i.e., $N = 7$ and $M = 20$. The properties matrix A is given in Table 1.

Table 1

The properties matrix *A* after identifying a unique product

	1	2	3	4	5	6	7	8	9	10	11	12	13	~~14~~	~~15~~	~~16~~	~~17~~	~~18~~	~~19~~	~~20~~	Σ
I	1	1	1	1	0	0	0	0	0	0	0	0	0	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	4
II	1	1	1	1	1	1	0	0	0	0	0	0	0	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	6
III	0	0	1	1	1	1	1	0	0	0	0	0	0	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	5
IV	0	0	0	0	1	1	1	1	1	1	0	0	0	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	6
V	0	0	0	0	0	0	0	1	1	1	1	1	1	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	6
VI	0	0	0	0	0	0	0	0	0	0	1	1	1	~~1~~	~~1~~	~~1~~	~~0~~	~~0~~	~~0~~	~~0~~	6
VII	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	~~1~~	7
Σ	2	2	3	3	3	3	2	2	2	2	2	2	2	2	2	2	1	1	1	1	

It is necessary to select a certain number of products that have all the twenty properties.

First, we demonstrate the capabilities of an exact algorithm based on the properties of Boolean polynomials [9, 10].

With the reduced problem dimension, the objective function (1) and the system of constraints (3) can be written in the expanded form

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \to \min,$$

$$x_1 + x_2 \geq 1,\ x_1 + x_2 \geq 1,$$

$$x_1 + x_2 + x_3 \geq 1,\ x_1 + x_2 + x_3 \geq 1,$$

$$x_2 + x_3 + x_4 \geq 1,\ x_2 + x_3 + x_4 \geq 1,$$

$$x_3 + x_4 \geq 1,\ x_4 + x_5 \geq 1,$$

$$x_4 + x_5 \geq 1,\ x_4 + x_5 \geq 1,$$

$$x_5 + x_6 \geq 1,\ x_5 + x_6 \geq 1,\ x_5 + x_6 \geq 1,$$

$$x_6 + x_7 \geq 1,\ x_6 + x_7 \geq 1,\ x_6 + x_7 \geq 1,$$

$$x_7 \geq 1,\ x_7 \geq 1,\ x_7 \geq 1,\ x_7 \geq 1.$$

In this case, the expression (4) takes the form

$$\left(x_1 + x_2\right)^2 \times \left(x_1 + x_2 + x_3\right)^2 \times \left(x_2 + x_3 + x_4\right)^2$$
$$\times \left(x_3 + x_4\right) \times \left(x_4 + x_5\right)^3 \times \left(x_5 + x_6\right)^3 \times \left(x_6 + x_7\right)^3 \times x_7^4 \geq 1.$$

The degree of the polynomial is 20, which matches the initial data. For further solution, we choose the Boolean polynomial terms of the maximum possible degree. Note that it is necessary to take only one term of the maximum degree from each cofactor of the Boolean polynomial. In this example, we have x_7^7, x_2^6, x_5^6, and x_3; another alternative is to choose x_7^7, x_2^6, x_5^6, and x_4. In other words, the admissible solutions are as follows:

- the first one: products II, III, V, and VII are selected as prototypes;
- the second one: products II, IV, V, and VII are selected as prototypes.

Even this elementary example elucidates the challenges in obtaining a solution from a Boolean polynomial: the procedure of extracting the terms of maximum degree will be difficult to formalize and quite complicated for programming. Therefore, we apply the heuristic algorithm described above.

Preliminary step. Let us build the properties matrix *A* given in Table 1.

Step 1. We check the existence of uncrossed-out rows in the properties matrix. There are such rows. Hence, we identify unique products. This is product VII, which has properties 17–20; it also closes properties 14–16. The corresponding columns and rows in Table 1 have been crossed out to reduce the problem dimension. We calculate the row and column sums; they are presented in Table 2.

Next, we find the row with the largest sum. In this case, there are several such rows: nos. 2, 4, 5, and 6. Selecting the second row, we delete this row and the associated columns from the properties matrix. The result is shown in Table 2.

Step 2. We check the existence of uncrossed-out rows in the properties matrix. There are such rows. Hence, we try to identify unique products. Such products are absent: all column sums differ from 1. Therefore, we find the rows with the largest sum. This is the fifth row with a sum of 6. The corresponding columns and rows in Table 3 have been crossed out to reduce the problem dimension. We calculate the row and column sums; they are presented in Table 3.

Step 3. We check the existence of uncrossed-out rows in the properties matrix. There are such rows. Hence, we try to identify unique products. Such products are absent: all column sums differ from 1. Therefore, we find the rows with the largest sum. There are two such rows: no. 3 and no. 4 with the same sum equal to 1. We include product III in the sample. The results are presented in Table 4.

Table 2

The properties matrix *A* after Step 1

	1	2	3	4	5	6	7	8	9	10	11	12	13	Σ
I	1	1	1	1	0	0	0	0	0	0	0	0	0	0
II	1	1	1	1	1	1	0	0	0	0	0	0	0	–
III	0	0	1	1	1	1	1	0	0	0	0	0	0	1
IV	0	0	0	0	1	1	1	1	1	1	0	0	0	4
V	0	0	0	0	0	0	0	1	1	1	1	1	1	6
VI	0	0	0	0	0	0	0	0	0	0	1	1	1	3
Σ	–	–	–	–	–	–	2	2	2	2	2	2	2	

Table 3

The properties matrix *A* after Step 2

	7	8	9	10	11	12	13	Σ
I	0	0	0	0	0	0	0	0
III	1	0	0	0	0	0	0	1
IV	1	1	1	1	0	0	0	1
V	0	1	1	1	1	1	1	0
VI	0	0	0	0	1	1	1	0
Σ	2	–	–	–	–	–	–	

Table 4

The properties matrix *A* after Step 3

	7	Σ
I	0	0
III	1	1
IV	1	1
VI	0	0
Σ	2	

Step 4. We check the existence of uncrossed-out rows in the properties matrix. There are no such rows, i.e., the solution has been obtained. It is necessary to choose products II, III, V, and VII for the new generation. According to the previous step, another solution is to include product IV instead of product III, i.e., the representatives of the new generation must be products II, IV, V, and VII.

Clearly, both solutions represent all the twenty properties of the original set of products in the model range. ♦

Thus, we have found the solution of the complete set covering problem. The solution of its incomplete counterpart causes certain difficulties: it is unclear what properties will be selected in the end (they are not specified in the original problem statement). In other words, the selected prototypes must possess not the entire set of properties of the product series but only some part of them, and this part has yet to be determined during the solution procedure. This circumstance prevents the application of the sequential approximation method: first, the complete set covering problem is considered; then the cardinality of the set is reduced by one, and the corresponding set covering problem is solved, etc., until reaching the required size of covering. In this case, the question is what properties should be discarded. Of course, the properties inherent in the earliest samples of products can be discarded, following a natural assumption that they have been implemented, in one form or another, in new samples or even have been replaced by advanced similar functions. But this problem turns on the assessment of product properties and their ranking by significance.

3. AN ALGORITHM FOR SOLVING THE PARTIAL SET COVERING PROBLEM

Consider the partial set covering problem for a bipartite graph. Let us define a bipartite graph $G(X, Y, W)$, where X is the set of vertices of the first layer (products), Y is the set of vertices of the second layer (properties), and W is the set of arcs. An arc $(i, j) \in W$ if product i has property j. We denote by $A \subseteq X$ a subset of X by $B(A) \subseteq Y$ a subset of Y containing all vertices adjacent to A. In other words, A covers $B(A)$, or the set of products A possesses in aggregate the properties $B(A)$.

We formulate the partial set covering problem as follows: it is required to find a set A of minimum cardinality such that $\left|B(A)\right| \ge m$.

Note that if $m = M$, we obtain the well-known bipartite graph covering problem.

A heuristic (greedy) algorithm yielding an upper bound includes the following steps.

Step 1. Define $i_1 \in X$ of the maximum degree. Remove it and the set $Y_1 \in Y$ of all vertices adjacent to i_1. If $\left|Y_1\right| \ge m$, the problem is solved. Otherwise, proceed to the next step.

Step k. Determine i_k of the maximum degree. Remove it and the set $Y_k \in Y$ of all vertices adjacent to i_k. If $\left|\bigcup_{S=1}^{k} Y_s\right| \ge m,$ the problem is solved. Otherwise, proceed to the next step.

Obviously, the problem will be solved in a finite number of steps not exceeding m. The resulting solution gives an upper bound H_{upp}.

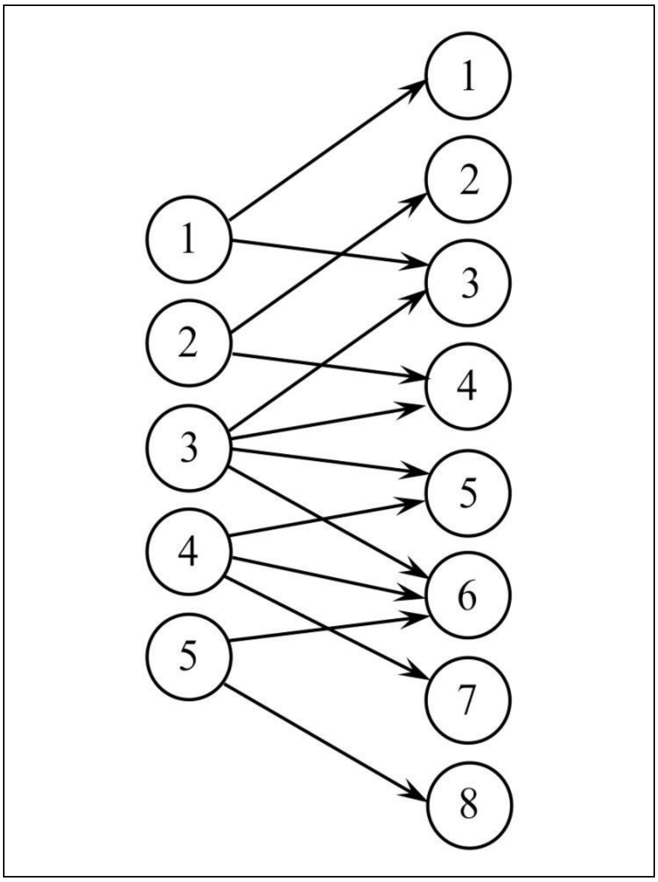

Fig. 1. The graph in Example 2.

Example 2. Consider the graph in Fig. 1. Let $m = 8$ (complete covering).

Step 1. Vertex $3 \in X$ has the maximum degree. We remove it and vertices 3–6 adjacent to it.

Step 2. The residual graph is shown in Fig. 2. Here, the solution is obvious. Taking all vertices 1–5, we obtain $A = (1, 2, 3, 4, 5)$ and $\left\|A\right\| = 5$. ♦

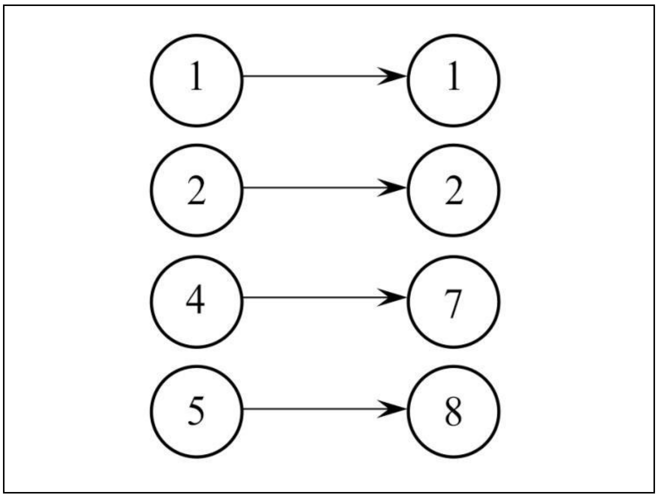

Fig. 2. The graph obtained at Step 2 of the greedy algorithm.

4. THE NETWORK PROGRAMMING METHOD: A LOWER BOUND

To obtain a lower bound, we apply the network programming method [15].

Let us formulate the generalized dual problem. Figure 3 shows the network representation of the original (primal) problem.

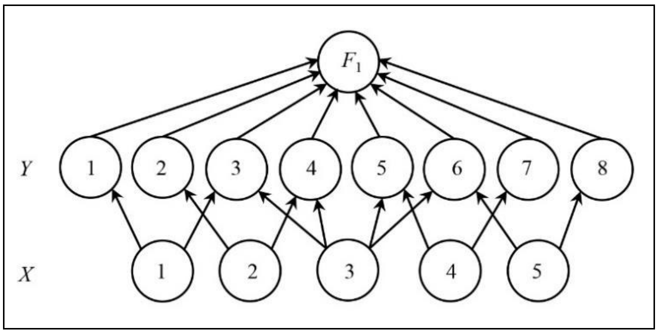

Fig. 3. The network representation of the original problem.

Following the network programming method, the weight of each arc (i, j), where $i \in X$ and $j \in Y$, is given by an arbitrary nonnegative number l_{ij} such that

$$\sum_{j \in P_i} l_{ij} = 1, \quad i = \overline{1, N}.$$

Here, P_i denotes the set of outgoing arcs of vertex $i \in X$.

As a result, we obtain M estimation problems of the form

$$L_i(x) = \min \sum_{i \in Q_i} x_i l_{ij}$$

subject to the constraints $x_i = (0, 1)$ and

$$\sum_{i \in Q_j} x_i \geq 1,$$

where Q_j is the set of incoming arcs from vertex $j \in Y$.

The solutions of these problems are

$$y_j = \min_{i \in Q_j} l_{ij}, \quad j = \overline{1, M}\ .$$

We rearrange the numbers y_j in ascending order:

$$y_{j1} \leq y_{j2} \leq \ldots y_{jm}.$$

Theorem 1. *The value*

$$H(y) = \sum_{k=1}^{m} y_{jk} \tag{6}$$

gives a lower bound for the original problem.

This theorem is a special case of the general theorem of network programming theory [15].

The generalized dual problem (GDP) is as follows: find the numbers (l_{ij}) maximizing the bound (6).

Example 3. Consider the graph in Fig. 1. The numbers l_{ij} are given in Table 5.

We have

$$y_1 = \frac{3}{4},\ y_2 = \frac{3}{4},\ y_3 = \frac{1}{4},\ y_4 = \frac{1}{4},\ y_5 = \frac{1}{4},$$

$$y_6 = \frac{1}{4},\ y_7 = \frac{3}{4},\ y_8 = \frac{3}{4}, \text{ and } H_{\text{low}} = 4.$$

Note that taking $m = 7$ yields $H(y) = 3\frac{1}{4}$. Anyway, this result is rounded to a lower bound of 4 as well due to the integer values of $F_1(x)$.

This estimate can be used in the branch-and-bound method. ♦

Let us consider an exhaustive search algorithm for solving the problem. It can be applied for small N. Defining a segment $[H_{\text{low}},\ H_{\text{upp}} - 1]$ of length $q = H_{\text{upp}} - H_{\text{low}} - 1$, we divide it into two parts: $r = \frac{1}{2}q$ (if q is even) or $r = \frac{1}{2}q$ and $r = \frac{1}{2}q + 1$ (if q is odd). We check all combinations of r elements in a total of N elements. Two cases are possible as follows:

- There exists a combination A such that $B(A) \geq m$. In this case, we divide the segment $[r - 1, H_{\text{low}}]$ into two parts and repeat the procedure.
- No combinations A with $B(A) \geq m$ are available. In this case, we divide the segment $[r, H_{\text{low}} - 1]$ into two parts and repeat the procedure. The optimal solution will be obtained in a finite number of steps. For the graph in Example 1, the resulting bounds are $H_{\text{upp}} = 5$ and $H_{\text{low}} = 4$. Therefore, it is enough to check the combinations of 5 and 4 elements; their number is 5. We find the optimal solution $A = (1, 2, 7, 8)$.

CONCLUSIONS

This paper has considered possible ways to select prototypes for forming a generation of new technological products. The algorithms proposed are based on the set covering problem. They provide solutions in the case of complete and partial set coverage.

The disadvantage of this problem statement is that all the properties possessed by the products are equally important, which, of course, is not true in practice. It would be correct to consider the significance of each property, but it will strongly depend on the target audience: one set of properties is important for product developers and another for ultimate consumers.

Table 5

Initial data for the generalized dual problem

i	1		2		3				4		5	
(i, j)	(1, 1)	(1, 3)	(2, 2)	(2, 4)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(4, 5)	(4, 7)	(5, 6)	(5, 8)
l_{ij}	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$

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A FUZZY COLD-START RECOMMENDER SYSTEM FOR EDUCATIONAL TRAJECTORY CHOICE

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Abstract. Several approaches to choosing an educational trajectory are considered, and the advantages of using recommender systems are determined. The cold start problem of recommender systems is formulated and solved by creating a hybrid recommender system that combines a rule-based fuzzy expert system and a recommender system with fuzzy collaborative filtering. As one application, the general approach is implemented for choosing the field of study when entering a higher education institution. A modification of Klimov's career guidance test is used as initial data. The rules for estimating the metrics and similarity of fuzzy triangular data are presented. The algorithms of a fuzzy expert system and a fuzzy recommender system with collaborative filtering are described in terms of the fuzzy representation accepted. The two approaches are combined by generating pseudo data using an expert system. This provides a solution of the cold start problem and yields a recommender system whose quality is gradually improved by substituting the values from real user queries into the database. The programs implementing these algorithms are tested to confirm the effectiveness of the fuzzy recommender system.

Keywords: expert system, recommender system, fuzzy description, fuzzy metric, collaborative filtering, cold start, educational trajectory.

INTRODUCTION

Recommender systems are intended to advise in decision-making when there are many alternatives to choose from, by analyzing the behavior of others who have made choices under similar conditions [1]. A recommender system selects and suggests decisions to the user based on the available knowledge about him, the decision space, and the interaction between the user and the decision space. Thus, the choice of a decision is determined by the properties of the user and the properties of the alternatives proposed. The algorithms of such systems differ in the type of data to generate recommendations. The methods implemented based on the data about the previous application of the system are called collaborative filtering. In collaborative filtering, a close group of users with the same private preferences as those of a particular user is identified. Then, the user's preferences are extended to the complete set of preferences of the entire close group. Content-based filtering rests on metadata, i.e., the descriptions of objects from the user's catalog. Depending on the algorithm, the system can generate recommendations by selecting objects similar to those previously chosen by the user, by matching objects with the data from the user's profile, or by searching for similar objects from the entire content. In fact, recommender systems are developed in order to create decision tools with the maximum possible consideration of the user's data-based preferences [2]. Due to the role of psychological factors in the choice process, the problem of designing perfect recommender systems is of great interest and leads to significant challenges.

The formal problem statement is as follows. Consider a set of users and a set of objects. For the set of users, the preference matrix for choosing objects is known. The problem of a recommender system is to fill the missing values in the preference vector of the user under study by analyzing the available data of the object rating matrix for the entire set of users. Thus, technically, tools are needed to fill the existing gaps in a matrix. Different algorithms serve for this purpose [3–5], including those exploring additional object and user data. In many practically important cases, the data underlying recommendation search are imprecise and fuzzy. It is natural to describe such data within probabilistic or fuzzy models. The advantages of probabilistic models [6] show up when working with big data. Under small initial samples, for solving the cold start problem, one has to be content with a priori expert assessments of probabilities. In this case, the approach based on fuzzy calculations turns out to be more suitable [7].

Fuzzy versions of clustering and factorization methods of the rating matrix [8–25] have been developed for fuzzy recommender systems. Fuzzy expert recommender systems are quite convenient for development and application [26–28], but they do not accumulate data during operation. According to the survey of available solutions, the choice of an appropriate algorithm for a recommender system essentially depends on a particular problem to be solved.

1. PROBLEM STATEMENT

Due to the diversity and complexity of large systems, various tools are needed to control them [29], including machine learning methods. In the recent decade, the problem of a more flexible response of the education system to global dynamic changes has arisen, followed by a steady trend to personalize the educational trajectory of students in higher education institutions (HEIs). The complexity and scale of this problem require developing intelligent control systems [30–33] with recommender systems that facilitate the rational choice of the fields of study by enrollees and their filling with courses (the profiles of educational programs) to link current learning with future professional careers. The real demand behind this task is that the labor market has many free vacancies due to the mismatch between the application forms submitted and the requirements of employers [34]. Therefore, deliberate efforts are made to develop user systems in order to determine the best educational trajectory for subsequent successful careers [35, 36]. Many factors and attributes can help in determining a career. They include skills, relationships, decision-making, home address, education, parents’ jobs, etc.

The problem of developing a recommender system for education has a characteristic feature as follows: there are fuzzy values among the initial parameters reflecting such factors as preferences, expectations, and other qualitative assessments [37]. There exist two different approaches to designing recommender systems with fuzzy logic. The first one is to develop rule-based expert systems [38, 39], whereas the second approach rests on big data [40–43]. Both approaches have their advantages and drawbacks. The rule-based method involves explicitly formulated rules with the experience of experts [44]. In this case, the system’s operation does not affect its quality. Methods for designing recommender systems based on machine learning assume that the decision rules are unknown in advance but require big data. In other words, a recommender system based on machine learning is unable to formulate relevant inference rules under small initial data in the database. This limitation is known as the cold start problem [45]. At the same time, as the data volume increases, the quality of such a recommender system will grow reflecting real correlations and relations. This paper aims to overcome the limitations of both approaches by combining them. At the initial stage, a recommender system will therefore be rule-based, generating an appropriate database. As the data volume increases, the simulated examples will be gradually replaced by real data, and the recommender system will be redesigned in accordance with machine learning results, thereby overcoming the cold start difficulties.

2. METHODS AND DATA

In the Russian multilevel education system, the problem of choosing an educational trajectory considering the future career arises several times: at the stage of determining the field of study and the profile of the educational program when entering an HEI, when choosing a master’s degree in the case of continuing higher education and, finally, when entering a graduate school. An individual educational trajectory can also be detailed using a set of elective courses. This paper proposes a recommender system [46], which can be applied at different education levels and adapt to the changing structure and priorities of Russian education. To provide a cold start for the system, a hybrid fuzzy recommender system is used, combining a rule-based expert system and a recommender system based on the collaborative filtering of user characteristics [9].

The approach is validated using a modification of Klimov’s career guidance test [44], where crisp estimates are replaced by fuzzy ones. This test is based on a classification of professional interests to determine a better field of study for an individual. According to this classification, all modern professions can be divided into five main types: “man–nature” (N, work with natural objects), “man–technology” (T, work with technical devices), “man–sign system” (S, work with abstract symbolic systems, models, and natural and artificial languages), “man–art” (A, creative work), and “man–man” (M, work involving direct communication and interaction with people). The table below shows the test [44] used by the recommender system.

Klimov’s career guidance test

No.	User characteristics	Type of professions				
		N	T	S	A	M
1	I get acquainted with people easily.	–	–	–	–	1
2	I can make something willingly and for a long time.	–	1	–	–	–
3	I like to go to museums, theaters, and exhibitions.	–	–	–	1	–
4	I take care of plants and animals willingly and constantly.	1	–	–	–	–
5	I can calculate and draw something willingly and for a long time.	–	–	1	–	–
6	I am happy to communicate with peers or kids.	–	–	–	–	1
7	I enjoy caring for plants and animals.	1	–	–	–	–
8	I usually make a few mistakes in writing.	–	–	1	–	–
9	My products usually arouse the interest of my friends and seniors.	–	2	–	–	–
10	People think I have artistic abilities.	–	–	–	2	–
11	I like to read about plants and animals.	1	–	–	–	–
12	I take part in plays and concerts.	–	–	–	1	–
13	I like to read about the design of mechanisms, devices, and machines.	–	1	–	–	–
14	I can spend much time solving puzzles, problems, and rebuses.	–	–	2	–	–
15	I settle disagreements between people easily.	–	–	–	–	2
16	People think I have a knack for technology.	–	2	–	–	–
17	People like my artwork.	–	–	–	2	–
18	I have an aptitude for working with plants and animals.	2	–	–	–	–
19	I can state my thoughts in writing clearly.	–	–	2	–	–
20	I rarely quarrel with people.	–	–	–	–	1
21	Even the unknowns approve the results of my technical creativity.	–	1	–	–	–
22	I learn foreign languages without much difficulty.	–	–	1	–	–
23	I often help even the unknowns.	–	–	–	–	2
24	I can spend a long time practicing music, drawing, reading books, etc.	–	–	–	1	–
25	I can influence the development of plants and animals.	2	–	–	–	–
26	I like to comprehend the design of mechanisms and devices.	–	1	–	–	–
27	I usually manage to convince people of the truth of my words.	–	–	–	–	1
28	I am eager to observe plants or animals.	1	–	–	–	–
29	I read popular science, critical literature, and journalism willingly.	–	–	1	–	–
30	I endeavor to understand the secrets of skill and try my hand at painting, music, etc.	–	–	–	1	–
Results		8	8	8	7	8

Each item in the table is assessed using a five-point fuzzy scale; the results are multiplied by the weight indicated in the corresponding column. A real number from 0 to 5 is indicated to reflect adequately the user’s estimate of his (her) answer to each test question. Klimov’s test is supplemented with the average grade points of the user’s high school diploma by the disciplines of interest in order to consider not only the user’s subjective estimates but also the available accurate and verifiable external data with a weight of 2:

– Type N: biology, geography, life safety, and informatics;

– Type T: algebra and elements of analysis, geometry, physics, and informatics;

– Type S: algebra and elements of analysis, geometry, Russian language, and informatics;

– Type A: Russian language, Russian literature, universal history, and history of Russia;

– Type M: social studies, universal history, history of Russia, and life safety.

