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CONTENTS

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ANALYSIS OF INFORMATION INCONSISTENCY IN BELIEF FUNCTION THEORY. PART II: INTERNAL CONFLICT[1]

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Abstract. Part II of the survey considers the measure of internal conflict in a body of evidence within belief function theory (the Dempster–Shafer theory of evidence). The concepts of non-conflict focal elements in a body of evidence and the basic requirements applied to measures of internal conflict are discussed. Some axiomatics of a measure of internal conflict based on strengthening desirable properties is studied. The general forms of measures of internal conflict that satisfy this system of axioms are presented and analyzed. Different methods for estimating internal conflict are considered: an entropy approach, methods based on auto-conflict calculation and contour function maximization, and metric and decompositional approaches. The decompositional approach assumes that the information source for a body of evidence with great internal conflict could be heterogeneous. This approach is considered in detail. Many illustrative examples are provided.

Keywords: belief function theory, combining rules, inconsistency of bodies of evidence, measure of internal conflict.

INTRODUCTION

This paper is a direct continuation of the publication [1], in which the main methods for analyzing the inconsistency of information between bodies of evidence in belief function theory (the Dempster–Shafer theory of evidence) were considered. However, one body of evidence can also provide inconsistent information. In this case, we speak of an internal conflict. An example of such evidence with a large internal conflict is as follows: the value of the company's stocks tomorrow will be in the range [0, 10] or [30, 35] with equal weights.

Different concepts describing internal conflicts in bodies of evidence (or the corresponding belief functions) were discussed by several authors in the 1980–1990s. Meanwhile, the idea of distinguishing between external and internal conflicts of evidence goes back to [2, 3].

This survey deals with the axiomatics and basic methods for estimating internal conflict in a body of evidence. The remainder of this paper is organized as follows. Section 1 introduces a basic background on belief function theory. For details, see part I in [1]. In Section 2, we discuss non-conflict focal elements and the basic requirements to measures of internal conflict. Section 3 presents the general forms of measures satisfying a given system of axioms. In Section 4, different methods for estimating internal conflict are considered: an entropy approach (Section 4.1), methods based on auto-conflict calculation and contour function maximization (Section 4.2), and metric and decompositional approaches (Sections 4.3 and 4.4, respectively). In the Conclusions, we summarize some findings of this study.

1. BASIC BACKGROUND ON BELIEF FUNCTION THEORY

For convenience, we recall in brief the Dempster–Shafer theory of evidence [4, 5]. For details, see Sections 1 and 2 of the paper [1].

Let $X=\{x_1,...,x_n\}$ be a finite set, and 2^X be the set of all subsets from *X.* In the Dempster–Shafer theory of evidence, a *basic belief assignment* (BBA, also termed a mass function) is a set function *m:*

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$2^X \to [0, 1]$ that satisfies the condition $\sum_{A\in 2^X} m(A) = 1$.

A subset $A \subseteq X$ is called a focal element of a BBA m if $m(A) > 0$. A pair $F = (\mathcal{A}, m)$ composed of the set of all focal elements $\mathcal{A} = \{A\}$ and a corresponding BBA $m(A)$, $A \in \mathcal{A}$, is called a body of evidence. We denote by $\mathcal{F}(X)$ the set of all bodies of evidence on X and by $\mathcal{P}(X)$ the set of all probability measures on X.

A body of evidence $F = (\mathcal{A}, m)$ can be bijectively described by the *belief function* $Bel(A) = \sum_{B\subseteq A} m(B)$ and the *plausibility function* $Pl(A) = 1 - Bel(A^c) = \sum_{A:B\cap A\neq\varnothing} m(B)$, where A^c indicates the complement of the set A. The function $Pl(x) = \sum_{A\in\mathcal{A}: x\in A} m(A)$, $x \in X$, is called the contour function of a body of evidence. The belief and plausibility functions will be denoted by Bel_F and Pl_F, respectively, whenever their dependence on the body of evidence $F = (\mathcal{A}, m)$ should be emphasized.

An order relation can be defined on the set of set functions g: $2^X \to \mathbb{R}$ as follows: $g_1 \le g_2$ if $g_1(A) \le g_2(A)$ $\forall A \in 2^X$.

A belief function (and the corresponding body of evidence) is said to be:

– *categorical* if it has only one focal element; the corresponding body of evidence will be denoted by $F_A = (A, 1)$;

– *vacuous* if the entire set X is the only focal element of this function, $F_X = (X, 1)$;

– *consonant* if its focal elements are nested, i.e., $\forall A, B \in \mathcal{A}$: $A \subseteq B$ or $B \subseteq A$;

– *simple* if the BBA has no more than two focal elements and, in the case of two focal elements, X is one of them;

– *dogmatic* if $X \notin \mathcal{A}$ (i.e., $m(X) = 0$).

Any body of evidence $F = (\mathcal{A}, m)$ can be represented as $F = \sum_{A\in\mathcal{A}} m(A) F_A$. A simple body of evidence can be represented as $F_A^{\omega} = (1-\omega) F_A + \omega F_X$, where $\omega \in [0, 1]$.

A body of evidence $F' = (\mathcal{A}', m')$ is called a specialization of a body of evidence $F'' = (\mathcal{A}'', m'')$ (and denoted by $F' \sqsubseteq F''$) if there exists a partition $\mathcal{A}' = \mathcal{A}'_1 \cup \ldots \cup \mathcal{A}'_k$, $\mathcal{A}'_i \cap \mathcal{A}'_j = \varnothing$ $\forall i \neq j$, $k = |\mathcal{A}''|$ such that $\bigcup_{A\in\mathcal{A}'_i} A \subseteq B_i$ and $\sum_{A\in\mathcal{A}'_i} m'(A) = m''(B_i)$, $\forall B_i \in \mathcal{A}''$, $i = 1, \ldots, k$. In other words, a body of evidence F' refines (specializes) a body of evidence F''. The latter body is called a generalization of a body of evidence F'.

The amount of ignorance in the information contained in a body of evidence $F = (\mathcal{A}, m)$ can be estimated using the so-called imprecision indices [6]. An example of such an index is the normalized generalized Hartley measure [7, 8] $H_0(F) = \sum_{A\in\mathcal{A}} m(A) \log_{|X|} |A|$, mostly used below.

Belief function theory provides well-developed tools to combine bodies of evidence. A combining rule is understood as a certain operation $\otimes: \mathcal{F}(X) \times \mathcal{F}(X) \to \mathcal{F}(X)$. The most widespread rules include the following [9]:

- The unnormalized Dempster rule $\otimes_{ND}$: $m_{ND}(A) = \sum_{B\cap C=A} m_1(B) m_2(C)$ $\forall A \in 2^X$.

The canonical measure of (external) conflict is the value $K = K(F_1, F_2) = m_{ND}(\varnothing) = \sum_{B\cap C=\varnothing} m_1(B) m_2(C) \in [0, 1]$. It characterizes the degree of conflict of information sources described by the bodies of evidence F_1 and F_2: the greater this value is, the more inconsistent information the sources will provide.

- The Dempster rule [4] $\otimes_D$: $m_D(A) = \dfrac{m_{ND}(A)}{1-K}$ $\forall A \in 2^X \setminus \varnothing$. If $K = 1$ (complete conflict), the Dempster combining rule becomes inapplicable.
- The disjunctive consensus rule $\otimes_{\cup}$ [10]:

$$m_{\cup}(A) = \sum_{B\cup C=A} m_1(B) m_2(C), \quad A \in 2^X. \qquad (1)$$

2. THE CONCEPT OF NON-CONFLICT FOCAL ELEMENTS AND MEASURE OF INTERNAL CONFLICT

The measure of internal conflict in a body of evidence is understood as a certain functional $Con_{int} : \mathcal{F}(X) \to [0, 1]$ that achieves maximum under complete conflict between its focal elements and minimum without any conflict.

By analogy with bodies of evidence (see Section 3 of [1]), the following degrees of non-conflict are considered for the focal elements of a given body of evidence $F = (\mathcal{A}, m)$:

1) strong non-conflict: $\bigcap_{A\in\mathcal{A}} A \neq \varnothing$,

2) (simple) non-conflict: $A \cap B \neq \varnothing$ $\forall A, B \in \mathcal{A}$.

(The corresponding information sources are said to be conflict-free.)

Remark 1. Focal elements satisfying condition 1) are said to be logically consistent [11], and those satisfying condition 2) are said to be pairwise consistent [12]. Logical consistency implies pairwise consistency; however, the converse fails. The paper [12] considered some properties of bodies of evidence satisfying the general *s*-consistency condition: $\bigcap_{i=1}^{s} A_i \neq \varnothing$ $\forall A_1,..., A_s \in \mathcal{A}$, where $2 \le s \le |\mathcal{A}|$.

Generally, the measure of internal conflict Con_{int} should satisfy the following conditions:

I1: $Con_{int}(F) = 0$ if the focal elements of a body of evidence F are logically (or weakly pairwise) consistent.

Under condition I1, we particularly have:

- $Con_{int}(F) = 0$ if F is a categorical body of evidence.
- $Con_{int}(F) = 0$ if F is a consonant body of evidence.
- $Con_{int}(F) = 0$ if F is a simple body of evidence.

I2: $Con_{int}(F_1) \ge Con_{int}(F_2)$ $\forall F_1, F_2 \in \mathcal{F}(X)$, $F_1 \sqsubseteq F_2$ (antimonotonicity with respect to specialization).

In addition, the measure of internal conflict of a "complex" body of evidence should not be smaller than the minimum measure of external conflict between its components ("elementary" bodies of evidence). This requirement can be formulated for the class $\mathcal{OR}$ of so-called optimistic combining rules for bodies of evidence.

A combining rule $\otimes$ is said to be optimistic (pessimistic) with respect to an imprecision index f if $f(F_1 \otimes F_2) \le f(F_i)$ ($f(F_1 \otimes F_2) \ge f(F_i)$, respectively), where $i = 1, 2$; see [6].

The following statement is true; for example, see the paper [13].

Proposition 1. $H_0(F_1 \otimes_{ND} F_2) \le H_0(F_i)$ *and* $H_0(F_1 \otimes_{\cup} F_2) \ge H_0(F_i)$, $i = 1, 2,$ $\forall F_1, F_2 \in \mathcal{F}(X)$.

Thus, the unnormalized Dempster rule $\otimes_{ND}$ is optimistic, and the disjunctive consensus rule is pessimistic with respect to the normalized generalized Hartley measure H_0. The same result holds for any linear strict imprecision index [6].

I3: $Con_{int}(F_1 \otimes F_2) \ge \min\{Con_{int}(F_1), Con_{int}(F_2)\}$ $\forall F_1, F_2 \in \mathcal{F}(X)$ $\forall \otimes \in \mathcal{OR}$.

For measures of internal conflict, desirable properties also include independence from the ordering of alternatives of the set X or some generalization of this property. Let $\varphi : X \to Y$ be a bijective mapping. Then we can consider the image of a body of evidence $F = (\mathcal{A}, m)$ under the mapping φ: $F^{\varphi} = (\mathcal{A}^{\varphi}, m^{\varphi})$, where $\mathcal{A}^{\varphi} = \{\varphi(A) : A \in \mathcal{A}\} \subseteq 2^Y$ and $m^{\varphi}(B) = \sum_{A:\varphi(A)=B} m(A)$ $\forall B \in \mathcal{A}^{\varphi}$.

I4: For any bijective mapping φ, $Con_{int}(F^{\varphi}) = Con_{int}(F)$ $\forall F \in \mathcal{F}(X)$.

3. AN AXIOMATICS FOR A MEASURE OF INTERNAL CONFLICT

An implicit-form axiomatics for measures of internal conflict appears when axiomatizing the so-called uncertainty measures within belief function theory [14] and imprecise probability theory [15].

An explicit-form axiomatics for measures of internal conflict was considered in [16]. More specifically, a system of axioms based on strengthening conditions I1–I4 was investigated:

B1: $Con_{int}(F) = 0 \Leftrightarrow$ the focal elements of a body of evidence $F = (\mathcal{A}, m)$ are in strong non-conflict, i.e., $\bigcap_{A \in \mathcal{A}} A \neq \varnothing$.

B2: $Con_{int}(F_1) \ge Con_{int}(F_2)$ $\forall F_1, F_2 \in \mathcal{F}(X)$, $Bel_{F_1} \ge Bel_{F_2}$.

B3: $Con_{int}(\alpha F_1 + (1-\alpha)F_2) \ge \alpha Con_{int}(F_1) + (1-\alpha)Con_{int}(F_2)$ $\forall \alpha \in [0, 1]$, $\forall F_1, F_2 \in \mathcal{F}(X)$.

B4: For any mapping $\varphi : X \to Y$, $Con_{int}(F^{\varphi}) \le Con_{int}(F)$ $\forall F \in \mathcal{F}(X)$; for an injective mapping φ, $Con_{int}(F^{\varphi}) = Con_{int}(F)$.

Axiom B2 strengthens property I2 since $Bel_{F'} \ge Bel_{F''}$ implies $F' \sqsubseteq F''$. However, the converse fails [10].

Axiom B3 strengthens property I3 for the case of linear combining rules since $\alpha Con_{int}(F_1) + (1-\alpha) \times Con_{int}(F_2) \ge \min\{Con_{int}(F_1, Con_{int}(F_2)\}$. (These rules are simultaneously optimistic and pessimistic with respect to the linear imprecision index.)

The axiom B4 strengthens property I4 for the case of non-injective mappings φ: if the images of different elements from the set X are the same element from the set Y, the measure of internal conflict in the body of evidence F^{φ} will not exceed that of the body of evidence F.

Then the measure of internal conflict can be extended from the set of probability measures $\mathcal{P}(X)$ to the set of all bodies of evidence $\mathcal{F}(X)$.

Theorem 1 [16]. *If a functional* $Con:\mathcal{P}(X)\to[0,1]$ *satisfies axioms* B1, B3, *and* B4 *on the set* $\mathcal{P}(X)$, *then the functional*

$$Con_{int}(F)=\inf\left\{Con(P):P\in\mathcal{P}_{Bel_F}\right\}$$

satisfies axioms B1–B4 *on the set* $\mathcal{F}(X)$, *where* Bel_F *is a belief function corresponding to the body of evidence* F, *and* $\mathcal{P}_{Bel_F}=\{P\in\mathcal{P}(X): Bel_F(A)\le P(A)\ \forall A\subseteq X\}$ *is the set of probability measures agreed with* Bel_F.

Theorem 1 allows determining the measure of internal conflict on the set $\mathcal{F}(X)$ given this measure on the set $\mathcal{P}(X)$. Since $P=\sum_{i=1}^{n}P(\{x_i\})F_{\{x_i\}}$,

$$Con(P)=f(P(\{x_1\}),...,\ P(\{x_n\})), \qquad (2)$$

where $f(t_1,...,t_n)$ is some function with $n=|X|$. The paper [16] found necessary and sufficient conditions on the function f under which the functional (2) satisfies axioms B1–B4 on the set $\mathcal{P}(X)$. In particular, the following proposition describes a wide class of such functions.

Proposition 2 [16]. *Assume that a function* $g:[0,1]\to[0,+\infty)$ *is concave,* $g(0)=g(1)=0$, *and* g *is strictly decreasing at the point* $t=1$. *Then the function* $f(t_1,...,t_n)=\sum_{i=1}^{n}g(t_i)$ *defines by formula* (2) *a measure of internal conflict on the set* $\mathcal{P}(X)$, *and this measure satisfies axioms* B1–B4.

Examples of the function g (a generating function for a measure of conflict on the set $\mathcal{P}(X)$) are:

– $g(t)=\begin{cases}-t\ln t, & t\in(0,1],\\ 0, & t=0,\end{cases}$ (in this case, $Con(P)=-\sum_{i=1}^{n}P(\{x_i\})\ln P(\{x_i\})$ is the Shannon entropy),

– $g(t)=t-t^2$, $t\in[0,1]$ (in this case, $Con(P)=E_I(P)$, where E_I is the entropy functional from the representation (6) in the paper [1]).

4. METHODS TO ESTIMATE INTERNAL CONFLICT

4.1. Entropy approach

In this case, the measure of internal conflict in a body of evidence $F=(\mathcal{A},m)$ should reflect the distribution of its mass function values on conflicting focal elements, i.e., on those focal elements that are not in strong or weak non-conflict. This definition of internal conflict was investigated in the early 1980s as a generalization of the Shannon entropy in the Dempster–Shafer theory [17]. As a rule, the entropy functional is the average value of the distribution of focal elements with respect to some conflict function:

$$\sum_{A\in\mathcal{A}}m(A)\theta\left(\psi(A)\right),$$

where $\theta:[0,1]\to[0,+\infty]$ is an increasing and convex function such that $\theta(0)=$ (e.g., $\theta(t)=-\log_2(1-t)$ for the functionals considered below), and $\psi:2^X\to[0,1]$ is a set function whose values $\psi(A)$, $A\in 2^X$, characterize the total mass of all focal elements in conflict with the set A. In particular, the following functionals are often considered:

- the *measure of dissonance* [18]

$$E(F)=-\sum_{A\in\mathcal{A}}m(A)\log_2 Pl(A)=-\sum_{A\in\mathcal{A}}m(A)\log_2\left(1-K(A)\right),$$

where $K(A)=\sum_{A\cap B=\varnothing}m(B)$ is the total mass of all focal elements in conflict with the set A by the non-intersection relation;

- the *measure of confusion* [19] as the average value of conflicting focal elements by the non-inclusion relation:

$$C(F)=-\sum_{A\in\mathcal{A}}m(A)\log_2 Bel(A)=-\sum_{A\in\mathcal{A}}m(A)\log_2\left(1-L(A)\right),$$

where $L(A)=\sum_{B\not\subseteq A}m(B)$ is the total mass of all focal elements in conflict with the set A by the non-inclusion relation;

- the *measure of discord* [20]

$$D(F)=-\sum_{A\in\mathcal{A}}m(A)\log_2\left(1-Conf(A)\right),$$

where $Conf(A)=\sum_{B\in\mathcal{A}}m(B)\dfrac{|B\setminus A|}{|B|}$ is the total weighted mass of all focal elements in conflict with the set A; obviously, $K(A)\le Conf(A)\le L(A)$;

- the *measure of strife* [21]

$$ST(F)=-\sum_{A\in\mathcal{A}}m(A)\log_2\left(1-CONF(A)\right),$$

where $CONF(A)=\sum_{B\in\mathcal{A}}m(B)\dfrac{|A\setminus B|}{|A|}$ is the total weighted mass of all focal elements in conflict with the set A.

Each entropy functional characterizes a certain type of conflict of focal elements. Only the entropy measure of dissonance satisfies conditions I1 and I2. Generally speaking, the other entropy measures under consideration do not satisfy these conditions.

Example 1. Let $X=\{x_1,...,x_5\}$. We find entropy measures of conflict for the bodies of evidence

$F_i(\alpha)=\alpha F_A+(1-\alpha)F_{B_i}$, $\alpha\in[0,1]$, $i=1,2,3$, and different mutual arrangements of the focal elements $A\in 2^X$ and $B_i\in 2^X$, $i=1,2,3$. In all cases, we consider $A=\{x_1,x_2\}$ and $|B_i|=3$, $i=1,2,3$.

1) $B_1=\{x_1,x_2,x_3\}$. In this case, $A\subseteq B_1$. Then $K(A)=K(B_1)=0$, $L(A)=1-\alpha$, $L(B_1)=0$, $Conf(A)=\frac{1}{3}(1-\alpha)$, $Conf(B_1)=0$, $CONF(A)=0$, and $CONF(B_1)=\frac{1}{3}\alpha$. Hence, we obtain the entropy measures $E(F_1(\alpha))=0$, $C(F_1(\alpha))=-\alpha\log_2\alpha$, $D(F_1(\alpha))=-\alpha\log_2\left(\frac{2}{3}+\frac{1}{3}\alpha\right)$, and $ST(F_1(\alpha))=-(1-\alpha)\log_2\left(1-\frac{1}{3}\alpha\right)$.

2) $B_2=\{x_2,x_3,x_4\}$. In this case, $A\cap B_2\neq\varnothing$, but $A\not\subseteq B_2$ and $B_2\not\subseteq A$. Then $K(A)=K(B_2)=0$, $L(A)=1-\alpha$, $L(B_2)=\alpha$, $Conf(A)=\frac{2}{3}(1-\alpha)$, $Conf(B_2)=\frac{1}{2}\alpha$, $CONF(A)=\frac{1}{2}(1-\alpha)$, and $CONF(B_2)=\frac{2}{3}\alpha$. Hence, we obtain the entropy measures $E(F_2(\alpha))=0$, $C(F_2(\alpha))=-\alpha\log_2\alpha-(1-\alpha)\log_2(1-\alpha)$, $D(F_2(\alpha))=-\alpha\log_2\left(\frac{1}{3}+\frac{2}{3}\alpha\right)-(1-\alpha)\log_2\left(1-\frac{1}{2}\alpha\right)$, and $ST(F_2(\alpha))=-\alpha\log_2\left(\frac{1}{2}+\frac{1}{2}\alpha\right)-(1-\alpha)\log_2\left(1-\frac{2}{3}\alpha\right)$.

Thus, the measure of dissonance is uninformative ($E(F_i)\equiv 0$, $i=1,2$) in the first two cases: it only considers the non-intersection relation of focal elements, absent in these cases.

3) $B_3=\{x_3,x_4,x_5\}$. In this case, $A\cap B_3=\varnothing$. Then $K(A)=L(A)=Conf(A)=CONF(A)=1-\alpha$ and $K(B_3)=L(B_3)=Conf(B_3)=CONF(B_3)=\alpha$. Hence, $E(F_3(\alpha))=C(F_3(\alpha))=D(F_3(\alpha))=ST(F_3(\alpha))=-\alpha\log_2\alpha-(1-\alpha)\times\log_2(1-\alpha)$. In this case, all entropy measures of conflict coincide, achieving maximum for any fixed $\alpha\in[0,1]$.

Note that $D(F_i(1-\alpha))=ST(F_i(\alpha))$, $i=1,2,3$, in all cases considered. In addition, the entropy conflict by any measure increases pointwise with increasing the "degree of non-intersection" of the focal elements (for example, $D(F_1(\alpha))\leq D(F_2(\alpha))\leq D(F_3(\alpha))$ $\forall\alpha\in[0,1]$). ♦

4.2. Methods based on auto-conflict calculation and contour function maximization

A body of evidence $F=(\mathcal{A},m)$ can be considered internally non-conflict if it has no conflict with itself by some measure of external conflict; see part I in [1]. For example, a body of evidence F can be non-conflict with itself by the canonical measure of conflict K: $K(F,F)=0$. The value $K(F,F)$ can be treated as a measure of internal conflict. The paper [22] introduced the so-called *auto-conflict* of order s: $Con_{aut,s}(F)=K(\underbrace{F,...,F}_{s})$. For $s=2$, such a measure will be simply called auto-conflict: $Con_{aut}(F)=Con_{aut,2}(F)$. The measure of auto-conflict Con_{aut} satisfies conditions I1 (in the case of simple non-conflict of focal elements) and conditions I2, I4, and I3 if $\mathcal{OR}=\{\otimes_{ND}\}$.

Another approach involves the strong non-conflict of focal elements. Clearly,

$$\Omega_{\mathcal{A}}=\bigcap\nolimits_{A\in\mathcal{A}}A\neq\varnothing \quad\Leftrightarrow$$
$$\exists x\in X: Pl(x)=\sum\nolimits_{A\in\mathcal{A}:x\in A}m(A)=1.$$

In other words, the logical consistency of a body of evidence ($\Omega_{\mathcal{A}}\neq\varnothing$) is equivalent to $\max_{x\in X}Pl(x)=1$ (the contour function achieves maximum equal to 1). Note that if $|\mathcal{A}|=s$ and $\Omega_{\mathcal{A}}\neq\varnothing$, then $Con_{aut,s}(F)=0$. This fact was used in [23] to introduce the measure of internal conflict $Con_{pl}(F)=1-\max\{Pl(x):x\in X\}$. In this case, the maximum of the contour function, $\max\{Pl(x):x\in X\}=1-Con_{pl}(F)$, is a measure of non-conflict. The measure Con_{pl} satisfies conditions I1–I4 (and condition I3 if $\mathcal{OR}=\{\otimes_{ND}\}$). Other properties of this measure were investigated in [2, 23].

Remark 2. As shown in [16], the measure of internal conflict $Con_{pl}(F)$ can be obtained by extending the measure of conflict (2) to the set $\mathcal{F}(X)$, where $f(t_1,...,t_n)=\min\{1-t_1,...,1-t_n\}$ and $n=|X|$; see Theorem 1. The resulting measure satisfies axioms B1–B4.

Example 2. For the bodies of evidence $F_i(\alpha)$, $i=1,2,3$, from Example 1, we obtain $Con_{pl}(F_1(\alpha))=Con_{pl}(F_2(\alpha))=1-\max_{1\leq k\leq 5}Pl(x_k)=0$ since $Pl(x_1)=Pl(x_2)=1$ (the first case) and $Pl(x_2)=1$ (the second case). In the third case, $Pl(x_1)=Pl(x_2)=\alpha$ and $Pl(x_3)=Pl(x_4)=Pl(x_5)=1-\alpha$. Therefore, $Con_{pl}(F_3(\alpha))=1-\max_{1\leq k\leq 5}Pl(x_k)=\min\{\alpha,1-\alpha\}$.

In this example, the measure of auto-conflict is given by $Con_{aut}(F_1(\alpha))=Con_{aut}(F_2(\alpha))=0$ and $Con_{aut}(F_3(\alpha))=2\alpha(1-\alpha)$. ♦

The measure of conflict Con_{pl} is easy to calculate and satisfies many desirable properties (particularly axioms B1–B4). As a result, it is popular in applications. At the same time—see Example 2—it becomes insensitive in the presence of disjoint focal elements.

4.3. Metric approach

In this case, the measure of external conflict in a body of evidence $F=(\mathcal{A},m)$ is given by

$$Con_{int}(F)=\inf_{F'\in\mathcal{V}(X)} d(F,F'), \qquad (3)$$

where d indicates some metric between bodies of evidence (see sub-subsection 4.3.1 in [1]), and $\mathcal{V}(X)$ is a set of bodies of evidence with zero internal conflict, i.e., the ones satisfying condition I1. This can be, e.g., the set of categorical or simple bodies of evidence. Such an approach was considered in [24] and was applied to estimate the reliability of expert weather forecasts. In the general case, such a measure may not satisfy all the desirable properties of measures of conflict. The result of calculating an internal conflict significantly depends on the choice of the set $\mathcal{V}(X)$. In addition, the solution procedure of the optimization problem (3) may have high computational complexity.

Example 3. We find the internal conflict in the bodies of evidence $F_i(\alpha)$, $i=1,2,3$, (Example 1) using formula (3), where the metric $d=d_J$ is given by

$$d_J(F_1,F_2)=$$
$$\sqrt{\tfrac{1}{2}\sum_{A,B\in 2^X\setminus\{\varnothing\}}\tfrac{|A\cap B|}{|A\cup B|}(m_1(A)-m_2(A))(m_1(B)-m_2(B))},$$
$$F_1=(\mathcal{A},m_1),\ F_2=(\mathcal{A}_2,m_2).$$

(For details, see the paper [25] and sub-subsection 4.3.1 of part I in [1].) Let $\mathcal{V}(X)$ be the set of simple bodies of evidence on the set X of the form $F_{\{x\}}^{\omega}=(1-\omega)F_{\{x\}}+\omega F_X$, where $\omega\in[0,1]$ and $x\in X$. In this case, $Con_{int}(F_i(\alpha))=\min\limits_{1\le i\le 5}\min\limits_{\omega_i\in[0,1]} d_J(F_i(\alpha),F_{\{x_i\}}^{\omega_i})$, $i=1,2,3$.

For the set $B_1=\{x_1,x_2,x_3\}$ ($A\subseteq B_1$), we particularly obtain

$$d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})=$$
$$\tfrac{1}{\sqrt{2}}\sqrt{h^2(\alpha,\omega_i)-\alpha(1-\omega_i)-\tfrac{2}{3}(1-\alpha)(1-\omega_i)},\ i=1,2,$$
$$d_J(F_1(\alpha),F_{\{x_3\}}^{\omega_3})=\tfrac{1}{\sqrt{2}}\sqrt{h^2(\alpha,\omega_3)-\tfrac{2}{3}(1-\alpha)(1-\omega_3)},$$
$$d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})=\tfrac{1}{\sqrt{2}}h(\alpha,\omega_i),\ i=4,5,$$

where

$$h(\alpha,\omega)=\sqrt{\alpha^2+(1-\alpha)^2+\omega^2+(1-\omega)^2-\tfrac{4}{5}\alpha\omega-\tfrac{6}{5}(1-\alpha)\omega}.$$

Now,

$$\min_{\omega_i\in[0,1]} d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})=d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})\Big|_{\omega_i=\frac{38-11\alpha}{60}}=$$
$$\tfrac{1}{30}\sqrt{\tfrac{3479}{4}\alpha^2-841\alpha+239},\ i=1,2,$$
$$\min_{\omega_3\in[0,1]} d_J(F_1(\alpha),F_{\{x_3\}}^{\omega_3})=d_J(F_1(\alpha),F_{\{x_3\}}^{\omega_3})\Big|_{\omega_3=\frac{38+4\alpha}{60}}=$$
$$\tfrac{1}{30}\sqrt{896\alpha^2-676\alpha+239},$$
$$\min_{\omega_i\in[0,1]} d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})=d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})\Big|_{\omega_i=\frac{8-\alpha}{10}}=$$
$$\tfrac{1}{10}\sqrt{99\alpha^2-84\alpha+36},\ i=4,5.$$

Consequently,

$$Con_{int}(F_1(\alpha))=\min_{1\le i\le 5}\min_{\omega_i\in[0,1]} d_J(F_1(\alpha),F_{\{x_i\}}^{\omega_i})=$$
$$\tfrac{1}{30}\sqrt{\tfrac{3479}{4}\alpha^2-841\alpha+239}.$$

For the set $B_2=\{x_2,x_3,x_4\}$ ($A\cap B_2\ne\varnothing$, but $A\not\subseteq B_2$ and $B_2\not\subseteq A$), we obtain $Con_{int}(F_2(\alpha))=Con_{int}(F_1(\alpha))$ $\forall\alpha\in[0,1]$; for the set $B_3=\{x_3,x_4,x_5\}$ ($A\cap B_3=\varnothing$),

$$Con_{int}(F_3(\alpha))=\min\Big\{\tfrac{1}{20}\sqrt{351\alpha^2-376\alpha+144},$$
$$\tfrac{1}{30}\sqrt{896\alpha^2-676\alpha+239}\Big\}.$$

As is easily seen, $Con_{int}(F_1(\alpha))=Con_{\mathrm{int}}(F_2(\alpha))\le Con_{\mathrm{int}}(F_3(\alpha))$ $\forall\alpha\in[0,1]$.♦

4.4. Decompositional approach

The decompositional approach proceeds from the assumption that an information source formed by a body of evidence with a great internal conflict could be heterogeneous. For example, information about the predictive stock value is obtained using several different techniques. In this case, a body of evidence $F=(\mathcal{A},m)$ can be treated as the result of combining several decomposed bodies of evidence $F_i=(\mathcal{A}_i,m_i)\in\mathcal{F}(X)$, $i=1,...,l$, using some combining rule $\otimes$: $F=F_1\otimes...\otimes F_l$. For a fixed combining rule $\otimes$ and a fixed measure of (external) conflict $Con_{ext}:\underbrace{\mathcal{F}(X)\times...\times\mathcal{F}(X)}_{l}\to[0,1]$ (see part I in [1]), the internal decomposition conflict Con_{dec} in the body of evidence F can be therefore estimated [26, 27] by the formula

$$Con_{dec}(F)=Con_{ext}(F_1,...,F_l)$$

provided that

$$F=F_1\otimes...\otimes F_l.$$

This equation has a set of solutions. Hence, we can formulate optimization problems on finding the greatest $\overline{Con_{dec}^{\otimes}}(F)$ and smallest $\underline{Con_{dec}^{\otimes}}(F)$ conflicts:

$$\overline{Con_{dec}^{\otimes}}(F)=\sup_{F=F_1\otimes...\otimes F_l} Con_{ext}(F_1,...,F_l),$$
$$\underline{Con_{dec}^{\otimes}}(F)=\inf_{F=F_1\otimes...\otimes F_l} Con_{ext}(F_1,...,F_l). \qquad (4)$$

Let $S_n=\left\{\mathbf{s}=(s_i)_{i=1}^n : s_i\ge 0\ \forall i=1,...,n,\ \sum_{i=1}^n s_i=1\right\}$ be an n-dimensional simplex.

Consider special cases of the problem.

Decomposition using Dempster rule. Let decomposition be performed using the Dempster rule $\otimes_D$. For $l = 2$, problems (4) take the following form:

find

$$K(F_1, F_2) = \sum_{\substack{B \cap C = \varnothing, \\ B \in \mathcal{A}_1, C \in \mathcal{A}_2}} m_1(B) m_2(C) \to \sup\ (\inf) \quad (5)$$

subject to the conditions

$$\mathbf{m}_1 = (m_1(B))_{B \in \mathcal{A}_1} \in S_{|\mathcal{A}_1|},\ \mathbf{m}_2 = (m_2(C))_{C \in \mathcal{A}_2} \in S_{|\mathcal{A}_2|}, \quad (6)$$

$$\left(1 - K_0(F_1, F_2)\right) m(A) = \sum_{\substack{B \cap C = A, \\ B \in \mathcal{A}_1, C \in \mathcal{A}_2}} m_1(B) m_2(C),\ A \in \mathcal{A}, \quad (7)$$

$$K(F_1, F_2) < 1. \quad (8)$$

These are quadratic programming problems under the linear (6) and quadratic (7), (8) constraints. Note that in the general statement (5)–(8), the measure of the decomposition conflict $\underline{Con}_{dec}^{\otimes_D}(F) = 0$ is achieved for the bodies of evidence $F_1 = F$ and $F_2 = F_X$. For two bodies of evidence satisfying conditions (6) and (7) (without condition (8)), the greatest value of the conflict $K(F_1, F_2) = 1$ is achieved, e.g., for the bodies of evidence $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1, 2$, in which $B \cap C = \varnothing \quad \forall B \in \mathcal{A}_1, \forall C \in \mathcal{A}_2$. The latter bodies are in no way related to the body F.

Decomposition using disjunctive consensus rule. Let decomposition be performed using the disjunctive consensus rule $\otimes_{\cup}$ (1). Then condition (1) will be applied instead of condition (7) for estimating internal conflict. Therefore, the problem is finding bodies of evidence with the greatest (smallest) canonical conflict (5) that satisfy conditions (1) and (6).

Remark 3. When using the disjunctive consensus rule, sometimes it seems convenient to consider an empty set a focal element in a body of evidence. This can be interpreted as $x \notin X$ and the value $m(\varnothing)$ as the degree of belief to $x \notin X$. The corresponding solutions will be called generalized and denoted by $\mathcal{CON}_{dec}^{\otimes_{\cup}}(F)$. Then the greatest value of the canonical conflict (5) satisfying conditions (1) and (6), $\overline{\mathcal{CON}}_{dec}^{\otimes_{\cup}}(F) = 1$, is achieved on the decomposition of the body of evidence F of the form $F_1 = F$, $F_2 = F_{\varnothing}$.

Clearly, in the general statement, the problem of finding the largest and smallest internal conflicts $\overline{Con}_{dec}^{\otimes}(F)$ and $\underline{Con}_{dec}^{\otimes}(F)$ often leads to trivial solutions.

At the same time, the assumption about the heterogeneous information source of a body of evidence with a great internal conflict implies the following: the bodies of evidence composing the initial body of evidence should be, in some sense, simpler than the latter. In addition, the combining method may impose restrictions on the decomposed set of bodies of evidence. In particular, we identify several constraints on the decomposable set of bodies of evidence:

- structural constraints,
- conflict constraints,
- constraints related to combining rules,
- mixed constraints.

Structural constraints imply that the decomposed set of bodies of evidence belongs to some class of simple-structure bodies of evidence. Examples of such classes are simple bodies of evidence (or their generalizations, see below), consonant bodies of evidence, and others.

For example, the paper [28] defined internal conflict as a conflict between the so-called *generalized simple BBAs* (the bodies of evidence $F_A^{\omega} = (1 - \omega) F_A + \omega F_X$, $\omega \in (0, \infty)$) into which an initial non-dogmatic body of evidence ($m(X) > 0$) is uniquely decomposed. (Shafer called such a decomposition canonical). If the initial body of evidence is dogmatic ($m(X) = 0$), then before the decomposition, the BBA should be discounted with a small parameter $\varepsilon > 0$: $m(X) = \varepsilon > 0$. The mass functions of the other focal elements are recalculated proportionally to the initial values. According to the report [29], a non-dogmatic body of evidence F can be decomposed into generalized simple BBAs in two stages. The first stage is calculating the *commonality function* $q(A) = \sum_{B \supseteq A} m(B)$. In the second stage, the weights ω_B of the bodies of evidence $F_B^{\omega_B}$ are calculated by the formula $\omega_B = \prod_{A \supseteq B} q(A)^{(-1)^{|A|-|B|+1}}$ for each subset $B \in 2^X \setminus X$. As a result, $F = \otimes_{B \in 2^X \setminus X} F_B^{\omega_B}$, where $\otimes = \otimes_{ND}$ is the unnormalized Dempster rule [29]. Another measure of internal conflict, $Con_{dec\_simple}(F) = \tilde{m}(\varnothing)$, where $\tilde{F} = (\mathcal{A}, \tilde{m}) = \otimes_{B \in 2^X \setminus \{\varnothing, X\}} F_B^{\omega_B}$, was proposed in [28]. Clearly,

$$Con_{dec\_simple}(F) = \sum_{\substack{B_{i_1} \cap \ldots \cap B_{i_k} = \varnothing, \\ B_{i_1}, \ldots, B_{i_k} \in 2^X \setminus \{\varnothing, X\}}} \prod_{s=1}^{k} \left(1 - \omega_{B_{i_s}}\right) \times \prod_{B \in \left(2^X \setminus \{\varnothing, X\}\right) \setminus \left\{B_{i_1}, \ldots, B_{i_k}\right\}} \omega_B. \quad (9)$$

Example 4. For $X = \{x_1, x_2\}$ and $F = \alpha F_{\{x_1\}} + \beta F_{\{x_2\}} + (1 - \alpha - \beta) F_X$, where $\alpha, \beta \geq 0$ and $\alpha + \beta < 1$, we obtain $q(\varnothing) = 1$, $q(\{x_1\}) = 1 - \beta$, $q(\{x_2\}) = 1 - \alpha$, and $q(X) =$

$1-\alpha-\beta$. Therefore, $\omega_\varnothing = \frac{(1-\alpha)(1-\beta)}{1-\alpha-\beta}$, $\omega_{\{x_1\}} = \frac{1-\alpha-\beta}{1-\beta}$, and $\omega_{\{x_2\}} = \frac{1-\alpha-\beta}{1-\alpha}$. Hence, $Con_{dec\_simple}(F) = \tilde{m}(\varnothing) = (1-\omega_{\{x_1\}})\,(1-\omega_{\{x_2\}}) = \frac{\alpha\beta}{(1-\alpha)(1-\beta)}$. ♦

For calculating the measure Con_{dec_simple} for the body of evidence from Example 1, we need the following result.

Lemma. *Let* $F = \alpha F_A + \beta F_B + (1-\alpha-\beta)F_X$, *where* $\alpha, \beta \in (0, 1)$, $\alpha+\beta<1$, *and* $A, B \in 2^X$. *The following statements are true:*

– *If* $A \subseteq B \subseteq X$, *then* $\omega_A = 1-\alpha$, $\omega_B = \frac{1-\alpha-\beta}{1-\alpha}$, *and* $\omega_D = 1 \quad \forall D \in 2^X \setminus \{\varnothing, A, B, X\}$.

– *If* $A \cap B \neq \varnothing$, $A \not\subseteq B$, *and* $B \not\subseteq A$, *then* $\omega_A = \frac{1-\alpha-\beta}{1-\beta}$, $\omega_B = \frac{1-\alpha-\beta}{1-\alpha}$, $\omega_{A\cap B} = \frac{(1-\alpha)(1-\beta)}{1-\alpha-\beta}$, *and* $\omega_D = 1 \quad \forall D \in 2^X \setminus \{\varnothing, A\cap B, A, B, X\}$.

– *If* $A \cap B = \varnothing$, *then* $\omega_A = \frac{1-\alpha-\beta}{1-\beta}$, $\omega_B = \frac{1-\alpha-\beta}{1-\alpha}$, *and* $\omega_D = 1 \quad \forall D \in 2^X \setminus \{\varnothing, A, B, X\}$.

Corollary. Let $F = \alpha F_A + \beta F_B + (1-\alpha-\beta)F_X$, where $\alpha, \beta \in (0,1)$ and $\alpha+\beta<1$. The following statements are true:

– $Con_{dec\_simple}(F) = 0$ if $A \cap B \neq \varnothing$.

– $Con_{dec\_simple}(F) = \frac{\alpha\beta}{(1-\alpha)(1-\beta)}$ if $A \cap B = \varnothing$.

Example 5. Consider the bodies of evidence $F_i(\alpha) = \alpha F_A + (1-\alpha)F_{B_i}$, $\alpha \in [0,1]$, $A = \{x_1, x_2\}$, $|B_i| = 3$, $i = 1, 2, 3$, on the set $X = \{x_1, ..., x_5\}$ (Example 1). First, we perform discounting with a small parameter $\varepsilon > 0$ to obtain the bodies of evidence $F_i(\alpha, \varepsilon) = \alpha(1-\varepsilon)F_A + (1-\alpha)(1-\varepsilon)F_{B_i} + \varepsilon F_X$, $i = 1, 2, 3$. According to the corollary, in the first case $B_1 = \{x_1, x_2, x_3\}$ (if $A \subseteq B_1$) and the second case $B_2 = \{x_2, x_3, x_4\}$ (if $A \cap B_2 \neq \varnothing$, but $A \not\subseteq B_2$ and $B_2 \not\subseteq A$), we have $Con_{dec\_simple}(F_i(\alpha, \varepsilon)) = 0$, $i = 1, 2$. In the third case $B_3 = \{x_3, x_4, x_5\}$ (if $A \cap B_3 = \varnothing$), we have $Con_{dec\_simple}(F_3(\alpha, \varepsilon)) = (1-\omega_{\{x_1,x_2\}})(1-\omega_{\{x_3,x_4,x_5\}}) = \frac{\alpha(1-\alpha)(1-\varepsilon)^2}{(\alpha(1-\varepsilon)+\varepsilon)((1-\alpha)(1-\varepsilon)+\varepsilon)}$. As $\varepsilon \to +0$, $Con_{dec\_simple}(F_3(\alpha)) = \begin{cases} 1, & \alpha \in (0, 1), \\ 0, & \alpha = 0 \vee \alpha = 1. \end{cases}$ ♦

This example shows that the measure of conflict Con_{dec_simple} is rather rough for dogmatic bodies of evidence. Moreover, decomposition into generalized simple BBAs has other disadvantages. First of all, the bodies of evidence F_A^ω, $\omega \notin [0, 1]$, require a certain interpretation. In this case, we cannot say that the initial body of evidence results from combining information from several other sources. In addition, the decomposition can contain up to $2^{|X|}-1$ generalized simple BBAs, different from the meaningless body of evidence F_X. Nevertheless (see the next example), the initial body of evidence may result from combining several more complex bodies of evidence than generalized simple BBAs without internal conflict.