As a result, the vectors of fuzzy data are formed, and the priority type of professions is selected based on the maximum-length vector. The length of fuzzy estimate vectors is adopted instead of the simple sum of features to increase the contribution of the most pronounced features to the resulting estimate. As in the paper [9], we use triangular fuzzy numbers $\tilde{a}$ with a membership function μ defined using a triplet of numbers (a^{-}, a, a^{+}) as

$$\mu_{\tilde{a}}(x) = \begin{cases} 0, \; x < a^{-}, \\ \dfrac{x - a^{-}}{a - a^{-}}, \; a^{-} \le x \le a, \\ \dfrac{a^{+} - x}{a^{+} - a}, \; a \le x \le a^{+}, \\ 0, \; a^{+} < x, \end{cases} \tag{1}$$

where $x \in X$.

When the fuzziness definitional domain exceeds the boundaries of the interval $\left[c,\ d\right]$, it is necessary to set $a^- = c$ (on the left) or $a^+ = d$ (on the right).

For triangular fuzzy numbers, arithmetic operations are defined by simple triplet transformation rules [47, 48]:

$$\tilde{a}+\tilde{b}=(a^-+b^-,\ a+b,\ a^++b^+),$$

$$\tilde{a}\cdot\tilde{b}=(a^-b^-,\ ab,\ a^+b^+),$$

$$\tilde{a}-\tilde{b}=(a^--b^+,\ a-b,\ a^+-b^-),$$

$$\frac{\tilde{a}}{\tilde{b}}=\left(\frac{a^-}{b^+},\frac{a}{b},\frac{a^+}{b^-}\right).$$

The distance between fuzzy numbers is given by [47]

$$d\left(\tilde{a},\ \tilde{b}\right)=\sqrt{\frac{1}{3}\left[(a^--b^-)^2+(a-b)^2+(a^+-b^+)^2\right]}. \tag{2}$$

The Euclidean distance between fuzzy vectors $\tilde{\mathbf{a}}$ and $\tilde{\mathbf{b}}$ [49] is calculated by analogy, i.e., the quadratic sum for one pair of triplets in (2) is replaced by the sum of the squared differences of the corresponding triplets of the fuzzy coordinates:

$$d\left(\tilde{\mathbf{a}},\ \tilde{\mathbf{b}}\right)=\sqrt{\frac{1}{3}\sum_{jk}\left(a_j^k-b_j^k\right)^2}. \tag{3}$$

In this formula, the subscript j is associated with vectors whereas the superscript k with triplet parameters. For crisp numbers, the expression (3) coincides with the conventional distance formula used in the Euclidean space. The relations presented above are sufficient for processing fuzzy test data and selecting a prioritized decision.

An expert system based on fuzzy parameters makes conclusions based on the user data but neglects the history of queries, which may contain information differing from expert assessments. One possible reason is a significant hidden factor, absent or disregarded when developing the expert system. Such a factor may systematically distort the result.

A fuzzy expert system with collaborative filtering based on the cosine similarity measure of fuzzy data is chosen to consider the information contained in the history of queries and the decisions made. The measure has the form

$$\operatorname{sim}\left(\mathbf{a},\ \mathbf{b}\right)=\frac{\sum_{jk}a_j^kb_j^k}{\sqrt{\sum_{jk}\left(a_j^k\right)^2\sum_{jk}\left(b_j^k\right)^2}}.$$

This criterion is similar to the classical Pearson correlation coefficient of vectors, coinciding with the latter in the case of crisp data. It takes the maximum value (equal to 1) when the vectors compared have the same direction. Due to this property, the search procedure in the database yields a precedent closest to the case under consideration. For small databases, this solution is the best one. Under big data, it is reasonable to cluster them and make a decision by comparing the case under consideration with the centers of clusters.

3. THE ALGORITHM AND NUMERICAL SIMULATION RESULTS

The software implementation of the fuzzy cold-start recommender system requires operation algorithms for the rule-based expert system and the recommender system with collaborative filtering as well as their coupling that generates pseudo data based on the expert system and given decision statistics.

The algorithm of the fuzzy expert system based on the user properties and rules includes the following steps.

Step 1. User data are entered.

Step 2. Crisp data are transformed into fuzzy data.

Step 3. Based on the fuzzy representation and rules, the weights of decisions are determined as the magnitudes of the fuzzy vectors of characteristic indicators related to different decisions.

Step 4. The weights of decisions are normalized and are compared, and the results are outputted in the form of ranked recommendation ratings.

To form the pseudo-data base, we use a random generation of five data blocks; in each block, a particular type of professional preferences dominates over the other decisions, which have a reduced but close proportion. The variations of the estimates of individual properties within a given type are smaller than the difference between the major type and the minor types for each block.

The algorithm of the fuzzy recommender system with the collaborative filtering of the user's query and the history of queries in the database includes the following steps.

Step 1. The database is formed and is loaded.

Step 2. The indicators are transformed into fuzzy characteristics.

Step 3. The current user's data are entered.

Step 4. The current user's crisp data are transformed into fuzzy data.

Step 5. The closest sample is searched in the fuzzy database.

Step 6. The decision for this sample is outputted as the recommended decision.

Running the collaborative filtering algorithm requires a relevant database. It is often unavailable in the initial stage of system operation, causing the well-known cold start problem. To solve this problem using the fuzzy rule-based expert system, we generate an initial pseudo-data base reflecting the observed decision statistics of users. Then, during the operation of the second recommender system with collaborative filtering, the pseudo-data are gradually replaced by the data taken from real examples. Thus, the relevance of the entire system is improved during operation.

Before the system operation, it is necessary to check the quality of the pseudo-data base by comparing the answers given by both recommender systems on the same test examples. *FuzzyExpert* and *FuzzyRecommend*, the programs implementing the fuzzy rule-based expert system and the fuzzy recommender system with collaborative filtering in MATLAB, have the same interface (a menu column).

Consider a numerical example. Let the input data matrix *A* with row distribution by the type of professions be

$$A = \begin{pmatrix} 2.00 & 2.00 & 4.00 & 2.00 & 2.00 & 3.00 & 5.00 \\ 4.00 & 4.00 & 4.00 & 4.00 & 5.00 & 4.00 & 5.00 \\ 5.00 & 4.00 & 4.00 & 4.00 & 0.50 & 4.00 & 4.75 \\ 3.00 & 1.00 & 2.00 & 0 & 4.00 & 1.00 & 4.75 \\ 3.00 & 2.00 & 3.00 & 3.00 & 2.00 & 3.00 & 5.00 \end{pmatrix}.$$

The first six columns contain 30 user answers to the questions of Klimov's test. The last column provides the average grade points of the user's high school diploma by the disciplines of interest (the five groups of professions with four related disciplines). The value of fuzziness, which determines the scatter of test parameter estimates in each direction, is taken equal to 1. The average grade points for the disciplines are treated as crisp values during processing. The weights of the estimates form the matrix

$$r = \begin{pmatrix} 1 & 1 & 1 & 2 & 2 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 & 2 & 1 & 1 \end{pmatrix}.$$

Based on the matrices *A* and *r*, the system calculates the preference vectors $B_{ij} = \left\{A_{ij} \cdot r_{ij}\right\}$ of the *i*th profession with projections $j = 1, 2, \ldots, 7$ by the number of indicators compared. Then the fuzzy triangular preference vectors $\tilde{R}_{ij} = \left\{B_{ij}^{-}, B_{ij}, B_{ij}^{+}\right\}$, where $B_{ij}^{\pm} = B_{ij} \pm \delta$ with $\delta = 1$, are formed. If the value of a triangular number boundary goes beyond the range [0, 5], it is replaced by the corresponding boundary value of the interval (0 or 5). Next, the lengths of the vectors $R_i = d\left(\tilde{\mathbf{B}}_i, 0\right)$ are calculated using formula (3), and the results are normalized by the total length, yielding the output vector $R_i / \sum_k R_k$ of recommendations in percentage. In the example under consideration, the recommendation vector is (15.4695; 26.4630; 24.3394; 14.2747; 19.4534). According to the simulation results, the user tested should choose professions related to technical devices or, with a slightly lower rating estimate, work with abstract symbolic systems, models, and natural and artificial languages.

These rating estimates are consistent with the results of individual interviews conducted by experts with different users. The fuzzy representation used instead of the initial estimates increases the consistency of the final results with the expert assessments by pushing the mean estimates apart and relaxing the limit estimates at the interval ends. Re-testing shows the robustness of prioritized choice with respect to the ranking of the results.

A pseudo-data base with one hundred entries was generated for the fuzzy recommender system with collaborative filtering. According to the numerical simulation results, this number of random entries ensures the robust identification of the priority profession; in the case of restricting the number of entries to a few tens, the generated data will not reproduce correct prioritization.

CONCLUSIONS

Due to the rich variety of the fields of study and the profiles of educational programs at an HEI, the choice of an appropriate educational trajectory is a serious task for enrollees. For effective assistance in this issue, it is necessary either to form a staff of expensive experts or to implement a special computer-based recommender system.

We have developed an approach to making recommendations on the choice of a general field of study based on Klimov's career guidance test. The initial data are treated as fuzzy values to be processed based on a fuzzy metric and fuzzy comparisons. The cold start problem has been dealt with using an expert estimation system algorithm with the integral length criterion of the fuzzy estimate vector. The fuzzy expert estimation program can be used separately and independently of databases. The cold start problem has been solved by generating a random pseudo-data base with the distribution of answers by priority.

The relevance of the pseudo-data base has been validated by comparing the results of fuzzy collaborative filtering with the answers produced by the fuzzy expert system. According to the numerical simulation results, the optimal size of the pseudo-data base in terms of fast operation with reliable answers is about one hundred entries.

During system operation with real users, pseudo-data are updated by replacing the entries with new data based on the tests passed. With regular updates, the database maintains correct operation and adapts to the changes in the user population.

Note an alternative approach to determining the educational trajectory using fuzzy artificial neural networks as follows: the data of students are clustered, and the neural network undergoes supervised learning at the stage of big data accumulation. Compared to the methodology of artificial neural networks, the approach presented in this paper has the advantage of possible operation under data volumes insufficient for machine learning during the cold start period.

We expect to apply this recommender system in Voronezh State Technical University (VSTU). The corresponding fields and profiles of study for VSTU enrollees are divided into five types:

– type N: geodesy and remote sensing, land management and cadastres, and environmental management and water use;

– type T: construction, radio engineering, and instrumentation;

– type S: economics, applied informatics, and computer and information sciences;

– type A: architecture, design of architectural environment, and reconstruction and restoration of architectural heritage;

– type H: journalism, advertising, and public relations.

The educational trajectory of an individual is further refined after entering an HEI (through choosing elective disciplines), after receiving a bachelor's or specialist degree when entering a master's program, and, finally, when entering a graduate school. This problem is topical due to organizational changes in the Russian higher education system. Further research is needed to develop intelligent support tools for forming an individual student's roadmap.

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SUSTAINABLE DEVELOPMENT OF FLOODPLAIN TERRITORIES OF REGULATED RIVERS.

PART I: Modeling Complex Structure Dynamics[1]

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Abstract. This two-part study presents an approach to designing a sustainable management system for the environmental socio-economic systems (ESESs) of floodplain territories based on modeling their structure dynamics and hydrotechnical projects on their hydrological regime stabilization. The objective of management is to achieve and maintain the optimal stationary complex structure of a floodplain territory, which is characterized by the best design-achievable correspondence between the functional purpose of its fragments and the nature of their spring flooding. The approach rests on the complex structure dynamics model of a floodplain territory that combines variable hydrological and permanent functional properties. This dynamic model, supplemented by an expert model of the socio-economic potentials of the floodplain territory state, yields optimal parameters of hydrotechnical and socio-economic projects. Implementing the approach for a particular floodplain ESES involves optimization, expert assessment, geoinformation and numerical hydrodynamic modeling, high-performance computing, and the statistical analysis of natural observation data and the results of computational experiments. The retrospective, modern, and forecasted complex structures of the northern part of the Volga–Akhtuba floodplain are numerically built considering the spatial heterogeneity of the riverbed degradation effect of the Volga. These numerical results are used to develop an algorithm for finding the parameters of hydrotechnical projects to ensure an optimal sustainable complex structure of the floodplain territory. The algorithm and the results of its numerical implementation will be presented in part II of the study.

Keywords: sustainable development, territorial structure control, hydrotechnical projects, high-performance computing, hydrodynamic modeling, Volga–Akhtuba floodplain.

INTRODUCTION

Floodplain environmental socio-economic systems (ESESs) are characterized by an increased dependence of their condition on the hydrological regime. The complex structure of the channel system and flood inundation zones determines the mosaic arrangement of the functional zones of floodplain territories. The efficient management of floodplain ESESs is based on an optimality measure of correspondence between the territorial distribution of water resources and the functional distribution of floodplain fragments.

The territorial distribution of water resources in floodplain ESESs is formed mainly by the dynamics of their flood inundation (the volume of spring floods, channel structure, and the topography of the territory). The construction of a hydroelectric power stations sys-

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tem on large rivers, in addition to hydroelectric power generation systems, creates an opportunity to regulate spring floods, which are transformed into the spring discharges of hydroelectric power stations (HPSs). This regulation provides favorable conditions for the operation of floodplain ESESs. However, the gradual erosion (depression) of the riverbed in significant (more than a hundred kilometers) channel zones in the downstream of HPSs due to the violation of the dynamic equilibrium between washout and deposition of bottom soil particles is the reason for the slow lowering of river levels. For example, the decreases in river levels over the years of HPS operation are as follows: for the Kama at the Votkinsk HPS, –1.1 m; for the Volga at the Nizhny Novgorod HPS, –1.3 m; for the Volga at the Volzhsk HPS, from –1.7 m to –1.8 m (low-water conditions) and from –1.25 m to –1.35 m (spring floods) [1–7]. This is the reason for the decreasing volume of flood waters entering the floodplains [1–8].

This decrease results in a progressive narrowing of the stable flooded area, which is the biotope of its environmental system, and the expansion of the unstable flooded area, least valuable for ESESs. Thus, unlike the floodplains of unregulated rivers suffering from floods, those of regulated ones suffer from anthropogenic aridization. Therefore, an urgent problem of their water resources management is to ensure the stability of ecosystems and sustainable socio-economic development under water scarcity.

In past decades, the problem of achieving a rational balance between socio-economic and environmental needs in river systems has been the subject of intensive research [7–26]. The rich list of recent-year publications can be divided into studies on monitoring, detection, and modeling of environmental and socio-economic problems and risks (e.g., see [7–18]) and studies on the design of decision systems [19–25] and risk management [26–29]. Note that the first group of studies involves relatively accurate quantitative (geoinformation, hydrodynamic, and statistical) methods and technologies, whereas the second group is mainly based on qualitative methods of management and expert assessment. Here, the objective reason is the multidimensional uncertainty in the sustainable management of floodplain ESESs.

The sustainable development of river systems and the closely related problem of ecological and economic management of regional ESESs are also the subject of many studies [30–35]. The authors [30–33] focused on the problem of reducing the quality of life and economic efficiency under the anthropogenic degradation of floodplain landscapes and ecosystems. The research topics were methods and technologies for identifying the parameters of sustainable development as well as ensuring their achievement and maintenance. In [33], the problem of sustainable development was analyzed using system compatibility indices; they are calculated based on the objective functions of actors and sustainability criteria for ecosystems. The resulting domain of admissible parameter values is equivalent to the domain of normative lossless actions for the economic agents of the corresponding ESES within its ecological and economic management system [34]. Note that, being adequate for stable systems, these approaches lose their efficiency when analyzing the floodplain ESESs of regulated rivers under developing natural and technogenic degradation. The identification and control of such parameters are significantly complicated due to the absence of long-term deterministic conditioning of flood inundation maps of the territory by the river hydrograph (the time dependence of water discharge through the river channel cross-section). Therefore, the main ground for the sustainable development of floodplains is the long-term stabilization of their hydrological regime.

This paper develops an approach to creating a sustainable management system for floodplain ESESs through optimizing hydrotechnical and socio-economic projects. The objective of management is to achieve and maintain the optimal stationary complex (C) structure of the floodplain territory, which determines the efficiency of the ESES. Optimality is characterized by the best design-achievable correspondence between the functional purpose of floodplain fragments and the nature of their spring flooding. The management problem is solved using geoinformation and numerical hydrodynamic modeling, dynamic programming, high-performance computing, heuristic optimization and expert assessment, and statistical analysis of natural observation data and the results of computational experiments.

The approach is implemented for the northern part of the Volga–Akhtuba floodplain (VAF) located within the Volgograd Region, occupying an area of 867 km^2 with a total length of large and small channels of about 800 km. The Volgo–Akhtuba floodplain has high natural diversity as well as favorable conditions for agriculture, limited housing construction, and ecological tourism. The creation of the Volga hydroelectric power stations system (especially, the Volzhsk HPS in 1961) was the main factor in the formation, operation, and subsequent degradation of the VAF [9, 10]. During spring flooding, over 70% of the VAF territory is flooded from the Akhtuba, a branch of the Volga. During the operation period of the Volzhsk HPS, the average share of water entering the Akhtuba from the Volga during spring flooding has decreased three times [35, 36]. A peculiarity of the Volzhsk HPS is that a significant part of the territory (37%) has the indeterminate cadastral type of land use.

Some elements of the approach and their implementation results were presented in the previous publications. For example, the paper [37] described the concept of the *C*-structure of a floodplain territory as a tool for analyzing its territorial potential. The strategic management problem of a floodplain territory was formulated as follows: achieve the C-structure maximizing the value of the aggregated territorial environmental socio-economic (ESE) potential. The ESE potential of a certain kind was defined as the weighted sum of the products of the area of territorial fragments attributed to that kind by the value of a function characterizing the measure of correspondence between the functional purpose of those fragments and the nature of their spring flooding. The problem was numerically investigated for the modern relief of the VAF territory controlled by a system of dams with variable cross-sections in large and small floodplain channels, and the corresponding simulation results were provided.

The paper [38] described a model of long-term natural and technogenic dynamics for the *C*-structure of the floodplain territory. This model is based on a simplified spatially homogeneous regression model of the technogenic depression of the floodplain channel. Also, the dynamics of the aggregated 12-element C-structure and three ESE potentials of the VAF territory were forecasted up to 2050.

In this paper, we present a long-term dynamics modeling method for the C-structure of the floodplain territories of regulated rivers that includes the following elements: an algorithm for building a set of flood modeling maps (FMMs), a regression model of the spatially heterogeneous depression of the main floodplain channel, an approximate algorithm for building retrospective and forecasted flood inundation maps, a 24-element model of the *C*-structure of the floodplain territory, and an algorithm for building retrospective and forecasted *C*-structures. This method is numerically implemented for the northern part of the Volga–Akhtuba floodplain and the corresponding results are presented. These numerical results are used to develop an algorithm for finding the parameters of hydrotechnical projects to ensure an optimal sustainable complex structure of the floodplain territory. The algorithm and the results of its numerical implementation will be presented in part II of the study.

1. THE METHODS AND TECHNOLOGIES OF THIS STUDY

1.1. Water Dynamics Modeling Tools

Calculations of water dynamics in river channels and the interfluve of the Volga and Akhtuba are based on shallow water equations that consider hydraulic resistance in the Manning model [39, 40]. The EcoGIS-Simulation software package is used for the numerical modeling of surface water movement [41, 42], with a digital elevation model (DEM) of the northern part of the Volga–Akhtuba floodplain [43, 44] as one module. The computational core of EcoGIS-Simulation involves CSPH-TVD, a numerical two-step algorithm for integrating hydrodynamic equations that combines the Lagrange and Euler approaches (Smoothed-ParticleHydrodynamics and TotalVariationDiminishing, respectively) [45, 46]. Parallel computations are performed on NVIDIA Tesla GPUs [39, 47].

1.2. The Set of Flood Modeling Maps

The calculations of water dynamics during the spring discharge of the HPS in year τ (see subsection 1.1) yield a series of flood inundation maps $\mathrm{K}_{\tau}^{\mathrm{in}}(t_k)=\mathrm{K}^{\mathrm{in}}(Q_\tau(t),\, t_k),\quad t\in[t_0,\, t_k],\ \ k=1,\ldots,\, k_{\max}$, where t_0 denotes the time instant of discharge start and $Q_\tau(t)$ is the hydrograph of the spring discharge of the HPS in year τ (Q_τ is the flow rate, and *t* indicates time, h). The digital flood inundation map of a territory is a two-dimensional array, where each element $(i,\, j),\ i=1,\ldots,N,\ j=1,\ldots,M,$ at each time instant t_k shows the water height $h_{ij}(t_k)$ and its velocity vector $v_{ij}(t_k)$. Due to the geoinformation and hydrodynamic modeling error ε_{g}, it is required to calculate the minimum flood height $h_{\min}$: under the inequality $h_{ij}(t_k)\geq h_{\min}$, the corresponding cell of the digital map is “flooded” at the time instant t_k. (The origin, algorithm, and results of calculating the values ε_{g} and $h_{\min}$ were described in [48].) With each map $\mathrm{K}^{\text{in}}(Q(t), t_k)$ we associate the map $\mathrm{K}(Q(t),t_k)$, where each cell contains the variable $m_{ij} = \{0, 1\}$, $i = 1,..., N$, $j = 1,..., M : m_{ij} = 0$ if $h_{ij}(t_k) < h_{\min}$, and $m_{ij} = 1$ if $h_{ij}(t_k) \geq h_{\min}$. In each of the numerous computational experiments with the real and model hydrographs $Q(t)$, there was a time instant $t^{\max}$ with the largest number of flooded cells in the map $\mathrm{K}(Q(t), t^{\max})$. Such maps are used in this paper.

When varying $Q(t)$ and the control vector u that changes the relief of the flooded territory or the safe flooding conditions, the computational complexity of building a large number of digital maps $\mathrm{K}^{\text{in}}(Q(t), t^{\max}, u)$ and the corresponding maps $\mathrm{K}(Q(t), t^{\max}, u)$ can be reduced by replacing, with an error $\varepsilon_c \leq \varepsilon_{\text{g}}$, each multistage hydrograph $Q(t)$ with a constant hydrograph $G^c = (Q^c, t^{\max})$, characterized by the constant flow Q^c and the duration $t^{\max}$. Note that $\varepsilon_c = \max(\varepsilon_1,..., \varepsilon_L)$, where the errors $\varepsilon_l (l = 1,..., L)$ characterizing the relative differences between the maps $\mathrm{K}_l^{(1)}(Q(t))$ and $\mathrm{K}_l^{(2)}(Q^c, t)$ are given by

$$\varepsilon_l = \left(\min\left(\sum_{j=1}^{M}\sum_{i=1}^{N} \overset{(1)}{m_{ijl}}, \sum_{j=1}^{M}\sum_{i=1}^{N} \overset{(2)}{m_{ijl}}\right)\right)^{-1} \sum_{j=1}^{M}\sum_{i=1}^{N}\left|\overset{(1)}{m_{ijl}} - \overset{(2)}{m_{ijl}}\right|,$$

where $\overset{(p)}{m_{ijl}} = 0$ if the cell (i, j), $i = 1,..., N$, $j = 1,..., M$, of the digital map $\mathrm{K}_l^{(p)} (p = 1, 2)$ is not flooded, and $\overset{(p)}{m_{ijl}} = 1$ otherwise. The hydrographs are replaced by a heuristic algorithm: on each time interval $[t_0, \tilde{t}]$, the step hydrograph $Q(t)$ is replaced by a constant hydrograph of equal volume with the corresponding flow rate $Q^c_{\tilde{t}}$. The value $t^{(1)} = \arg\max_{\tilde{t}}[Q^c_{\tilde{t}}]$ preliminarily estimates the maximum flood time instant. This value is refined by calculating the relative difference function $\varepsilon(\eta)$ between the maps $\mathrm{K}^{(1)}(Q(t))$ and $\mathrm{K}^{(2)}_{\eta}(Q^c_{t^{\max}_{\eta}}, t^{\max}_{\eta})$, where $t^{\max}_{\eta} = t^{(1)}\left(1 + \eta(t_1 - t_0) \times (t^{(1)} - t_0)^{-1}\right)$, and t_1 is the root of the equation $Q^c_{t^{(1)}} = Q(t_1)$. The most accurate estimate is $t^{\max}_{\eta^*}$, where $\eta^* = \arg\min_{\eta} \varepsilon(\eta)$. Let us denote $t^{\max}_{\eta^*} \equiv t^{\max}$.