Example 6. Consider two bodies of evidence, $F_1 = \alpha F_{\{x_2,x_3\}} + (1-\alpha)F_{\{x_1,x_2,x_3\}}$ with $\alpha \in (0,1)$ and $F_2 = \beta F_{\{x_1,x_4\}} + (1-\beta)F_{\{x_1,x_2,x_4\}}$ with $\beta \in (0,1)$, on the set $X = \{x_1, x_2, x_3, x_4\}$. These bodies are consonant: each has no conflict with itself. The canonical measure of conflict is $K = K(F_1, F_2) = \alpha\beta$. Combining these bodies of evidence using the unnormalized Dempster rule yields $F = F_1 \otimes_{ND} F_2 = (1-\alpha)\beta F_{\{x_1\}} + \alpha(1-\beta)F_{\{x_2\}} + (1-\alpha)(1-\beta)F_{\{x_1,x_2\}}$. Decomposing this body of evidence into generalized simple BBAs and calculating the corresponding measure of conflict, we obtain $Con_{dec\_simple}(F) = \frac{\alpha\beta(1-\alpha)(1-\beta)}{(1-\alpha+\alpha\beta)(1-\beta+\alpha\beta)} < K$ (Example 4). In other words, the conflict between the initial consonant bodies of evidence will be greater than the one from decomposing the combined body of evidence into generalized siymple BBAs. ♦

A decompositional approach to estimating the internal conflict in bodies of evidence, close to [28], was considered in the paper [30]. The cited authors studied the conflict function $f_\varnothing\left(\left\{B_{i_1}, ..., B_{i_k}\right\}\right) = \prod_{s=1}^{k}\left(1-\omega_{B_{i_s}}\right)\prod_{B\in\left(2^X\setminus\{\varnothing,X\}\right)\setminus\left\{B_{i_1},...,B_{i_k}\right\}}\omega_B$ on sets of disjoint subsets $\left\{B_{i_1}, ..., B_{i_k}\right\}$, $B_{i_1} \cap ... \cap B_{i_k} = \varnothing$ (see formula (9)) and the *local conflict function* $\bar{f}_\varnothing(A) = \sum_{A\in\left\{B_{i_1},...,B_{i_k}\right\},\, B_{i_1}\cap...\cap B_{i_k}=\varnothing} \frac{1}{\left|\left\{B_{i_1}, ..., B_{i_k}\right\}\right|} f_\varnothing\left(\left\{B_{i_1}, ..., B_{i_k}\right\}\right), \quad A \subsetneq X$.

These functions were used in [30] to choose the least-conflict information sources for combining in the robot localization problem.

Conflict constraints mean that the decomposed set of bodies of evidence belongs to the class of ones with a smaller internal conflict than the original body of evidence by another (non-decomposition) measure of conflict.

Example 7. Let $X = \{x_1, x_2, x_3\}$ and $F = \alpha F_{\{x_1\}} + \beta F_{\{x_2\}} + \gamma F_{\{x_3\}} + (1-\alpha-\beta-\gamma)F_{\{x_2,x_3\}}$, where $\alpha, \beta, \gamma > 0$, $\alpha + \beta + \gamma < 1$. Consider the decomposition of the body of evidence F using the unnormalized Dempster rule: $F = F_1 \otimes_{ND} F_2$. Assume that the decomposition belongs to the class of non-conflict bodies of evidence (zero auto-conflict: $Con_{aut}(F_i) = 0$, $i = 1, 2$). As is easily shown, the unique decomposition has the form

$$\begin{cases} F_1 = \lambda_1 F_{\{x_2,x_3\}} + \lambda_2 F_{\{x_1,x_2\}} + \lambda_3 F_{\{x_2\}}, \\ F_2 = \mu_1 F_{\{x_2,x_3\}} + \mu_2 F_{\{x_1,x_3\}} + \mu_3 F_{\{x_3\}}. \end{cases}$$

Due to $F = F_1 \otimes_{ND} F_2$, the nonnegative coefficients λ_i, μ_i, $i = 1,2,3,$ satisfy the system of equations

$$\begin{cases} \lambda_1 + \lambda_2 + \lambda_3 = 1, \quad \mu_1 + \mu_2 + \mu_3 = 1, \\ \lambda_1\mu_1 = 1-\alpha-\beta-\gamma, \quad \lambda_2\mu_2 = \alpha, \\ (\lambda_2 + \lambda_3)\mu_1 = \beta, \quad \lambda_1(\mu_2 + \mu_3) = \gamma. \end{cases}$$

This system is solvable if $\alpha(1-\alpha-\beta-\gamma) = \beta\gamma$, and the solution is given by $\lambda_1 = 1-\alpha-\beta$, $\mu_1 = 1-\alpha-\gamma$, $\lambda_2 = \frac{\alpha}{\alpha+\gamma}$, $\mu_2 = \alpha+\gamma$, and $\lambda_3 = \mu_3 = 0$. Then $Con_{dec}(F) = K(F_1, F_2) = 0$. However, for example, $Con_{aut}(F) = \alpha(1-\alpha) + \beta\gamma$. ♦

Constraints related to combining rules. The choice of a combining rule imposes constraints on the set of admissible bodies of evidence. This is due to the different nature of these rules. For example, the conjunctive rule is optimistic, and the disjunctive rule is pessimistic. Constraints on the set of admissible bodies of evidence, agreed with the nature of combining rules, can be defined, e.g., using imprecision indices [6]. From now on, we will use the normalized generalized Hartley measure $H_0(F) = \sum_{A\in\mathcal{A}} m(A)\log_{|X|}|A|$ as an imprecision index. Nevertheless, all results are valid for a broader class of such indices, particularly for strict linear imprecision indices [26, 27].

Recall that the Dempster rule is optimistic (Proposition 1). When decomposing a body of evidence F into two bodies of evidence $F_i = (\mathcal{A}_i, m_i) \in \mathcal{F}(X)$, $i = 1,2$, the problem of estimating its internal conflict can be formulated as follows: find the greatest (smallest) value of the functional $K(F_1, F_2)$ under the constraints (6)–(8) and the conditions

$$H_0(F) \le H_0(F_i),\ i = 1,2. \qquad (10)$$

We denote their solutions by $\underline{Con}^{\otimes_D}_{dec\_gen}(F)$ and $\overline{Con}^{\otimes_D}_{dec\_gen}(F)$, respectively. Note that the bodies of evidence $F_1 = F$ and $F_2 = F_X$ satisfy conditions (10) because $H_0(F_X) = 1$. Therefore, $\underline{Con}^{\otimes_D}_{dec\_gen}(F) = 0$. Then the problem is finding bodies of evidence with the greatest canonical conflict (5) that satisfy conditions (6)–(8) and (10).

Besides the lower-bound constraints (10), the upper-bound constraints can be introduced on the amount of ignorance in the information contained in the decomposed bodies of evidence: $H_0(F_i) \le H_{\max}$, $i = 1,2$, where $H_{\max}$ is the maximum admissible level of ignorance.

If the body of evidence F is decomposed using the disjunctive consensus rule, conditions (1) are replaced for conditions (7) in the internal conflict problem. Moreover, for the disjunctive consensus rule and any linear imprecision index (particularly H_0), we have the relation

$$H_0(F) \ge H_0(F_i),\ i = 1, 2. \qquad (11)$$

(For details, see Proposition 1.)

Thus, in this case, the problem is finding bodies of evidence with the greatest (smallest) canonical conflict (5) that satisfy conditions (1), (6), and (11). We denote by $\underline{Con}^{\otimes_\cup}_{dec\_gen}(F)$ and $\overline{Con}^{\otimes_\cup}_{dec\_gen}(F)$ the solutions of the corresponding problems.

Example 8. Let $X = \{x_1, x_2\}$ and $F = \alpha F_{\{x_1\}} + \beta F_{\{x_2\}} + (1-\alpha-\beta)F_X$, where $\alpha, \beta \ge 0$, $\alpha + \beta < 1$. According to [26], $\overline{Con}^{\otimes_D}_{dec\_gen}(F) = \frac{\alpha\beta}{(1-\alpha)(1-\beta)}$; if $\sqrt{\alpha} + \sqrt{\beta} \le 1$, then $\underline{Con}^{\otimes_\cup}_{dec\_gen}(F) = 2\sqrt{\alpha\beta}$. For $\sqrt{\alpha} + \sqrt{\beta} > 1$, the corresponding decomposition problem for finding the measure of conflict $\underline{Con}^{\otimes_\cup}_{dec\_gen}$ has no solution. (However, it has a generalized solution; see Remark 3.) Note that $\overline{Con}^{\otimes_D}_{dec\_gen} = Con_{dec\_simple}$ on the set $X = \{x_1, x_2\}$ (Example 4). ♦

The paper [31] established some properties for the measures of conflict obtained by decomposition with constraints related to combining rules. In particular, it was shown therein that $\overline{Con}^{\otimes_D}_{dec\_gen}(F) = 1$ in the case of complete conflict of focal elements and $\overline{Con}^{\otimes_D}_{dec\_gen}(F) = 0$ for simple bodies of evidence.

The general disadvantage of the decompositional approach is high computational complexity. However, it is compensated by good interpretability in the case of a heterogeneous information source.

CONCLUSIONS

This paper has reviewed current research on the inconsistency (conflict) of information in a single source within belief function theory. In particular, the following aspects can be highlighted:

- There are several requirements to measures of internal conflict: minimum under some degree of nonconflict of focal elements, antimonotonicity with respect to specialization, nondecrease under optimistic combining, and nonincrease under mappings of the basic set.
- These properties underlie the axiomatics of a measure of internal conflict. A general form of such a measure is found; on the set of probability measures, it coincides with some entropy functional (particularly with the Shannon entropy for an appropriately chosen generating function).
- There are several methods for estimating internal conflict: an entropy approach, methods based on auto-conflict calculation and contour function maximization, and metric and decompositional approaches.

The internal conflict estimation methods considered differ in the conditions to satisfy desired properties, their sensitivity and computational complexity, and the estimation model: the average mass distribution of conflicting focal elements, the distance to the set of non-conflicting bodies of evidence, auto-conflict, the measure of logical consistency of focal elements, the heterogeneity of information sources, etc.

Of course, there are still open problems in estimating the internal conflict of bodies of evidence:
– exploring the properties of internal conflict measures based on a particular model;
– finding a general form of a measure of internal conflict for other systems of axioms;
– studying measures of conflict for bodies of evidence defined on a metrical space;
– and others.

Applied problems related to the estimation of internal conflict are topical. Among them, we mention reducing the internal conflict in a body of evidence (including evidence obtained by expert data processing). This problem can be solved by generalizing the initial body of evidence (see condition I2) or by decomposing it into internally non-conflicting bodies of evidence (see subsection 4.4).

REFERENCES

1. Lepskiy, A., Analysis of Information Inconsistency in Belief Function Theory. Part I: External Conflict, *Control Sciences*, 2021, no. 5, pp. 2–16.
2. Daniel, M., Conflicts within and between Belief Functions, in *Information Processing and Management of Uncertainty* (*IPMU 2010*)*: Lecture Notes on Artificial Intelligence*, Hüllermeier, E., Kruse, R., and Hoffmann, E., Eds., vol. 6178, Berlin–Heidelberg: Springer-Verlag, 2010, pp. 696–705.
3. Daniel, M., Non-conflicting and Conflicting Parts of Belief Functions, in *Proc. of the 7th Intern. Symp. on Imprecise Probabilities: Theories and Applications* (*ISIPTA*'11), Coolen, F., de Cooman, G., Fetz, T., and Oberguggenberger, M., Eds., Innsbruck: Studia Universitätsverlag, 2011, pp. 149–158.
4. Dempster, A.P., Upper and Lower Probabilities Induced by a Multivalued Mapping, *Annals of Mathematical Statistics*, 1967, vol. 38, pp. 325–339.
5. Shafer, G., *A Mathematical Theory of Evidence*, Princeton: Princeton University Press, 1976.
6. Bronevich, A. and Lepskiy, A., Imprecision Indices: Axiomatic, Properties and Applications, *Int. J. of General Systems*, 2015, vol. 44, no. 7-8, pp. 812–832.
7. Higashi, M. and Klir, G.J., Measures of Uncertainty and Information Based on Possibility Distributions, *Int. J. General Systems*, 1983, no. 9, pp. 43–58.
8. Dubois, D. and Prade, H., A Note on Measures of Specificity for Fuzzy Sets, *Int. J. of General Systems*, 1985, no. 10, pp. 279–283.
9. Sentz, K. and Ferson, S., Combination of Evidence in Dempster–Shafer Theory, *Report SAND 2002-0835*, Sandia National Laboratories, 2002.
10. Dubois, D. and Prade, H., A Set-Theoretic View on Belief Functions: Logical Operations and Approximations by Fuzzy Sets, *Int. J. of General Systems*, 1986, no. 12, pp. 193–226.
11. Destercke, S. and Burger, T., Toward an Axiomatic Definition of Conflict between Belief Functions, *IEEE Transactions on Cybernetics*, 2013, vol. 43, no. 2, pp. 585–596.
12. Pichon, F., Jousselme, A.-L., and Ben Abdallah, N., Several Shades of Conflict, *Fuzzy Sets and Systems*, 2019, vol. 366, pp. 63–84.
13. Lepskiy, A., General Schemes of Combining Rules and the Quality Characteristics of Combining, in *Belief Functions: Theory and Applications* (*BELIEF 2014*)*, Lecture Notes on Artificial Intelligence*, Cuzzolin, F., Ed., vol. 8764, Springer-Verlag, 2014, pp. 29–38.
14. Harmanec, D., Toward a Characterization of Uncertainty Measure for the Dempster–Shafer Theory, *Proc. of the 11th Intern. Conf. on Uncertainty in Artificial Intel.*, Montreal, Canada, 1995, pp. 255–261.
15. Bronevich, A. and Klir, G.J., Measures of Uncertainty for Imprecise Probabilities: An Axiomatic Approach, *Int. J. of Approximate Reasoning*, 2010, vol. 51, pp. 365–390.
16. Bronevich, A. and Lepskiy, A., Measures of Conflict, Basic Axioms and Their Application to the Clusterization of a Body of Evidence, *Fuzzy Sets and Systems*, 2021. https:doi.org/10.1016/j.fss.2021.04.016
17. Klir, G.J., *Uncertainty and Information: Foundations of Generalized Information Theory*, Hoboken, NJ: Wiley-Interscience, 2006.
18. Yager, R.R., Entropy and Specificity in a Mathematical Theory of Evidence, *Int. J. of General Systems*, 1983, vol. 9, no. 4, pp. 249–260.
19. Höhle, U., Entropy with Respect to Plausibility Measures, *Proc. of the 12th IEEE Intern. Symposium on Multiple-Valued Logic*, Paris, 1982, pp. 167–169.
20. Klir, G.J. and Ramer, A., Uncertainty in the Dempster–Shafer Theory: A Critical Re-examination, *Int. J. of General Systems*, 1990, vol. 18, no. 2, pp. 155–166.
21. Klir, G.J. and Parviz, B., Probability-Possibility Transformations: A Comparison, *Int. J. of General Systems*, 1992, vol. 21, no. 3, pp. 291–310.

22. Osswald, C. and Martin, A., Understanding the Large Family of Dempster–Shafer Theory's Fusion Operators – A Decision-Based Measure, *Proc. Int. Conf. on Information Fusion*, Florence, Italy, 2006.
23. Daniel, M., Properties of Plausibility Conflict of Belief Functions, in *Artificial Intelligence and Soft Computing* (*ICAISC* 2013): *Lecture Notes on Artificial Intelligence*, Rutkowski, L., Korytkowski, M., Scherer, R., et al., Eds., vol. 7894, Berlin–Heidelberg: Springer-Verlag, 2013, pp. 235–246.
24. Bronevich, A.G. and Spiridenkova, N.S., Measuring Uncertainty for Interval Belief Structures and Its Application for Analyzing Weather Forecasts, in *Advances in Intelligent Systems and Computing*, Kacprzyk, J., Szmidt, E., Zadrożny, S., Atanassov, K., and Krawczak, M., Eds., vol. 641, Cham: Springer, 2018, pp. 273–285.
25. Jousselme, A.-L., Grenier, D., and Bossé, E., A New Distance between Two Bodies of Evidence, *Information Fusion*, 2001, no. 2, pp. 91–101.
26. Lepskiy, A., On Internal Conflict as an External Conflict of a Decomposition of Evidence, in *Belief Functions: Theory and Applications* (*BELIEF 2016*)*, Lecture Notes on Artificial Intelligence*, Vejnarová, J. and Kratochvil, V., Eds., vol. 9861, Springer-Verlag, 2016, pp. 25–34.
27. Lepskiy, A., Decomposition of Evidence and Internal Conflict, *Procedia Computer Science*, 2017, vol. 122, pp. 186–193.
28. Schubert, J., The Internal Conflict of a Belief Function, in *Belief Functions: Theory and Applications* (*BELIEF 2012*)*, Advances in Intelligent and Soft Computing*, Denoeux, T. and Masson, M.H., Eds., vol. 164, Berlin–Heidelberg: Springer-Verlag, 2012, pp. 169–177.
29. Smets, P., The Canonical Decomposition of a Weighted Belief, *Proc. of the 14th Intern. Joint Conf. on Artificial Intel.*, 1995, pp. 1896–1901.
30. Roquel, A., Le Hégarat-Mascle, S., Bloch, I., and Vincke, B., Decomposition of Conflict as a Distribution on Hypotheses in the Framework on Belief Functions, *Int. J. of Approximate Reasoning*, 2014, vol. 55, pp. 1129–1146.
31. Lepskiy, A.E., Decompositional Approach for Evaluation of Internal Conflict in the Framework of the Evidence Theory, *Fuzzy Systems and Soft Computing*, 2020, vol. 15, no. 1, pp. 43–63.

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STRESS TESTING OF NONFINANCIAL ORGANIZATIONS: AN ANALYTICAL APPROACH TO SOLVING THE REVERSE PROBLEM

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Abstract. This paper considers an approach to stress testing of nonfinancial organizations. It includes the problem statement and a methodology for solving the reverse problem. The mathematical model is based on open-source data (the financial statements of companies). The relevance of this approach is increasing due to different-nature crises (economic crisis, the COVID-19 pandemic, etc.). The resilience of companies (especially "backbone" ones) to shock situations is tested, and preventive management measures are developed. The direct problem statement involves determining the company's financial model parameters that ensure a nonnegative level of cash balance in the forecast period. The reverse problem is to find the characteristics of the financial and economic state of the enterprise that correspond to different critical combinations of its financial result parameters. We develop an original stress-testing methodology that significantly reduces the labor intensity and computational complexity compared to stress-testing technologies for financial institutions. A new analytical model is used. The model results are illustrated by an example: stress testing of a backbone enterprise in the real economy sector, which was significantly affected by restrictive measures in the COVID-19 pandemic. Model calculations employ open data from the organization's financial statements.

Keywords: stress testing, critical combinations of parameters, risk management, financial forecasting, modeling, reverse problem, cash flow, operational efficiency, COVID-19.

INTRODUCTION

The large-scale economic crisis caused by restrictive measures under the COVID-19 pandemic provoked a sharp sales slowdown in many markets for goods and services of the real sector of the global economy, resulting in mass bankruptcies in some industries, a request from business for financial support of the companies most affected by the crisis,[1] and emergency measures of governments. The Russian Government's Decree[2] provides several financial support measures for business: subsidies for reimbursement of costs, deferral of taxes and advance payments, state guarantees to restructure the existing loans or issue new loans, and bond issues. First and foremost, these measures apply to the so-called backbone enterprises.[3]

These efforts of the Government are intended to improve the sustainability of the Russian economy. For a backbone organization to obtain state measures of financial support, a prerequisite is to analyze its financial and economic activities, assess its financial stability (stress test) in accordance with the procedure

[1] URL: https://www.cbr.ru/covid/ (Accessed May 25, 2021).

[2] URL: http://government.ru/docs/39665/ (Accessed May 25, 2021).

[3] URL: https://data.economy.gov.ru/ (Accessed May 25, 2021).

established by the Ministry of Economic Development of the Russian Federation.[4]

Financial resilience assessment methods (stress testing) have been known in the theory and practice of risk management for more than two decades. However, they are more widespread in the banking sector of the economy, which is under strong pressure from regulators along with environment factors. Stress-testing procedures are almost not used in the real economy sector due to the complexity of mathematical models involved. The complexity and ambiguity of stress testing for nonfinancial organizations lies in the need to develop:

– scenarios of the behavior of enterprises' economic indicators in a stress situation (crisis),

– forecasting models.

We propose a stress-testing scheme, which consists of the following main measures:

– obtaining data on the financial and economic state of a managed object;

– developing stress-testing scenarios;

– adjusting the mathematical model of the managed object affected by these scenarios;

– conducting computational experiments with the mathematical model;

– analyzing the results of mathematical modeling;

– developing action plans to prevent possible critical situations diagnosed by stress testing.

Within the proposed approach, it is possible to correct the scenarios and parameters of the mathematical model with the subsequent computational experiments and the analysis of the received results by decision-makers. An important and labor-intensive stage of these studies is simulating changes in the enterprise production process under different stress-testing scenarios.

We develop an original methodology that significantly reduces the labor intensity and computational complexity of the experiments by using a new mathematical model in analytical form. This model describes the dynamics of the financial and economic state of the enterprise under different values of the control parameters of the models and disturbances.

The computational complexity is reduced by developing the following:

- A model to forecast the dynamics of the enterprise's financial and economic state in the analytical form, used to solve the direct problem. This problem consists in forecasting the values of the enterprise's financial and economic indicators on different periods and determining the critical combination of parameters that destabilize or completely halts the enterprise.
- Models and methods for solving the reverse problem, which consists in determining the states of the enterprise corresponding to different critical combinations of parameters. The labor intensity is significantly reduced by using a mathematical model in analytical form.

1. RELATED LITERATURE

In a general scientific sense, a stress test is a form of testing to determine the stability of a system under some external disturbances.

Research on stress testing is based on studies of the financial stability of a business organization. M.E. Zmijewski [1] surveyed studies to determine financial instability indicators (problems, *distress*, and bankruptcy). In particular, the bankruptcy indicators (financial coefficients) were justified mostly using a comparative analysis of financial indicators of sampled companies with and without financial problems (*distressed*).

In corporate finance, the concept of stress test appeared in the late 1990s. The prerequisites were the advances in the theory and practice of corporate credit risk analysis, bankruptcy probability modeling, and research based on large empirical data arrays of American companies carried out by leading rating agencies (Moody's, S&P, and Fitch). Several generations of approaches to the corresponding problems were surveyed in [2–4].

A stress test checks the financial position endurance of an organization under a "severe yet probable shock."[5] Stress testing of an organization is an alternative to financial forecasts. Stress-testing models determine sensitivity to single risk factors and different combinations of critical factors. A stress test typically includes four elements:

- A set of risks to be tested.
- A scenario in which the risks are implemented. These can be scenarios of economic recession, growth of unemployment, or fall of real estate prices on the stress test horizon. As a rule, the stress test horizon is 2–5 years.
- Models describing the effect of risks on the tested parameters. Stress-testing models for financial

[4] Order of the Ministry of Economic Development of the Russian Federation of May 13, 2020, No. 276 "On Approval of the Procedure for Assessing Financial Stability (Stress Test) of Backbone Enterprises of the Russian Economy Applying for State Support Measures in 2020." URL:https://www.garant.ru/products/ipo/prime/doc/73936434/#1000 (Accessed May 25, 2021).