The maps $\mathrm{K}(Q^c, t^{\max})$ are built using the set of FMMs, which contains an array of floodplain inundation maps with constant hydrographs $G^c_{ij} = (Q^c_i, t_j)$ and the modern topography of channel bottom and flooded territory. This set is formed so that the relative differences between the maps $\mathrm{K}(G^c_{ij})$ and $\mathrm{K}(G^c_{i+1,j})$ as well as between the maps $\mathrm{K}(G^c_{ij})$ and $\mathrm{K}(G^c_{i,j+1})$ do not exceed ε_{g}. The map $\mathrm{K}(G^c_{ij})$ $(Q^c_i \leq Q^c \leq Q^c_{i+1}, t_j \leq t^{\max} \leq t_{j+1})$ is selected as the map $\mathrm{K}(Q^c, t^{\max})$ for calculating target flooding and environmental safety conditions. The map $\mathrm{K}(G^c_{i+1,j+1})$ is selected as the map $\mathrm{K}(Q^c, t^{\max})$ for calculating socio-economic safety conditions.

1.3. The Regression Model of Main Channel Depression and the Approximate Algorithm for Building Floodplain Inundation Maps

The progressive natural-technogenic depression of the main channel is modeled by the regression dependence $h(Q^c, \tau, L)$, $(\tau \in [\tau_0, \tau_0 + N]$, in years) of the water level h on the hydrograph Q^c at the distance L from the downstream of the HPS according to the long-term measurements of water levels $h_i(Q(t_j), \tau, L_i)$, $i = 1,..., I$, $j = 1,..., J$, $\tau \in [\tau_0, \tau_0 + N]$, at I gauging stations located at the distances L_i from the downstream of the HPS. The regression equation describing the spatial heterogeneity of the channel depression effect has the form

$$h(w) = (a, w) + w^{\mathrm{T}} A w + b, \;\; w = (Q^c, \tau, L)^{\mathrm{T}},$$
$$a = (a_1, a_2, a_3), \;\; A = \left\|a_{ij}\right\|^{3}_{i,j=1}. \qquad (1)$$

The expression (1) allows finding a virtual hydrograph $Q^c_2 = \varphi(Q^c_1, \tau_1, \tau_2, L)$, a function of the variable L, which is a solution of the equation $h(Q^c_1, \tau_1, L)$

$= h(Q_2^c, \tau_2, L)$. For any $t^{\max}$, the virtual hydrograph $(\varphi(Q_1^c, \tau_1, \tau_2, L), t^{\max})$ provides flooding of the territory under the channel state in year τ_2 that is equivalent to flooding of the territory with the constant hydrograph $(Q_1^c, t^{\max})$ under the channel state in year τ_1.

The L-variable hydrograph is approximated by an L-stepped hydrograph to build a rather accurate retrospective or forecasted flood inundation map of the territory in year τ_2 as the composition (gluing) of separate fragments from the set of flood modeling maps (based on the relief in year $\tau_1 = 2022$) with different values of the constant flow Q_j^c and the modeling error $\varepsilon_c = \max(\bar{A}, \varepsilon_{\mathrm{g}})$, where $\bar{A}$ denotes the average approximation error of (1). The map fragments approximating those of the flood inundation map $\tilde{\mathrm{K}}_k(\varphi(Q_1^c, \tau_1, \tau_2, L), t^{\max})$: $\tilde{\mathrm{K}}(\varphi(Q_1^c, \tau_1, \tau_2, L), t^{\max})$, $L_k \le L \le L_k + \Delta L$, $k = 1, \ldots, K-1$, $L_1 = 0$, are the corresponding fragments $\mathrm{K}_k(G_{ij}^c)$ of the maps from the set of FMMs: $Q_i^c \le \varphi(Q_1^c,\ \tau_1,\ \tau_2,\ L) \le Q_{i+1}^c$, $t_j \le t^{\max} \le t_{j+1}$, $L_k \le L \le L_k + \Delta L$. Here, $L_{\max}$ is the length of the modeled part of the channel; ΔL_k, $k = 1, \ldots, K-1$, are the roots of the equations $\varphi(Q_1^c, \tau_1, \tau_2, L_k + \Delta L_k) - \varphi(Q_1^c, \tau_1, \tau_2, L_k) = Q_{i+1}^c - Q_{i+1}^c$, providing the required accuracy; the values K and ΔL are determined by the relations $\sum_{k=1}^{K} \Delta L_k = L_{\max} = K \Delta L$.

1.4. The Complex Structure Model of a Floodplain Territory

The elements of the primary functional (F) structure are the sets of local fragments of a territory with the same cadastral type of land use (F-kinds). The elements of aggregated F-structures are their unions into groups (F-types) on various grounds. The F-structure of a floodplain territory is built based on its digital cadastral map.

The elements of the hydrological (H_1) structure are the sets of local fragments of the territory with the same frequency ranges of their flood inundation (H_1-kind). The number of frequency ranges and their boundaries are determined, on the one hand, by the objectives of the study and, on the other hand, by the possibility of their identification with a given accuracy. The enlarged frequency ranges form H_1-types. The complex structure of a floodplain territory is the superposition of its H_1- and F-structures. The elements of the C-structure are the sets of its local fragments with the same H_1-kind (type) and the same F-kind (type). The existence of the structure $H_1(\tau)$ is determined by the stability of the relief, channel structure, and distribution function of the annual flood hydrograph volume calculated for samples. The size Θ of such samples (the number of consecutive years of observations corresponding to the interval $\left[\tau - \frac{\Theta}{2}, \tau + \frac{\Theta}{2} - 1\right]$) is determined based on the requirements for the identification accuracy. An algorithm for calculating the minimum value Θ was described in [37]. In this study, we use the enlarged frequency ranges (H_1-types) characterizing the following territories: stable flooded (with a frequency equal to or exceeding some threshold $n^{\lim}$), unstable flooded (with a frequency below $n^{\lim}$), and non-flooded (never flooded during the observation period). The ESE potentials of F-structure elements are estimated using characteristic functions reflecting the measure of correspondence between their F-kind (type) and the H_1-kind (type). The characteristic functions are built by experts.

If the H_1- and F-structures contain n and m elements, respectively, they will form the C-structure of nm elements. An algorithm for building a 12-element C-structure based on a three-element H_1-structure and a four-element F-structure (social, environmental, economic, and uncertain territories) was described in detail in [37], including the resulting C-structure as well. However, this F-structure has a disadvantage: it provides no classification for the territorial fragments of floodplains. Therefore, in this paper, we adopt the aggregated functional structure F_1 as a set of eight typical elements, each being described by one of the three typical characteristic functions: social (S, φ_3), environmental (En, φ_1), economic (Ec, φ_3), environmental-economic (EnEc, φ_1), social-environmental (SEn, φ_1), socio-economic (SEc, φ_3), environmental socio-economic (EnSEc, φ_1), and indeterminate (I, φ_2). The latter type means no cadastral land use. The characteristic functions φ_1, φ_2, and φ_3 have the following form:

$$\varphi_1(n)=\begin{cases}0,\ 0\le n<n^{\lim},\\ 1,\ n^{\lim}\le n\le 1;\end{cases}$$

$$\varphi_2(n)=1;$$

$$\varphi_3(n)=\begin{cases}0,\ 0<n\le 1,\\ 1,\ n=0.\end{cases}$$

The superposition of the F_1-structure and the three-element H_1-structure forms the 24-element C-structure $K_{24}(\tau)$, used below to solve the management problem.

1.5. The Dynamics Model of the *H*1-structure

As mentioned above, the channel depression effect leads to changes in the H_1-structure over time. The dynamics model of this structure is a set of algorithms for building a sequence $H_1(\tau),\ \ \tau=\tau_0,\ldots,\ \tau_0+N,\ldots,T$. This sequence consists of real ($H_1^r(\tau)$) and model ($H_1^{\mathrm{mod}}(\tau)$) structures. The real structures $H_1^r(\tau)$, corresponding to the case $\tau_0+\frac{\Theta}{2}\le\tau\le\tau_0+N-\frac{\Theta}{2}+1$, are built using a set of flood inundation maps for the observation period with the hydrographs G_θ^c, $\theta\in\left[\tau-\frac{\Theta}{2},\ \tau+\frac{\Theta}{2}-1\right]$, obtained by the algorithm from subsection 1.3. (On the stable flood inundation map $\mathrm{K}_\tau^{n^{\lim}}$ of the structure $H_1^r(\tau)$, a flooded cell is the one flooded on at least $n^{\lim}\Theta$ maps of the set containing Θ maps.)

The model structures $H_1^{\mathrm{mod}}(\tau)$ are formed for the cases where the sampling interval, fully or partially, exceeds the boundaries of the observation period ($\tau<\tau_0+\frac{\Theta}{2}$ or $\tau>\tau_0+N-\frac{\Theta}{2}+1$). In the paper [37], these structures were built using flood inundation maps calculated considering the spatially homogeneous depression effect of the main channel in year τ with the virtual model hydrographs $G_i^c,\ i=1,\ldots,\Theta$, whose parameters were randomly selected from the general population.

If the algorithm [37] is applied to build a large number of the model structures $H_1^{\mathrm{mod}}(\tau)$ corresponding to different hydrotechnical projects considering the spatially heterogeneous depression effect of the main channel, the computational complexity of the algorithm for solving the management problem will increase significantly. Therefore, in this study, a less computationally intensive algorithm is used to build approximate model structures. This algorithm is to find the generalized hydrograph $\widehat{G}_o^c=(\widehat{Q}_o^c,\ \widehat{t}_o)$ and the nearest hydrograph from the set of FMMs $G_{ij}^c=(Q_i^c,\ t_j),\quad Q_i\le\widehat{Q}_o^c\le Q_{i+1},\quad t_j\le\widehat{t}_o\le t_{j+1}$, whose map best approximates the set of stable flood inundation maps $\mathrm{K}_\tau^{n^{\lim}}$ in the virtual structures $H_1^v(\tau)$ generated for the entire observation period $\tau_0+\frac{\Theta}{2}\le\tau\le\tau_0+N-\frac{\Theta}{2}+1$ using the flood inundation maps based on the modern relief of the main channel with the hydrographs $G_\theta^c,\quad\theta\in\left[\tau-\frac{\Theta}{2},\ \tau+\frac{\Theta}{2}-1\right]$. (These structures would coincide with the real ones in the absence of channel depression.)

The hydrograph $\widehat{G}^c=(\widehat{Q}^c,\ \widehat{t})$ is found as follows. For the parameters Q_τ^c and $t_\tau^{\max}$ of the set of hydrographs $G_\tau^c\ (\tau=\tau_0,\ldots,\ \tau_0+N)$, the linear regression $Q^c=at^{\max}+b$ is constructed. For each fixed $\tau=\tau_0+\frac{\Theta}{2},\ldots,\ \tau_0+N-\frac{\Theta}{2}+1$ and each hydrograph $G_\theta^c,\ \theta\in\left[\tau-\frac{\Theta}{2},\ \tau+\frac{\Theta}{2}-1\right]$, the virtual hydrograph $\tilde{G}_\theta^c(\tau)$ of the volume $V_\theta=Q_\theta^c\,t_\theta^{\max}$ is calculated so that the parameters $\tilde{Q}_\theta^c$ and $\tilde{t}_\theta^{\max}$ lie on this regression. These hydrographs are ordered by volume. The approximate steady-state flood inundation map $\tilde{\mathrm{K}}_\tau^{n^{\lim}}$ is the one of the hydrograph $\tilde{G}_\tau=(\tilde{Q}_\tau^c,\ \tilde{t}_\tau^{\max})$ with the serial number $\left\lceil n^{\lim}\Theta\right\rceil+1$. The error ε_τ of determining the map $\tilde{\mathrm{K}}_\tau^{n^{\lim}}$ is the relative share of cells whose flooding character differs in the maps $\tilde{\mathrm{K}}_\tau^{n^{\lim}}$ and $\mathrm{K}_\tau^{n^{\lim}}$ (see the algorithm in subsection 1.2). If the average value of these errors does not exceed ε_{m}, then the weighted average values of the parameters $\widehat{Q}_o^c$ and $\widehat{t}_o$

of the generalized hydrograph $\widehat{G}_o^c$ are determined using the linear regression and the equation $\widehat{Q}^c\widehat{t} = \left(N-\Theta\right)^{-1}\sum_{\tau}\tilde{Q}_{\tau}^c\,\tilde{t}_{\tau}^{\max}$. The hydrograph closest to $\widehat{G}_o^c$ from the set of FMMs $G^{\mathrm{st}} = (Q^{\mathrm{st}}, t^{\mathrm{st}}) = (Q_i^c, t_j)$, $Q_i \le \widehat{Q}_o^c \le Q_{i+1}, t_j \le \widehat{t}_o \le t_{j+1}$, is the desired result. This approximation has the error $\varepsilon_s = (N-\Theta)^{-1}\sum_{\tau}\varepsilon_{\tau}$. To build the retrospective and forecasted structures $H_1^{\mathrm{mod}}(\tau)$ using the set of FMMs, it is necessary to calculate the variable hydrograph $G_{\tau}^{\mathrm{st}}(L) = (\varphi(Q^{\mathrm{st}}, 2022, \tau, L), t^{\mathrm{st}})$ and form flood inundation maps from the fragments of maps of this set (see the algorithm in subsection 1.3). The resulting approximate structures $H_1^v(\tau)$, $H_1^r(\tau)$, and $H_1^{\mathrm{mod}}(\tau)$ are used to build the corresponding approximate 24-element complex structures $K_{24}^v(\tau)$, $K_{24}^r(\tau)$, and $K_{24}^{\mathrm{mod}}(\tau)$. The hydrological structures built from the flood inundation maps calculated for floodplain channel bed reliefs with projected dams are called the projected structures $K_{24}^{pr}(\tau)$.

In addition to the structure H_1, the algorithm for optimizing hydrotechnical projects involves the hierarchical hydrological structure H_2 of the floodplain territory (a set of its fragments, i.e., zones flooded from channel systems formed by branches from the principal main channel. This algorithm will be presented in part II of the study. One part of the zones is formed by dead-end branches; the other part, by medium main channels (MMC). The territorial fragments flooded from separate channels form microzones. The error of determining the zone boundary, equal to the volume of transboundary water flows divided by the volume of water entering the zone from its channels, may exceed ε_{g}. In this case, the zone joins the neighbor one, forming a macro-zone. The algorithm and the numerical results of building the structure H_2 of the VAF were described in [49].

2. THE RETROSPECTIVE, MODERN, AND FORECASTED *C*-STRUCTURES OF THE NORTHERN PART OF THE VOLGA–AKHTUBA FLOODPLAIN

According to the results of computational experiments, the error of open-data satellite measurements for VAF relief elevations (with an absolute value of 0.5 m) causes the hydrodynamic modeling error of digital flood inundation maps. The relative value of the latter error is calculated by the algorithm from subsection 1.2 and does not exceed $\varepsilon_{\mathrm{g}} = 0.05$ [48]. To implement the algorithms described above, the set of FMMs with the hydrographs of the Volzhsk HPS $G_{ij}^c = (Q_i^c,\ t_j)$, $13\,000 \le Q_i^c \le 28\,000\ \mathrm{m^3/s}$, $0 \le t_j \le 960\ \mathrm{h}$, was created. This set contains over 3500 maps built with a relative error not exceeding $\varepsilon_{\mathrm{g}} = 0.05$.

The coefficients of the regression (1) for the Volga levels downstream of the Volzhsk HPS were calculated based on the official hydrological data on water level dynamics at four gauging stations: the downstream of the Volzhsk HPS, Volgograd (the river port), Svetly Yar, and Cherny Yar (see the website of PJSC RusHydro). Figure 1 shows the water level dynamics at these gauging stations under low-water flow values. Also, the linear trends of water level changes for $Q^c = 4\,000, 5\,000, 6\,000\ \mathrm{m^3/s}$ and the average annual water level decreases $\delta\eta$ are presented. For the downstream of the Volzhsk HPS, $\delta\eta = -0.0201\ \mathrm{m}$; for Svetly Yar (65 km from the Volzhsk HPS), $\delta\eta = -0.0108\ \mathrm{m}$. In the vicinity of Cherny Yar, the bottom depression almost disappears. These estimates give a total depression of about 1.25 m for the downstream of the Volzhsk HPS on the time horizon 1961–2023.

The statistically significant coefficients of the regression (1) describing the depression of the Volga channel downstream of the Volzhsk HPS are $a_1 = 3.00\cdot10^{-4}$, $a_2 = -2.25\cdot10^{-2}$, $a_3 = -5.35\cdot10^{-4}$, $a_{23} = a_{32} = 1.17\cdot10^{-7}$, and $b = 32.54$. The average approximation error is $\bar{A} = 0.097$.

Thus, the depression of the Volga channel downstream of the Volzhsk HPS is described by the regression (1) of the form

$$h(Q^c, \tau, L) = a_1Q^c + a_2\tau + a_3L + 2a_{23}\tau L + b. \quad (2)$$

The function $\varphi(Q_1^c, \tau_1, \tau_2, L)$ (see subsection 1.3) built using the regression (2) has the form

$$\varphi(Q_1^c, \tau_1, \tau_2, L) = Q_1^c - \frac{(a_2 + 2a_{23}L)}{a_1}(\tau_2 - \tau_1). \quad (3)$$

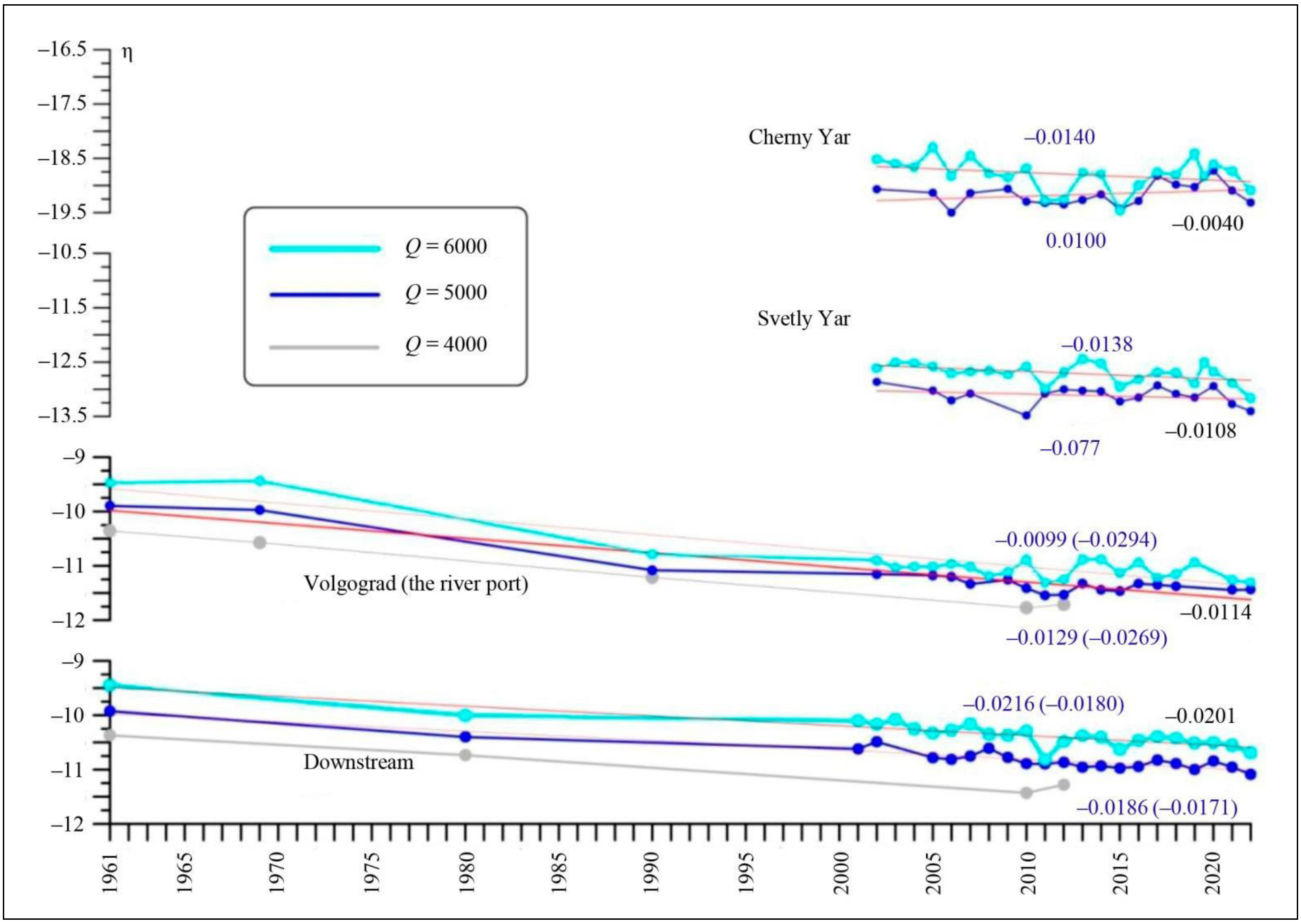

Fig. 1. Measurements at four gauging stations downstream of the Volzhsk HPS dam. The blue color indicates the average annual water level decrease at a gauging station considering all data. Similar values for the period since 2001 are given in brackets. The black color indicates the values averaged over *Q*.

In view of (3), the virtual discharges—the basis of the algorithms for building the retrospective $K^r_{24}(1975)$, modern $K^r_{24}(2005)$, and forecasted $K^{\mathrm{mod}}_{24}(2052)$ *C*-structures (see subsections 1.3–1.5)—are calculated by the formulas

$$Q^c_{1975}(Q^c_{2022},L)=Q^c_{2022}-3525-0.0367L,$$
$$Q^c_{2005}(Q^c_{2022},L)=Q^c_{2022}-1275-0.0133L, \qquad (4)$$
$$Q^c_{2052}(Q^c_{2022},L)=Q^c_{2022}+2250-0.0235L.$$

When gluing the flood inundation maps from the set of FMMs to ensure the required accuracy $\varepsilon_c=\max(0.05;\,0.097)=0.097$ (see the algorithm in subsection 1.3), each map within the territory was built from nine fragments of the maps from the set of FMMs, whose parameters were found by formulas (4). The length of each map fragment along the Volga channel was $\Delta L=7500\ \mathrm{m}$. The difference in the Volga discharge values for neighbor map fragments was $\Delta Q=250\ \mathrm{m^3/s}$.

When building all *C*-structures, the sample size $\Theta=30$ and $n^{\lim}=0.85$ were selected. The regression equation $Q=237t+23117$ was used to find the hydrograph $\hat{G}^c=(\hat{Q}^c,\hat{t}_c)$ in accordance with the algorithm from subsection 1.5. In turn, this hydrograph served to build the approximate projected structures $H_1^{\mathrm{mod}}(2052)$ of the Volzhsk HPS. The resulting parameters $\tilde{Q}^c_\tau$ and $\tilde{t}^{\max}_\tau$ of the approximate hydrographs $\tilde{G}_\tau$ of the Volzhsk HPS, used to build the generalized hydrograph $\hat{G}^c_o$ and the nearest hydrograph from the set of FMMs G^c, as well as the relative approximation errors ε_τ, $\tau=1975,\ldots,2005$, are presented in the table below. The hydrograph parameters calculated from these data are $\widehat{G}^c_o = (24\,302{,}5)$ and $G^{\text{st}} = (24\,250{,}5)$. The stable decrease in the value ε_τ as τ increases can be explained by the transition of the Volzhsk HPS in recent decades to a relatively constant planned two-stage hydrograph of spring discharges.

The value $Q^{\text{st}} = 24\,250\ \text{m}^3/\text{s}$ was used in formulas (4) to find map fragments from the set of FMMs when building maps and calculating the areas of the elements of the retrospective $K^r_{24}(1975)$, modern $K^r_{24}(2005)$, and forecasted $K^{\text{mod}}_{24}(2052)$ *C*-structures of the VAF (see Figs. 2 and 3). In Fig. 3, the 12 zones of the structure H_2 are also marked by solid lines. Colors indicate stable flooded (green and yellow), unstable flooded (blue and orange), and non-flooded (purple and red) structural elements. Figure 4 shows the maps of the stable flooded territory corresponding to these structures. According to the analysis of these figures, the VAF territory is mainly occupied by the land of the environmental, social-environmental, environmental-economic, and indeterminate cadastral types. Direct comparison of the structures $K^r_{24}(1975)$, $K^r_{24}(2005)$, and $K^{\text{mod}}_{24}(2052)$ in Figs. 2–4 demonstrates the progressive natural-technogenic degradation of the VAF stable flooded territory (a biotope of its floodplain ecosystem) and *C*-structure (a determinant for the efficiency of its socio-economic system). For example, the forecasted decrease in the stable flooded area for 77 years is 62%, including 44% for environmental, 78% for environmental-economic, and 60% for indeterminate types.

The parameters of approximate hydrographs of the Volzhsk HPS for building the generalized hydrograph and approximation errors

τ	$\tilde{Q}^c_\tau$	$\tilde{t}^{\max}_\tau$	ε_τ
1975	24539	6	0.0837
1976	24539	6	0.0835
1977	24539	6	0.0705
1978	24302	5	0.0644
1979	24302	5	0.0641
1980	24302	5	0.0640
1981	24302	5	0.0640
1982	24302	5	0.0637
1983	24302	5	0.0554
1984	24302	5	0.0552
1985	24539	6	0.0552
1986	24302	5	0.0548
1987	24302	5	0.0528
1988	24302	5	0.0522
1989	24539	6	0.0511
1990	24539	6	0.0508
1991	24302	5	0.0458
1992	24302	5	0.0457
1993	24302	5	0.0457
1994	24302	5	0.0455
1995	24302	5	0.0408
1996	24302	5	0.0408
1997	24302	5	0.0405
1998	24302	5	0.0377
1999	24302	5	0.0375
2000	24302	5	0.0375
2001	24302	5	0.0371
2002	24302	5	0.0327
2003	24302	5	0.0325
2004	24302	5	0.0316
2005	24302	5	0.0307

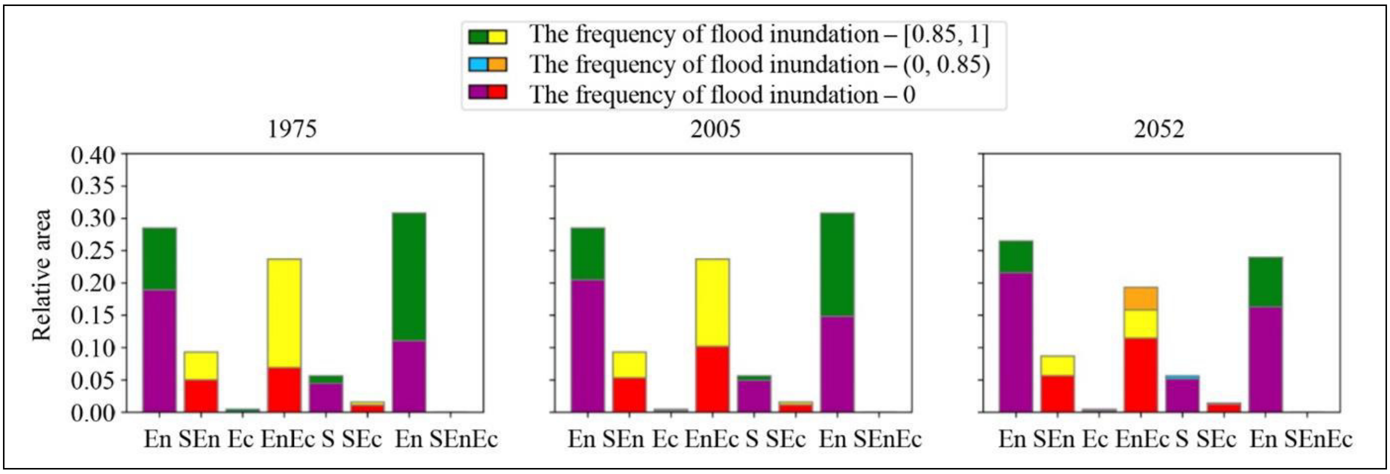

Fig. 2. The diagrams of areas of environmental (En), social-environmental (SEn), economic (Ec), environmental-economic (EnEc), indeterminate (I), and social-environmental (SEn) elements of the *C*-structures $K^r_{24}(1975)$, $K^r_{24}(2005)$, and $K^{\text{mod}}_{24}(2052)$ of the VAF territory.

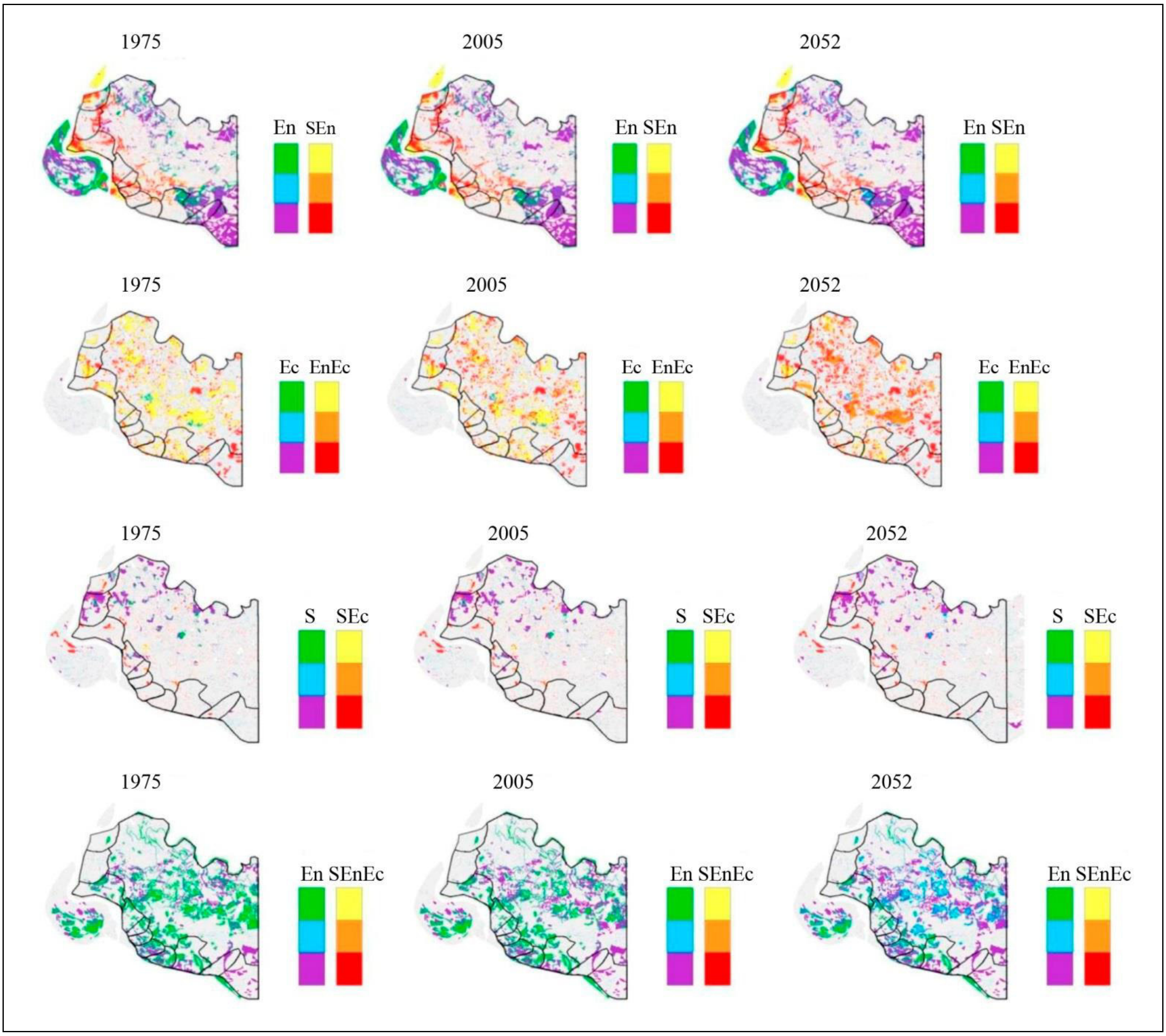

Fig. 3. The maps of environmental (En), social-environmental (SEn), economic (Ec), environmental-economic (EnEc), indeterminate (I), and social-environmental (SEn) elements of the *C*-structures $K^r_{24}(1975)$, $K^r_{24}(2005)$, and $K^{\mathrm{mod}}_{24}(2052)$ of the VAF territory.

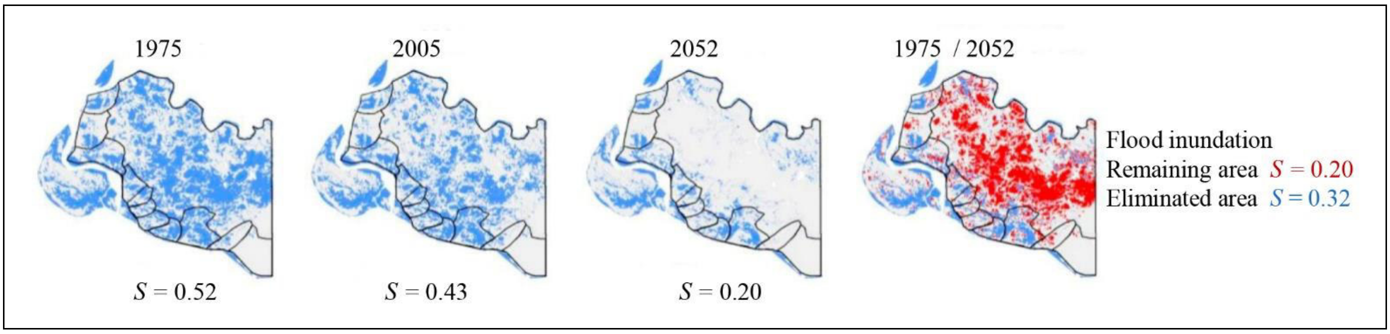

Fig. 4. The maps and relative areas of stable flooded territory in the *C*-structures of the VAF: retrospective $K^r_{24}(1975)$, modern $K^r_{24}(2005)$, and forecasted $K^{\mathrm{mod}}_{24}(2052)$.

CONCLUSIONS

This paper has presented a long-term dynamics modeling method for the C-structure of the floodplain areas of regulated rivers that includes the following elements: an algorithm for building a set of flood modeling maps (FMMs), a regression model of the spatially heterogeneous depression of the main floodplain channel, an approximate algorithm for building retrospective and forecasted flood inundation maps, a 24-element model of the *C*-structure of the floodplain territory, and an algorithm for building retrospective and forecasted C-structures. Implementing this method for a particular floodplain territory involves optimization, expert assessment, geoinformation and numerical hydrodynamic modeling, high-performance computing, and the statistical analysis of natural observation data and the results of computational experiments.

The method has been numerically implemented for the northern part of the Volga–Akhtuba floodplain are the corresponding results have been provided. They demonstrate the progressive natural-technogenic degradation of the VAF stable flooded territory (a biotope of its floodplain ecosystem) and C-structure (a determinant for the efficiency of its socio-economic system). According to these results, hydrotechnical projects are needed to stabilize the hydrological structure of the VAF. Due to the territorial distribution and variability of the parameters of hydrotechnical projects, it is topical to find the optimal *C*-structure.

The models and the results of their numerical implementation are used to develop an algorithm for finding the parameters of hydrotechnical projects to ensure an optimal sustainable C-structure of the floodplain territory. This algorithm and the results of its numerical implementation for the northern part of the Volga–Akhtuba floodplain will be presented in part II of the study.

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A RANK-EXPERT DEVIATION FUNCTION TO CLASSIFY COMPLEX OBJECTS[1]

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Abstract. This paper proposes a novel function for classifying environmental, social, and socio-environmental objects. It is based on the sum of rank deviations between a given object and a reference object considering the significance of the object's characteristics (factors). Characteristics are estimated using weight coefficients, which are provided by expertise or another method. A verbal numerical scale is developed to assess the proximity of objects by the numerical value of the deviation function. As is demonstrated below, this function is not a metric in the geometric sense but a proximity function defined in multidimensional scaling theory. As illustrative examples, the values of the deviation function are calculated for two applications: an environmental problem of comparing the vulnerability of territories to accidental oil spills and an economic problem of choosing real estate objects to purchase. A recommended sequence with a set of procedures based on the deviation function is presented to solve these problems.

Keywords: rank, object, classification, verbal numerical scale.

INTRODUCTION

Classification is one of the main problems of science. The goal of classification is to arrange objects so that those belonging to the same group can be considered close in their qualities. The result of classification is always a grouping of objects according to their properties.

The methodology of classifications varies from one science to another. We can mark off social and natural sciences, where a unified approach is difficult to develop. For example, researchers distinguish from 8 to 22 types of civilizations [1]. It is even harder to classify mixed objects consisting of social, natural, anthropogenic, and other components, e.g., different types of settlements. Nevertheless, mixed objects can be classified by comparing their components in some commensurable terms, such as scores, ranks, distances in hyperspaces, etc.

There exist no unified universal methods of classification. But they are actively developed by representatives of various sciences. Carl Linnaeus contributed much to developing the classifications of natural objects [2].

1. THE IDEOLOGY OF OBJECT COMPARISON

The methodology proposed below involves the component-wise comparison of objects. Their belonging to the same class is determined by a function, i.e., the total value of the deviations between the characteristics (indicators) of the corresponding components. This idea is not novel and is adopted in many applications. One of the simplest and most efficient functions is the well-known metric of Richard Hamming [3], originally developed by him within coding theory. Later, it found application in many fields of science and technology. In addition, there are several modifications, e.g., the weighted Hamming metric [4].

Consider the Hamming metric in detail. It has the general form

$$d\left(x_i,\ x_j\right)=\sum_{i,j=1}^{N}\left|x_i-x_j\right|,$$

where x_i and x_j are the coordinates of the vectors of

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objects under comparison, $i, j = \overline{1, N}$; N denotes the number of object characteristics. The simplest method for determining the proximity of objects is to represent differences as binary relations: if the difference between the corresponding characteristics is below some threshold (e.g., 10%), it takes value 1, and value 0 otherwise. Next, the unities are summed up; if the result equals or exceeds some percentage of the total number of object characteristics (e.g., 90%), such objects are considered close and belong to the same class. The criteria of proximity and belonging (in the current example, 10% and 90%, respectively) are set by researchers arbitrarily depending on the conditions of a particular problem. There are no unified recommendations here, and it seems impossible to provide them. Meanwhile, the approach is quite simple.

Similar ideas are also employed in multidimensional scaling methods to analyze data by reducing their dimension, implemented by comparing objects in different ways. They are widely used in sociology and psychology, and their founder is L. Guttman [5, 6].

Currently, due to advances in machine learning, artificial intelligence, and cybernetics, the clustering problem [7] occupies a special place in science. This problem is similar to classification, in which the classes must be determined by an algorithm.

Nevertheless, various classification methods are still being developed. Among them, we note ATOVIC (Amended fused TOPSIS-VIKOR for classification) [8], a combination of two multicriteria choice methods modified for classification. In this method, an ideal object and the so-called negative object in terms of class belonging are determined for each class. Next, the proximity of each ideal object and each negative object to each class is checked using the Minkowski and Chebyshev metrics, and the belonging of each object is decided accordingly [9]. There exist other metric methods as well; *k* nearest neighbors [10] is the most popular algorithm. It has numerous modifications and can be used with different metrics.

In addition, we mention the naive Bayes classifier, which is based on the Bayes theorem. This classifier uses various probability statistics for decision-making [11].

The practical application of the Hamming metric (including our experience) demonstrated its effectiveness. At the same time, several disadvantages were revealed [12] (a common situation when a methodology developed for solving particular problems is translated to other objects). These disadvantages appear in other metric methods as well; the main one is that even a very high percentage of the matched pairs of characteristics with an equally high criterion of single pair matching (which can be set even below 5%) does not guarantee the belonging of objects to the same class.

For example, let some territories be geographically and ecologically classified by the degree of their pollution. Such an object is often described by selecting a finite number of most typical indicators (characteristics). In this case, almost all indicators, except for one, may have very close results (even completely coincide), giving all formal grounds to assign the same class to the objects under comparison. Nevertheless, a single mismatch may be so large (by orders of magnitude) that the objects cannot belong to the same class under any conditions. This situation occurs for accidental spills and large toxic releases causing environmental disasters. In such cases, the Hamming metric becomes inappropriate because it leads to erroneous results. This metric and its analogs have other drawbacks; see the presentation below.

Despite the considerations above, the principle of comparing and classifying objects based on the analysis of the deviations of their characteristics seems to be justified and methodologically correct. Based on this principle, we propose a function to compare and classify a wide range of objects.

2. THE RANK-EXPERT DEVIATION FUNCTION

To determine whether a given object belongs to a particular class, it must be compared with another object from this class. For such comparisons, we introduce the notion of a reference object, interpreted as an object characterizing a given class to the greatest extent. (How should a reference object be formalized? This issue will be discussed in a separate publication, as it requires a special methodology depending on the object's complexity and the availability of information.) From the viewpoint of computational procedures, a reference is also an object; therefore, we will consider a more general case, i.e., the comparison of two objects.

Deviations can be measured by several methods. The most natural approach is a simple difference expressed in percentage or Boolean symbols. Before that, it is possible to convert all characteristics into points using special scales and obtain differences in unified units. Indicators are often normalized to bring them to a dimensionless form. Other methods are also used to make object characteristics comparable.

This paper proposes a method based on ranking scales with one important property: verbal definitions have no sign, and negative and positive values, both quantitative and qualitative, can be therefore treated as unambiguous. This property is crucial for some characteristics. For example, when assessing the climate comfort for living, air temperature is estimated approximately in the range from –50°C to +40°C; when assessing the efficiency of industrial enterprises by their financial result, which can be both negative and positive, the degree of their unprofitability or profitability is estimated.

We now proceed to the function proposed in this paper. Consider two objects *x* and *y* with the sets of characteristics $\bar{\mathbf{x}} = \left(x_1, \ldots, x_N\right)$ and $\bar{\mathbf{y}} = \left(y_1, \ldots, y_N\right)$, respectively, where $\bar{\mathbf{x}}, \bar{\mathbf{y}} \in \mathbb{R}^N$. In other words, the characteristics belong to the set of *N*–dimensional vectors, and their elements belong to the set of real numbers. Let the characteristics be represented in their "natural" units of measurement (e.g., physical). Then the vector of their deviations, $\bar{\mathbf{z}} \in \mathbb{R}_+^N \cup \{\mathbf{0}\}$, with nonnegative coordinates has the form

$$\bar{\mathbf{z}} = \left|\bar{\mathbf{x}} - \bar{\mathbf{y}}\right| = \left(z_1, \ldots, z_N\right). \tag{1}$$

However, such a vector quantity is not very informative and can only indicate the proximity of the values of objects' characteristics. For example, assume that the values of one characteristic of objects differ by 0.01 units, all others being equal. Does it mean that the objects are close to each other? Not necessarily; information about only two values is insufficient to assess the nature and range of variation of any characteristic.

One method for solving this problem is ranking: the values of characteristics are arranged in a certain order (in our case, by increasing the deviation of their values). Both quantitative and qualitative characteristics are subject to ranking, and this procedure can be therefore used to measure deviations by the difference in ranks between the objects under comparison. In this case, it is possible to obtain a quantitative estimate and also a verbal assessment of the degree of deviation for each characteristic ("small," "considerable," etc.), i.e., to determine its significance. It can be useful in some particular optimization problems, e.g., in the analysis of alternatives.

This approach requires developing special verbal numerical ranking scales for each characteristic. However, the corresponding costs are covered by the possibility of their independent application in other problems.

We introduce a nonnegative matrix of private ranking scales, $\mathbf{H} \in \mathbb{M}_{N \times M}\left(\mathbb{R}_+ \cup \{0\}\right)$. In this matrix, row $i = \overline{1,\ N}$ corresponds to the scale of the characteristic with the same number:

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & \cdots & h_{1,M} \\ \vdots & \ddots & \vdots \\ h_{N,1} & \cdots & h_{N,M} \end{pmatrix},$$

where $M \in \mathbb{N}$ is the maximum rank value, $M \geq 1$. Then the number of gradations or intervals of the ranking scale equals $(M+1)$. (For $M = 1$, we obtain the binary scale.) According to the aforesaid, the scale values must be arranged in ascending order, from the left column of the matrix **H** to the right one: for each *i*, $0 \leq h_{i,1} < \ldots < h_{i,M}$. Different methods are used to construct ranking scales. Based on our experience, it seems preferable to design them by an expert survey within the theory of fuzzy sets.

Different objects can be described by a different number of characteristics; moreover, the same object may have different lengths of components depending on its level of scrutiny or the conditions imposed on the accuracy of its classification. Therefore, the following question arises naturally: what shall we do if the number of gradations is not the same for each component? In this case, a technically simple scale synchronization procedure can be proposed: scale factors are introduced to bring all scales to the same number of gradations. But such situations should be quite justifiably avoided, and objects are often described using scales with the same number of gradations. For example, a single verbal scale of five gradations was developed for all seven characteristics of the atmosphere [13] to classify the atmospheric pollution potential. This verbal scale has been used in practice for several decades.

We define the vector of rank deviations $\bar{\mathbf{r}} = \left(r_1, \ldots, r_N\right)$ of objects *x* and *y*, $\bar{\mathbf{r}} \in \mathbb{N}_0^N$, such that $0 \leq r_i \leq M$, $i = \overline{1,\ N}$. Then

$$r_i = \sum_{j=1}^{M} \theta(z_i - h_{i,j}), \tag{2}$$

where θ denotes the Heaviside function:

$$\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0, \end{cases} \quad x \in \mathbb{R}.$$

For each *i*, formula (2) yields the number of values $h_{i,j}$ strictly less than z_i. This number is the deviation rank, which varies from 0 to *M* inclusive, where *M* is the number of columns in the matrix *H* and (*M* + 1) is the number of gradations of private ranking scales. This expression can also be written in vector form when replacing the Heaviside function with the function of counting all nonnegative elements of the vector.

All object’s characteristics are unequal by their role in the object’s functioning or intended purpose. This value inequality of characteristics is considered through weight coefficients expressed in unit fractions or percentage. We introduce the vector $\bar{\mathbf{k}} = \left(k_1,\ldots,k_N\right)$, $\bar{\mathbf{k}} \in \mathbb{R}_+^N \cup \left\{\mathbf{0}\right\}$, of weight coefficients such that $\sum\limits_{i=1}^{N} k_i = 1$.

Then the rank-expert deviation function is defined as a mapping from the Cartesian product of the two sets of N-dimensional nonnegative real vectors into the set of nonnegative real numbers, i.e., $R : \left(\mathbb{R}_+^N \cup \left\{\mathbf{0}\right\}\right) \times \left(\mathbb{R}_+^N \cup \left\{\mathbf{0}\right\}\right) \to \mathbb{R}_+ \cup \left\{0\right\}$ and is given by

$$R\left(\bar{\mathbf{k}},\ \bar{\mathbf{r}}\right) = \frac{\bar{\mathbf{k}} \cdot \bar{\mathbf{r}}}{M} = \sum_{i=1}^{N} \frac{k_i r_i}{M}, \qquad (3)$$

where $\cdot$ denotes the inner product of vectors. Note that in formula (3), the vector of rank deviations $\bar{\mathbf{r}}$ is also normalized by the maximum rank value. Therefore, the range of R is the set of unit fractions, which can be represented in percentage terms for convenience.

The function described above is not a metric: it satisfies neither the identity axiom nor the triangle inequality. Therefore, this function does not define the distance between objects in some space in the conventional geometric sense. Within the theory of multidimensional scaling, this problem is solved using proximity functions defined as follows [6, *p. 39*]. Consider three vectors of object’s characteristics, a_i, a_j, and a_k. A proximity function $s\left(a_i, a_j\right)$ is a function such that, for all i, j, k, the following relations (axioms) are satisfied: $s(a_i,\ a_i) \geq s(a_i,\ a_j)$; $s(a_i,\ a_j) = s(a_j,\ a_i)$; for large values $s\left(a_i, a_j\right)$ and $s\left(a_j, a_k\right)$, the value $s\left(a_i, a_k\right)$ has at least the same order [6, *p. 39*].

These relations are the weakened axioms from the definition of a metric. As is easily checked, they hold for the rank-expert deviation function defined above, so it represents a proximity function. In particular, the affine transformation, often used in multidimensional scaling methods for scaling and dimension reduction, can be applied to this function as well.

One of the main advantages of the rank-expert deviation function is the possibility to establish equality criteria for values. For example, the objects whose measured physical quantities differ at most by the measurement error should be assumed equal. For such objects, the first gradation of the private ranking scale (the first column of the matrix H) should be not less than the measurement error. In the general case, however, it can be 0, meaning the absence of deviation only if the characteristics are exactly equal. This situation applies, e.g., to qualitative variables.

In this context, the following questions seem natural. Why is it necessary to use private ranking scales? Is it prohibited to take the ratio of characteristics in percentage? For example, formulas (1) and (2) can be in theory replaced by

$$z_i = 1 - \frac{\min(x_i,\ y_i)}{\max(x_i,\ y_i)},\ \ i = \overline{1,\ N}. \qquad (4)$$

We provide the answer with an illustrative example. Let objects be some water masses characterized by the concentration of suspended solids in water, measured in mg/l. It has a rather wide range of variation, taking values in a few mg/l and in a few hundredths of mg/l. If two objects under comparison have concentrations of 0.01 mg/l and 0.02 mg/l, respectively, their difference by formula (4) is 50%; for objects with values of 2 mg/l and 0.01 mg/l, respectively, it equals 99.5%. In this case, the objects with values of 0.01 mg/l and 0.02 mg/l may differ insignificantly within a particular problem or can be taken as equal under a measurement accuracy of 0.01 mg/l. Therefore, in this example, the 50% difference in characteristics becomes incorrect.