[5] URL: https://www.cbr.ru/finstab/stress_testing/chto-takoe-stress-testirovanie/what_is_stress_testing/ (Accessed April 20, 2021).

institutions determine relationships between macroeconomic indices, market indicators (interest rates, bond yields, stock prices, etc.), and financial parameters (e.g., ratings of corporate borrowers) that affect the volume of additional provisions on loans.

- Measurement of results. In most cases, the financial result on the stress test horizon is assessed, the final capital adequacy indicator is compared with the standard, and the capital deficit is calculated. In several stress tests, the liquidity shortage is also assessed.

There are two basic approaches, *bottom-up* and *top-down* stress tests. According to the first approach, the organization applies a stress-testing methodology independently (initiative stress testing). The regulator performs the top-down stress test using supervisory or publicly available information on individual organizations according to a single definite scenario. The bottom-up stress test is performed by financial institutions using internal data and models but with the same scenario defined by the regulator. The stress-testing principles for financial institutions are comprehensively described in the Requirements of the Basel Committee on Banking Supervision (BCBS).[6]

In October 2009, *The Financial Stability Board* released the report "*Risk Management Lessons from the Global Banking Crisis of* 2008."[7] It paid special attention to stress testing, especially emphasizing the importance of *reverse stress tests*. Unlike standard stress tests, they identify a set of scenarios (combinations of risk factors) in which an organization will lose financial stability. That is, *reverse stress testing* (the reverse problem) analyzes financial stability from the other side, defining limiting values of risk factors under which an organization will be bankrupt. For this purpose, the boundary values of financial stability indices are identified under which an organization becomes bankrupt; then, the events that can lead to such values of the indices are determined.

There are few publications on stress-testing methods for companies of the real economy sector, although this topic is studied within risk management, a rather rapidly developing discipline. In the western stress-testing practice of nonfinancial organizations, the following models were described and are widely used:

– The credit risk model (SEBRA-model), developed by the Norwegian Bank to forecast the annual probability of borrower firm bankruptcy.

– Moody's Analytics RiscCalc model.[8] The model is based on financial analysis indices (financial statements) with an additional correction for the probability of bankruptcy [5]. The model forecasts the probability of bankruptcy and the expected losses of credit institutions.

The paper [6] considered the Ooghe–Joos–De Vos model of forecasting the bankruptcy of firms (eight factors for one year and six factors for three years). Also, the authors studied the accuracy of bankruptcy forecasting by the first-kind error of credit risk (bankrupt firms were defined as non-bankrupt) and the second-kind error of commercial risk (non-bankrupt firms were defined as bankrupt). In this model, both errors belonged to the range 14–33% for a sample of 280 000 firms in Belgium (6.5% bankrupts).

Among the research works included in the Russian Science Citation Index (RSCI) database[9] as of May 1, 2021, only nine publications on the banking sector and one publication on nonfinancial organizations were found with the keyword "stress test." The paper [7] surveyed different approaches to forming stress tests in the financial sector.

Mainly regression models are used for stress testing, whereas simulation models are not. The authors [8] briefly discussed publications on stress testing of enterprises. In particular, three assessment methods (econometric, discriminant, and mixed) and five approaches according to the set of variables (market, microeconomic (balance), macroeconomic, hybrid, and rating) were distinguished. The paper [8] considered an approach to stress testing of companies of the Russian economy's real sector based on enterprise statements, micro-sector indices, and forecasts of sectoral indices within macroeconomic forecasts. In addition, possible approaches to assessing the probability of the organization's default (bankruptcy) were surveyed.

The papers [9, 10] considered financial coefficients developed based on organizations' financial statements by the correlation with organizations' defaults.

In [11], the Altman models (five factors) were applied for a sample of enterprises in the Orenburg region. As shown therein, the accuracy of bankruptcy forecasting for four years is 20%. The Olson bankruptcy forecasting models (nine factors) show an accuracy of about 20% for one year.

The principles of corporate finances include [12] the concepts of risk probability and the definition of possible damage. It is reasonable to estimate damage using mathematical models.

[6] URL: https://www.bis.org/bcbs/index.htm (Accessed May 25, 2021).
[7] URL: https://www.sec.gov/news/press/2009/report102109.pdf (Accessed June 1, 2021).
[8] URL: https://www.moodysanalytics.com/product-list/riskcalc (Accessed January 5, 2021).
[9] URL: https://elibrary.ru (Accessed May 2, 2021).

In topical problems of stress testing, it is necessary to analyze the effect of inner and outer environment factors on the target values of financial state indices. As we believe, the concept of critical (emergency) combinations of events [13, 14] should be used here. This concept involves the analysis of the effect of individual factors considering their causal relations with other factors. Relatively insignificant individual events may occur in a definite order, producing a synergistic effect. With the concept of critical combinations, synergy can be taken into account. The development of such undesirable effects is prevented using a set of measures determined by the algorithms proposed within the concept [15]. The effect of critical combinations of factors should be studied starting with the partial effect of individual factors considering possible significant causal relations of a given factor with other variables.

The paper [16] surveyed methods of single-factor stress tests for banks. In [17], a regression model was considered to identify the factors affecting the financial stability of enterprises (their dependence on the sources of funding) in the Ukrainian food sector on open panel data.

According to Order of the Ministry of Economic Development of the Russian Federation No. 276 of May 13, 2020 (footnote no. 4), monthly scenarios for assessing the stability of Russian backbone organizations should be formed, and the risk category of the organization should be determined. The list of Russian economy's sectors most affected by the deteriorated situation due to the spread of a new coronavirus infection was developed.[10] Twelve sectors were identified, whose activities were paralyzed under the current restrictions; particularly transportation, leisure and entertainment, hotel business and public catering, and population consumer services. Besides, the Ministry of Economic Development of the Russian Federation developed the list of 1392 backbone enterprises as of May 20, 2021.[11]

As we believe, this work was not continued due to the absence of reliable tools for stress testing of nonfinancial organizations. Hence, the approach proposed below is of scientific and practical significance.

This paper develops a methodology for evaluating the parameters of a mathematical model of the stress-testing procedure under which a nonfinancial organization will have a nonnegative cash balance on different management periods under significant environment disturbances. Unlike the existing approaches, this methodology uses the forecasting model of the enterprise's financial and economic state in the analytical form. As a result, the reverse problem is solved in analytical form, and the time complexity of the developed algorithms is significantly reduced.

2. PROBLEM STATEMENT AND GENERAL SOLUTION APPROACH

The model of forecasting the financial state of an enterprise is the key to stress testing [18]. However, as practice shows, this formal apparatus does not yield a mathematical model for stress testing of enterprises. Such a model determines conditions for a nonnegative cash balance at the end of the forecast period under known constraints on the enterprise's material and financial resources.

It is necessary to develop the enterprise's management model with a cash flow criterion under significant adverse impacts from the outer environment (e.g., lockdown and drop in consumer demand, also called knockdown for business).

The stress-testing procedure of nonfinancial organizations is generally represented by the information-logical scheme in the figure below.

We believe that stress testing of nonfinancial organizations is one of the most important problems.

Let $F(x, u, t)$ be a main financial and economic criterion of the enterprise's activity, where x denotes a vector of parameters, u is a vector of control parameters, and t indicates time.

One significant index can be identified for analyzing the critical factors of the enterprise's activity, namely, the cash forecast at the end of the forecast period:

$$F(x, u, t) = Cash_t .$$

The critical importance of this criterion is clear: with $Cash_t \geq 0$, the enterprise continues to operate (at least by financial conditions), whereas with $Cash_t < 0$, it terminates the activity due to real insolvency.

Let $F(x, u, t) = Cash_t < 0$ be the solution of the direct forecasting problem. The forecasted value $Cash_t$ may be less than zero. In reality, however, there is no passive asset balance (including cash). In other words, such a forecast is not feasible. A negative cash flow value may indicate an imbalance in the organization's financial plans and its possible insolvency (bankruptcy). The common reasoning is that in the case of passive cash balance, the enterprise gets into debt. This means two operations: increases in borrowed funds (debts) and cash (to a nonnegative value).

[10] Decree of the Government of the Russian Federation of April 03, 2020, No. 434 (as amended on October 16, 2020) "On Approval of the List of Russian Economy's Sectors Most Affected by the Deteriorated Situation due to the Spread of a New Coronavirus Infection." URL: https://base.garant.ru/73846630/ (Accessed May 25, 2021).

[11] URL: https://data.economy.gov.ru/analytics/facilities/industry (Accessed May 20, 2021).

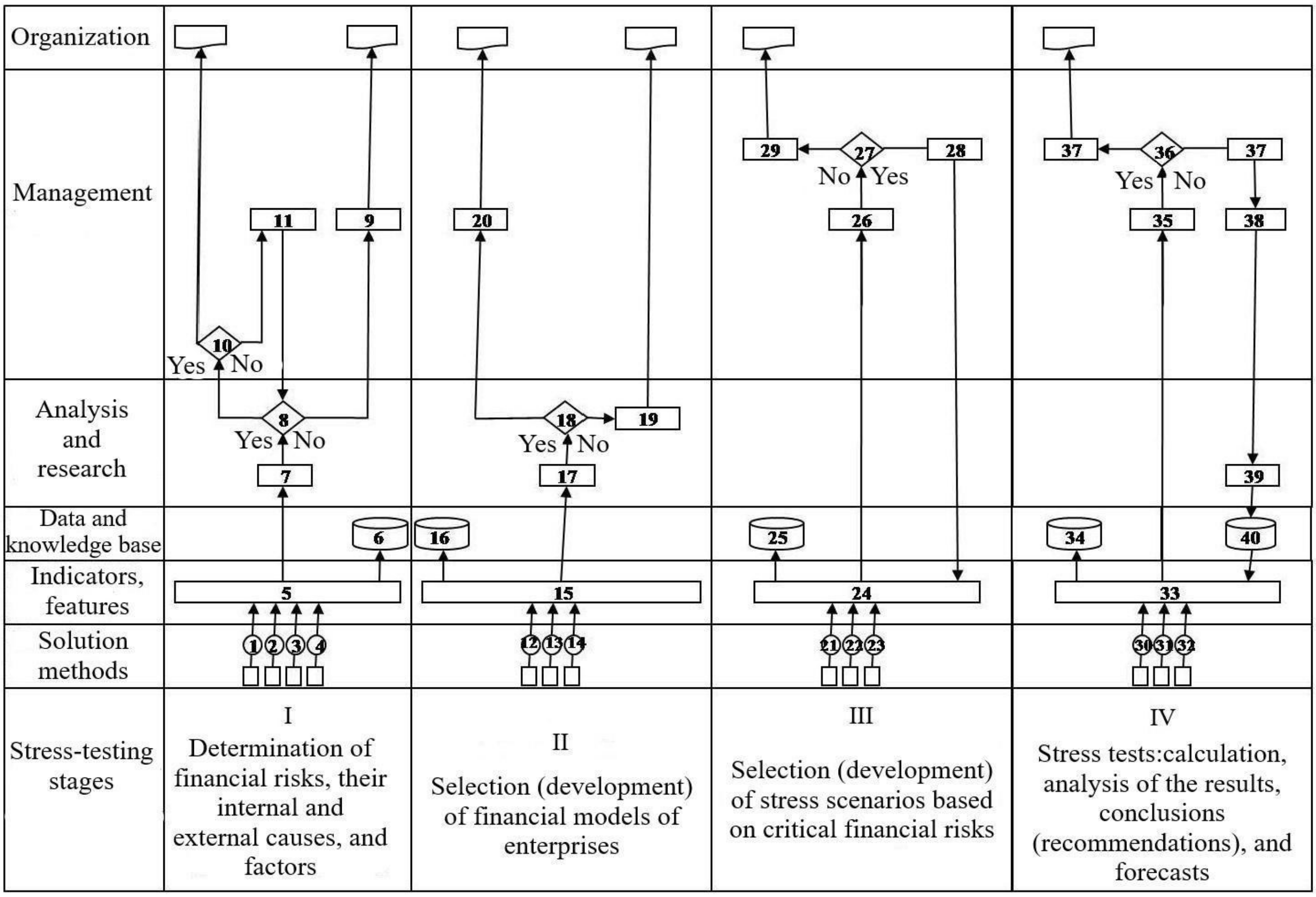

Fig. The information-logical scheme of stress-testing procedures of nonfinancial organizations: 1–4 — parametric method, regression analysis, retrospective analysis, the method of similar companies; 5 — risk analysis method selection based on current indicators and attributes; 6 — fixation of the results in company's data and knowledge base; 7 — justification and testing of the selected risks, justification of the complete set of factors affecting them; 8 — Are the analysis results satisfactory?; 9 — return to Block 5; 10 — Does the management approve the selected risks and factors?; 11 — return to the analysis of the first stage results (Block 8); 12 — historical data analysis; 13 — expert methods; 14 — methods for determining the stability of the model parameters to stress factors and model horizons; 15 — selection of the organization's financial model; 16 — entering information into the data and knowledge base; 17 — determining the model accuracy on the horizons; 18 — Is the model accuracy satisfactory?; 19 — return to horizons determination, etc.; 20 — approval by the head of the analytical department; 21–23— methods for developing stress scenarios; 24 — selection of the basic set of stress scenarios; 25 — entering information into the data and knowledge base; 26 — assessment of the scenarios by the organization's management; 27 — Does the set of scenarios correspond to the current completeness and accuracy requirements?; 28 — selection of relevant procedures and responsible executors for finalizing the set of scenarios; 29 – – preparation of intermediate results; 30 — modeling; 31 — analysis of the results; 32 — formation of conclusions; 33 — formation of options to answer the main questions based on the conclusions; 34 — entering information into the data and knowledge base; 35 — presentation of the results; 36 — Are the answers to the questions yielded by stress testing satisfactory?; 37 — preparation of the obtained solutions for implementation; 38 — detailed formulation of claims to the results of stress testing; 39 — analysis of updated requirements; 40 — updating the data and knowledge bases.

The problem is to develop a mathematical model for stress-testing of enterprises and determine conditions for a nonnegative cash balance at the end of the forecast period under known constraints on the enterprise's material and financial resources.

The general-form solution is not unique and is defined by some hypersurface U of the parameters $\boldsymbol{u}$. For the sake of simplicity, we fix all parameters except the change in revenue (denoted by u_1). The reverse problem has the following statement: find the limiting value of the parameter u_1 such that

$$F(x, u_1, t) = Cash_t = 0.$$

Due to the analytical form of the financial state forecasting model proposed below, we solve the reverse problem in the analytical form. Hence, the calculations and possible studies are considerably simplified.

3. MATHEMATICAL MODEL FOR STRESS TESTING

The list of upper-level financial and economic indices is determined by the financial statements of the enterprise. This choice of indices is dictated by the open-source data on financial statements. For real administration, the enterprise's management uses a much wider list of managerial statement indices, which are not available to third-party users. The approach presented below considers this fact. The directions of changes in the parameters are determined at the upper level using open-source data. Then, they are specified at the lower level using the data available to management.

We propose an approach to constructing a mathematical model. Its main idea can be stated as follows.

- The enterprise's financial state is described by combining its balance sheet at a particular date and its income and expenditure account (gains for a particular period).
- The growth drivers are distinguished. In this paper, the revenue (sales volume) is used.
- The management parameters are forecasted. Since production technologies and business management processes are rather inertial,[12] the specific characteristics of the financial state are constant or change insignificantly on the forecast period. This approach is called the percentage-of-sales method [19, 20].
- The forecast of the financial and economic indices has the general form

$$x_t = x_{t-1}(x_{t-1}, u),$$

where x is the vector of model variables corresponding to the enterprise's financial and economic parameters, and u denotes the control vector.

Different forecasting sub-models are used to forecast different indices [18]. For example, the accounts receivable are forecasted by the formula

$$Y_t = l_{Yt}\, S_t$$

with the following notations: Y is an index from the vector x corresponding to the accounts receivable (in thousand rubles (RUB)); S gives the revenue (in thousand RUB per year); l_Y denotes the turnover of the accounts receivable (in years). In the formulas below, the indices measured in monetary units are specified in thousand RUB, and the turnover indices are specified in years.

The turnover is determined by the average turnover coefficient over the previous periods:

$$l_{Yt} = \frac{1}{t-1}\sum_{j=1}^{t-1}\frac{Y_j}{S_j}.$$

It is possible to use a linear dependence of the form $l_{Yt} = A_Y S_t + B_Y$, where A_Y and B_Y denote linear regression coefficients. Linear regression is one of the simplest approaches. Using more complex regressions for economic calculations is not excluded. (However, we have not encountered organizations practicing such regressions in the real economy sector.)

The inventory turnover is determined by technological and managerial business processes: technological operations, internal production movements, procurement processes, and inventory delivery. Such processes are inertial and are quite stable for enterprises with established technologies. Hence, the following assumption seems natural: if the business processes have no changes, the turnover period will remain the same (or slightly vary) for the next forecast period.

- The relation between the balance sheet and the income and expenditure account is considered: undistributed profit increases equity capital.
- The cash flow is forecasted using the indirect method.[13] A passive cash balance is formed when the forecasted assets exceed the forecasted liabilities. This indicates the enterprise's insolvency.
- Measures for eliminating the cash deficit in real management are considered and implemented. In the stress-testing model, we fix the limit revenue causing insolvency. Also, we suppose that the management parameters cannot increase the system's efficiency "in the stress time" and are therefore constant.

In real management, several parameters often vary simultaneously. To forecast the production-oriented behavior of an enterprise, we should set scenarios for varying the parameters of sales, production, purchases, working capital requirements, etc. Different combinations of the parameters form a Cartesian set, and their total number is given by $N_i \times N_j \times N_{lm} \times N_f \times N_d$, where: N_i is the number of product types; N_j is the number of resource types; N_l is the number of currencies; N_m is the number of equipment types; N_f is the number of fixed cost items; N_d is the number of credits.

Direct calculations can be labor-, time-, and computationally intensive. Most important, they are not provided with information in the operational mode. One way to find a critical combination of the parameters is Monte Carlo analysis with a given probability distribution for each parameter. Note that many enterprises have no information about the probability distribution of parameters. Acquiring and processing this information is a rather labor-intensive procedure, and such an approach is not implementable in practice.

This paper considers another approach: using the total values of indices and studying their variation scenarios.

We write the mathematical model for calculating the cash balance at the end of the forecast period:

$$Cash_t = Cash_{t-1} + CF_t, \quad (1)$$

where $Cash$ is the cash balance, CF denotes the cash flow, and t indicates the period.

The cash flow is determined indirectly by the formula

$$CF = Pr + Am - I + \Delta D, \quad (2)$$

[12] If financial state characteristics change significantly, sub-models are required to forecast these parameters.

[13] In this paper, the indirect cash flow method is important due to interconnecting the cash flow with changes in balance sheet items.

where: *Pr* denotes the undistributed profit; *Am* is the amortization; *I* means the investments; ΔD is the change in debt. (The time variable *t* is omitted to simplify the expression within a single period.)

The undistributed profit for the forecast period is given by

$$Pr = (S(1-v) - FC - r_D D)(1-r_\tau)(1-r_u), \quad (3)$$

where: *v* denotes the share of variable costs; *FC* is the fixed costs without interest; *D* means the debt (loans); r_D is the interest rate on loans; r_τ is the income tax rate; r_u is the share of net income for consumption (dividends).

The net investments are defined as the gains on fixed assets at residual value considering the amortization and changes in the net working capital:

$$I - Am = \Delta FA + \Delta WC = n_{FA} FA + l_{WC} \Delta S, \quad (4)$$

where: *FA* denotes the fixed assets; *WC* is the net working capital (the difference between the current assets and current liabilities); n_{FA} means the increase in the residual value of the fixed assets; l_{WC} indicates the turnover of the net working capital.

The mathematical model with the total values of the indices takes the form

$$Cash_t = Cash_{t-1} + (S(1-v) - FC - r_D D) \times \\ \times (1-r_\tau)(1-r_u) - (\Delta FA + \Delta WC) + \Delta D.$$

4. STRESS TEST AS A SCENARIO OF VARYING SEVERAL PARAMETERS

Stress-testing scenarios are difficult to formulate: it is necessary to describe variations of target values and performance indices, consider the impact of a development program, etc. However, many parameters remain invariable (have not enough time to vary) in the shock situation, which simplifies the problem statement and calculations.

In what follows, we consider a one-factor stress test (variation of one parameter). However, the forecasting model proposed below is suitable to analyze and perform calculations for more parameters. The multifactor case goes beyond the scope of this paper.

Let us introduce the coefficients of stress test influence on each parameter. The stress test situation assumes negative influences, leading to decreased revenue, cost savings, reduced investments, etc. Therefore, as a rule, these coefficients belong to the segment [0, 1]. If the indices increase, the corresponding coefficient will be greater than unity.

We consider the following scenario of varying the factors:

- The main parameter is decreased revenue: a drop in the demand and a reduction in sales ($k_S < 1$).
- There is no decrease in the direct (variable) costs of sold products: the specific variable costs are constant ($k_v = 1.0$).
- The wage fund is maintained at the same level due to the no-dismissal requirement (or reduced at most by 10%): $k_W = 0.9$–1.0 if the number of employees is reserved, or $k_W = 0.5$–0.9 if the number of employees is reduced.
- There is no decrease in the overhead (fixed) costs of sold products: the fixed costs (administrative and business expenses) remain the same ($k_{FC} = 1.0$).
- There is no change in the accounting policy: the balances of other income and expenses remain the same: $k_{OthInc} = 1.0$. (Note that they are quite large sums comparable to sales profit, usually without a detailed breakdown of the items.)
- The inventory is changing under two factors:

  – The replenishment of unsold goods due to inertial production. The production response coefficient is $k_p = 1$ for inertial productions with the previous production program or $k_p = 0$ for organizations with the rapidly restructured purchases, production, and sales.