3. A UNIVERSAL CLASSIFICATION SCALE FOR OBJECTS

A special scale is needed to assess the proximity of objects under comparison. We propose a possible solution based on the following considerations. According to the aforesaid, the function achieves maximum for the largest difference between the objects, i.e., when the deviation ranks of each characteristic take their maximum value M. The minimum value of the function is 0. (In this case, one object strictly matches another, and all deviations are 0.) Consequently, all possible deviations lie in the range from 0 to 1, or 100%. As a result, the following estimation scale can be constructed (Table 1).

Other modifications of this scale are possible, but it is a matter of discussion. (Note that this scale should be designed by fuzzy set methods.) The graphical form of this scale is presented in Fig. 1. The private ranking scales making up the matrix H have the same form as well.

Table 1

The universal scale for assessing the proximity of objects by the rank-expert deviation function

No.	Interval, %	The degree of proximity
1	0–5	Coincidence
2	6–10	Minor discrepancy
3	11–20	Slight discrepancy
4	21–30	Moderate discrepancy
5	31–40	Notable discrepancy
6	41–55	Substantial discrepancy
7	56–70	Significant discrepancy
8	71–100	Very significant discrepancy

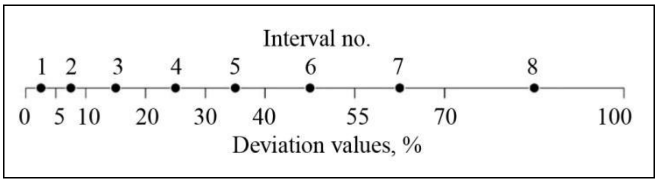

Fig. 1. The universal scale for assessing the proximity of objects. Numbers above the scale indicate the intervals according to Table 1.

4. NUMERICAL EXAMPLES

Example 1. This example primarily demonstrates the technical aspects of applying the calculation procedures: the techniques for selecting the private ranking scales and weight coefficients are beyond the scope of this paper.

Consider an environmental problem of comparing the vulnerability of territories to accidental oil spills [14]. In this case, the objects are the domains with the available values of indicators. We adopt three indicators:

- the forecasted (simulation-based) pollution area for a particular sector of the terrain (m^2), used to assess the degree of damage and the scope of restoration works;
- the average surface slope (%), affecting the rate of pollution spreading on the ground surface and the shape of the oil spot;
- the share of waterbodies (%), where oil and oil products are transported by water streams to significant distances, thereby increasing the pollution area (banks and coastal territories) and environmental damage.

For these indicators, private ranking scales with four gradations were developed and weight coefficients were determined. Thus, $N = 3$ and $M = 4$; the input data for the calculations are provided in Table 2.

Note that all values in this table, except for columns 1 and 4, are given in the indicator units. Columns 5–8 of Table 2 present the matrix of private ranking scales H. Next, Table 3 shows the vector of the deviations of object's characteristics (formula (1), column 2), its transformation into the vector of rank deviations (formula (2), columns 3–6 and 7), and the scalar product (formula (3), column 8). The transformation into the ranking scales is carried out by counting positive values in columns 3–6, which is the rank value (column 7).

The value of the rank-expert deviation function is 0.725; it means that the objects differ from each other by 72.5%. According to the universal classification scale, it corresponds to a very significant discrepancy by the degree of proximity (see Table 1).

Table 2

Input data for calculating the rank-expert deviation function

Characteristic	i	$\bar{x}$	$\bar{y}$	$\bar{k}$	$h_{i,1}$	$h_{i,2}$	$h_{i,3}$	$h_{i,4}$
Column no.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Pollution area, m^2	1	2376.7	1831.6	0.5	10	25	100	200
Surface slope, %	2	1.05	1.13	0.3	0.05	0.25	0.5	1
The share of waterbodies, %	3	2	0.24	0.2	0.01	0.1	1	5

Table 3

Calculation results for the rank-expert deviation function

i	$\bar{z}$	$z_i - h_{i,1}$	$z_i - h_{i,2}$	$z_i - h_{i,3}$	$z_i - h_{i,4}$	r_i	$r_i \cdot k_i / M$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	545.1	535.1	520.1	445.1	345.1	4	0.5
2	0.08	0.03	–0.17	–0.42	–0.92	1	0.075
3	1.76	1.75	1.66	0.76	–3.24	3	0.15
The value of the rank-expert deviation function							$R = 0.725$

Such tables can be constructed for each sector of the terrain of the object or a separate alternative. Then we obtain some set of objects for classification, which is a primary goal when studying objects and alternatives, e.g., in industrial safety problems and choice problems (see below). ♦

Example 2. As another example, we solve a classical economic problem of choosing real estate objects (apartments in a building). There are three alternatives (*a*, *b*, and *c*) and five different characteristics of apartments. We denote their vectors by $\bar{\mathbf{a}}$, $\bar{\mathbf{b}}$, and $\bar{\mathbf{c}}$ (Table 4, columns 3–5). The easiest way to form a reference object ($\bar{\mathbf{e}}$) is to take the minimum or maximum possible values of the characteristics. For example, for the depreciation of the building (in %), the acceptable value of the reference is 0: the less the depreciation is, the better the alternative will be (Table 4, column 2). We form the vector of weight coefficients (the fastest method is simple ranking) and arrange the characteristics in descending order of their significance by number *i*: 1, 2, 3, 4, and 5. As a result, for this problem, $k_i = (6 - i)/15$ (Table 4, column 6). Next, we construct the matrix of private ranking scales $\mathbf{H}$ with four gradations or intervals for each characteristic. Then $N = 5$ and $M = 3$. In this case, to simplify the example, the scales are formed by questioning one expert (the purchaser or his or her representative). There may be several experts; as we believe, the most appropriate number is about 10 people. Also, the scale can be designed by other methods. Thus, all the initial data necessary for the rank-expert deviation function have been formed (see Table 4).

Let us proceed to the calculations, i.e., the sequential application of this function to compare the reference with each alternative. For convenience, the first few steps of calculations are omitted; they are carried out by analogy with the previous example (see Table 3, columns 2–6). Table 5 shows the rank values (columns 2, 4, and 6), their normalization (columns 3–5), and the values of the rank-expert deviation function for each alternative.

In this case, the best alternative is *b* as the one with the smallest value of the rank-expert deviation function $R = 0.553$. (It differs least from the reference.) ♦

Table 4

Input data for calculating the rank-expert deviation function

Characteristic	i	$\bar{e}$	$\bar{a}$	$\bar{b}$	$\bar{c}$	$\bar{k}$	$h_{i,1}$	$h_{i,2}$	$h_{i,3}$
Column no.	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Value, thousand rubles/m^2	1	70	80	100	150	0.33	5	10	50
Distance to the city center, km	2	1	15	5	6	0.27	5	10	15
The depreciation of the building, %	3	0	30	60	10	0.2	10	25	50
The number of parking lots near the building, pcs.	4	1000	500	50	100	0.07	100	500	1000
Apartment area, m^2	5	60	63	78	55	0.13	5	10	25

Table 5

Calculation results for the rank-expert deviation function

Alternative	i	$\bar{a}$		$\bar{b}$		$\bar{c}$	
		r_i	$r_i \cdot k_i / M$	r_i	$r_i \cdot k_i / M$	r_i	$r_i \cdot k_i / M$
Column no.	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Value, thousand rubles/m^2	1	2	0.220	2	0.220	3	0.330
Distance to the city center, km	2	2	0.180	0	0.000	1	0.090
The depreciation of the building, %	3	2	0.133	3	0.200	1	0.067
The number of parking lots near the building, pcs.	4	2	0.047	2	0.047	2	0.047
Apartment area, m^2	5	0	0.000	2	0.087	1	0.043
The value of the rank-expert deviation function R		0.580		0.553		0.577	

5. APPLICATIONS OF THE RANK-EXPERT DEVIATION FUNCTION

The function proposed above serves to classify objects with characteristics (indicators) expressed in a variety of quantitative and qualitative values. First of all, such objects include social and economic, geographical, and ecological objects. They are formalized by considering many factors from various fields of knowledge: natural, technical, social, military, geopolitical, and others. Ranking scales make such characteristics commensurable. It is possible to use available verbal numerical scales provided that they correspond to the problem conditions.

Generally speaking, the rank-expert deviation function and its components can be treated as one element of classification technologies [15] and can be used for such complex problems as the geographical and ecological zoning of territories [14] and others. We will demonstrate it on some intentionally simplified examples from real life.

Consider again the vulnerability of territories to accidental oil spills along a pipeline route (see Example 1 in Section 4). Let us divide the entire route into equally long sectors. The length and width of the sectors are of no fundamental importance: due to the capabilities of GIS technologies, they can be set as small as desired. The rank-expert deviation function calculated for each sector (see Table 4) is mapped (Fig. 2) to assess the vulnerability of the entire object, to develop scientifically grounded recommendations for industrial environmental monitoring, and to select optimal locations for emergency spill response equipment (barrier booms, sorbents, etc.). For example, such equipment should be placed in the sector of the terrain with the highest vulnerability (according to Fig. 2, in the northeastern sector with $R = 0.87$).

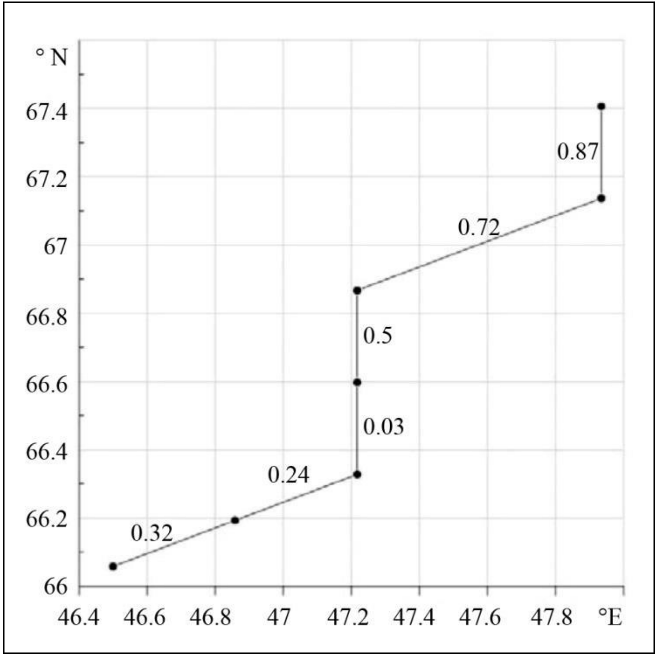

Fig. 2. The value diagram of the rank-expert deviation function for the fragment of a linear object (oil pipeline).

CONCLUSIONS

An object classification methodology based on the rank-expert deviation function has been proposed. It represents a set of procedures performed in the following recommended sequence.

1) The goal of classification is determined.

2) According to this goal, object's characteristics are selected for an appropriate formalization of the object; the number of such characteristics is not limited but must be the same for all objects.

3) Private verbal numerical ranking scales are constructed (if available, selected) for each characteristic of the object. All of them must have the same number of gradations or be preliminarily synchronized using a correction factor.

4) A reference object with the most appropriate characteristics is designed for each class.

5) The weight coefficients of the characteristics are calculated.

6) After determining the initial data, the rank-expert deviation function is calculated in three steps.

6.1) The vector of the deviations of characteristics is constructed.

6.2) This vector is transformed into the vector of rank deviations.

6.3) The vector of rank deviations is normalized, and the inner product of the normalized one with the vector of weight coefficients is obtained.

7) The verbal numerical scale is used to find the degree of proximity of the object under comparison and the reference objects; if necessary, a sequential comparison with other reference objects is made.

This function may serve to solve other problems, e.g., simple comparison of two objects with specified characteristics in the analysis of alternatives, scenario design problems, and object optimization.

However, all values of the rank-expert deviation function may fall into the same gradation, i.e., the object is homogeneous. This situation sometimes occurs in practice. In such cases, additional classification rules are developed to differentiate the object under consideration. It often suffices to add one or two conditions; e.g., among equally important alternatives, priority can be given to the one where the greatest contribution is made by the characteristic with the highest weight factor and the minimum deviation from the reference.

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A NUMERICAL AGGREGATION METHOD FOR FINITE-STATE MACHINES USING ALGEBRAIC OPERATIONS

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Abstract. This paper considers the problem of synthesizing finite-state machines (FSMs) based on algebraic methods. The aggregation operations of FSMs are numerically implemented using symbolic matrices that describe their functioning. An algebra is defined for these matrices as follows: the carriers are matrix elements and special symbols, and the signature includes two operations serving to determine actions over these symbols. As a result, it becomes possible to define an algebra of symbolic matrices whose signature includes three operations. The classical operations over FSMs are represented in matrix form based on the algebra of symbolic matrices. Next, special operations over FSMs are constructed involving classical operations over them. Special operations are constructed considering the constraints and requirements of the subject area. A numerical example of FSM synthesis—the joint activity of two functional groups in an emergency zone—is provided.

Keywords: synthesis of automata, algebra of automata, symbolic matrices.

INTRODUCTION

The theory of finite-state machines (FSMs, also termed finite automata in the literature) is an effective apparatus for modeling the functioning of objects and systems [1]. Presently, an urgent problem is to describe the functioning of several interacting objects or systems of different nature. This problem can be solved by aggregating the corresponding FSMs [2–4]. Therefore, it is necessary to introduce operations on the set of FSMs [2, 3, 5] in order to aggregate FSMs under the constraints and requirements of the subject area. Among them, we mention the following possibilities:

1) the functioning of the general FSM with synchronous state changing if the aggregated FSMs simultaneously change the states;

2) the functioning of the general model with asynchronous state change if the state change instants do not coincide;

3) the initiation of state change of one FSM model by the output action of another FSM model;

4) the elimination of certain combinations of states from the general model since the objects or systems modeled cannot simultaneously be in these states.

An example of a subject area with such constraints is the process of emergency response, which involves joint actions of several functional groups [6, 7].

Aggregation operations allow synthesizing FSMs, yielding an algebra [8] of FSMs $\mathcal{A} = \langle \mathcal{M}, \mathcal{S} \rangle$, where $\mathcal{M}$ and $\mathcal{S}$ denote the set of FSMs (the carrier) and the set of operations over them (the signature), respectively.

Synthesis of FSMs is a traditional problem of automata theory. As a rule, parallel and (or) sequential composition operations are used [2–5, 9]. However, when implementing these operations in practice, one should consider the constraints of the subject area.

To solve modeling problems using an algebra $\mathcal{A}$, it is necessary to develop a numerical method for the software implementation of FSM aggregation operations considering additional requirements. For this purpose, the matrix representation of FSMs [10] can be adopted; as a result, the operations over FSMs are reduced to matrix operations.

In the works [11–14], FSM synthesis procedures were described by tables or transition matrices with additional conditions for identifying their elements to ensure the commutativity of aggregation operations. This fact increases the computational complexity of the numerical synthesis method of an FSM.

In this regard, we develop below a numerical method with the sets of input and output symbols of given FSMs as the input and output symbols of the general FSM synthesized from them. In this case, the matrices describing the functioning of FSMs consist of sets. Such matrices are further called symbolic matrices; modern programming languages allow handling their elements.

In addition, the FSM models of objects and systems in many subject areas are synthesized only with the joint use of parallel and sequential composition operations involving the same actions.

The numerical method proposed in this paper excludes duplication when executing aggregation operations considering the requirements of the subject area.

1. THE REPRESENTATION OF FINITE-STATE MACHINES BY SYMBOLIC MATRICES

Let a Mealy FSM $A=\left(X, Y, Q, \lambda, \mu\right)$ be given with the following notations: X is an input alphabet; Y is an output alphabet; Q is the set of states; $\lambda : X\times Q\rightarrow Q$ is the transition function; finally, $\mu : X\times Q\rightarrow Y$ is the output function. The functions λ and μ can be fully characterized by the operator F of the form

$$\left\{Fq^{i}=\left\{q^{i_1}\left(x^{j_1}/y^{k_1}\right),\ldots, q^{i_l}\left(x^{j_l}/y^{k_l}\right),\ldots,\right.\right.$$
$$\left.\left.q^{i_{n_i}}\left(x^{j_{n_i}}/y^{k_{n_i}}\right)\right\}, i=\overline{1,|Q|}\right\}.$$

The expression $q^{i_l}\left(x^{j_l}/y^{k_l}\right)\in Fq^{i}$ means that if the FSM is in a state q^{i} and the input symbol is x^{j_l}, it will pass to a state q^{i_l} with an output symbol y^{k_l}.

The functions λ and μ can be described by two square symbolic matrices

$$R_A^X=\left(r_{ij}^x\right)_{i,j=\overline{1,|Q|}} \text{ and } R_A^Y=\left(r_{ij}^y\right)_{i,j=\overline{1,|Q|}},$$

respectively, where

$$r_{ij}^x=\begin{cases} x \text{ if } q^{j}(x/y)\in Fq^{i} \\ \theta \text{ otherwise,} \end{cases}$$

and

$$r_{ij}^y=\begin{cases} y \text{ if } q^{j}(x/y)\in Fq^{i} \\ \theta \text{ otherwise.} \end{cases}$$

Accordingly, the operator F can be described by a unique connection matrix $R_A=\left(R_A^X/R_A^Y\right)$ expressed through the matrices R_A^X and R_A^Y as follows:

$$R_A=\begin{pmatrix} x_A^{11}/y_A^{11} & \cdots & x_A^{1|Q|}/y_A^{1|Q|} \\ \vdots & \ddots & \vdots \\ x_A^{|Q|1}/y_A^{|Q|1} & \cdots & x_A^{|Q||Q|}/y_A^{|Q||Q|} \end{pmatrix}. \tag{1}$$

The symbol θ in these matrices characterizes the impossibility of transition from a state q^{i} to a state q^{j} if the FSM A is partial. The elements of the matrices R_A^X and R_A^Y are sets, whereas the elements of the matrix R_A are the ordered pairs of sets.

Let us illustrate the matrix representation of FSMs by a simple example.

Consider FSMs $A_1=\left(X_{A_1}, Y_{A_1}, Q_{A_1}, F_{A_1}\right)$ and $A_2=\left(X_{A_2}, Y_{A_2}, Q_{A_2}, F_{A_2}\right)$, where:

- The input alphabets are $X_{A_1}=\left\{x_{A_1}^1, x_{A_1}^2\right\}$ and $X_{A_2}=\left\{x_{A_2}^1, x_{A_2}^2\right\}$.
- The output alphabets are $Y_{A_1}=\left\{y_{A_1}^1, y_{A_1}^2\right\}$ and $Y_{A_2}=\left\{y_{A_2}^1, y_{A_2}^2\right\}$.
- The sets of states are $Q_{A_1}=\left\{q_{A_1}^1, q_{A_1}^2\right\}$ and $Q_{A_2}=\left\{q_{A_2}^1, q_{A_2}^2\right\}$.
- The operators are given by

$$F_{A_1}=\left\{\begin{matrix} F_{A_1}q_{A_1}^1=\left\{q_{A_1}^1\left(x_{A_1}^1/y_{A_1}^1\right), q_{A_1}^2\left(x_{A_1}^2/y_{A_1}^2\right)\right\}, \\ F_{A_1}q_{A_1}^2=\varnothing \end{matrix}\right\}$$

$$\text{and } F_{A_2}=\left\{\begin{matrix} F_{A_2}q_{A_2}^1=\left\{q_{A_2}^2\left(x_{A_2}^1/y_{A_2}^1\right)\right\}, \\ F_{A_2}q_{A_2}^2=\left\{q_{A_2}^2\left(x_{A_2}^2/y_{A_2}^2\right)\right\} \end{matrix}\right\}.$$

The matrices of input and output connections have the form

$$R_{A_1}^X=\begin{pmatrix} x_{A_1}^1 & x_{A_1}^2 \\ \theta & \theta \end{pmatrix},\ R_{A_2}^X=\begin{pmatrix} \theta & x_{A_2}^1 \\ \theta & x_{A_2}^2 \end{pmatrix},$$

$$R_{A_1}^Y=\begin{pmatrix} y_{A_1}^1 & y_{A_1}^2 \\ \theta & \theta \end{pmatrix},\ R_{A_2}^Y=\begin{pmatrix} \theta & y_{A_2}^1 \\ \theta & y_{A_2}^2 \end{pmatrix}.$$

According to (1), the connection matrices of the FSMs A_1 and A_2 are given by

$$R_{A_1}=\begin{pmatrix} x_{A_1}^1/y_{A_1}^1 & x_{A_1}^2/y_{A_1}^2 \\ \theta & \theta \end{pmatrix},\ R_{A_2}=\begin{pmatrix} \theta & x_{A_2}^1/y_{A_2}^1 \\ \theta & x_{A_2}^2/y_{A_2}^2 \end{pmatrix}.$$

To introduce actions over symbolic matrices, it is necessary to define operations over the elements of these matrices (symbols), which are arbitrary sets.

2. ALGEBRAS OF SYMBOLS AND SYMBOLIC MATRICES

We denote by $\mathcal{M}$ the set of square symbolic matrices and by M the set of their elements. Assume that the set M additionally includes a special element ε, an analog of the unit element, which cannot be an input or output symbol of FSMs by definition.

We define an algebra $\mathcal{A}_1=\langle M,\cdot,\vee\rangle$ as follows.

Let $c_1, c_2\in M$; the operations $\cdot$ and $\vee$ are executed according to the following rules:

$$\begin{gathered}\forall c_1, c_2\notin\{\theta,\varepsilon\}\ c_1\cdot c_2=\{c_1, c_2\};\\ \forall c_1\ \theta\cdot c_1=c_1\cdot\theta=\theta;\\ \forall c_1, c_2\neq\theta\ \varepsilon\cdot c_1=c_1\cdot\varepsilon=c_1;\\ \forall c_1\ \theta\vee c_1=c_1\vee\theta=c_1.\end{gathered}$$

The expression $c_1\vee c_2$ means that:

– If one of the input symbols c_1 or c_2 arrives at the FSM input, it is used for transition.

– If both symbols arrive simultaneously, the operation $c_1\cdot c_2$ is executed.

Assume that the set $\mathcal{M}$ contains an identity matrix of dimensions $k\times k$:

$$E_k=\begin{pmatrix}\varepsilon & \cdots & \theta\\ \vdots & \ddots & \vdots\\ \theta & \cdots & \varepsilon\end{pmatrix}.$$

We define an algebra $\mathcal{A}_2=\langle\mathcal{M},\cdot,\times,\cup\rangle$ as follows.

Let $c_1\in M$ and $V, W\in\mathcal{M}$ be given elements such that

$$V=\left(v^{ij}\right)_{i,j=\overline{1,n}}\ \text{and}\ W=\left(w^{ij}\right)_{i,j=\overline{1,m}}.$$

Then the operations $\cdot$, $\vee$, and $\times$ are executed according to the following rules:

$$c_1\cdot V=\begin{pmatrix}c_1\cdot v_{11} & \cdots & c_1\cdot v_{1n}\\ \vdots & \ddots & \vdots\\ c_1\cdot v_{n1} & \cdots & c_1\cdot v_{nn}\end{pmatrix};\qquad (2)$$

$$V\times W=\begin{pmatrix}v_{11}\cdot W & \cdots & v_{1n}\cdot W\\ \vdots & \ddots & \vdots\\ v_{n1}\cdot W & \cdots & v_{nn}\cdot W\end{pmatrix};\qquad (3)$$

if $n=m$, then

$$V\cup W=\begin{pmatrix}v_{11}\vee w_{11} & \cdots & v_{1n}\vee w_{1n}\\ \vdots & \ddots & \vdots\\ v_{n1}\vee w_{n1} & \cdots & v_{nn}\vee w_{nn}\end{pmatrix}.\qquad (4)$$

The identity matrix is necessary to define the sum operation over FSMs, so it has been introduced separately.

The algebra $\mathcal{A}_2$ is convenient for handling input and output connection matrices R_A^X and R_A^Y of FSMs. However, to simplify the description of certain operations over FSMs, we extend the operations $\times$ and $\cup$ of the algebra $\mathcal{A}_2$ to the connection matrices R_{A_1} and R_{A_2}.

Let $*\in\{\times,\cup\}$; then

$$R_{A_1}*R_{A_2}=\left(\left(R_{A_1}^X*R_{A_2}^X\right)/\left(R_{A_1}^Y*R_{A_2}^Y\right)\right).$$

We illustrate the execution of the signature operations $\{\cdot,\times,\cup\}$ on an example of the FSMs A_1 and A_2. Since the operation $\cdot$ is executed when implementing the operation $\times$, we calculate $R_{A_1}^X\times R_{A_2}^X$ and $R_{A_1}^X\cup R_{A_2}^X$ only.

Due to (2) and (3), the matrix of output connections $R_{A_1}^X\times R_{A_2}^X$ has the form

$$R_{A_1}^X\times R_{A_2}^X=\begin{pmatrix}x_{A_1}^1 & x_{A_1}^2\\ \theta & \theta\end{pmatrix}\times\begin{pmatrix}\theta & x_{A_2}^1\\ \theta & x_{A_2}^2\end{pmatrix}$$

$$=\begin{pmatrix} x_{A_1}^1\cdot\begin{pmatrix}\theta & x_{A_2}^1\\ \theta & x_{A_2}^2\end{pmatrix} & x_{A_1}^2\cdot\begin{pmatrix}\theta & x_{A_2}^1\\ \theta & x_{A_2}^2\end{pmatrix}\\ \theta\cdot\begin{pmatrix}\theta & x_{A_2}^1\\ \theta & x_{A_2}^2\end{pmatrix} & \theta\cdot\begin{pmatrix}\theta & x_{A_2}^1\\ \theta & x_{A_2}^2\end{pmatrix}\end{pmatrix}$$

$$=\begin{pmatrix}\theta & \left\{x_{A_1}^1, x_{A_2}^1\right\} & \theta & \left\{x_{A_1}^2, x_{A_2}^1\right\}\\ \theta & \left\{x_{A_1}^1, x_{A_2}^2\right\} & \theta & \left\{x_{A_1}^2, x_{A_2}^2\right\}\\ \theta & \theta & \theta & \theta\\ \theta & \theta & \theta & \theta\end{pmatrix}.$$

In view of (4), the connection matrix $R_{A_1}^X \cup R_{A_2}^X$ has the form

$$R_{A_1}^X \cup R_{A_2}^X = \begin{pmatrix} x_{A_1}^1 & x_{A_1}^2 \\ \theta & \theta \end{pmatrix} \cup \begin{pmatrix} \theta & x_{A_2}^1 \\ \theta & x_{A_2}^2 \end{pmatrix}$$

$$= \begin{pmatrix} x_{A_1}^1 \vee \theta & x_{A_1}^2 \vee x_{A_2}^1 \\ \theta \vee \theta & \theta \vee x_{A_2}^2 \end{pmatrix} = \begin{pmatrix} x_{A_1}^1 & x_{A_1}^2 \vee x_{A_2}^1 \\ \theta & x_{A_2}^2 \end{pmatrix}.$$

Similarly, calculating $R_{A_1}^Y \times R_{A_2}^Y$ and $R_{A_1}^Y \cup R_{A_2}^Y$ and using (1), we obtain

$$R_{A_1} \times R_{A_2}$$

$$= \begin{pmatrix} \theta & \{x_{A_1}^1, x_{A_2}^1\} / \{y_{A_1}^1, y_{A_2}^1\} & \theta & \{x_{A_1}^2, x_{A_2}^1\} / \{y_{A_1}^2, y_{A_2}^1\} \\ \theta & \{x_{A_1}^1, x_{A_2}^2\} / \{y_{A_1}^1, y_{A_2}^2\} & \theta & \{x_{A_1}^2, x_{A_2}^2\} / \{y_{A_1}^2, y_{A_2}^2\} \\ \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta \end{pmatrix},$$

$$R_{A_1} \cup R_{A_2} = \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta \end{pmatrix}$$

$$\cup \begin{pmatrix} \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}$$

$$= \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & x_{A_1}^2 / y_{A_1}^2 \vee x_{A_2}^1 / y_{A_2}^1 \\ \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}.$$

3. THE MATRIX REPRESENTATION OF CLASSICAL OPERATIONS OVER FINITE-STATE MACHINES

Consider Mealy FSMs $A_1 = \left(X_{A_1}, Y_{A_1}, Q_{A_1}, F_{A_1}\right)$ and $A_2 = \left(X_{A_2}, Y_{A_2}, Q_{A_2}, F_{A_2}\right)$ and the corresponding connection matrices R_{A_1} and R_{A_2}.

The following operations over FSMs are used to aggregate FSM models. Their matrix description is based on the operations of the algebra $\mathcal{A}_2$ introduced above.

We define the connection matrix R_Π of the FSM $\Pi = A_1 \times A_2$ as follows:

$$R_\Pi = R_{A_1} \times R_{A_2}. \tag{5}$$

The FSM Π describes a parallel synchronous change in the states of the FSMs A_1 and A_2 since its matrix is composed of the pairs of element sets $\left\{x_{A_1}^{k_1}, x_{A_2}^{k_2}\right\} \Big/ \left\{y_{A_1}^{l_1}, y_{A_2}^{l_2}\right\}$ such that one element of each set belongs to A_1 and the other to A_2. Consequently, each such pair describes a simultaneous change in the states of the FSMs A_1 and A_2 that make up the FSM Π.

We define the connection matrix R_Σ of the FSM $\Sigma = A_1 + A_2$ as follows:

$$R_\Sigma = \left(R_{A_1} \times E_{\left|Q_{A_2}\right|}\right) \cup \left(E_{\left|Q_{A_1}\right|} \times R_{A_2}\right). \tag{6}$$

The FSM Σ describes an asynchronous change in the states of the FSMs A_1 and A_2 since its matrix is composed of the elements $x_{A_i}^k / y_{A_i}^l$, $i = 1, 2$: they describe a change in the state of only one component (A_1 or A_2) of the FSM Σ.

The composition $\circ$ of FSMs is used to initiate the functioning of one FSM by means (output symbols) of another FSM. The connection matrix R_{K} of the FSM $\mathrm{K} = A_1 \circ A_2$ is defined as follows:

$$R_{A_1} \circ R_{A_2} = \left(r_{ps}\right)_{p,s=\overline{1,\left|Q_{A_1}\right| \cdot \left|Q_{A_2}\right|}}, \tag{7}$$

where $r_{p,s} = \begin{cases} \left\{x_{A_1}, x_{A_2}\right\} / \left\{y_{A_1}, y_{A_2}\right\} \\ \quad \text{if } y_{A_1} = x_{A_2} \text{ or } y_{A_2} = x_{A_1} \\ \theta \text{ otherwise.} \end{cases}$

The union $\cup$ of FSMs is necessary to describe their functioning in different modes. The connection matrix R_C of the FSM $C = A_1 \cup A_2$ is defined as follows:

$$R_C = R_{A_1} \cup R_{A_2}. \tag{8}$$

The aggregation of FSM models involves several operations over automata; for example, it is necessary to construct an FSM model that represents both synchronous and asynchronous changes in the states of FSMs. Note that these operations are executed with the same actions (e.g., constructing the set of states). Therefore, it is reasonable to combine classical operations over FSMs into groups to avoid any duplication of these actions.

Consider the operations $+$ and $\circ$ on an example of the FSMs A_1 and A_2 introduced in Section 1. (The operations $\times$ and $\cup$ have been described in Section 2; see formulas (5) and (8).)

Due to (6), the connection matrix describing the FSM $A_1 + A_2$ has the form

$$\left(R_{A_1} \times E_{\left|Q_{A_2}\right|}\right) \cup \left(E_{\left|Q_{A_1}\right|} \times R_{A_2}\right)$$

$$= \left(\begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta \end{pmatrix} \times \begin{pmatrix} \varepsilon & \theta \\ \theta & \varepsilon \end{pmatrix}\right)$$

$$\cup \left(\begin{pmatrix} \varepsilon & \theta \\ \theta & \varepsilon \end{pmatrix} \times \begin{pmatrix} \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}\right)$$

$$= \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & \theta & x_{A_1}^2 / y_{A_1}^2 & \theta \\ \theta & x_{A_1}^1 / y_{A_1}^1 & \theta & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta \end{pmatrix}$$

$$\cup \begin{pmatrix} \theta & x_{A_2}^1 / y_{A_2}^1 & \theta & \theta \\ \theta & x_{A_2}^2 / y_{A_2}^2 & \theta & \theta \\ \theta & \theta & \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & \theta & \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}$$

$$= \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & x_{A_2}^1 / y_{A_2}^1 & x_{A_1}^2 / y_{A_1}^2 & \theta \\ \theta & x_{A_1}^1 / y_{A_1}^1 \vee x_{A_2}^2 / y_{A_2}^2 & \theta & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta & \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & \theta & \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}.$$

For a more visual representation of the connection matrix describing the FSM $A_1 \circ A_2$, we impose an additional condition, i.e., $y_{A_1}^1 = x_{A_2}^1$, $y_{A_1}^2 = x_{A_2}^1$, and $y_{A_1}^1 = x_{A_2}^2$. Then, according to (7), the connection matrix of the FSM $A_1 \circ A_2$ has the form

$$R_{A_1} \circ R_{A_2}$$

$$= \begin{pmatrix} \theta & \left\{x_{A_1}^1, x_{A_2}^1\right\} / \left\{y_{A_1}^1, y_{A_2}^2\right\} & \theta & \left\{x_{A_1}^2, x_{A_2}^1\right\} / \left\{y_{A_1}^2, y_{A_2}^1\right\} \\ \theta & \left\{x_{A_1}^1, x_{A_2}^2\right\} / \left\{y_{A_1}^1, y_{A_2}^2\right\} & \theta & \theta \\ \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta \end{pmatrix}.$$

4. THE MATRIX REPRESENTATION OF COMBINED OPERATIONS OVER FINITE-STATE MACHINES

In applications, under the requirements of the subject area, it is often necessary to use the aggregation operations of FSMs jointly. For example, the following combined operations describe requirements 1–3 (see the Introduction) and serve to model emergency response processes.

We define the operation $\otimes$ over FSMs A_1 and A_2 as follows:

$$\Psi = A_1 \otimes A_2 = \left(A_1 \times A_2\right) \cup \left(A_1 + A_2\right).$$

Its matrix representation has the form

$$R_\Psi = \left(R_{A_1} \times R_{A_2}\right) \cup \left[\left(R_{A_1} \times E_{\left|Q_{A_2}\right|}\right) \cup \left(E_{\left|Q_{A_1}\right|} \times R_{A_2}\right)\right]. \quad (9)$$

The elements of the matrix R_Ψ are pairs of the form $\left\{x_{A_1}^{k_1}, x_{A_2}^{k_2}\right\} / \left\{y_{A_1}^{l_1}, y_{A_2}^{l_2}\right\}$ or $x_{A_i}^k / y_{A_i}^l$, $i = 1, 2$. In other words, they characterize both synchronous and asynchronous changes in the states of the FSMs A_1 and A_2.

We define the operation $\odot$ over FSMs A_1 and A_2 as follows:

$$\Phi = A_1 \odot A_2 = \left(A_1 \circ A_2\right) \cup \left(A_1 + A_2\right).$$

Its matrix representation has the form

$$R_\Phi = \left(R_{A_1} \circ R_{A_2}\right) \cup \left[\left(R_{A_1} \times E_{\left|Q_{A_2}\right|}\right) \cup \left(E_{\left|Q_{A_1}\right|} \times R_{A_2}\right)\right]. \quad (10)$$

The elements of the matrix R_Φ are pairs of the form $\left\{x_{A_1}^{k_1}, x_{A_2}^{k_2}\right\} / \left\{y_{A_1}^{l_1}, y_{A_2}^{l_2}\right\}$, where $y_{A_1}^{l_1} = x_{A_2}^{k_2}$ or $y_{A_2}^{l_2} = x_{A_1}^{k_1}$, or of the form $x_{A_i}^k / y_{A_i}^l$, $i = 1, 2$. In other words, they characterize both the initiation of the functioning of one FSM by means (output symbols) of another FSM and an asynchronous change in the states of the FSMs A_1 and A_2.

Let an FSM $A_3 = \left(X_{A_3}, Y_{A_3}, Q_{A_3}, F_{A_3}\right)$ be obtained from FSMs A_1 and A_2 by transformations using the operations $\otimes$ and $\odot$. We denote by Ξ_{A_3} the set of all inadmissible states of the FSM A_3. The filtering operation ∇ of the FSM A_3 over the set Ξ_{A_3}, ∇A_3, is defined as follows:

$$Q_{A_3} = Q_{A_3} \setminus \Xi_{A_3}.$$

To derive the matrix representation of the filtering operation, we write the connection matrix of the FSM A_3:

$$R_{A_3} = \left(r_{ij}\right)_{i,j=1,\overline{\left|Q_{A_3}\right|}}.$$

Then the filtering operation has the form

$$R_{\nabla A_3} = M_{i_1,\dots,i_n}^{i_1,\dots,i_n}, \tag{11}$$

where $M_{i_1,\dots,i_n}^{i_1,\dots,i_n}$ is a submatrix of the matrix R_{A_3}; $i_1,\dots, i_n$ specify the rows and columns excluded from the matrix R_{A_3}; $i_k = i$ if $q_{A_3}^i \in \Xi_{A_3}$ (i.e., this state belongs to the set of inadmissible states Ξ_{A_3}). The filtering operation can be also used to reduce the sets X_{A_3} and Y_{A_3}: if some input symbols in the set X_{A_3} do not belong to the matrix $R_{\nabla A_3}$ after the filtering operation, they are removed from the set X_{A_3} and the result is the set X_{A_3}. A similar procedure is performed for the set Y_{A_3}, yielding the set Y_{A_3}. The operator F_{A_3} also turns into the operator F_{A_3}, which is defined by the matrix $R_{\nabla A_3}$.

Here is an example of the filtering operation over FSMs A_1 and A_2. According to (9), the matrix describing the functioning of the FSM $A_1 \otimes A_2$ has the form

$$R_{A_1} \otimes R_{A_2} = \begin{pmatrix} \theta & \left\{x_{A_1}^1, x_{A_2}^1\right\}/\left\{y_{A_1}^1, y_{A_2}^1\right\} & \theta & \left\{x_{A_1}^2, x_{A_2}^1\right\}/\left\{y_{A_1}^2, y_{A_2}^1\right\} \\ \theta & \left\{x_{A_1}^1, x_{A_2}^2\right\}/\left\{y_{A_1}^1, y_{A_2}^2\right\} & \theta & \left\{x_{A_1}^2, x_{A_2}^2\right\}/\left\{y_{A_1}^2, y_{A_2}^2\right\} \\ \theta & \theta & \theta & \theta \\ \theta & \theta & \theta & \theta \end{pmatrix}$$

$$\cup \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & x_{A_2}^1 / y_{A_2}^1 & x_{A_1}^2 / y_{A_1}^2 & \theta \\ \theta & x_{A_1}^1 / y_{A_1}^1 \vee x_{A_2}^2 / y_{A_2}^2 & \theta & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta & \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & \theta & \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}$$

$$= \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & \left\{x_{A_1}^1, x_{A_2}^1\right\}/\left\{y_{A_1}^1, y_{A_2}^2\right\} \vee x_{A_2}^1 / y_{A_2}^1 & x_{A_1}^2 / y_{A_1}^2 & \left\{x_{A_1}^2, x_{A_2}^1\right\}/\left\{y_{A_1}^2, y_{A_2}^1\right\} \\ \theta & \left\{x_{A_1}^1, x_{A_2}^2\right\}/\left\{y_{A_1}^1, y_{A_2}^2\right\} \vee x_{A_1}^1 / y_{A_1}^1 \vee x_{A_2}^2 / y_{A_2}^2 & \theta & \left\{x_{A_1}^2, x_{A_2}^2\right\}/\left\{y_{A_1}^2, y_{A_2}^2\right\} \vee x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta & \theta & x_{A_2}^1 / y_{A_2}^1 \\ \theta & \theta & \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}.$$

Then, due to (11), the connection matrix of the FSM $\nabla\left(A_1 \otimes A_2\right)$ is given by

$$\nabla\left(R_{A_1} \otimes R_{A_2}\right) = \begin{pmatrix} x_{A_1}^1 / y_{A_1}^1 & \left\{x_{A_1}^1, x_{A_2}^1\right\}/\left\{y_{A_1}^1, y_{A_2}^2\right\} \vee x_{A_2}^1 / y_{A_2}^1 & \left\{x_{A_1}^2, x_{A_2}^1\right\}/\left\{y_{A_1}^2, y_{A_2}^1\right\} \\ \theta & \left\{x_{A_1}^1, x_{A_2}^2\right\}/\left\{y_{A_1}^1, y_{A_2}^2\right\} \vee x_{A_1}^1 / y_{A_1}^1 \vee x_{A_2}^2 / y_{A_2}^2 & x_{A_1}^2 / y_{A_1}^2 \\ \theta & \theta & x_{A_2}^2 / y_{A_2}^2 \end{pmatrix}.$$

5. NUMERICAL EXAMPLE

As an example, we consider the joint functioning of a police station and a radiation, chemical, and biological monitoring post in an emergency zone. The activity of these units includes the following responsibilities:

- The police station organizes the interaction and general governance of subordinate forces and means; its actions are modeled by an FSM $A_1 = \left(X_{A_1}, Y_{A_1}, Q_{A_1}, F_{A_1}\right)$;
- The radiation, chemical, and biological monitoring post detects the factual contamination of objects and terrain; its actions are modeled by an FSM $A_2 = \left(X_{A_2}, Y_{A_2}, Q_{A_2}, F_{A_2}\right)$.

As an emergency evolves, these units may be in the following states (Table 1).

Table 1

The states of the FSMs A_1 and A_2 (the actions of functional groups)

Notation	Description
$q^1_{A_1}$	Daily activities
$q^2_{A_1}$	Report to the management, refinement of the incoming information about an emergency, and staff preparation for appropriate measures
$q^3_{A_1}$	Control of the units carrying out their activities
$q^4_{A_1}$	Actions in accordance with the current situation
$q^5_{A_1}$	Measures to finalize the emergency
$q^1_{A_2}$	Waiting for the post to be formed
$q^2_{A_2}$	Measurements in the controlled area
$q^3_{A_2}$	Report to the police station, measures to localize the center of contamination and eliminate the consequences of radiation, chemical, and biological effect

The operators F_{A_1} and F_{A_2} are described in Tables 2 and 3, respectively.

Problem. It is required to determine an FSM describing the joint functioning of the police station and the radiation, chemical, and biological monitoring post. The following combinations of states are inadmissible: $q^1_{A_1}$ and $q^2_{A_2}$; $q^1_{A_1}$ and $q^3_{A_2}$; $q^2_{A_1}$ and $q^2_{A_2}$; $q^2_{A_1}$ and $q^3_{A_2}$; $q^3_{A_1}$ and $q^1_{A_2}$; $q^3_{A_1}$ and $q^3_{A_2}$; $q^4_{A_1}$ and $q^1_{A_2}$; $q^5_{A_1}$ and $q^2_{A_2}$; $q^5_{A_1}$ and $q^3_{A_2}$. The FSMs can initiate the functioning of each other as follows: the output symbols of the FSM A_1 coincide with the input symbols of the FSM A_2, i.e., $y^2_{A_1} = x^1_{A_2}$, $y^4_{A_1} = x^4_{A_2}$, and $y^5_{A_1} = x^5_{A_2}$; the output symbol of the FSM A_2 coincides with the input symbol of the FSM A_1, i.e., $y^3_{A_2} = x^3_{A_1}$.

Table 2

The operator F_{A_1}

F_{A_1}	$q^1_{A_1}$	$q^2_{A_1}$	$q^3_{A_1}$	$q^4_{A_1}$	$q^5_{A_1}$
$q^1_{A_1}$	–	$x^1_{A_1} / y^1_{A_1}$	–	–	–
$q^2_{A_1}$	–	–	$x^2_{A_1} / y^2_{A_1}$	–	–
$q^3_{A_1}$	–	–	–	$x^3_{A_1} / y^3_{A_1}$	–
$q^4_{A_1}$	–	–	$x^4_{A_1} / y^4_{A_1}$	–	$x^5_{A_1} / y^5_{A_1}$
$q^5_{A_1}$	$x^6_{A_1} / y^6_{A_1}$	–	–	–	–

Table 3

The operator F_{A_2}

F_{A_2}	$q^1_{A_2}$	$q^2_{A_2}$	$q^3_{A_2}$
$q^1_{A_2}$	–	$x^1_{A_2} / y^1_{A_2}$	–
$q^2_{A_2}$	$x^5_{A_2} / y^5_{A_2}$	–	$x^2_{A_2} / y^2_{A_2} \vee x^3_{A_2} / y^3_{A_2}$
$q^3_{A_2}$	–	$x^4_{A_2} / y^4_{A_2}$	–

Based on the problem statement, we obtain the set $\Xi = \left\{\left\{q^1_{A_1}, q^2_{A_2}\right\}, \left\{q^1_{A_1}, q^3_{A_2}\right\}, \left\{q^2_{A_1}, q^2_{A_2}\right\}, \left\{q^2_{A_1}, q^3_{A_2}\right\}, \left\{q^3_{A_1}, q^1_{A_2}\right\}, \left\{q^3_{A_1}, q^3_{A_2}\right\}, \left\{q^4_{A_1}, q^1_{A_2}\right\}, \left\{q^5_{A_1}, q^2_{A_2}\right\}, \left\{q^5_{A_1}, q^3_{A_2}\right\}\right\}$. Since $X_{A_1} \cap Y_{A_2} \neq \varnothing$, the operation $\odot$ is used: the desired FSM has the form $A_3 = A_1 \odot A_2$.

The connection matrices of the FSMs A_1 and A_2 are given by

$$R_{A_1} = \begin{pmatrix} \theta & x^1_{A_1} / y^1_{A_1} & \theta & \theta & \theta \\ \theta & \theta & x^2_{A_1} / y^2_{A_1} & \theta & \theta \\ \theta & \theta & \theta & x^3_{A_1} / y^3_{A_1} & \theta \\ \theta & \theta & x^4_{A_1} / y^4_{A_1} & \theta & x^5_{A_1} / y^5_{A_1} \\ x^6_{A_1} / y^6_{A_1} & \theta & \theta & \theta & \theta \end{pmatrix},$$

$$R_{A_2} = \begin{pmatrix} \theta & x^1_{A_2} / y^1_{A_2} & \theta \\ x^5_{A_2} / y^5_{A_2} & \theta & x^2_{A_2} / y^2_{A_2} \vee x^3_{A_2} / y^3_{A_2} \\ \theta & x^4_{A_2} / y^4_{A_2} & \theta \end{pmatrix}.$$

Due to the cumbersome calculations, we will provide only the final result of the operations. After filtering, the set of states has the form

$$Q_{A_3} = \left\{ \left\{ q^1_{A_1}, q^1_{A_2} \right\}, \left\{ q^2_{A_1}, q^1_{A_2} \right\}, \left\{ q^3_{A_1}, q^2_{A_2} \right\}, \right.$$
$$\left. \left\{ q^2_{A_1}, q^3_{A_2} \right\}, \left\{ q^4_{A_1}, q^3_{A_2} \right\}, \left\{ q^4_{A_1}, q^2_{A_2} \right\}, \left\{ q^5_{A_1}, q^1_{A_2} \right\} \right\}.$$

For the sake of simplification, let us introduce the notations $\left\{ q^i_{A_1}, q^j_{A_2} \right\} = q^{ij}_{A_3}$, $\left\{ x^i_{A_1}, x^j_{A_2} \right\} = x^{ij}_{A_3}$, and $\left\{ y^i_{A_1}, y^j_{A_2} \right\} = y^{ij}_{A_3}$. Then, according to (10) and (11),

$$R_{A_3} = \nabla \left(R_{A_1} \odot R_{A_2} \right)$$
$$= \begin{pmatrix} \theta & x^1_{A_1} & \theta & \theta & \theta & \theta \\ \theta & \theta & x^{21}_{A_3} / y^{21}_{A_3} & \theta & \theta & \theta \\ \theta & \theta & \theta & x^{33}_{A_3} / y^{33}_{A_3} & x^3_{A_1} / y^3_{A_1} & \theta \\ \theta & \theta & x^{44}_{A_3} / y^{44}_{A_3} & \theta & x^4_{A_2} / y^4_{A_2} & \theta \\ \theta & \theta & x^4_{A_1} / y^4_{A_1} & \theta & \theta & x^{55}_{A_3} / y^{55}_{A_3} \\ x^6_{A_1} & \theta & \theta & \theta & \theta & \theta \end{pmatrix}.$$

Consequently, the operator F_{A_3} takes the form presented in Table 4.

Table 4

The operator F_{A_3}

F_{A_3}	$q^{11}_{A_3}$	$q^{21}_{A_3}$	$q^{32}_{A_3}$	$q^{43}_{A_3}$	$q^{42}_{A_3}$	$q^{51}_{A_3}$
$q^{11}_{A_3}$	–	$x^1_{A_1}$	–	–	–	–
$q^{21}_{A_3}$	–	–	$x^{21}_{A_3} / y^{21}_{A_3}$	–	–	–
$q^{32}_{A_3}$	–	–	–	$x^{33}_{A_3} / y^{33}_{A_3}$	$x^3_{A_1} / y^3_{A_1}$	–
$q^{43}_{A_3}$	–	–	$x^{44}_{A_3} / y^{44}_{A_3}$	–	$x^4_{A_2} / y^4_{A_2}$	–
$q^{42}_{A_3}$	–	–	$x^4_{A_1} / y^4_{A_1}$	–	–	$x^{55}_{A_3} / y^{55}_{A_3}$
$q^{51}_{A_3}$	$x^6_{A_1}$	–	–	–	–	–

Thus, the resulting FSM A_3 interprets the joint activity of functional groups involved in emergency response. It includes an asynchronous change in the states of the FSMs A_1 and A_2 and the initiation of the functioning of A_1 by A_2: if the FSM A_2 passes from one state to another when receiving a certain symbol at the input, corresponding to the output symbol that coincides with the input symbol of the FSM A_1, then A_1 makes a state transition. The FSM A_2 is initiated by the FSM A_1 in a similar way.