  – Turnover reduction.
- The accounts receivable can vary in the shock state (coming in a sharply reduced amount): the buyers also experience a drop in sales and cash receipts. This paper considers a “moderate” shock scenario: the accounts receivable decrease proportionally to the revenue.
- The accounts payable to counterparties are paid proportionally to the costs.
- The short- and long-term loans are not repayable: $k_D = 1.0$. (For the sake of simplicity, we combine them.) This assumption is important since banks should (may) demand early repayment of loans in case of violating banking covenants.
- The investment program is implemented in a reduced amount with the coefficient k_{FA} (in the case of no changes, $k_{FA} = 1$).

In the shock scenario, the environment parameters vary sharply, and the organization has no time to change the internal management parameters and business processes. Hence, the organization has to meet the stress test criteria without internal changes.

This paper considers the scenario of declining revenue during the analyzed period. In practice, there may be more complex scenarios: revenue decline is followed by a reduction in costs and optimization of assets. Moreover, at many enterprises, production and shipment of goods are completed by the end of the year, and the annual financial statements "ignore" the intraannual gains on current assets (inventory, advances given, and debtors). These current assets are "frozen" for declining revenue, and the consequences become more serious.

5. A MODEL VARIATION OF MANAGEMENT PARAMETERS REQUIRED FOR NON-RISKY DEVELOPMENT

The system of equations (1)–(5) with the scenario parameters of stress testing takes the following form.

The retained earnings vary according to the equation

$$Pr_C = (k_S S(1-k_v v) - k_{FA} FC - k_D r_D D)\times \times(1-r_\tau)(1-r_u), \quad (6)$$

where: k_S is the revenue variation coefficient; k_v is the variation coefficient of the specific variable costs; k_{FC} is the variation coefficient of the fixed costs; k_D is the variation coefficient of interest on loans; C is the stress test subscript.

The gains on (non-current and current) assets are determined by the investments.

The inventory is recorded on actual costs. The inventory at the end of period (t – 1) is described by the equation

$$Inv_{t-1} = l_{Inv}\, TC_{t-1},$$

$$VC = v\, S,$$

$$TC = VC + FC = v\, S + FC,$$

where: Inv denotes the inventory; l_{Inv} is the inventory turnover; TC is the total costs (without interest on loans); VC is the variable costs.

The inventory at the end of period t, considering overstocking, is described by the equation

$$Inv_t = k_{Inv}\, l_{Inv}\, TC_t - k_p(TC_t - TC_{t-1}),$$

where: k_{Inv} is the inventory variation coefficient; k_p is the production response coefficient.

The investments in the inventory (a working capital component) are described by the equation

$$\Delta Inv_C = Inv_t - Inv_{t-1} = k_{Inv}\, l_{Inv}\, TC_t - l_{Inv}\, TC_{t-1} - k_p(TC_t - TC_{t-1}). \quad (7)$$

For inertial production, $k_p = 1$. For $k_{Inv} = 1$, we obtain $\Delta Inv_C = Inv_t - Inv_{t-1} = (l_{Inv} - 1)(TC_t - TC_{t-1})$.

The accounts receivable at the end of periods (t – 1) and t are described by the equations

$$AR_{t-1} = l_{AR}\, S_{t-1},$$

$$AR_t = k_{AR}\, l_{AR}\, S_t,$$

where: AR denotes the accounts receivable; k_{AR} is the variation coefficient of the accounts receivable; l_{AR} denotes the accounts receivable turnover.

The investments in the accounts receivable (a working capital component) are described by the equation

$$\Delta AR_C = k_{AR}\, l_{AR}\, S_t - l_{AR}\, S_{t-1} = l_{AR}(k_{AR}\, S_t - S_{t-1}). \quad (8)$$

The accounts payable at the end of periods (t – 1) and t are described by the equations

$$AP_{t-1} = l_{AP}\, TC_{t-1},$$

$$AP_t = k_{AP}\, l_{AP}\, TC_t,$$

where: AP denotes the accounts payable; k_{AP} is the variation coefficient of the accounts payable; l_{AR} is the accounts payable turnover.

The investments in the accounts payable (a working capital component) is described by the equation

$$\Delta AP_C = k_{AP}\, l_{AP}\, TC_t - l_{AP}\, TC_{t-1} = l_{AP}(k_{AP}\, TC_t - TC_{t-1}). \quad (9)$$

The working capital varies according to the equation

$$\Delta WC_C = l_{Inv}(k_{Inv}\, TC_t - TC_{t-1}) - k_p(TC_t - TC_{t-1}) + l_{AR}(k_{AR}\, S_t - S_{t-1}) - l_{AP}(k_{AP}\, TC_t - TC_{t-1}),$$

or

$$\Delta WC_C = (k_{AR}\, l_{AR} S_t - l_{AR} S_{t-1}) + \left(k_{Inv} l_{Inv} - k_{AP} l_{AP} - k_p\right)\times TC_t - \left(l_{Inv} - l_{AP} - k_p\right)TC_{t-1}. \quad (10)$$

The investment program is implemented with the coefficient k_{FA} (in the case of no changes, $k_{FA} = 1$):

$$\Delta FA_C = k_{FA}\, n_{FA}\, FA_{t-1},$$

where k_{FA} is the variation coefficient of the investments in the non-current assets.

The investments in non-current and current assets minus accounts payable are equal to

$$I_C = k_{FA} n_{FA} FA_{t-1} + (k_{AR}\, l_{AR} S_t - l_{AR} S_{t-1}) + \left(k_{Inv} l_{Inv} - k_{AP} l_{AP} - k_p\right)TC_t - \left(l_{Inv} - l_{AP} - k_p\right)TC_{t-1}. \quad (11)$$

The debt varies according to the equation

$$\Delta D_C = (k_D - 1)D_{t-1}, \quad (12)$$

where k_D is the share of the existing loans.

For the sake of brevity, we introduce the notations $r_{\tau u} = (1-r_\tau)(1-r_u)$, $l_{TC1} = \left(l_{Inv} - l_{AP} - k_p\right)$, and $l_{TC} = \left(k_{Inv} l_{Inv} - k_{AP} l_{AP} - k_p\right)$.

Due to formulas (5)–(12), the cash balance nonnegativity condition is written as

$$Cash_t = Cash_{t-1} + (k_D - 1)D_{t-1} - k_{FA} n_{FA} FA_{t-1} + (S_t(1-k_v v) - k_{FC} FC_t - k_D r_D D_{t-1})r_{\tau u} - [\,(k_{AR}\, l_{AR} S_t - l_{AR} S_{t-1}) + l_{TC}(k_v v S_t + FC_t) - l_{TC1} TC_{t-1}] \geq 0,$$

or

$$Cash_t = Cash_{t-1} + (k_D - 1)D_{t-1} - k_{FA} n_{FA} FA_{t-1} + S_t\left[(1-k_v v) r_{\tau u} - k_{AR}\, l_{AR} - l_{TC} k_v v\right] - (k_{FC} FC_t + k_D r_D D_{t-1}) r_{\tau u} - l_{TC} FC_t + l_{AR} S_{t-1} + l_{TC1} TC_{t-1} \geq 0.$$

The financial forecasting model yields an analytical solution of the reverse problem: using this model, we determine the limiting values of the scenario parameters that still satisfy the stress test conditions.

For the one-factor test, the limiting variation of revenue for $Cash_t = 0$ and $S_t = k_S S_{t-1}$ is given by

$$k_S{}^d = -\frac{Z}{S_{t-1}[(1-k_v v) r_{\tau u} - k_{AR}\, l_{AR} - l_{TC} k_v v]},$$

$$Z = Cash_{t-1} + (k_D - 1)D_{t-1} - k_{FA} n_{FA} FA_{t-1} - (k_{FC} FC_t + k_D r_D D_{t-1}) r_{\tau u} - l_{TC} FC_t + l_{AR} S_{t-1} + l_{TC1} TC_{t-1},$$

where the superscript d indicates no deficit.

The parameters $Cash_{t-1} + (k_D - 1)D_{t-1} - k_{FA} n_{FA} FA_{t-1}$ consider cash carryover, debt repayment, and net investments in non-current assets. The parameters $(1-k_v v) r_{\tau u} - k_{AR}\, l_{AR} - l_{TC} k_v v$ show the impact of the revenue on cash carryover through the variable costs and the variable share of investments.

6. AN EXAMPLE OF CALCULATIONS FOR A BACKBONE ENTERPRISE

As an illustrative example, we performed armchair calculations for a leading Russian machine-building enterprise using a simulation model of financial statements forecasting. This enterprise is a backbone organization of the automotive sector[14] (Taxpayer Identification Number 6320002223). The data for the calculations were obtained from the Federal State Statistics Service (Rosstat)[15] for the period 2012–2018 (open source) and from the Federal Tax Service[16] (FTS) for the period 2019–2020.

We used the simulation model with slightly simplified formulas to reduce the number of parameters. The cost of goods sold was chosen as a variable part of the expenses for profit forecasting. Administrative and business expenses and the balance of other income and expenses were considered fixed expenses. The company's official statement of financial results is given in the "Actual" column; see the table below. The calculation for the next year (2020) was based on the initial data of the previous year (2019). An aggregate

Forecast of financial statement indicators under the stress test, million rubles

Items of income and expenses	Actual data for 2019	Variation, %	Income statement forecast	Increase in profit
Revenue	292 010	–22.2	227 165	–4 543
Cost of goods sold	–271 533	–	–211 251	0
Administrative and business expenses	–16 353	–	–16 353	0
Interests, net	–5 306	–	–5 306	0
Other income and expenses	–481	–	–481	0
Income tax	–423	–	0	423
Net profit	385	–	–3 734	–4 120
Items of assets and liabilities	Actual data for 2019	Variation, %	Balance forecast	Increase in cash
Non-current assets	93 704	0.0	93 704	0
Inventory	15 316	–22.2	11 915	3 401
Trade receivables	19 179	–22.2	14 920	4 259
Other current assets	344	–	344	0
Cash and short-term investments	5 253	–100.0	0	5 253
Accounts payable	41 335	–22.2	32 156	–9 179
Other current liabilities	5 465	–	5 465	0
Long-term loans	81 350	–	81 350	0
Short-term loans	4 876	–	4 876	0
Common stock	55 750	–	55 750	0
Retained earnings	–143 636	–	–147 370	–3 734
Other equity items	88 659	–	88 659	0
Total loss (+)/deficit (-) in financing	**0**	–	**0**	**0**

[14]URL: https://www.rusprofile.ru/accounting?ogrn=1026301983113 (Accessed August 14, 2021).
[15]URL: https://rosstat.gov.ru/opendata/7708234640-7708234640bdboo2018 (Accessed May 20, 2021).
[16]URL: https://bo.nalog.ru/ (Accessed May 20, 2021).

forecast of the financial statement indicators and cash flow is shown in the table. The upper part contains the forecasted income and expenses; the lower part, the forecasted assets and liabilities. The table columns provide the following information:

1. the names of items,
2. the actual data for 2019 according to the financial statements,
3. the variations of the efficiency parameters in the stress test,
4. the calculation of items under the efficiency variation,
5. the estimated contribution of items to net income and cash flows.

Let us give comments on some items. The stress test scenario considered revenue reduction by a given percentage. Other efficiency parameters for income and expenses were not varied. The result was almost no deficit: a negative cash flow reduced the cash carryover to zero.

The decrease in the revenue corresponding to zero cash balance was $k_s^d = 22.2\%$ in the one-factor stress test without efficiency variation. Moreover, the limiting revenue decline for the break-even operation was about $\Delta S_{\text{be}} = -4.0\%$. The enterprise turned out sensitive to declining revenue by the profit criterion and not so sensitive by the cash balance criterion in the shock scenario of the stress test. Note that the negative cash flow in this example mostly consists of the losses from operating activities. If the company manages to make administrative decisions to cut the costs, suspend investments, and reduce inventory in time, the results of the one-year stress test will be little affected by the revenue variation.

7. MANAGEMENT: DEVELOPING A SYSTEM OF COMPENSATING MEASURES

This section is not part of the stress-testing procedure itself, but in most cases, stress testing is performed to understand the depth of business problems and develop compensating measures.

The general areas of efficiency improvement ("anticipatory management") are well known and can be implemented with weak signals on an impending crisis. As a rule, compensating measures are applied in the accelerated mode when a crisis occurs. Some of them will be delayed and have small efficiency. In addition, "surgical" measures can be applied. A list of possible measures and the practice of their application (without claiming to be complete) were described in [19–22]. Generally speaking, the financial result and cash flow are improved using a set of measures:

– increasing sales (by the marginal profit criterion),
– decreasing technological costs,
– reducing purchase prices,
– decreasing overhead costs,
– shortening the financial and production cycle,
– optimizing the investment program,
– optimizing the assortment, including structural changes in product shares and financing conditions.

Methods to form and select promising efficiency improvement projects were described in [23, 24]. Support measures can be simulated using the TEO Invest system [25]. The paper [26] presented a list of energy-saving technologies to reduce costs.

According to experimental evidence in the successful implementation of innovative development programs, it is possible to increase revenue by 20% and reduce specific costs and turnover by 10% and 20%, respectively (in a calendar year). Note that many enterprises "know" the areas of increasing efficiency. However, the corresponding methods are not fully implemented, especially those for managing working capital, fixed capital, and risk: the enterprises focus on the profit criterion.

With stress test scenarios, an enterprise can concentrate efforts primarily on the change areas with the greatest impact on the target criteria and potential.

According to the actual performance in 2020, the enterprise's revenue declined from 292 to 257 billion RUB (12.0%). This value is smaller than the result of one-factor stress testing. A set of measures was implemented to improve the economic situation. In particular:

– The cost of goods sold decreased by 13.1% compared to the base year. The share of variable expenses decreased by 1.1% of the revenue.

– The administrative and business expenses decreased by 12.9%.

– The negative balance (difference) of other income and expenses increased by 7.5 times.

– The net interests decreased by 13.7%.

As a result, the total retained earnings constituted 0.3% of the revenue.

More significant changes were observed in assets and liabilities:

– The inventory increased by 1.8 billion RUB, and the inventory turnover grew from 20 to 26 days.

– The accounts receivable increased by 18.2 billion RUB, and their turnover increased from 24 to 52 days.

– The non-current assets increased by 9.7 billion RUB. During the crisis year, the company implemented an investment program.

– The accounts payable increased by 30.1 billion RUB, and their turnover increased from 55 to 109 days.

– The loans increased by 41.4 billion RUB.

– The cash balance increased by 42.8 billion RUB.

The increase in loans is close to the cash balance variation. The increase in accounts payable is close to the increase in accounts receivable, inventory, and investments in non-current assets. The company was able to operate successfully in the crisis year of 2020.

CONCLUSIONS

This paper has presented an approach to stress testing of nonfinancial organizations. A stress-testing model based on financial forecasting has been proposed in analytical form. The reverse problem of determining the "critical" variation of organization's revenue for deficit-free operation (with other constant parameters) has been formulated and solved.

Stress-testing scenarios with industrial enterprise peculiarities have been considered.

Calculations based on the financial statements of a leading Russian company have been executed. According to the calculation results, the safety margin for declining revenue is below 10% and can be recognized as critical.

Stress-testing procedures for nonfinancial organizations are reasonable for understanding the company's safety margin and strategic business risks and developing a set of compensating measures to ensure the company's financial and economic stability.

This paper has proposed a method for determining one parameter and estimating its variations necessary to ensure a nonnegative forecast of the enterprise's cash balance.

However, varying a single management parameter is a particular case of varying multiple ones. Therefore, a topical problem is determining combinations of several parameters whose relatively small variations will jointly produce a similar result. This problem can be solved using critical combinations of events in future works.

According to the actual changes in the illustrative example above, the one-factor stress test begins studies of admissible parameters and their variations. The proposed model can analyze the effects of economic crises, particularly the crisis caused by the COVID-19 pandemic.

REFERENCES

1. Zmijewski, M.E., Methodological Issues Related to the Estimation of Financial Distress Prediction Models, *Journal of Accounting Research*, 1984, no. 22, pp. 59–82.
2. Altman, E. I., Resti A., and Sironi, A., Analyzing and Explaining Default Recovery Rates, *A Report Submitted to International Swaps and Derivatives Dealers' Association*, London, 2001.
3. Nehrebecka, N., COVID-19: Stress-Testing Non-financial Companies: A Macroprudential Perspective. The Experience of Poland, *Eurasian Economic Review*, 2021, vol. 11, pp. 283–319.
4. Greene, J.R., Krouskos, S., Hood, J., Basnayake, H., and Casey, W., *The Stress Test Every Business Needs: A Capital Agenda for Confidently Facing Digital Disruption, Difficult Investors, Recessions, and Geopolitical Threats*, Wiley, 2018.
5. Korableva, I., Measuring the Default Risk of Private Russian Firms Using the Moody's Analytics RiscCalc Model, *Risk Menedzhment*, 2011, no. 3 (189), pp. 54–55. (In Russian.)
6. Ooghe, H., Balcaen, S., Camerlynck, J. The Ooghe-Joos-DeVos Failure Prediction Models: A Cross-Industry Validation. *Brussels economic review*, 2003, no. 46(1), 39-70.
7. Andrievskaya, I.K., Stress Testing: A Survey of Methodologies, *Upravl. Kreditn. Organiz.*, 2007, no. 5, p. 88.
8. Salnikov, V.A., Mogilat, A.N., and Maslov, I.Yu., Stress Testing for Russian Real Sector: First Approach, *Journal of the New Economic Association*, 2012, no. 4(16), pp. 46–70. http://www.econorus.org/repec/journl/2012-16-46-70r.pdf.
9. Pomazanov, M.V. and Kolokolova, O.V., Developing a Formula for the Probability of Company's Bankruptcy Based on Accounting Statements Indicators, *Operativn. Upravl. Strategich. Menedzhm. Kommerch. Banke*, 2004, no. 6. (In Russian.)
10. Kossova, T.V. and Kossova, E.V., Assessing the Credit Risk of Companies in the Russian Corporate Sector Based on Predicting the Probability of Default on Obligations, *Problemy Analiza Riska*, 2011, vol. 8, no. 2, pp. 68–78. (In Russian.)
11. Evstropov, M.V., To the Question of Possibilities Appraisal of Enterprises Bankruptcy Forecasting in Russia, *Vestn. Orenburg State Univ.*, 2008, no. , pp. 25–32. (In Russian.)
12. Brealey, R.A. and Myers, S.C., *Principles of Corporate Finance*, 7th ed., Irwin: McGraw-Hill, 2003.
13. Bogomolov, A.S., Prevention of Accidental Combinations of Events in the Control of Human-Machine Systems, *Izv. Saratov Univ. Math. Mech. Inform.*, 2019, vol. 19, no. 2, pp. 196–206.
14. Bogomolov, A.S., Ivashchenko, V.A., Kushnikov, V.A., et al., Modeling Complex for Critical Events Combinations Analysis in Aviation Transport Systems, *Control Sciences*, 2018, no. 1, pp. 74–79. (In Russian.)
15. Rezchikov, A.F. and Bogomolov, A.S., The Problem of Preventing the Development of Emergency Combinations of Events in Large-Scale Systems, *Proceedings of 12th International Conference on Management of Large-Scale System Development* (*MLSD*'2019), Moscow, Trapeznikov Institute of Control Sciences RAS, 2019, pp. 1066–1069. (In Russian.)
16. Samsonenko, F.A. and Khamidullin, F.F., Stress Testing: a Brief Review of Methods, *Vestn. TISBI*, 2017, no. 2, pp. 186–190. (In Russian.)
17. Lukianenko, I.G. Methods of Stress Testing of Financial Stability of Ukrainian Enterprises, Business Inform, 2013, no. 11, pp. 57–64.
18. Dranko, O.I., A Financial Prediction Model and Home Investment Scenarios, *Control Sciences*, 2007, no. 1, pp. 37–40. (In Russian.)
19. Irikov, V.A., Novikov, D.A., and Trenev, V.N., *Tselostnaya sistema gosudarstvenno-chastnogo upravleniya innovatsionnym razvitiem kak sredstvo udvoeniya tempov vykhoda Rossii iz krizisa i postkrizisnogo rosta* (An Integral Public-Private Management System for Innovation Development as a Means of Doubling the Pace of Russia's Recovery from Recession and

Post-crisis Growth), Moscow: Trapeznikov Institute of Control Sciences RAS, 2009. (In Russian.)

20. Balashov, V.G., Irikov, V.A., and Ivanova, S.I., *Rost i proryv v lidery: praktikum po preodoleniju krizisa* (Growth and Breakthrough to the Leaders: an Aid on Overcoming the Crisis), Moscow: Delo, 2009. (In Russian.)
21. Loginovskii, O.V., Hollay, A.V., Dranko, O.I., et al., *Effektivnoe upravlenie organizatsionnymi i proizvodstvennymi strukturami* (Effective Management of Organizational and Production Structures), Moscow: INFRA-M, 2020. (In Russian.)
22. Golov, O.E. and Irikov, V.A., *Instrumenty i primery povysheniya rezul'tativnosti i effektivnosti upravleniya soglasovannym razvitiem regiona i otraslei* (Tools and Examples of Increasing the Efficiency of Managing the Coordinated Development of the Region and Industries), Ufa: Belaya Reka, 2015. (In Russian.)
23. Dranko, O.I., Otarashvili, Z.A., and Sushkov, D.V., Cost Management under Program of Innovations, *Control Sciences*, 2012, no. 6, pp. 26–31. (In Russian.)
24. Burkova, I.V., Rossikhina, L.V., Meshcheryakova, T.V., and Burkov, V.N., Information Technologies for Development Management Systems, *Automation and Remote Control*, 2018, vol. 79, no. 10, pp. 1813–1822.
25. Akinfiev, V.K. and Tsvirkun, A.D., *Metody i instrumental'nye sredstva upravleniya razvitiem kompanii s slozhnoi strukturoi aktivov* (Methods and Tools for Managing the Development of Companies with a Complex Structure of Assets), Moscow: Trapeznikov Institute of Control Sciences RAS, 2020. (In Russian.)
26. Burkov, V., Burkova, I., Averina, T., and Perevalova, O., Integrated Technology for Creating Development Management Systems in the Field of Energy Saving, in *Advances in Intelligent Systems and Computing*, 2021, vol. 1258, pp. 588–600.