Note that if the operations $\circ$ and $+$ are used separately, we have to calculate the sets of states for each operation and then filter each of them. The operation $\odot$ allows avoiding duplication when calculating the set of states and when executing the filtering operation ∇.

CONCLUSIONS

This paper has considered the representation of an FSM by a symbolic matrix. This representation method allows reducing operations over FSMs to operations over the corresponding symbolic matrices. A new algebra of symbols included in such matrices has been introduced. The carrier of the algebra has been supplemented with the special symbols θ and ε; due to their properties, the synthesis of FSMs is described correctly. Also, a new algebra of symbolic matrices has been considered to construct operations over FSMs in matrix form.

For a large number of FSM synthesis problems, parallel and sequential composition operations are performed jointly. In view of this fact, a new operation combining the two types of composition has been proposed to avoid certain duplication of actions. Special operations have been introduced to aggregate an FSM model considering possible constraints of the subject area. These operations have been given the matrix representation as well.

A numerical example of FSM aggregation has been provided. The corresponding FSM describes the joint actions of functional groups in an emergency situation.

The matrix representation of operations derived above will be further used in a computational experiment: this representation simplifies the software implementation of the FSM synthesis procedure.

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THE FUNCTIONAL VOXEL METHOD APPLIED TO SOLVING A LINEAR FIRST-ORDER PARTIAL DIFFERENTIAL EQUATION WITH GIVEN INITIAL CONDITIONS[1]

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Abstract. This paper considers an approach to solving the Cauchy problem for a linear first-order partial differential equation by the functional voxel (FV) method. The approach is based on the principles of differentiation and integration developed for functional voxel modeling (FVM) and yields local geometrical characteristics of the resulting function at linear approximation nodes. A classical approach to solving the Cauchy problem for a partial differential equation is presented on an example, and an FV-model is built as a reference for further comparison with the FVM results. An algorithm for solving differential equations by FVM means is described. The FVM results are visually and numerically compared with the accepted reference. Unlike numerical methods for solving such problems, which give the values of a function at approximation nodes, the FV-model contains local geometrical characteristics at the nodes (i.e., gradient components in the space increased by one dimension). This approach allows obtaining an implicit-form nodal local function as well as an explicit-form differential local function.

Keywords: functional voxel modeling, partial differential equation, Cauchy problem, local function, local geometrical characteristics.

INTRODUCTION

Continuous processes in control systems can be often described by differential equations with initial conditions. For example, under a known input signal, the output signal is determined by the solution of the Cauchy problem for an ordinary differential equation.

The resulting function for a partial differential equation is not difficult to obtain and has long been provided by both analytical and numerical computer methods. However, the resulting function obtained manually or by means of an analytical computer-based calculator is a formulaic expression [1–5], whereas numerical methods produce numerical values at approximation grid nodes [6–9]. In this case, due to the absence of an analytical expression, the researcher cannot obtain, e.g., functions of partial derivatives for the available solution function, etc. The functional voxel (FV) method [10, 11] fills a given area of an analytical function with local functions describing a linear law for each minimal neighborhood of the area obtained during linear discretization. Hence, it becomes possible to apply not just the value at a point but the corresponding analytical expression in further calculations, with all the ensuing advantages.

The paper [12] considered the principles of differentiation and integration by functional voxel modeling (FVM) means. The transition to the FV-model of partial derivatives and back to the FV-model of the antiderivative is quite simple: an infinitesimal neighborhood of a point in a given (m – 1)-dimensional domain is described by the linear equation $n_1 x_1 + n_2 x_2 + \ldots + n_m x_m + n_{m+1} = 0$, where the coefficients are the components of the unit vector of the gradient with dimension (m + 1) increased by one. For computer representation, each component is encoded by a numerical value of the color palette, forming a sepa-

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[1] This study was carried out within the scientific program of the National Center for Physics and Mathematics, direction No. 9 “Artificial Intelligence and Big Data in Technical, Industrial, Natural, and Social Systems.”

rate (m – 1)-dimensional image M_i. As a result, to describe and store a given area of the space E^3 (i.e., $u = f(x, y)$) on the computer, one needs to form four 2D bitmap images (M_1, M_2, M_3, M_4).

In this case, it suffices to set initial conditions (formulate the Cauchy problem) to construct the FV-model of an antiderivative. Based on the results of [12], we apply the FV method and implement an FV-model for solving a partial differential equation with given initial conditions.

1. PROBLEM STATEMENT

To demonstrate the algorithm, let us consider an example of solving a homogeneous partial differential equation of the form [13]

$$\frac{\partial u}{\partial x} + \left(e^{-x} - y\right)\frac{\partial u}{\partial y} = 0 \qquad (1)$$

with the initial condition

$$u(0, y) = 3y + 2. \qquad (2)$$

The differential equation (1) has the analytical solution

$$u(x, y) = 3(e^x y - x) + 2. \qquad (3)$$

Here,

$$\frac{\partial u}{\partial x} = 3(e^x y - 1), \qquad (4)$$

$$\frac{\partial u}{\partial y} = 3e^x. \qquad (5)$$

Figure 1 shows the graph of function (3) on the domain $x \in [0, 1]$, $y \in [0, 1]$, obtained by conventional visualization in MathCAD with a sampling step of 1/30.

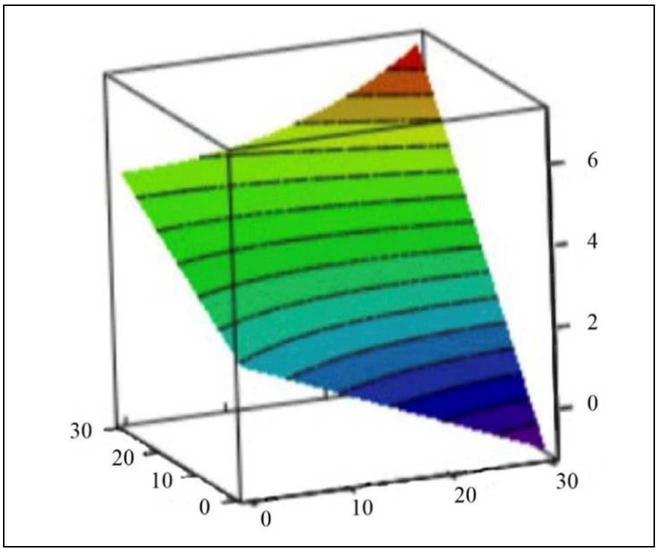


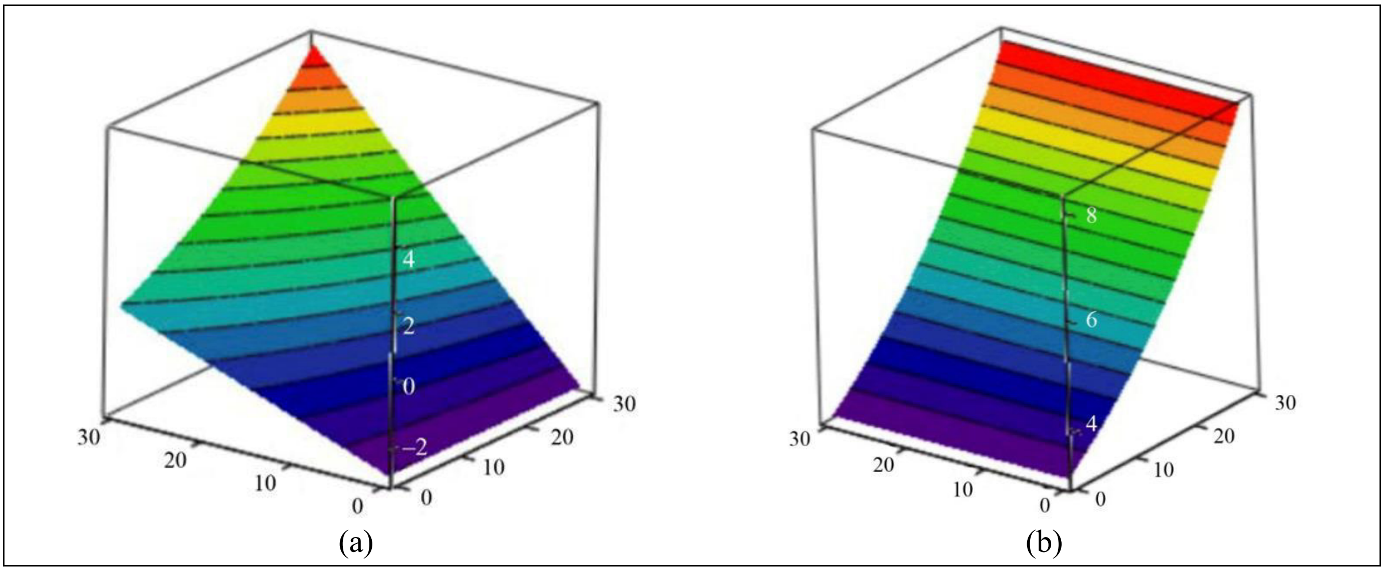
Fig. 1. The graph of function (3) in MathCAD.

By visual analysis of Fig. 1, it is possible to determine approximate values at the corner points of the surface segment under consideration. Their precise values are (0; 0; 2), (1; 0; –1), (0; 1; 5), and (1; 1; 7.1584).

Figures 2a and 2b present the graphs of functions (4) and (5), respectively, as the partial derivatives of function (3). We calculate their values at the corner points of the surface segments:

– for function (4), (0; 0; –3), (1; 0; –3), (0; 1; 0), and (1; 1; 5.1584);

– for function (5): (0; 0; 3), (0; 1; 3), (1; 0; 8.15484), and (1; 1; 8.15484).



(a) (b)

Fig. 2. The graphs of functions (4) and (5) in MathCAD.

The computer algorithm for obtaining the domains of local functions can be described as follows.

Step 1. A rectangular grid is applied to a given domain of the function for further linear approximation. The dimension of the grid space coincides with that of the function domain. The grid step is determined by the ratio of the size of the function domain to the size of the corresponding sides of the graphical image.

Step 2. During linear approximation on the function domain, the current grid element (the node and its nearest neighbors) is sequentially determined, i.e. the simplest element of the corresponding dimension is formed. For example, for a function of two variables $u = f(x, y)$, we have a triangular approximation element in which the neighbors are the nearest grid nodes shifted parallel to the axes Ox and Oy. For a function of three variables $u = f(x, y, z)$, the approximation element is a tetrahedron with a node and neighbors shifted parallel to the axes Ox, Oy, and Oz, respectively, etc.

Step 3. The local equation for the selected approximation element is obtained using the determinant of the matrix consisting of the homogeneous coordinates of the triangle nodes and the variable point on the domain. For example, in the case $u = f(x, y)$, the determinant of the matrix of dimensions 4 × 4 has the form

$$\begin{vmatrix} x & y & u & 1 \\ x_1 & y_1 & u_1 & 1 \\ x_2 & y_2 & u_2 & 1 \\ x_3 & y_3 & u_3 & 1 \end{vmatrix} = ax + by + cu + d = 0. \qquad (6)$$

Note that the matrix determinant allows obtaining such an equation for any matrix dimensions. The coefficients a, b, c, and d represent the components of the gradient vector increased by one dimension; the original function $u = f(x, y)$ can be replaced by a local function of the form

$$u = -\frac{a}{c}x - \frac{b}{c}y - \frac{d}{c}$$

since the plane given by equation (6) passes through the node under consideration.

Step 4. There is no sense in storing the local function for each point of the domain on the computer. It suffices to store the coefficients a, b, c, and d in the form of four bitmap images. This representation provides visual clarity of the data, which will be further used to assess the solution, and their compact storage.

To proceed, we normalize each coefficient by the length $N = \sqrt{a^2 + b^2 + c^2 + d^2}$ of the gradient vector, obtaining the components of the unit normal:

$$\vec{n} = (n_1, n_2, n_3, n_4),$$

where $n_1 = a / N$, $n_2 = b / N$, $n_3 = c / N$, and $n_4 = d / N$.

The color at the image point is defined as

$$M_i = \frac{P(1 + n_i)}{2},$$

where $P = 256$ and $i = 1, ..., 4$.

The inverse transition from the color value M_i to the component n_i is performed by the formula

$$n_i = \frac{2M_i - P}{P}.$$

Further, applying the FV method to function (3), we obtain a computer FV representation, i.e., the domain of local functions of the form $n_1 x + n_2 y + n_3 u + n_4 = 0$, where n_1, n_2, n_3, and n_4 are the coefficients of the local function (local geometrical characteristics). They are displayed on the computer by *M*-images M_1, M_2, M_3, and M_4 (Fig. 3) with a resolution of 400 × 400. In [10], *M*-images were understood as image models displaying in tone or color one of the local geometrical characteristics of the FV-model. The accuracy of representing a numerical value by half-tint is provided in the RGB format (256 color grades). To increase clarity, we demonstrate the *M*-images for the color palette *P* = 16 777 214 (256 × 256 × 256) color grades in Fig. 4. The resulting patterns characterize the transition from red color grades through green color ones to blue color grades, providing higher visibility due to the resulting patterns for comparing the result with the reference. In our case, the reference is the *M*-images in Figs. 3 и 4.

At this stage, we assume that there is sufficient initial information for the numerical and visual experiment.

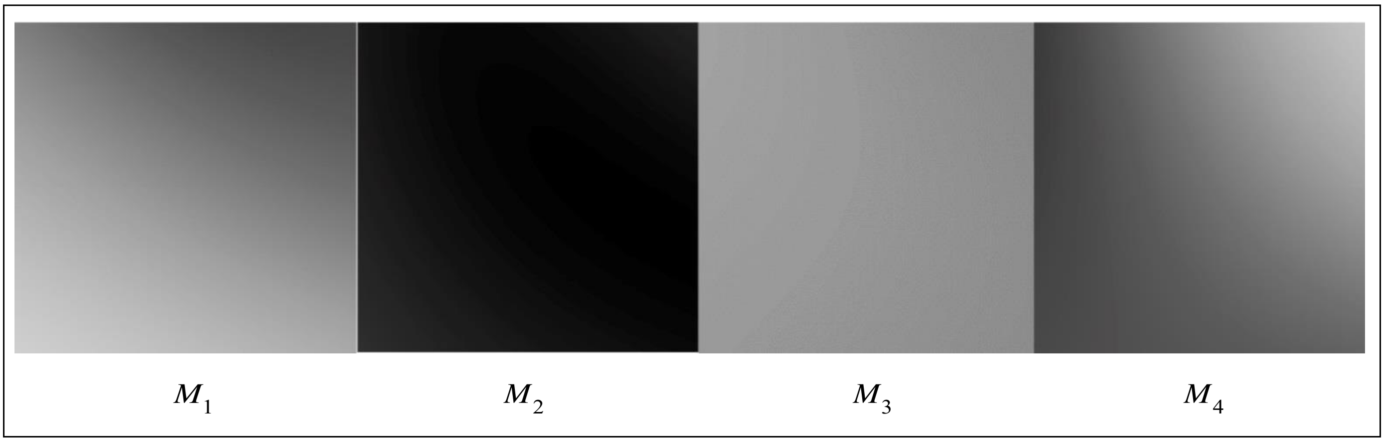


Fig. 3. The graphical representation of the local geometrical characteristics of function (3) (256 greyscale grades).

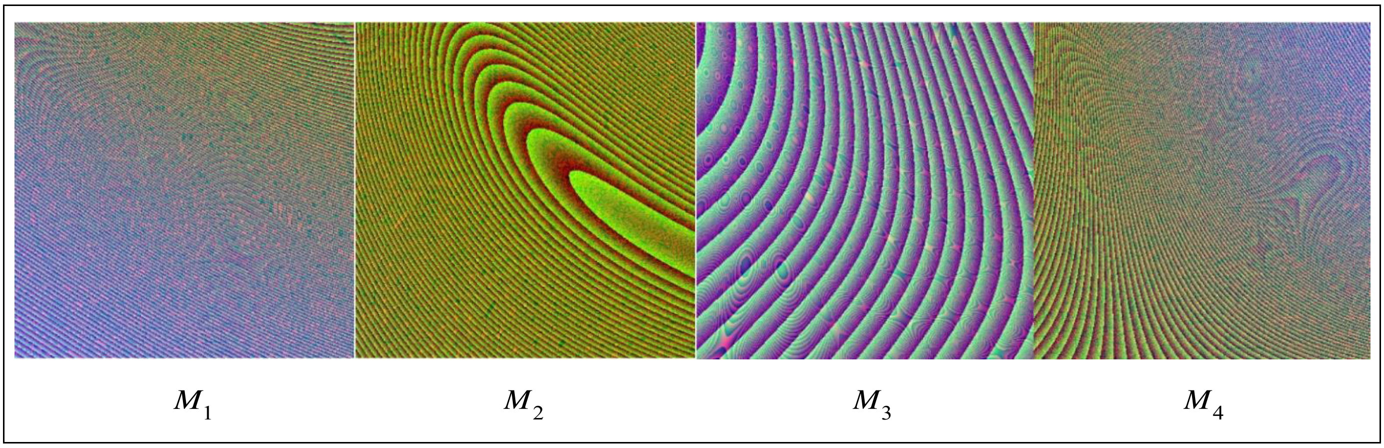


Fig. 4. The graphical representation of the local geometrical characteristics of function (3) (16 777 214 color grades).

2. THE ALGORITHM FOR BUILDING AN FV-MODEL TO SOLVE DIFFERENTIAL EQUATIONS

The paper [8] presented an algorithm for obtaining the FV-model of the antiderivative of a function by given FV-models of its partial derivatives. Note that it suffices to determine the local geometrical characteristics at one approximation point to calculate the function values at the other points of the triangular element of the approximation grid. Hence, local geometrical characteristics can be further found in the entire solution domain.

To apply this algorithm, we express the partial derivatives of a given function to obtain their exact values at the point under consideration.

In the example above, the initial condition is function (2). It represents the cross-section of the desired surface of function (3) for $x = 0$.

Therefore,

$$\frac{\partial u}{\partial y} \approx \frac{\Delta u}{\Delta y}, \ \Delta y = h,$$

$$\Delta u = u_{i+1}(0,\ (i+1)h) - u_{i+1}(0,\ ih),$$

$$i = [0 \dots n],$$

where h is the approximation step.

Figure 2b shows the numerical data confirming the validity of this solution. Clearly, $\partial u / \partial y = 3$ at the point (0, 0) and $\partial u / \partial y = 8.15484$ at the point (49, 0). Along the axis Oy, the value of the derivative exponentially increases.

For $x = 0$, the partial derivatives can be defined as follows:

$$\frac{\partial u}{\partial x} \approx \frac{(e^{-x} - y)\Delta y}{\Delta u}.$$

With the local function of the FV-model written as $ax + by + cu + d = 0$, we obtain

$$\frac{\partial u}{\partial x} = -\frac{a}{c}, \quad \frac{\partial u}{\partial y} = -\frac{b}{c},$$

where the coefficient c can be replaced by the approximation value C (Fig. 5):

$$C = x_1\left(y_2 - y_3\right) - x_2\left(y_1 - y_3\right) + x_3\left(y_1 - y_2\right).$$

Performing the transition to the components of the gradient vector, we have

$$A = -\frac{a}{c}C, \quad B = -\frac{b}{c}C,$$
$$D = -Ax_1 - By_1 - Cu_1.$$

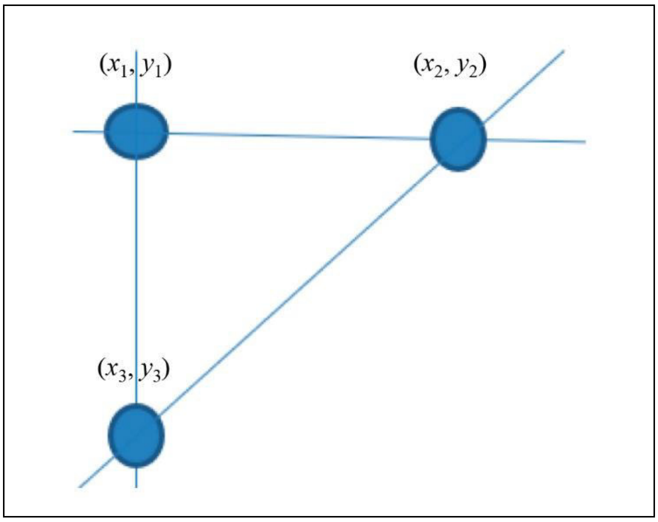


Fig. 5. Approximation nodes.

At the first calculation step, the value u_1 is given by formula (3); in other cases, the values are obtained through the next coefficients for the local function:

$$u_i = -\frac{A}{C}x_i - \frac{B}{C}y_i - \frac{D}{C}.$$

The algorithm for solving the differential equation includes the following basic steps:

1. Select the next triangular approximation element $(x_0, y_0, x_1, y_1, x_2, y_2)$.
2. Calculate the coefficient

$$C = x_0\left(y_1 - y_2\right) - x_1\left(y_0 - y_2\right) + x_2\left(y_0 - y_1\right).$$

3. For $x = 0$, calculate one of the derivatives according to given conditions (in the example above, $\partial u / \partial x = ((3y_2 + 2) - (3y_0 + 2)) / \Delta y$). For other values x, the derivative is determined by the relation $\partial u / \partial x = \Delta u / \Delta y$, where $\Delta u = (u_1 - u_0)$ and

$$u_0 = -\frac{A_0}{C_0}x_0 - \frac{B_0}{C_0}y_0 - \frac{D_0}{C_0},$$

$$u_1 = -\frac{A_1}{C_1}x_1 - \frac{B_1}{C_1}y_1 - \frac{D_1}{C_1}.$$

4. Calculate the second derivative based on the first derivative (in the example, $\partial u / \partial x = -(e^{-x_0} - y_0)(\Delta u / \Delta y)$).
5. Calculate the coefficients A_i, B_i, and D_i.
6. Pass to the $(i + 1)$th triangular element.

For each node of the approximation triangular grid, the local geometrical characteristics are successively calculated by FVM [6, 7], and the solution domain of the desired differential equation is filled with the local functions $n_1x + n_2y + n_3u + n_4 = 0$. On the computer, such a domain is represented by the corresponding images M_1, M_2, M_3, and M_4; see Fig. 6 (256 greyscale grades) and Fig. 7 (16 777 215 RGB color grades).

The result in Figs. 6 and 7 is visually comparable with that in Figs. 3 и 4. This confirms the adequacy of the algorithm. The numerical estimates of the nodal values of the function and its partial derivatives at the corner points of the domain $x \in [0, 1]$, $y \in [0, 1]$ are presented in the table.

The nodal values of function (3) and its partial derivatives (numerical estimates)

x	y	u	$\frac{\partial u}{\partial x}$	$\frac{\partial u}{\partial y}$
0	0	2.0000	–3.0000	3.0000
0	399	5.0000	0.0000	3.0000
49	0	0.0235	–2.0279	7.9164
49	399	7.1486	4.9453	7.9164

Let us compare the points for the corresponding M-images with the accepted references. According to the comparison results, among 640 054 points of the image, the number of points with a value differing at most by unity is, respectively, $M_1 = 9515$, $M_2 = 3254$, $M_3 = 2116$, and $M_4 = 6086$ (not more than 1.5%).

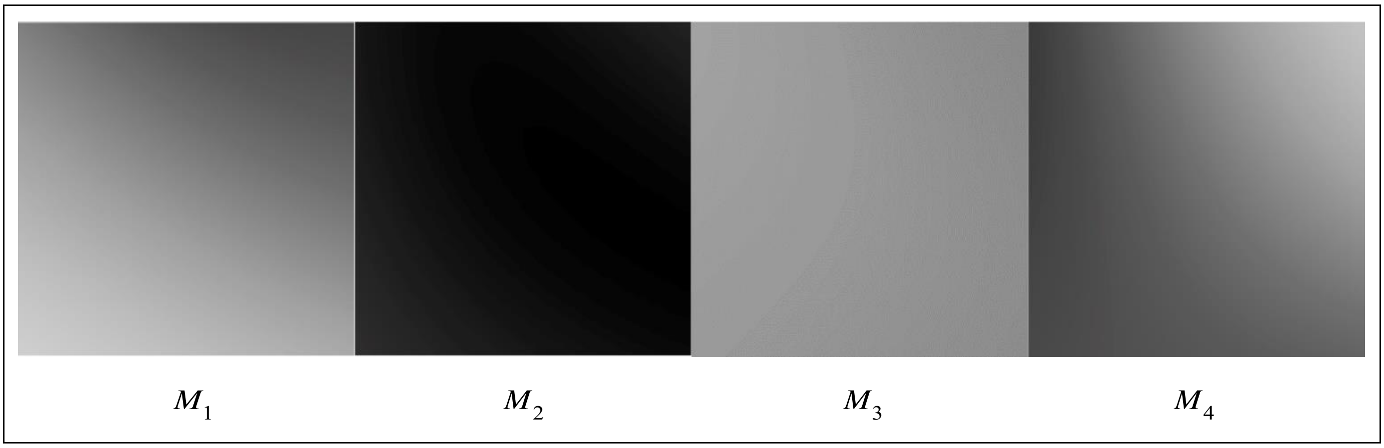


Fig. 6. The graphical representation of the local geometrical characteristics of the differential equation solution (256 greyscale grades).

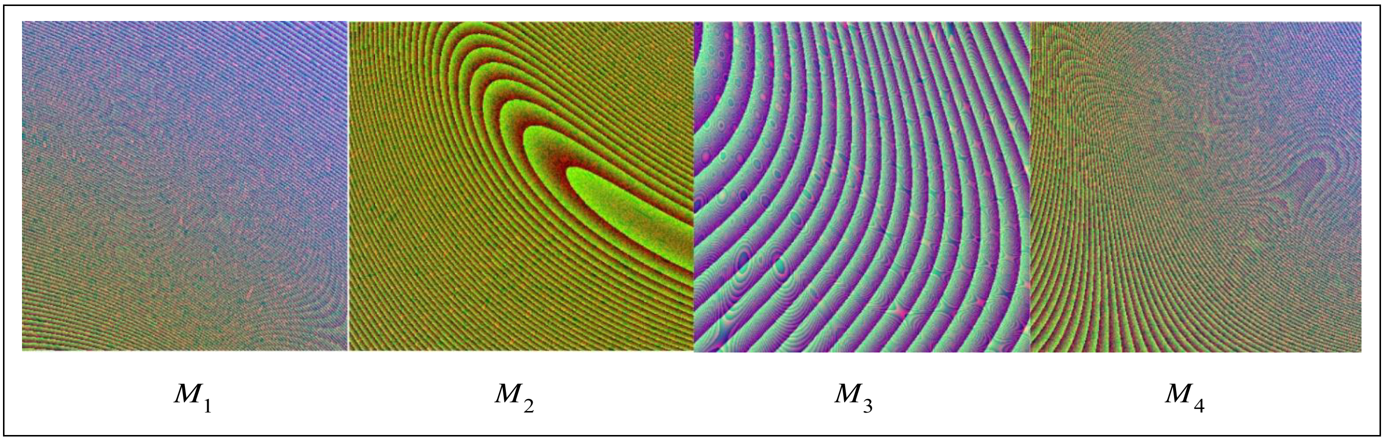


Fig. 7. The graphical representation of the local geometrical characteristics of the differential equation solution (16 777 215 color grades).

The resulting solution is a linear local function represented by the local geometrical characteristics for the points of the selected domain:

$$n_1 x + n_2 y + n_3 u + n_4 = 0.$$

Expressing $u\ (x,\ y)$, we obtain the local differential equation

$$u = -\frac{n_1}{n_3} x - \frac{n_2}{n_3} y - \frac{n_4}{n_3} \text{ or } u = \frac{\partial u}{\partial x} x + \frac{\partial u}{\partial y} y - \frac{n_4}{n_3}.$$

CONCLUSIONS

This paper has considered an approach to solving a linear first-order partial differential equation using the functional voxel method. An algorithm for solving such differential equations based on the proposed approach has been presented. The numerical simulation results have confirmed the adequacy of this algorithm.

In future studies, this algorithm will be compared with well-known numerical methods in terms of the growing error of the function value at the approximation grid nodes with different steps. Also, the accuracy of local geometrical characteristics will be compared with the approximation accuracy of the analytical solution.

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16TH INTERNATIONAL CONFERENCE ON MANAGEMENT OF LARGE-SCALE SYSTEM DEVELOPMENT (MLSD'2023)

The 16th International Conference on Management of Large-Scale System Development (MLSD'2023) was held on September 26–28, 2023. This conference is organized annually by the Trapeznikov Institute of Control Sciences, the Russian Academy of Sciences (ICS RAS), with the support of the IEEE Russia Section.

The MLSD'2023 program included one plenary session and 16 sections in the following areas.

Section 1. Management problems of large-scale system development, including multinational corporations, state holdings, and state corporations.

Section 2. Methods and tools for managing investment projects and programs.

Section 3. Management of development of a digital economy. Design offices both situational and expected analytical centers, institutes of development of large-scale systems.

Section 4. Simulation and optimization in the problems of development management of large-scale systems.

Section 5. Nonlinear processes and computing methods in the problems of management of large-scale systems.

Section 6. Management of development of banking and financial systems.

Section 7. Management of fuel, power, infrastructure, and other systems.

Section 8. Management of transport systems.

Section 9. Managing the development of aerospace and other large-scale organizational-technical complexes.

Section 10. Managing the development of regional, urban, and municipal systems.

Section 11. Management of objects of nuclear power and other objects of increased danger.

Section 12. Information support and software management systems for large-scale production.

Section 13. Methodology, methods, software, and algorithmic support of intellectual processing of large volumes of information.

Section 14. Monitoring in the management of large-scale systems.

Section 15. Management of large-scale systems advancement in healthcare, medico-biological systems, and technologies.

Section 16. Managing the development of social systems.

MLSD conferences are intended to discuss research in the theory and applications of computer control and management for developing large-scale manufacturing, transport, energy, financial, and social systems. Leading scientists from academia, research institutes, universities, and governmental and commercial organizations are invited to participate in the conference. The statistics of MLSD conferences are presented in the figure below. Three hundred ninety-three participants attended MLSD'2023.

The original proceedings of MLSD'2023 (20 plenary and 224 sectional papers) have been published in Russian and indexed by the RSCI. One hundred fifty-five papers have been extended and published electronically in English in the IEEE *Xplore* digital repository[1] (Scopus indexing).

The main problem-thematic vector of contributions at MLSD'2023 was determined by plenary papers devoted to new formal statements for the strategic management problems of large-scale system development in modern conditions of increased sanctions pressure as well as the approaches, principles, and technologies to solve them.

The tone was set by the plenary paper of Dr. Sci. (Eng.), Prof. *A.D. Tsvirkun* (ICS RAS), "Problems of Managing the Development of Large-Scale Systems in Modern Conditions." He posed the problem of developing methodological foundations to form an integral planning model for the long-term growth trends of the national economy in the current unfavorable geopolitical situation. The author's approach is to plan the macroeconomic development of Russia's large-scale systems that form the gross domestic product, national income, aggregate demand, aggregate supply, general price level, unemployment rate, interest rate, currency exchange rate, state budget, and the balance

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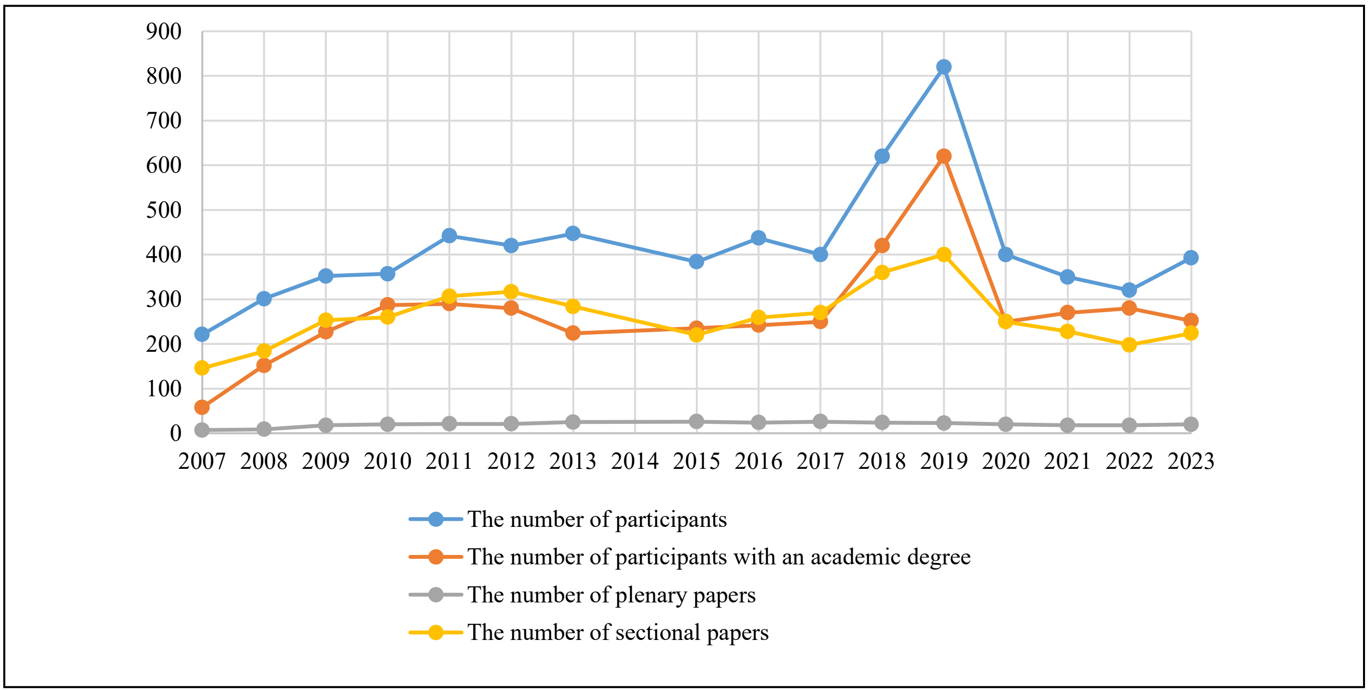
[1] https://ieeexplore.ieee.org/xpl/conhome/10303766/proceeding



of payments. The plenary paper introduced a set of models for analyzing and selecting development scenarios for Russia and a digital platform for managing the development of large-scale production and transport systems.

A.D. Tsvirkun's main conclusions and recommendations are as follows. State programs and large business projects should be implemented after thorough elaboration in the RAS and the Institute of Applied Economic Research (RANEPA). It is necessary to develop a project (topic) and create the Association of Management of Large-Scale System Development (the Center of Competencies), uniting experts in the most important areas of the management of large-scale system development, including investment, information technology, and creation of a digital environment for investment planning. This project aims to design a complex analysis methodology and management tools for large-scale system development for solving strategic tasks and problems in order to manage large-scale production and transport projects in Russia, including emergency conditions. It is advisable to maintain relations with similar international organizations and conferences, as well as hold seminars and schools for experts and young scientists on topical issues arising in the management of large-scale system development.

The energy sector is basic for the Russian economy. Its sustainable development and reliable operation largely determine national security. Academician of the RAS *S.P. Filippov* and Cand. Sci. (Econ.) *F.V. Veselov* (The Energy Research Institute RAS) reflected this problem in the paper "Features and Means of Forming Technological Forecasts of Energy Sector in Modern Conditions." The authors described the future technological image of the energy sector meeting heterogeneous requirements, including the availability and reliability of energy supply, technological independence, and the global competitiveness of energy production. In addition, the significant role of environmental and climatic requirements due to the decarbonization of the economy was emphasized.

Under the sanctions, it is of great interest to manage the development of Russia's production infrastructure according to the concept of fast-growing organizations and industries. In his paper "Fast-Growing Organizations in Russia: Growth versus Sustainability," Dr. Sci. (Eng.) *O.I. Dranko* analyzed the prospects for practical implementation of this concept. The author developed a technology for evaluating fast-growing segments of the economy based on organizations' statistical data and financial statements. Fast-growing industries were defined by the average annual growth rates exceeding those for the entire economy. Fast-growing organizations were defined by the average annual growth rate of more than 50% per year in current prices. About 2000 fast-growing organizations in Russia with revenues of more than 1 billion rubles were identified. The total revenue of the sample of fast-growing organizations in 2022 was about 20 trillion rubles. The analysis demonstrated the contribution of fast-growing organizations to the growth of the Russian economy under various development scenarios. The big data processing methods applied by the author quickly decompose information from the macro level (the GDP of the country) to the revenues of individual organizations. However, for the sample of fast-growing organizations, the calculations revealed an increase in the debt burden compared to the complete list of organizations. They did not reveal changes in current liquidity. Therefore, the class of fast-growing organizations can be attributed to organizations with higher but not critical financial risks. The studies showed the average factual values of risk indicators and can be used to formulate the risk management problem under fast growth.

An interesting aspect of the consequences of the sanctions policy was mentioned by Dr. Sci. (Econ.) *O.S. Sukharev* (the Institute of Economics RAS) in his paper "Industrial Development of Russia: Analysis of Prospects." The author defined sanctions pressure as force majeure circumstances generating crises in the system of large-scale management, including its content, speed, the efficiency and accuracy of decisions, and non-standard institutional constraints. In such circumstances, forecast assessments become inadequate. According to the conclusions, it is necessary to analyze industrial development at the sectoral and system-wide levels, including assessing the impact of macroeconomic instruments. He considered industry to be a large-scale multi-sector system with branching and significant links (the industrial base of the economy) that requires a special methodology of macroeconomic analysis and system management. The main outcome of the paper is the fundamental possibility to relax the blow to the Russian economy provoked by the decline in net exports due to the reduction in sales of raw material components and to deploy new productions and technologies based on the closed cycle principle, which is used in innovation systems to ensure technological sovereignty.

An essential problem in the new theory of managing the development of large-scale systems in force-majeure circumstances is the organization of purposeful (targeted) activity. This problem was considered in the paper "Control of the Purposeful Process of Complex Systems Design" by Corresponding Member of the RAS *A.F. Rezchikov* (ICS RAS), Cand. Sci. (Eng.) *E.V. Kushnikova* and *O.V. Kushnikov* (Saratov State Technical University), and Dr. Sci. (Eng.) *A.S. Bogomolov* (Federal Research Center Saratov Scientific Center RAS). The authors represented the achievement of design objectives as a controlled process and defined its input and output coordinates invariant to the activity's nature, duration, and scale. When achieving design objectives, the values of the input coordinates of this process are repeatedly changed by the decision maker, which may cause significant deviations from the desired results. The paper proposed a problem statement for determining the perturbations of the control coordinates without deviations from the planned results.

In the paper "Hierarchical Games in Deep Learning," Cand. Sci. (Phys.–Math.) *M.A. Gorelov* and Dr. Sci. (Eng.) *F.I. Ereshko* (Federal Research Center "Computer Science and Control" RAS) demonstrated a formal tool to uniformly generate and consider different statements of deep learning and strategic planning problems. In deep learning, it is topical to find weight coefficients under uncertain factors affecting the entire network, and the game-theoretic approach provides an apparatus to write solution algorithms in various enumeration schemes in the state space or gradient algorithms. In strategic planning, the weight parameters of convolutions are usually given by industrial processes, and the most relevant problems are to form activation functions (in this interpretation, for the players' strategy) and the interaction graph, i.e., the architecture of an artificial neural network.

The main results of this work are models of hierarchical systems in the multistep case, with different degrees of awareness of the Principal and subsystems. The authors investigated control procedures for the system of players in a peculiar architecture of links, characteristic of artificial neural networks. In addition, the problem of calculating the Principal's maximum guaranteed result was formulated, a fundamentally new problem for hierarchical games.

Dr. Sci. (Econ.), Corresponding Member of the RAS *V.I. Suslov* and his colleagues *Yu.S. Ershov* (the Institute of Economics and Industrial Engineering, Siberian Branch RAS) and Cand. Sci. (Econ.) *N.M. Ibragimov* (Novosibirsk State University) presented the paper "Interregional Models in the Study of Spatial Economy of Russia." The authors introduced an original method of studying the Russian economy from a spatial perspective based on the principle of sequential detailing of the object (from point to space). In addition, they described a model-software complex replacing the optimization interregional inter-industry model with modified search algorithms for equilibrium and the system core within the spatial "input-output" model. This approach generates forecasting information using not only the national, sectoral, and regional retrospective trends but also the commissioning dates and capacities of individual large enterprises.

The paper by Dr. Sci. (Econ.) *V.G. Varnavskii* (ICS RAS) "Using Input-Output Tables for Assessment Structural Changes in the Russian Transport Sector" was devoted to a methodological approach to studies describing structural shifts in international production linkages and foreign trade at the level of products and sectors. The author considered an indicator of an exporting country engaged in the importing country's production (the so-called production participation index proposed by him previously) and global input-output models to assess multiplicative economic effects and the involvement of countries, sectors, and products in global value chains. The paper provided an algorithm for analyzing key aspects of structural changes in the Russian transport sector in 2000–2018 based on the Inter-country Input-Output Tables (ICIO) for 2021. According to the conclusions, the integration of the Russian transport sector into global value chains has approached the optimal level.

The same topic was touched on by Dr. Sci. (Eng.) *V.V. Tsyganov* (ICS RAS) in his plenary paper "From a Complex of Models to a Platform for Strategic Management on Transport Infrastructure." The author proceeded from the assumption that the socio-economic development of Russia under sanctions is impossible without advanced transport infrastructure development in new and promising directions of traffic flows. The accelerated changes, scale, number, and complexity of transport infrastructure links determine the relevance of a strategic management platform project to be scientifically justified and implemented. V.V. Tsyganov described the objectives, tasks, principles, and main processes for the strategic management platform of transport infrastructure as well as the processes of coordinating strategic documents by the objectives, tasks, timing of activities, and the amount of resources required. A central mechanism of this platform is a simulation complex for the consequences of management decisions using artificial intelligence technologies and mathematical analysis methods. Note that the approach under consideration has been tested for strategic sustainable operation planning of the Russian economic complex, including Siberia, the Far East, and the Arctic zone of Russia. The formal apparatus to implement this project was illustrated by the mathematical modeling and forecasting of the consequences of one-time adjustments of transport infrastructure as one example.

Dr. Sci. (Eng.) *V.K. Akinfiev* (ICS RAS) presented the paper "The Choice of Optimal Options for the Development of Transport Corridors in the Context of the Reorientation of Export Flows of Russian Companies." He considered the problem of choosing the optimal option for developing transport infrastructure under sanctions and the closure of some export markets for industrial companies from Russia. The development of transport corridors will allow companies to reorient their export flows to other markets without losing financial stability. The problem under consideration was stated as a dynamic production and transportation problem with mixed variables and the choice of options for developing transport infrastructure. The solution of this problem yields the optimal options for developing transport infrastructure and the optimal production and supply volumes of companies considering the capacity dynamics of export markets and other constraints under different initial data scenarios and assumptions.

The paper "Theory and Practice of Decision-Making in the Implementation of Large-Scale Transport Projects" by Dr. Sci. (Econ.) *V.N. Livschits*, Cand. Sci. (Econ.) *I.A. Mironova*, Cand. Sci. (Econ.) *T.I. Tishchenko*, and Cand. Sci. (Econ.) *M.P. Frolova* (Federal Research Center "Computer Science and Control" RAS) was devoted to an approach to implementing a large-scale transport project. Its socio-economic efficiency was assessed in accordance with the federal and regional regulatory legal acts of Russia.

An investment project was defined as a justification of economic feasibility, the volume and timing of capital investments, as well as a description of practical actions to implement investments (a business plan). Planned and draft decisions were divided into three categories as follows:

- small, which have no appreciable effect on the structural characteristics of the country's economy, individual sectors, or individual regions, as well as do not affect the prices of essential resources, inflation rates, bank interest rates, currency exchange rates, etc.;
- large-scale, which may significantly affect the above characteristics (e.g., exploitation of new coal deposits may affect the price of coal on the national and world markets);
- global, which may fundamentally change the structure of the national economy and affect the indicators of socio-economic development of a country (or even several countries).

The authors considered the problems of assessing the socio-economic efficiency of a large-scale investment project to develop a transport infrastructure network. An approach based on a vector criterion was proposed. It reflects the properties of investment projects and expands the range of information available to the decision-maker.

Currently, online platforms for communication and opinion exchange are becoming an information resource to support decision-making when managing the development of large-scale systems. Examples include government websites for public interaction, Internet projects for informing the public and involving citizens in the discussions of legal and regulatory documents, and social media, where users themselves create the content. Social media can play a special role in force majeure circumstances. This topic was covered in the plenary paper “The Impact of Online Social Network Algorithms on User Opinion Formation” by Dr. Sci. (Eng.) *D.A. Gubanov* and Dr. Sci. (Phys.–Math.) *A.G. Chkhartishvili* (ICS RAS). The authors considered the formation model of information cascades in which the agents’ opinions (concerning some issue) are not observed whereas the observable actions of agents reflect their opinions only partially. The actions performed by agents (writing comments) influence the opinions of agents acting subsequently, thus forming an information cascade of opinions and actions. According to computational experiments, such a cascade is significantly affected by the algorithm showing previous actions to the network agent: in reverse chronological order, in descending order of likes, or first the comments with a given position. This is especially true when agents read a few comments (perhaps due to their cognitive constraints). Therefore, relatively simple changes in the algorithms of an online social network may exert an indirect but decisive impact on the opinions and preferences of network users (in the final analysis, on their actions).

The paper “Modernizing the Solution of Practical Problems and Methods of Program-Target Planning and Management of Breakthrough Development” by Cand. Sci. (Econ.) *D.R. Gonchar* (Federal Research Center “Computer Science and Control” RAS) and Dr. Sci. (Eng.) *V.A. Irikov* (ICS RAS) discussed the issues of implementing the initiative of the President of the Russian Federation on Russia’s transition to breakthrough development. The authors proposed a set of tools and technologies tested both on pilots and on small and medium series of controlled objects. Even in the difficult conditions of new challenges, their implementation will solve the previously considered untreatable problems (mainly due to the system approach, modernization, and the identification and mobilization of new opportunities unutilized so far).

Dr. Sci. (Eng.) *V.V. Kul’ba* (ICS RAS) and Dr. Sci. (Eng.) *V.I. Medennikov* (Federal Research Center “Computer Science and Control” RAS) presented the paper “Management of Scientific Agricultural Digital Ecosystems Based on Ecosystem Classification.” They proposed a classification of ecosystems and developed a scientific digital ecosystem. This ecosystem provides a digital tool for collecting, accumulating, and using scientific knowledge and for measuring the state of the most essential ecosystems in nature (agricultural ecosystems).

The paper “Soft Dependencies between Projects in Program Management” by Dr. Sci. (Eng.) *V.N. Burkov*, Dr. Sci. (Eng.) *I.V. Burkova*, and Dr. Sci. (Eng.) *A.V. Shchepkin* (ICS RAS) was devoted to program management problems under soft dependencies between projects. Implementing soft dependencies reduces the time or costs of the next project. The following problem was stated and solved: determine the set of executable soft dependencies under given conditions. The paper considered the problem of finding the minimum program duration under soft dependencies between projects. An algorithm for determining the critical path was proposed.

Let us emphasize two thematically close plenary papers: “Development of Control Methods for Nonlinear Processes in Continuous Media” by Dr. Sci. (Phys.–Math.) *A.G. Kushner* (ICS RAS and Moscow State University) and “A Model of the Wave Displacement of Hard-to-Recover Oil Fields Reservoirs by Active Reagents” by Cand. Sci. (Eng.) *A.V. Akhmetzyanov* and Dr. Sci. (Eng.) *A.V. Samokhin* (ICS RAS). These papers give an example of advanced computer modeling tools supporting breakthrough technologies of innovative development at the sectoral level.

In particular, A.G. Kushner presented some results on controlling processes in continuous media. These results refer to many physical processes: thermodynamics, filtration, and motion of media with molecular structure. The unified approach described in the paper is based on the geometric theory of nonlinear differential equations and contact and symplectic geometries. The results have been applied to controlling oil and gas field exploitation processes and phase transition control.

In their plenary paper, A.V. Akhmetzyanov and A.V. Samokhin investigated the effect arising in the production of hard-to-recover oil field reservoirs using active reagents. High-frequency nonlinear wave oscillations accelerate diffusion and increase the oil recovery factor of deposits. The authors showed the results of numerical experiments using a mathematical model of cylindrical waves in the direction of filtration fluid flow; the effectiveness of cyclic wave control actions was demonstrated as well.

The development of domestic advanced software for large-scale industrial processes is of great importance for improving national security. Some results in this sphere were presented in the papers “An Integration Platform for the Operator System (Process Control System)” (Dr. Sci. (Eng.) *A.G. Poletykin*, *N.E. Megazetdinov*, Cand. Sci. (Eng.) *E.F. Jharko*, Cand. Sci. (Phys.–Math.) *V.G. Promyslov*, Cand. Sci. (Eng.) *M.E. Byvaikov*, Cand. Sci. (Eng.) *V.N. Ste-*

panov, Cand. Sci. (Eng.) *A.A. Baibulatov*, Cand. Sci. (Phys.–Math.) *K.V. Semenkov*, and *K.V. Akaf'ev* (ICS RAS)) and "Implementation of Advanced Planning and Scheduling Systems for Oil Refineries, Petrochemical Complexes and Their Associations" (Dr. Sci. (Eng.) *A.S. Khokhlov*, Cand. Sci. (Eng.) *D.Yu. Mishutin*, and Cand. Sci. (Eng.) *E.S. Baulin* (OOO Center for Digital Technologies, Skolkovo)).

This brief overview of the plenary papers demonstrates the active and constructive nature of the concepts, methods, and models of strategic management for developing Russia's large-scale systems under increased sanctions pressure.

The conference results were summarized at the closing session. The chairs of the conference sections underlined the diversity and relevance of the conference problems. Following the established tradition, they also recommended the most interesting papers for publication, in an extended form, in leading peer-reviewed journals of ICS RAS: *Automation and Remote Control*, *Advances in Systems Science and Applications*, *Control Sciences*, and *Large-Scale Systems Control*.

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