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THE TECHNOLOGICAL CORE MODEL OF A LARGE-SCALE ECONOMIC SYSTEM: OPTIMAL CHARACTERISTICS

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Abstract. This paper considers the technological core model of an economic system and mathematical methods of its analysis. As a formalized criterion for the effectiveness of structural innovations, an indicator of productivity is proposed. The problem of finding an equilibrium state that optimizes the productivity of the technological core of the economy is formally stated. The method of equivalent transformation of the model considering the achieved value of indicators is developed. Several propositions on the properties of the equilibrium state are proved. A multi-stage process for calculating the trajectory that brings the economic system closer to the equilibrium state is constructed. The developed model uses the intersectoral balance of national accounts of the economy. The model is analyzed by determining the preferred structure of outputs at the development stages of the economic system's technological core. The phased process of changing the structure of outputs that asymptotically brings the technological core to the productivity maximum is calculated on an example of Russia's data. The results allow assessing the potential growth of economic productivity within the existing technological order by eliminating structural disproportions.

Keywords: structural disproportions, technological core of the economy, productivity optimum, plans for phased development, equilibrium state.

INTRODUCTION

Despite the existing high potential for development, the modern economy of Russia faces crisis phenomena. At the macrolevel, they include low GDP growth rate, the economy's critical dependence on oil and gas exports, unstable and undervalued exchange rates, a small share of the manufacturing sector, the economy's dependence on external sanctions, and ineffective management mechanisms. These factors cause the incomplete realization of the economy's potential. The results below demonstrate the opportunities for improving the economy's effectiveness based on the planned structural modification of its technological core.

This paper analyzes the effective use of the existing technological potential of an economic system and ways of its development [1, 2]. Comparing such indicators as labor productivity and economic growth rates for developed countries shows that similar technological processes can yield different results. To a certain extent, this difference can be explained by the structural features of the economies of these countries. As noted in [2], there is an increasing awareness that the economy's structure causes the main limitations of economic growth in Russia; the matter concerns an ineffective structure of production, an unproductive structure of incomes, an outdated structure of exports, and an irrational regional structure of the distribution of productive forces. A possible approach to accelerate economic growth is finding a preferred structure of economic activities and ways to implement this structure. According to numerical calculations, such a possibility is justified.

The analysis method under consideration uses the technological core model of a multisectoral economic system. The technological core of an economic system will be understood as a set of economic activities available for observation and measurement and the costs incurred to achieve the results of these activities, sufficient to represent the state of this system adequately. An example of the technological core is the set of factors used by the Federal State Statistics Service (Rosstat) to form the intersectoral balance in the national accounting system [3, 4]. The technological core model describes a static picture of the impact of economic activities on the volume of goods and services supplied. The parameters of this impact characterize the achieved level of technological development of the economy and the possible limits of its use. Technological core subset models can have several types, including goods and services, final consumption expenditures, accumulation, and net exports. The interpretation of the results will differ, depending on the model type. The main data source, the intersectoral balance, reflects the volume of goods and services consumed by different sectors and activities [5].

The important point is that the analyzed technological links form a stable Schur matrix of specific costs of full rank [6]. This feature allows determining the productivity potential (the excess of output over costs in the autonomous mode) and suggesting a way to improve the effectiveness of the economy's technological core by varying the output and prices. It is also possible to find "bottlenecks" in the technological interaction system: to identify the services, outputs, or sectors currently limiting GDP growth (there will be no growth in other sectors within their growth) and determine the potential effect of increasing production in these critical sectors. These questions are settled by optimizing an objective function of technological core productivity subject to different-type constraints [7]. In the resulting solutions, the potential demand is balanced by the supply [8], and the optimal outputs therefore form an equilibrium.

The development of new technologies is a problem with a high degree of uncertainty. Only a small share of innovations seems effective and is embedded in the economy's technological structure. New technologies are included in the economy's structure after assessing their potential. Moreover, the model allows determining the admissible output increase for sectors significant by non-economic criteria.

The reproduction model [9] of a multiproduct system is adopted for defining the multiplier of output (and an indicator of economic system productivity) as a function of structural proportions of outputs and prices for the produced goods and services of sectors. This indicator reflects the ratio of output and costs. Maximizing it, we find the potential of the technological core and the balanced structure of outputs and prices in the reproduction mode.

The inertial character of economic processes should be considered when implementing the calculated parameters of the output structure in practice. For this purpose, we describe a procedure for calculating the indicative forecast of output indicators [9]. Several propositions on the properties of the applied calculation procedures are formulated and proved. The results calculated for the structure of Russia's intersectoral balance are presented to demonstrate the prospects of the proposed approach.

1. INDICATORS OF STABLE DEVELOPMENT

We consider models and methods of management describing stable self-sufficient development of the economy. For this purpose, we use the closed input-output model of the Leontief type [3].

The *productivity* of a single-product (scalar) model of an economic system is defined as $\pi = Y/Z$, where Z denotes the total intermediate costs (input), and Y is the gross value added (GDP). We denote by V the gross output and by $a = Z/V$ the materials consumption. If the gross output is represented as the sum $V = Y + Z$, then the productivity equals

$$\pi = (V - Z)/Z = 1/a - 1. \tag{1}$$

Throughout the paper, all indicators are written in value form at comparable prices of the base year. In the scalar model, economic productivity depends only on the materials consumption parameter: the lower it is, the higher the productivity will be.

For the multiproduct (vector) model of the economy, different configurations of the input $\mathbf{Z}$ and output $\mathbf{V}$ vectors yield different productivity. Note that the relative values of their components are important. Therefore, we can pose the following problem: choose the model parameters with the highest productivity by varying the structure of these vectors.

The productivity potential of a multiproduct economic system is defined as the maximum productivity under the natural constraints imposed on the output $\mathbf{V}$ and input $\mathbf{Z}$ vectors:

$$\pi^* = \max_{\mathbf{V},\mathbf{Z}} \pi.$$

When passing to the multiproduct economy model, we assume that the direct costs Z_{ij} of sector j for the output of products or services of type i and the outputs V_j of products and services of type j are given. These data are used to calculate the specific cost coefficients

$$a_{ij} = Z_{ij} / V_j, \; i = 1,\ldots,n \;\; j = 1,\ldots,n\,.$$

which form the technological matrix $\mathbf{A}$. Here n is the number of sectors. The total costs of sector i have the form

$$Z_i = \sum_{j=1}^{n} a_{ij} V_j, \; i = 1,\ldots, n.$$

By analogy with formula (1), we define the productivity of sector i as the value-added share in the intermediate consumption value. We define the productivity of the economic system's core as the minimum of all sectoral productivities:

$$\pi = \min_i Y_i / Z_i = \min_i \{(V_i - Z_i) / Z_i\}.$$

The input-output model can be described by an equality expressing the balance between the outputs (supply) and total costs (demand) of all sectors:

$$V_i(t) = \gamma_i \sum_{j=1}^{n} a_{ij} V_j(t),\ i = 1, ..., n, \qquad (2)$$

where γ_i denotes the output multiplier of sector i, $\gamma_i \geq 1$.

We formulate an optimization problem for the output structure V_i: maximize the lower bound of the output multipliers

$$\gamma = \min_i \gamma_i,$$

i.e., find

$$\gamma^* = \max_{\gamma, \mathbf{V}} \gamma, \qquad (3)$$

subject to the technological output balance constraint corresponding to condition (2). According to this constraint, direct costs include the costs of all activities and cannot be less than the volume of a certain regulated share of outputs:

$$V_i(t) \geq \gamma \sum_{j=1}^{n} a_{ij} V_j(t). \qquad (4)$$

The relations (3) and (4) represent a bilinear programming problem. The outputs satisfying condition (4) are called balanced. Thus, the problem reduces to determining a balanced output vector $\mathbf{V}$ with the maximum lower bound of the multiplier γ.

If problem (3), (4) has a solution ($\mathbf{V}$, γ), the maximum of the productivity indicator can be calculated as $\pi^* = \gamma - 1$. It represents the value-added share in the intermediate consumption value in the optimal balanced mode of technological development of the economic system. Since $\pi \leq \pi^*$, the inequality $\gamma \geq 1/a$ always holds. In the optimal balanced mode, we have

$$a = 1/\gamma.$$

Now we give an illustrative example of the two-dimensional case of this optimal output structure problem. In this case, the balance condition (4) takes the form

$$\begin{aligned} V_1 &\geq \gamma(a_{11}V_1 + a_{12}V_2), \\ V_2 &\geq \gamma(a_{21}V_1 + a_{22}V_2). \end{aligned} \qquad (4`)$$

Let ($\mathbf{V}$, γ) be the solution of problem (3), (4`) under the natural assumptions $\gamma a_{11} < 1$ and $\gamma a_{22} < 1$. (They are necessary for the technological core to be productive.) Suppose that the specific costs $a_{ij}$ and the components $V_i$ of the output vector are positive. We solve the system of inequalities (4`), which is satisfied under the condition

$$(1 - \gamma a_{11})(1 - \gamma a_{22}) \leq \gamma^2 a_{21} a_{12}.$$

An admissible solution of this system consists of an eigenvector $\mathbf{V}^*$ and the multiplier $\gamma^* = 1/a^*$ corresponding to an eigenvalue a^* of the matrix $\mathbf{A} = [a_{ij}],\ i, j = 1, 2$. It will be called the eigenvalue multiplier. This multiplier is given by

$$\gamma^* = \left((a_{11} + a_{22})/2 \pm \sqrt{(a_{11} - a_{22})^2/4 + a_{12}a_{21}}\right) / (a_{11}a_{22} - a_{21}a_{12}) = \frac{2}{(a_{11} + a_{22}) \mp \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}.$$

The model parameters make economic sense if

$$1 < \gamma < \min(1/a_{11}, 1/a_{22}).$$

We denote by $\gamma_{\min}$ and $\gamma_{\max}$ the minimum and maximum values, respectively, of the eigenmultiplier γ^*.

In the case of strong intersectoral links,

$$a_{11}a_{22} < a_{21}a_{12},$$

the eigenmultipliers are real and have different signs, whereas the constraints (4`) hold if $\gamma_{\min} \leq \gamma \leq \gamma_{\max}$. Then we obtain

$$\gamma_{\max} = \frac{2}{(a_{11} + a_{22}) + \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}} <$$

$$\frac{2}{(a_{11} + a_{22}) + \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22})}} < \frac{1}{\max(a_{11}, a_{22})},$$

$$\gamma_{\min} < 0,$$

$$\gamma_{\max} > 0.$$

Since $\gamma_{\max} < 1/\max(a_{11}, a_{22})$, condition (3) is satisfied, the relations (4`) hold on the strict equality, and $\gamma^* = \gamma_{\max}$.

If

$$a_{11}a_{22} = a_{21}a_{12},$$

we have

$$\gamma^* = 1/(a_{11} + a_{22}),$$

and the constraints (4`) are satisfied for $\gamma \leq \gamma^*$. Problem (3), (4`) has the solution $\gamma = \gamma^*$, and the relations (4`) hold on the strict equality.

If

$$a_{11}a_{22} > a_{21}a_{12}$$

(a small impact of intersectoral links), the values $\gamma_{\min}$ and $\gamma_{\max}$ are positive,

$$\gamma_{\max} = \frac{2}{\left(a_{11}+a_{22}\right)-\sqrt{(a_{11}+a_{22})^2-4\left(a_{11}a_{22}-a_{12}a_{21}\right)}} > \frac{1}{\max(a_{11},\,a_{22})}.$$

In this case,

$$\gamma_{\min} = \frac{2}{\left(a_{11}+a_{22}\right)+\sqrt{(a_{11}+a_{22})^2-4\left(a_{11}a_{22}-a_{12}a_{21}\right)}} < \frac{1}{\max(a_{11},\,a_{22})},$$

and the constraints (4`) are satisfied under the conditions $\gamma \le \gamma_{\min}$ and $\gamma \ge \gamma_{\max}$. Since $\gamma_{\max} \ge 1/\max(a_{11},\ a_{22})$, which is characteristic for weak intersectoral links, the solution is $\gamma^* = \gamma_{\min}$.

Thus, in each case, the solution (whenever exists) coincides with the eigenmultiplier; moreover, when solving the technological core productivity problem (3), (4`), we can use the equality constraints

$$V_1 = \gamma(a_{11}V_1 + a_{12}V_2),$$
$$V_2 = \gamma(a_{21}V_1 + a_{22}V_2),$$

instead of condition (4`).

If the multiplier values γ^* are real and $\gamma_{\min} > 1$, then the matrix $\mathbf{A} = \left[a_{ij}\right],\ i,\ j = 1,\,2,$ is stable (Schur matrix) [6].

To find the eigenvector $\mathbf{V}^*$, we solve the system of inequalities (4`) with an additional normalization condition, e.g., in the form of bounds:

$$\mathbf{V}_i' \le \mathbf{V}_i \le \mathbf{V}_i'',\ i = 1,\,2\,.$$

It is convenient to construct models and analyze calculation results using dimensionless relative prices. In this form, the models and interpretation of the results become more compact and clear.

2. EQUILIBRIUM OUTPUT INDICES

Consider the operations of transforming the technological matrix in current prices to that in relative prices and conversely.

We denote by $\mathbf{D}(\mathbf{X})$ a diagonal matrix with diagonal elements $X_1, X_2, ..., X_n$: $\mathbf{D} = \mathrm{diag}(\mathbf{X})$. Also, we denote by $\mathbf{C}(\mathbf{X})$ a diagonal matrix with diagonal elements $1/X_1, 1/X_2, ..., 1/X_n$.

As outputs change, specific cost estimates $\boldsymbol{a_{ij}}$ also change. To fix the changes in the outputs $\boldsymbol{V_i}$ at the previous step, we recalculate the direct cost coefficients:

$$\bar{a}_{ij} = a_{ij}\ V_j\,/\,V_i\ \ i = 1,\ldots,n,\ \ j = 1,\ldots,n,$$

or

$$\bar{\mathbf{A}} = \mathbf{D}(\mathbf{V})\mathbf{A}\,\mathbf{C}(\mathbf{V})\,. \tag{5}$$

Proposition 1. *If* λ *is some eigenvalue,* $\mathbf{V}$ *is the corresponding eigenvector of the matrix* $\mathbf{A}$*, and all* $X_i \ne 0,\ i = 1,..,n$*, then the transformed matrix* $\bar{\mathbf{A}}$ *has the same eigenvalues, and the eigenvector* $\mathbf{v}$ *equals the original one up to the expansion transformation* $\mathbf{D}(\mathbf{X})$:

$$\mathbf{v} = \mathbf{D}(\mathbf{X})\mathbf{V}\,.$$

All propositions are proved in the Appendix.

Obviously, the inverse to the transformation $\mathbf{D}(\mathbf{X})$ has the form

$$\mathbf{V} = \mathbf{D}(\mathbf{X})^{-1}\mathbf{v} = \mathbf{C}(\mathbf{X})\mathbf{v}\,.$$

The values v_i are interpreted as the proportions (indices) of the outputs (products and services). Thus, the transformation (5) does not change the eigenvalue of the specific cost matrix (technological matrix $\mathbf{A}$) when passing to the scale of output proportions $\bar{\mathbf{A}}$, and the corresponding eigenvector is subjected to the expansion $\mathbf{D}\,(\mathbf{X})$. This transformation of the technological matrix will be called a *deformation.*

Assume that the eigenvector of outputs $\mathbf{V}^0$ in the absolute scale is used for passing to the relative scale (price proportions) by deforming the technology matrix:

$$\mathbf{A}^1 = \mathbf{C}(\mathbf{V}^0)\mathbf{A}\mathbf{D}(\mathbf{V}^0)\,. \tag{6}$$

In this case, the eigenvector of the technological matrix becomes a unit vector after the deformation. In other words, the vectors $\mathbf{V}^0$ and $\mathbf{v}^0$ describe an equilibrium state of the outputs in different scales.

We formulate the following problem: find the output structure maximizing the multipliers under balanced outputs and costs. In the relative scale of output proportions, this optimization problem for the structure of outputs v_i is written as

$$\max_{v_i} \gamma, \tag{7}$$

subject to the technological output constraint

$$v_i(t) \ge \gamma \sum_{j=1}^{n} \bar{a}_{ij} v_j(t),\ i = 1,\ldots,n, \tag{8}$$

where t is the current time instant. Condition (8) is equivalent to condition (4): multiplying the former on the left by the positive matrix $\mathbf{D} = \mathbf{D}(\mathbf{V})$, we obtain

$$\mathbf{v} = \mathbf{D}\mathbf{V} \ge \mathbf{D}\gamma\mathbf{A}\mathbf{V} = \gamma\mathbf{D}\mathbf{A}\mathbf{C}\mathbf{D}\mathbf{V} = \ \gamma\bar{\mathbf{A}}\mathbf{v}\,.$$

For calculations, we choose a lower bound μ on the output indices. Then the corresponding constraint has the form

$$v_i(t) \ge \mu > 0,\ i = 1,\ldots,\ n. \tag{9}$$

This constraint is normalizing: it sets the scale of the indices without affecting their relations. Its economic meaning is as follows: for an equilibrium output vector, there should be no excessive drop in the outputs of sectors that are little involved in technological chains but have noneconomic importance (social sphere, security, ecology, etc.)

The solution under consideration has an economic interpretation if $\bar{\mathbf{A}}$ is a Schur matrix (its maximum eigenvalue is less than 1 by magnitude, i.e., the multiplier value $\gamma > 1$), and its elements and all components of its eigenvector are nonnegative.

Proposition 2. *Let* $\bar{\mathbf{A}}$ *be a positive Schur matrix with real eigenvalues. Then the positive solution of problem* (7), (8) *is implemented on the equality*

$$v_i(t) = \gamma^* \sum_{j=1}^{n} \bar{a}_{ij} v_j(t),\ i = 1,\dots,n,$$

representing the eigenvector of this matrix and the multiplier γ^* *corresponding to the eigenvalue*

$$a^* = 1/\gamma^* > \max_i \{a_{ii}\}.$$

The solution implemented on the equality will be called an equilibrium; the corresponding state of the system, a technological equilibrium. Only the equilibrium states are economically reasonable: if the strict equality does not hold, then

$$\exists i : \overline{\Delta V_i} = V_i - \gamma \sum_{j=1}^{n} a_{ij} V_j > 0$$

(surplus output of the products).

The eigenvector of the matrix $\bar{\mathbf{A}}$ does not necessarily satisfy the optimality condition (7), or additional constraints can be imposed on the outputs besides (8) such that the equilibrium condition is violated. In this case, the productivity of the technological core turns out below the potential value $\pi^* = \gamma^* - 1$.

The indicator of technological effectiveness of the economic system, $u = \pi/\pi^*$, shows the degree of its closeness to the state of technological equilibrium. Obviously,

$$0 \le u \le 1,\ \max u = 1.$$

3. INDICATIVE PLAN-FORECAST FOR JOINT DEVELOPMENT OF SECTORS

If the optimal outputs differ significantly from the current ones, it is impossible to implement a jump or change the structure of outputs according to conditions (7)–(9) in a short period. We define a framework (directive) optimal output trajectory corresponding to additional implementability conditions by imposing extra constraints on changes in the outputs at planning stages. The feasibility of such constraints should be ensured by organizational capabilities and the availability of resources to increase the output of the corresponding sectors. The calendar duration of each planning stage also depends on these capabilities.

To determine a more rational plan for the development of sectors, we use the local problem by supplementing the expressions (6)–(9) with a constraint on the admissible change in the output indices at a rate $0 > \theta > 1$ per plan stage:

$$v_i(t) \le \theta v_i(t-1),\ i = 1,\dots, n.$$

Repeating the procedures of finding the optimal solution and recalculating the direct costs matrix from stage to stage, we obtain an indicative multistage plan-forecast for the joint development of the sectors in the economy's technological core. The absolute and relative outputs are used in the indicative plan calculation procedure. Let $\mathbf{V}^1$ be the vector of current outputs. At the first stage, the deformation of the technological matrix is applied for passing to the relative outputs $\mathbf{v}^1$:

$$\mathbf{A}^2 = \mathrm{diag}(\mathbf{V}^1)^{-1} \mathbf{A}\, \mathrm{diag}(\mathbf{V}^1).$$

The corresponding transformation implements the inverse transition for the vectors of intermediate calculations $\mathbf{v}$:

$$\mathbf{V} = \mathrm{diag}(\mathbf{V}^1)\, \mathbf{v}.$$

Then the vector of relative outputs $\mathbf{v}^k$ are found from the optimization problems

$$\max_{\gamma^k, \mathbf{v}^i} \gamma^k, \tag{10}$$

where $k = 1, 2, \dots$ denotes the stage number, with the technological output constraint

$$\mathbf{v}^k \le \gamma^k \bar{\mathbf{A}}^k \mathbf{v}^k \tag{11}$$

and the relative output growth condition with a rate $\theta > 1$ per plan stage,

$$\mu \mathbf{I} \le \mathbf{v}^k \le \theta \mathbf{I}, \tag{12}$$

where $\mathbf{I}$ denotes a unit vector of compatible dimension. Then the indicative plan-forecast for the joint development of sectors can be calculated using the following propositions.

Proposition 3. *The sequence* $\mathbf{V}^k$ *and the estimate* γ^k *tend to the solution of problem* (7)–(9) *in a finite number of steps.*

Remark. Let $\mathbf{V}^*$ be the solution of problem (7)–(9). When deforming the technological matrix $\mathbf{A}$ by the matrix $\mathbf{D} = \mathrm{diag}(\mathbf{v}^*)$, the solution of the planning problem becomes trivial: $\mathbf{v} = \gamma \mathbf{I}$, where $\mathbf{v} = \mathbf{D}^{-1}\ \mathbf{V} = \mathbf{CV}$. In other words, when the technological optimum is achieved, the structure of outputs becomes equilibrium, and there are no further changes.

Proposition 4. *Assume that there exists a solution of the local problems for* $k \ge 1$. *Then the indicative outputs can be obtained at stage* i *in absolute units of the form*

$$\mathbf{V}^k = \prod_{j=k}^{1} \operatorname{diag}(\mathbf{v}^j) \cdot \mathbf{V}^0 . \qquad (13)$$

Proposition 5. *If the coefficient* $\theta > 1$ *is constant for all stages, then the relative growth of outputs becomes the same starting from some stage. This property is similar to the highway property of optimization models of economic dynamics* [9].

Consider the optimum vector of outputs for problem (10)–(12). Note that the output structure $\mathbf{v}^k$ obtained by solving this problem differs at the initial stages from the equilibrium and is not stable for a fixed technological matrix **A.** However, changes in the output structure should lead to changes in this matrix in accordance with formula (5):

$$\bar{\mathbf{A}}^k = \mathbf{D}(\mathbf{v}^k)\mathbf{A}\,\mathbf{C}(\mathbf{v}^k) .$$

The resulting output structure tends to a stable equilibrium structure between planning stages; see the remark to Proposition 3.

The transformation (13) is applied to pass from the obtained indicators to the value-form outputs.

4. CALCULATION OF INDICATIVE PLAN-FORECAST

Consider the numerical technological core model (10)–(12). As a data source, let us use Russia's intersectoral balance for 2016; see [5]. It reflects the volume of goods and services consumed by different sectors and activities. (The base input-output tables are developed quinquennially for the years ending with 1 and 6.) An Excel library was used for numerically solving the corresponding mathematical programming problems. Its analog was described in [10].

Figure 1 shows the graph of the coefficient of productivity for the technological core when balancing the outputs at successive indicative planning stages. The upper limit of change in the output proportions was increased with a rate of $\theta = 1.5$ per stage. The productivity estimate can be written as $\pi = (\gamma - 1) \cdot 100\%$.

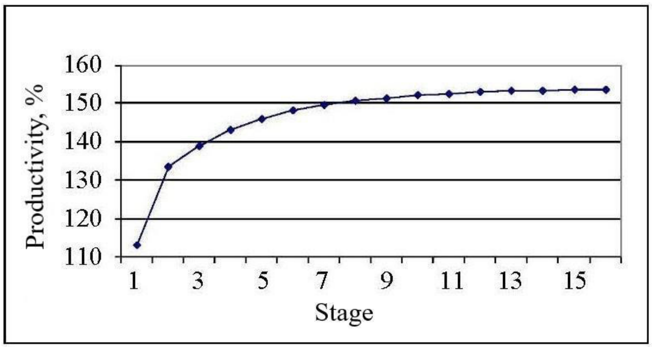

Fig. 1. The coefficient of technological core productivity when optimizing output proportions at successive indicative planning stages.

Next, Fig. 2 presents the indicative dynamics of output proportions for some sectors when solving the problem at successive indicative planning stages based on Russia's economic data. For a considerable part of economic activities, the output indices immediately reach the level $\theta = 1.5$ and remain there at all subsequent stages. The graphs correspond to the first eight rows of 98 types of economic activities in the intersectoral balance table for which the estimated outputs differ from $\theta = 1.5$ for $k > 1$.

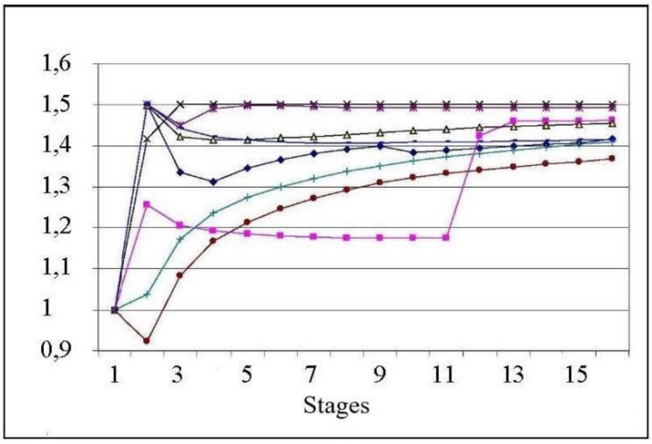

Fig. 2. Indicative dynamics of output proportions for some sectors at successive indicative planning stages for Russia's economic data:

- agricultural output;
- services related to hunting, trapping, and breeding of wild animals;
- fish and other fishery and fish-farming products; services related to fishery and fish-farming;
- oil, including oil derived from bituminous minerals; oil shale (bituminous) and bituminous sandstone;
- other mining products;
- meat, meat products, and other processed animal products;
- fish and fish products, processed and canned;
- fruit, vegetables, and potatoes, processed and canned.

Figures 1 and 2 illustrate the highway property [11] of the development model of the Russian economy's technological core: on a sufficient horizon, the output indices reach a constant level. According to formula (13), the outputs in value terms grow at rates v_i, $i = 1,..., n$.

The above calculations allow estimating the growth potential of the technological core of the economic system. They demonstrate that changes in the output structure may significantly increase the indicator of productivity. To estimate the realizability of the existing potential of the technological core, we should supplement the balancing conditions with constraints on labor and raw materials, final consumption costs, accumulation, stock formation, and export-import flows. Thus, the obtained dynamics of output indices can be treated as their upper bound.

CONCLUSIONS

According to the Rosstat data and the calculation results, the state of the Russian economy is not equilibrium because the real productivity of the technological core (112%, see the first point in Fig. 1) is noticeably lower than the potential value (above 152%, see the asymptotic value of productivity). This observation gives hope in increasing the indicator of productivity in real conditions significantly. Such a possibility should be implemented by elaborating strategic plans for the development of the economy. Along with the choice of priority areas of technological core development, it requires adequate methods for forecasting multisectoral dynamics considering the main aspects of economic activity: stock formation, accumulation, final consumption of the state and households, and export-import flows [12–14]. New-level planning also involves appropriate organizational mechanisms [15].

In addition to the applied aspect, the results of this paper illustrate the specifics of the proposed methodology with relevant calculation and analysis tools. The class of problems discussed above has several specific features. Real interest in macroeconomic studies of this kind is connected with high-dimensional models. (The list of economic activities analyzed can be counted in hundreds.) Besides, the practical application of technological core models requires effective algorithms for solving the mathematical programming problems of the considered type [16–18] and linguistic calculation management tools, which should be integrated into a working environment [10, 19]. Free access to actual verified data and modern information technology, including the corresponding computational environment and interface devices, is necessary as well. For example, the paper [20] described a similar open-access toolkit (Thread Pool Executor of Akka) to process high-dimensional problems.

APPENDIX

P r o o f of Proposition 1. Let λ and $\mathbf{x}$ be an eigenvalue and the corresponding eigenvector of the matrix **A**, respectively:

$$\mathbf{Ax} = \lambda\mathbf{x}.$$

Multiplying both sides of this equation on the left by the matrix $\mathbf{D}$, we obtain

$$\mathbf{DAx} = \mathbf{D}\lambda\mathbf{x}.$$

Since $\mathbf{CD} = \mathbf{E}$ is an identity matrix, it follows that $\mathbf{x} = (\mathbf{CD})\mathbf{x}$ and

$$\mathbf{DA}(\mathbf{CD})\mathbf{x} = \lambda(\mathbf{Dx}), \text{ or}$$

$$\mathbf{DAC}(\mathbf{Dx}) = \lambda(\mathbf{Dx}).$$

In other words, $\mathbf{Dx}$ is an eigenvector of the matrix $\mathbf{DAC}$, and λ is its eigenvalue. ♦

P r o o f of Proposition 2. Let $\mathbf{V}^*$ be an eigenvector of matrix **A**. Since the number of inequalities in the constraint coincides with the dimension of the output vector, the solution of the bilinear programming problem

$$\max_{\gamma, Vi} \gamma, \qquad \text{(A.1)}$$

$$V_i \ge \gamma \sum_{j=1}^{n} a_{ij} V_j, \qquad \text{(A.2)}$$

is achieved on the equality

$$V_i = \gamma^* \sum_{j=1}^{n} a_{ij} V_j, \; i = 1, ..., n, \qquad \text{(A.3)}$$

where $\gamma^* = 1/a^*$, and a^* is the eigenvalue of the matrix **A**.

Indeed, considering the condition $\mathbf{V} > 0$ and its corollary $\gamma < 1/\max_i \{a_{ii}\}$, we eliminate the variables V_i from (A.2), arriving at the following inequality for the characteristic polynomial $L(\gamma)$ of degree n:

$$L(\gamma) = (1-\gamma a_{11})(1-\gamma a_{22})...(1-\gamma a_{nn}) - a_{12}a_{21}(1-\gamma a_{33})... - a_{13}a_{31}(1-\gamma a_{22})... \ge 0.$$

As a result, we obtain the optimization problem

$$\max \gamma,$$
$$L(\gamma) \ge 0,$$
$$1 \le \gamma < 1/\max_i \{a_{ii}\}.$$

This problem has a unique finite solution γ^* coinciding with a root of the polynomial $L(\gamma)$ if the value γ^* satisfies (A.3). The converse is true as well (otherwise, the matrix **A** would have more than n eigenvalues).

Assume on the contrary that Proposition 2 is false: the maximum γ^* is reached on the half-interval $1 \le \gamma^* < 1/\max_i \{a_{ii}\}$ on the strict inequality $L(\gamma) > 0$. In the neighborhood of γ^*, the analytic function $L(\gamma)$ can be approximated by a segment of the Taylor–Lagrange series:

$$L(\gamma) = L(\gamma^*) + L'(\gamma^*)\left(\gamma - \gamma^*\right) + L''\left(\gamma^* + \theta\left(\gamma - \gamma^*\right)\right)\left(\gamma - \gamma^*\right)^2$$
$$0 < \theta < 1.$$

Hence, in this neighborhood, there exist an admissible point ($\gamma^* + \delta$) and a constant $\varepsilon > 0$ such that $L(\gamma^* + \delta) > 0$ and

$$0 < \delta < \min\left\{ L(\gamma^*) / \left(\left|L'(\gamma^*)\right| + \varepsilon\right), \sqrt{L(\gamma^*) / \left(\left|L''(\gamma^*)\right| + \varepsilon\right)}, 1/\max_i \{a_{ii}\} - \gamma^* \right\}.$$

In other words, if $L(\gamma^*) > 0$, the value γ^* cannot deliver maximum to the polynomial $L(\gamma)$ under the condition $L(\gamma) \geq 0$. Therefore, the maximum value γ is achieved at the root of the polynomial $L(\gamma)$, and the solution of problem (A.1), (A.2) is reached on equality (A.3). ♦

P r o o f of Proposition 3. Consider an auxiliary bilinear programming problem of the form

$$\max_{\mathbf{v}} \gamma$$

subject to the output constraint

$$\mathbf{v} \geq \gamma \mathbf{A}^0 \mathbf{v}$$

and the relative output growth condition with an unbounded rate per one planning stage:

$$\mathbf{I} \leq \mathbf{v}.$$

Let $\mathbf{v}^*$ be the solution of this problem. Assume that $\theta^* = \max_i v_i^*$. Then the proposition holds for $\theta = \theta^*$, and

$$\mathbf{V}^1 \leq \theta^* \mathbf{V}^0.$$

For $1 < \theta = \theta_1 < \theta^*$, the planning problem will be solved in two stages: for $\theta = \theta_1$ and $\theta = \theta_2 = \theta^* / \theta_1$. Solving the planning problem in the two stages, we also obtain the solution $\mathbf{v}^*$. At the last stage, we have the conditions of the previous problem for one stage:

$$\mathbf{I} \leq \mathbf{v}^1 \leq \theta_1 \mathbf{I},\ \mathbf{v}^2 \leq \theta^* / \theta_1 \mathbf{I},$$

$$\mathbf{V}^2 \leq \theta^* \mathbf{V}^0.$$

Following similar considerations, we can divide the interval $\left[1, \theta^*\right]$ into any finite number of segments and the solution of the planning problem into the corresponding number of stages. ♦

P r o o f of Proposition 4. For $i = 1$, we have $\mathbf{D} = \mathrm{diag}(\mathbf{v}^1)$, $\mathbf{V}^0$ is the initial output vector, and $\mathbf{V}^1$ is the output vector after the first stage. Then $\mathbf{V}^1 = \mathbf{D}\mathbf{V}^0$. For $i > 1$, we obtain $\mathbf{V}^i = \mathrm{diag}(\mathbf{v}^i)\mathbf{V}^{i-1}$ by induction. Therefore, $\mathbf{V}^i = \mathrm{diag}(\mathbf{v}^i) \prod_{j=i-1}^{1} \mathrm{diag}(\mathbf{v}^j)\ \mathbf{V}^0$. ♦

P r o o f of Proposition 5. Since the value $\gamma^* = \max_{\mathbf{v}} \gamma$ is bounded, the output multiplier will reach the constant level γ^* starting from some step k^*. Moreover, under the hypotheses of Proposition 3, the inequality constraints

$$\mathrm{v}^k(t) \leq \theta \cdot \mathrm{I}$$

where $1 < \theta \leq \theta^*$ and $\theta^* = \max_i v_i^*$, will be satisfied on the equality

$$\mathrm{v}^k(t) = \theta \cdot \mathbf{I},\ k \geq k^*. ♦$$

REFERENCES

1. *Indikativnoe planirovanie i provedenie regional'noi politiki* (Indicative Planning and Implementation of Regional Policy), Abdikeev, M.N., et al., Moscow: Finansy i Statistika, 2007. (In Russian.)
2. Uzyakov, M.N., Problems of Economic Measurements and Possibilities of Structural Analysis, *Studies on Russian Economic Development*, 2020, vol. 31, no. 1, pp. 1–2.
3. Leontief, W.W., *Essays in Economics. Theories, Theorizing, Facts, and Politics*, New York: Oxford University Press, 1966.
4. Sayapova, A.R., World Input Output Tables as a Tool for Assessing the "Point of Growth" of the National Economy, *Scientific Articles - Institute of Economic Forecasting RAS*, 2019, vol. 17, pp. 27–39. (In Russian.)
5. *Basic Input-Output Tables of the Russian Federation for 2016* (*in Current Prices, mil. rub.*). Published by the Federal State Statistics Service of the Russian Federation, January 30, 2020. https://rosstat.gov.ru/storage/mediabank/baz-tzv-2016.xlsx. (In Russian.)
6. Polyak, B.T., Khlebnikov, M.V., and Rapoport, L.B., *Matematicheskaya teoriya avtomaticheskogo upravleniya* (Mathematical Theory of Automatic Control), Moscow: URSS, 2019. (In Russian.)
7. Gusev, V.B., Equilibrium Models of the Multi-Resource Self-Developing Systems, *Control Sciences*, 2007, no. 3, pp. 18–25. (In Russian.)
8. Samuelson, P.A., *Economics*, New York: McGraw-Hill, 1989.
9. Gusev, V.B., Models of Autonomous Control in the Developing Systems, *Control Sciences*, 2018, no. 6, pp. 2–17. (In Russian.)
10. Mason, A.J., OpenSolver - An Open Source Add-in to Solve Linear and Integer Progammes in Excel, in *Operations Research Proceedings 2011*, Klatte, D., Lüthi, H.-J., and Schmedders, K., Eds., Berlin–Heidelberg: Springer, 2012, pp. 401–406. http://dx.doi.org/10.1007/978-3-642-29210-1_64.
11. Dorfman, R., Samuelson, P.A., and Solow, R.M., *Linear Programming and Economic Analysis*, New York: McGraw-Hill, 1958.
12. Gusev, V.B., The Sufficient Conditions for Stable Development During the Diversification of the Economy, *Drukerovskij Vestnik*, 2015, no. 3 (7), pp. 91–98. (In Russian.)
13. *Odnoproduktovaya model' dolgosrochnogo prognoza VVP* (A Single-Product Model of Long-Run GDP Forecasting), Antipov, V.I., et al., Moscow: Trapeznikov Institute of Control Sciences RAS, 2005. (In Russian.)
14. *Prikladnoe prognozirovanie natsional'noi ekonomiki* (Applied Forecasting of the National Economy), Budanov, I.A., Ivanter, V.V., Korovin, A.G., and Sutyagin, V.S., Eds., Moscow: Ekonomist, 2007. (In Russian.)
15. Antipov, V.I., *GOSPLAN. Vchera, segodnya, zavtra* (State Planning: Yesterday, Today, Tomorrow), Moscow: Kontseptual, 2019. (In Russian.)
16. Polyak, B.T., *Introduction to Optimization*, Optimization Software, 1987.
17. Fox, W.P. and Burks, R., Mathematical Programming: Linear, Integer, and Nonlinear Optimization in Military Decision-Making, in *Applications of Operations Research and Management Science for Military Decision Making*, New York: Springer, 2019, pp. 137–191.

18. Bergstra, J., Bardenet, R., Bengio, Y., and Kégl, B., Algorithms for Hyper-parameter Optimization, *Proceedings of the 24th International Conference on Neural Information Processing Systems* (*NIPS'11*), Red Hook, NY, USA: Curran Associates Inc., 2011, pp. 2546–2554.
19. Doumic, M., Perthame, B., Ribes, E., et al., Toward an Integrated Workforce Planning Framework Using Structured Equations, *European Journal of Operational Research*, 2017, vol. 262, no. 1, pp. 217–230.
20. Hai, T.N., Tien, V.D., and Csaba, R., Optimizing the Resource Usage of Actor-Based Systems, *Journal of Network and Computer Applications*, 2021, vol. 190:103143. https://doi.org/10.1016/j.jnca.2021.103143.

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AUTONOMOUS COLLECTIVE ADJUSTMENT OF VEHICLES MOTION ON A HIGHWAY

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Abstract. This paper describes a fully autonomous decentralized method for synchronizing the interaction of vehicles moving on a highway. The method synchronizes the vehicles using simultaneous signal transmission from a group of transmitters to a group of receivers. With this feature, data exchange speed is increased, and the computing abilities of vehicles are connected into a moving computing cluster. The autonomous system operates without external controllers. Due to decentralization, the group of vehicles implements the synchronization process without any system control center. The group members interconnect via wireless optical and radio communication channels. There are two interacting stages of the synchronization process. The first stage is intended to perform decentralized coordination and information exchange within the group and determine the location, speed, and motion direction of the group members. The first stage passes initial information to the second stage. The second stage provides much more accurate vehicle tracking data and simultaneous information exchange between the groups of transmitters and receivers. Message transmission is synchronized very precisely (up to a single bit). In particular, necessary information about n vehicles is quickly acquired and transmitted to all receivers using one common message containing no more than n digits. Thus, the provided solution allows collecting the necessary information for vehicle coordination on highway sections, combining every vehicle's computing capability into one mobile computing cluster.

Keywords: autonomous vehicles, vehicle synchronization, decentralized object control, group interaction of mobile objects, fast in-network computing.

INTRODUCTION

Due to the rapid development of unmanned, highly automated vehicles, the need for automatic motion tracking and adjustment means has increased. Relevant research and development are mainly carried out within three approaches with numerous publications. Tracking means are developed using:

1) satellite navigation aids,
2) radars and lidars installed on the vehicle,
3) vision aids (see the surveys [1, 2]).

Solutions of the second and third approaches operate autonomously. They determine the distance to other vehicles within the line of sight with accuracy and speed sufficient for the object's safe motion. (From now on, we will use the word "object" instead of "car" and "vehicle" whenever possible.) However, these approaches do not determine the mutual location of all objects on a highway section. The solutions of the first approach determine the mutual location of all objects, but they are not autonomous and require interaction with satellites or special ground stations. The mutual location of moving objects is determined based on known coordinates, but accuracy is lower than in the second and third approaches. All approaches can be used jointly.

The methods proposed below for tracking and adjusting the motion of objects on a highway are autonomous, decentralized, and add new capabilities for the above three research approaches. Compared to the first approach, the dependence on signals from external sources is excluded; the accuracy of determining the location of objects is commensurate with those of the second and third approaches. For the second and third approaches, a new capability is that each object can now determine the current location of all objects on a highway section. A new feature also distinguishes the proposed methods from all approaches. Simple, computer-free communication facilities allow object computers to perform important distributed calculations for estimating the states of objects directly in the network. These operations are performed during data exchange between objects, causing no additional delays. Moreover, their duration does not depend on the number of objects participating in the operation. The new capabilities are divided into two groups as follows.

The first group performs motion adjustment, common for all objects on a given highway section. The objects act collectively, and each object simultaneously provides information about its state and actions to the entire group. These actions are executed with speed sufficient for tracking the motion of objects on a highway. Information about the group's current state allows making better private decisions and coordinating them. These tasks are performed by the synchronization process SP_0 proposed below.

The second group applies higher requirements to the interaction of objects. Each moving object has a control computer, and all objects interact via their computers. The group of objects should also be treated as a computer cluster operating in the hard real-time mode with the following features. Distances between cluster members are continuously changing while solving a current task. The composition of objects on a highway section (cluster members) changes in fractions of a second. The computers within a section have a history of their actions, which needs to be considered. The motion control task consists of small subtasks performed by cluster members in the hard real-time mode. The resources of such a cluster need to be shared with fast access to parts. For such a cluster, it is possible to perform the fast distributed collection of information about the state of n objects and deliver only one common message to all objects. This message combines same-name digits from the messages of n objects; see Section 5.

The group of computers acts as a single cluster, restricting the length of highway sections for their operation. Actions at hundreds of meters are significantly less flexible than those at tens of meters. This feature will be considered below. Thus, the second capability is that the objects act as a single mobile computer system.

The capabilities mentioned are achieved by introducing accurate synchronization of object actions. In this case, object coordinates are determined with an error not exceeding those of the above methods within the range of allowed transport speeds. Objects act synchronously and exchange information about the object location directly when determining the object coordinates. All active equipment can be installed directly on the object. Objects exchange data about their current location at high speed. Hence, their computer resources can be used as a common resource for solving a single motion safety task. All these tasks are solved based on the synchronization process SP.

The main purpose of this paper is to supplement the known solutions by acquiring timely data on the state of a distributed group of mobile objects and pooling the resources of computers controlling the movement of objects. Algorithms with such capabilities are not developed in the paper.

The system combining highway sections and the objects on them has a variable structure. The motion conditions on adjacent sections at different points may vary significantly over time.

The proposed solutions are based on the paper [3], where the interaction of mobile objects of a more general form was considered, and on the earlier paper [4]. However, they contain new capabilities considering the specifics of the above tasks.

The remainder of this paper is organized as follows. Sections 1–3 consider synchronization of object actions with accuracy sufficient to adjust the motion of objects based on their states. In Section 4, we propose a synchronization method under which the computers of objects operate as a single cluster. Section 5 presents distributed cluster operations executed at a rate independent of the number of their participants. In Section 6, the effect of the environment's state on the accuracy of the proposed synchronization processes is studied.

1. PARTICIPANTS OF MOTION ADJUSTMENT PROCESSES

Let us address the figure below.

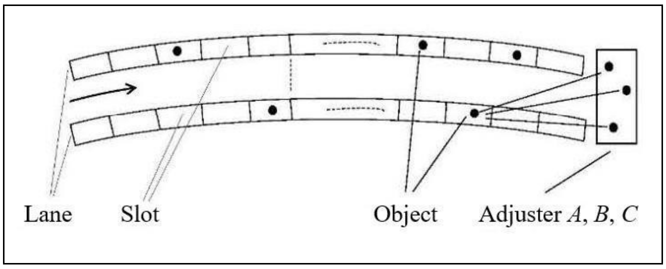

Fig. Highway section with motion adjustment.

The highway on which the objects move is divided into sections of a length L meters (possibly individual for each section). A group containing no more than n objects can be located within a section simultaneously; their speed and location vary over time. The highway has p lanes in one direction. The object's length is at least l meters. In front of each object, there should be a free section (interval) of a length d without other objects for safety purposes. (In what follows, $d = 2/3l$.) At the end of each section, there is a passive or active motion adjuster containing a source of signals. In the simplest case, we have a passive adjuster with at least three passive optical retroreflectors spaced sufficiently to perform trilateration with a required accuracy. They return the light signal coming to them from the moving object. Trilateration and an example of its application in robotics were described in the standard [5]. Each retroreflector is equipped with a light filter that passes a particular frequency band. We denote by A, B, and C the filters and their frequencies. The relative position of the retroreflectors in the adjuster is fixed.

Such passive equipment on the highway is sufficient to execute the synchronization process SP_0; see Sections 2 and 3. In the paper, additional complication of the equipment on the highway is introduced only when necessary. For example, an active repeater of optical signals in the adjuster is used in Section 4 to operate the objects in the mobile cluster mode (to execute the synchronization process SP, more accurate than SP_0). If an active adjuster is used, the objects will send both optical and radio signals to it. Also, we show the possibility of using an arbitrary mobile object as an adjuster and leaving only passive equipment on the highway.

The objects contain a computer that controls their motion, an optical pulse source that routes the object signals to a passive adjuster, and a receiver for the reflected signals, acting independently of the radio transceivers for the signals exchanged between the objects.

Each object is provided with a highway map in advance. The map captures static information about the current highway section, such as markings, warning signs, and the adjuster's coordinates and interaction features. The signal frequencies allowed for objects in the current highway section are included in this information to simplify interference control. The map also contains similar data for the next highway section in the motion direction.

The object's computer plots the following dynamic information on the map:

– the current coordinates of this object and all objects that have transmitted their coordinates;

– the object's permanent individual registration number;

– the location of the object body on the map (see subsection 2.2).

Each lane on the highway section map is divided into slots sl of the length $(l + d)$, where the parameters l and d are defined above. As a result, the map is covered by a grid of slots, and a vehicle can occupy one or more slots.

Each object transmits and receives radio signals on the frequencies allowed for the current section. In addition, it receives radio signals of the objects located on the next section.

With messaging rights, an object can transmit broadcast, group, or individual messages. (An individual message is addressed to particular objects.) New object coordinates received in the message simultaneously correct the maps for all objects.

2. DETERMINING THE LOCATION OF OBJECTS IN THE PROCESS SP_0

2.1 Distance to the motion adjuster: alternating determination by the objects

Each object interacting with the motion adjuster knows the highway section length L. Hence, each object also knows the signal interval T_1 within the section. For determining the distance, objects need to be assigned sequential numbers; they are described below. Objects wait for the absence of signals transmitted by them and the motion adjuster during the interval T_1 and transmit a special radio signal S of a duration not less than T_1. At the instant S^* of its completion, process 1 is executed: the next (first, second, etc.) object is allocated. Then this object determines its coordinates using process 2. Other objects wait for the completion of the first object; the next object sends its signals, and so on until all other objects on the section complete similar operations. The first object can determine its coordinates again only after all objects on the section have done so. During the process SP_0, the objects alternately determine their coordinates. However, they must not limit the speed allowed for highways.

Any object starts the next message transmission after all objects in the section have completed message transmission. Hence, the object may change its position significantly during this time, which is not considered by the process SP_0. This fact is the main restriction for the accuracy of the process SP_0. In Sections 3 and 4, we take some measures to reduce the mentioned pause.

In this section, only the passive motion adjuster is used. For the sequential numbering of objects, we first adopt the license plates of the cars. For example, over 50 million cars are registered in Russia. Representing their license plates as binary numbers requires $a = 26$ digits. This numbering option is the simplest but slowest one. In other sections, the numbering method will be accelerated.

Further, the adjuster measures the object's coordinates using the following information: the highway section length L; the number of lanes in the corresponding direction, p; the object's minimum length l; the object speed v; the maximum possible number of objects on the specified highway section, n; the speed of light c; the number of binary digits required to identify each object, $a = 26$.

We use the relations

$$T_1 = L/c,\ T_2 = (2a + 3)\ T_1,\ n = 3pL/5l,\ T_3 = nT_2, \quad (1)$$

where T_1 is the time required for the signal to pass through a section of the length L (particularly for the signal to reach the retroreflector from the farthest object). The time T_2 is determined by the time required to select the smallest number in the group of 2^a-digit numbers. We add two times to T_2: the time $2T_1$ to measure the distance to the retroreflector and the time T_1 to transmit data to other objects. A large amount of data can be transmitted in the time T_1. The formula for n gives the maximum number of objects located simultaneously on a highway section with p lanes. Here, the factor 5/3 at l considers the minimum distance d to the next object (2/3 of the car body length as recommended in the literature). After the time T_3, all objects on the section complete the distance measurements, and the objects start a new cycle of determining distances to the adjuster. If the number of objects is less than n, part of the interval T_3 will not be used. Suppose that the speed of objects on the highway is $v \leq 180$ km/h (50 m/s).

All objects on the section must start measuring times and distances with a minimum variation over time. To do this, they apply process 1.

Process 1 (identifying the object with the current smallest number)

Step 1. The object transmits radio signals with the highest bit of its sequential number (the most significant bit among the ones not transmitted in this process earlier), which contains a digits. The value "1" is transmitted by the frequency signal f_1; the value "0," by the frequency signal f_0.

Step 2. If the object that transmitted signal f_1 in Step 1 receives the signal f_0 from other objects, it stops executing process 1. The remaining objects proceed to Step 3.

Step 3. The object checks whether there are bits of the sequential number not transmitted in Step 1. If any, the object returns to Step 1; otherwise, process 1 is complete.

Remark to Step 1. The original version of this simple process is the decentralized priority Control (DPC) method: the object with the highest current priority gets the right to transmit the message. For the wire bus, this method was developed in 1970 at the Institute of Automation and Remote Control (now Trapeznikov Institute of Control Sciences) of the USSR Academy of Sciences [6]. DPC capabilities were extended in the monograph [7]. This method was applied in process control systems. Its wireless version was described in the paper [3].

Process 2, following process 1, defines the object's coordinates.

Process 2 (determining the object's coordinates).

Step 1. The source of the object's optical signals simultaneously sends signals in frequency bands A, B, and C to the retroreflectors A, B, and C for the adjuster. The retroreflectors A, B, and C return the signals to the objects. Each of these signals is received by the corresponding receiver at the object. Timers are associated with the source and receivers of the signals. When sending a signal, the source starts all timers simultaneously. When a reflected signal is received, the receiver stops the corresponding timer.

Remark to Step 1. With the simultaneous transmission of signals to the three retroreflectors A, B, and C, the process SP eliminates the influence of object motion on the accuracy of measurements when computers execute fast processes; see Section 5. In this case, the distances to the reflectors determined during the last distance measurement will slightly differ from the real ones.

Step 2. Given the signal velocity c and timer counts, the object's computer determines the distances to the retroreflectors; using trilateration, it calculates the object's coordinates relative to the adjuster position.

For the receivers to be unaffected by source signals, the timers start when the transmission is complete and stop when the reflected signals are received.

Step 3. The object transmits radio signals with its coordinates (and additional information if necessary) simultaneously to all objects. Using the special radio signal rs^*, the object informs the other objects about completing its measurement.

The object transmits its coordinates to other objects through radio signals in the extra time T_1. This transmission can be combined with measuring the distance of another object to the adjuster when sending optical signals to the latter.

Variant of Step 3. Sending the signals A, B, and C, the object transmits a radio signal to all objects about sending the signal. When each of them returns, the object sends appropriate radio signals to other objects. Based on the received data, the latter objects calculate the coordinates of the object that sent the signals.

Remark to process 2. At any instant, this process involves a single signal source and a single adjuster that transmit a single signal. Therefore, the adjuster's useful signal arrives at the receivers before its external reflections and does not affect the measurement. In Step 3, the receiver receives a message with binary zeros and ones sent by different frequencies.

First of all, we make sure that the ordering of objects' actions does not limit the motion speed on the highway. Also, the arrival of new objects on the highway section with lower sequential numbers compared to the existing ones must not prohibit the latter from determining their coordinates. Let us show that these conditions are satisfied.

First, we discuss the sequential ordering of object actions. Let the event S^* (the disappearance of the signal S) occur on some highway section. In this case, nearby objects will start transmitting the highest bit of their sequential number. In the time T_1, these signals will leave the section. In the time T_1 after the event S, the most distant objects from this event will send their signals with their highest bits. After the time T_1, these signals will also leave the section, and the next bit of the sequential number can be transmitted, and so on.

Each section contains at most $n = 3pL/5l$ objects simultaneously. According to the relations (1), measuring their coordinates will require T_3 seconds, where

$$T_3 = nT_2 = n(2a + 3)T_1 = 3\,p(2a + 3)L^2/5lc. \qquad (2)$$

Given the speed v, in the time T_3 the object will move to the distance $\Delta S = vT_3$ meters. For the parameter values $L = 50$ m, $l = 2$ m, $p = 10$, $a = 26$, and $v = 50$ m/s, we have $\Delta S = 7.125$ cm. As a result, the location of the objects on the maps will not change: the objects will be shifted by less than 3% of the slot length.

Similarly, when a new object enters a highway section with a speed v, measurements for all objects will be completed before the new object occupies one of the section slots.

The obtained result is acceptable for the measurement of coordinates. Moreover, it can be improved by excluding the dependence on the number a (changing the sequential numbering of objects). This issue will be considered in Section 3.

2.2 Motion rules for objects on a route section and when entering a new section

Let us return to the map defined in Section 1.1. The standard slot-to-slot motion of an object is within the lane it occupies. There must be a free section of the lane with a length of at least $(l + d)$ meters in front of the object. The standard motion does not need to be coordinated with other objects. Moving to another lane requires coordination with an object in this lane. The details of their interaction are a matter of particular implementation. However, the actions outlined above allow an object to send its request and repeatedly exchange motion details.

Two consecutive measurements (or two simultaneous measurements taken from different locations on an object's body) determine its orientation and position on the lane. The object transmits these data to other objects.

Thus, within the interaction described in Section 2, objects timely inform their neighbors on the highway section about their position on the highway on time, but motion control is not considered.

The object has a map of the next highway section. Hence, transition to the next section means changing the signal frequencies and using the new section map.

3. ACCELERATING THE PROCESS SP_0

Here, the measurement accuracy of object's coordinates is improved by eliminating the dependence of the exponential a on the number of vehicle license plates. This approach reduces the time between successive measurements of the distance to the motion adjuster.

The measurement order is dictated by the map, more precisely by the current highway section divided into the n numbered slots (see above). The measurement process starts in the same way as before. However, after the signal S signal finishes, all section *slots* get the right to perform the measurement one by one. If an object occupies several slots, it confirms the occupation of each such slot. If there is no object in the slot, the time allocated to the slot remains unoccupied. Since all objects have the same maps, the measurement process is accelerated if necessary by considering only the slots occupied by objects. An object on the map is also marked with an individual vehicle number.

As a result, we obtain the relations $T_1 = L/c$, $T_2 = 3T_1$, $n = 3pL/5l$, and $T_3 = 3nL/c$. Now in the time T_3, the object will move by $\Delta S = 3vnL/c$ meters. For the example from subsection 2.1, $\Delta S = 0.375$ cm. The environment's state can affect the accuracy of the measurement; see Section 6.

Slots allow using several adjusters on a highway section. For a slot or a group of slots, an appropriate adjuster with the clearest signal will be allocated. Different adjusters will respond to different sets the frequency bands A, B, and C. The object map will indicate which object should be used for a particular slot.

On the ordered access of new objects to a highway section. The sequential process of determining the position of objects on the highway section has been presented above. Now we consider the ordered access of objects to a new section.

Let an access zone be a subsection at the end of a highway section immediately before a motion adjuster. The access zone length L^* is the slot length, i.e., $1.67l$

meters. It can contain at most $n^* = p$ objects, one object in each lane. The objects in the access zone are separated from the nearest adjuster by the distance L^*. Small values of L^* significantly accelerate determining the coordinates of the objects entering the next section.

We attribute an access zone to the current and the next highway sections simultaneously. Before entering an access zone, an object performs measurements using the nearest adjuster of the current section. An object in the access zone gets the right to enter the first slot of the next section, entering it similarly to the actions of all objects of this section to move to the next slot. That is, the number of section slots is increased by the number of access zone slots.

Objects may use sequential numbers or slot numbers on the object maps to enter an access zone. Then they apply processes 1 and 2. As a result, they receive entry numbers on the next slot starting with one; see subsection 2.1. Their subsequent motion will be tracked using the methods of Section 2 or 3.

The necessity to order object actions also arises when entering the highway from the outside. Thus, we have determined the coordinates of objects using license plates or maps available to the objects.

4. ORGANIZATION OF THE PROCESS *SP*

In the process *SP*, synchronization is accelerated by making the time T_3 independent of the number n of objects. As a result, the time required in the process SP_0 to measure the signal transfer time to the repeater for one object becomes enough to determine such times for all objects. During this time, all objects move to a smaller distance. The synchronization accuracy accelerates the objects control process in the mobile computer cluster mode. Another necessary condition for this mode is satisfied. In the distributed system, the messages of a group of source objects must be synchronized to arrive at a group of receivers in a given order (for example, simultaneously). The way to do this is to replace the source group with a single source, which forwards the source group messages to receivers without delay. As such a source, here we use a unified repeater and signal processor [4]. Then, the sources located at different distances from the repeater consider this difference, delaying the transfer of messages to the repeater. Upon receiving the messages of objects synchronized via delays, the repeater sends them without delay to all objects simultaneously. Synchronization is achieved. There may be several repeaters. In this case, the general task of motion control can be divided into interacting but asynchronously executed subtasks. Let us describe the process *SP* in detail.

Suppose that a repeater is a stationary active motion adjuster containing signal receivers and sources. We denote it by *CR*. Also, assume that the process SP_0 was executed: the objects determined the distance to the adjuster, and a separate communication channel, different from the one occupied by the process SP_0, was allocated for the actions of the process *SP*.

In addition, each object has the current distances between all objects and the adjuster on its electronic map. Suppose that the process SP_0 was executed, and the signal transmission times from the objects to the adjuster *CR* are known.

We introduce the concept of a logical scale as a bit sequence *LS* where each bit is allocated to a separate map slot.

In the synchronization process *SP*, the scale *LS* is sent using all objects on the section, e.g., in the following way. The objects send to the adjuster *CR* the signal *S* discussed above; the adjuster *CR* returns it to the objects at another frequency in the form of a new signal *Scr*. The object detecting Scr^* (the instant of completing the signal *Scr*) sends a delayed pulse signal *s* to arrive at the adjuster *CR* in the middle of its logical scale bit. Free space is left in the bit on both sides of the signal *s*.

An object sends a signal to the adjuster *CR* with the delay $D = T_1 - T_i$. Here T_1 is the signal transfer time within the highway section, and T_i is the signal transfer time between object *i* and the adjuster *CR*. With such a delay, the same-named bits of the scales of all objects will arrive at the adjuster *CR* at the specified instants, forming a common scale.

CR retransmits the scale *LS* to all objects in a similar scale LS^* in which the signals *s* are replaced by the signals s^* of a different frequency. Due to the motion of objects, the bit position of signal s^* shifts, and the object corrects the distance to the adjuster *CR*.

Let us estimate the influence of motion on the accuracy of time measurements by objects. The time Δt occupied by a scale bit completely depends on the speed of objects. The motion of objects must not lead to the transfer of the signal s^* from the bit belonging to a particular object to the neighbor bit belonging to another object. When sending the signal to the adjuster *CR* and receiving the response signal s^*, the object can move by $\Delta s = 2vL/c$ meters. Due to the distance change, the signal s^* in the scale will shift by the time $\Delta t = 2vL/c^2$ seconds, and the duration of the scale bit must not be less than this value. For the values *L* and *c* in the examples above, we have $\Delta t = 0.056$ ps. Thus, scale transmission can be performed at very high speeds. All *n* bits of the scale will be transmitted in 8.4 ps; the scale can be transmitted even 396 times while an object exchanges a single pair of s/s^* with the adjuster *CR*. The dependence of the distance measurement time on the number n of objects is preserved but decreased many times, becoming insignificant for the process *SP*.

Up to this point, we have adopted the active stationary motion adjuster *CR* in the process *SP*. Let us combine the passive stationary adjuster with the mobile one (*mCR*). Assume that an object was allocated to act as *mCR* before the process *SP* starts. The object *mCR* determines its current coordinates by measuring the distance to the adjuster *CR*. The coordinates of *mCR* become known to all objects. Then they perform the *SP* actions by exchanging signals with it instead of the stationary adjuster.

This section completes the presentation of synchronization methods for the interaction of objects on a highway. The following sections will discuss the application of the process *SP* and the effect of external interference on the processes SP_0 and *SP*.

Concluding the description of synchronization processes, we note two well-known approaches to develop technical means for implementing the processes considered in Sections 1–4.

A more complex version of synchronization than estimating an individual signal shift can be performed by sending special synchronization messages. For example, very accurate stationary measurements were implemented in the White Rabbit project for physics experiments at CERN [8–10]. Here, the nonius range measurement method was applied, replacing a separate signal with a more complex message. For fiber-optic communication lines, the synchronization accuracy of message transmission over the line is better than 100 ps over a distance of more than 16 km when the temperature varies within the range 12.5–85.0ºC. Special synchronization correction devices are built into the network hardware.

Active optical phased arrays with high-speed beam travel [11] are gradually replacing mechanized object detection. With such an approach, the object determines the position of the adjuster quickly, uses the energy of the object signals more efficiently, and reduces the interference caused by the reflection of the object signal from external objects.

5. APPLICATION OF THE PROCESS *SP* BY OBJECTS

The solutions outlined in this section are important for managing the behavior of mobile objects equipped with increasingly powerful computers. As a result, an object can solve increasingly complex control tasks considering the current behavior of all objects. However, the objects are connected through a very loaded resource (shared communication channel). Methods are required for objects to execute distributed computational and control operations with low channel load. Such methods will accelerate the operations; see their description below.

This section does not discriminate between the adjusters *CR* and *mCR*. Therefore, we introduce the generalized abbreviation *aCR* (*active CR*) for them.

5.1 Management of synchronous messaging by objects

Assume that an object sends a command via the adjuster *aCR* to all objects: permits the synchronous transmission of their messages. Upon receiving the command, the objects willing to transmit a message respond by sending a logical scale to the adjuster *aCR*, placing “1” in their digit of the scale (the signal f_1). The scales of all objects should come to the adjuster *aCR* with combining the same-name bits. For this purpose, each object O_i sends its scale with the delay $D_i = L_{max} - L_i$, where L_i is its distance to the adjuster *aCR*, and L_{max} denotes the highway length. The resulting scale comes to the adjuster *aCR*, and it sends it without delay to all objects, replacing the signals f_1 with the signals f_1^*. Upon receiving all scale bits from the adjuster *aCR*, each object O_i sends its message to the adjuster *aCR* with the delay D_i. The adjuster *aCR* uses radio signals to send the received total message to all objects. Objects may need to transmit messages one after another. In this case, it suffices to transmit them considering the messages transmitted by the preceding objects. Therefore, we have a time gain: the objects’ messages are transmitted as a single message at a rate depending only on the selected transmission frequency (and not on the distance between the objects and the adjuster *aCR*).

For the distributed computations shown below, the objects’ messages must be transmitted with the simultaneous arrival of the same-name bits to the adjuster *aCR*.

5.2 Eliminating access conflicts to the adjuster *aCR*

In subsection 5.1, the objects are synchronized by a command of one object. However, self-synchronization of message sources is also necessary. Assume that the objects determined the distance to the adjuster *aCR*. Suppose that there are currently no signals, and the objects can start message transmission without a special command. Then the objects start transmitting messages to the adjuster *aCR* and receive the transmission result from it. If a conflict occurs, then at least one bit of the message will simultaneously have the signals f_1* and f_0* created in the adjuster *aCR* from the signals f_1 (“1”) and f_0 (“0”) sent by the objects. The appearance of these signals is perceived as a synchronization start command, and the synchronization process described in subsection 5.1 is executed. The access conflict is therefore eliminated.

5.3 Simultaneous data acquisition on the state of all objects

The paper [4] considered distributed computational processes in which computations are performed directly in message transmission without delay. Their applicability to motion-tracking tasks requires a more detailed study of the needs in particular situations. Therefore, we will present only general (typical) examples.

Estimating the state of all parameters for all objects (bitwise logical AND and OR operations). Let the state of each object be described by the same set of quantitative parameters. (For example, a group of parameters estimating the conditions of the vehicle engine, brakes, stability, etc.) Objects transmit all parameters in the form of a logical scale: a binary sequence in which each parameter has a separate bit. If a parameter value meets the specified requirements, the object bit is assigned "1" by transmitting the frequency signal f_1. Otherwise, "0" is transmitted by the signal f_0. Objects transmit scales when they receive a corresponding command through the adjuster in the process *SP*. As a result, all scales of the objects arrive at the adjuster with combining the same-named bits. All objects receive the combined scale. If the command was to perform bitwise logical AND over the scales, then the presence of the signals f_1 and f_0 (or f_0 only) in the combined scale bit is considered "0"; otherwise, "1." If the command was to perform bitwise logical OR over the scales, then the presence of the signals f_1 and f_0 (or f_1 only) in the combined scale bit is considered "1"; otherwise, "0." Thus, the simple synchronous retranslation of scales performs without delay the specified calculations in a time independent of the number of participating objects.

Finding the maximum (minimum). Each object has measured values of all parameters mentioned above. For all objects, it is required to find the maximum (minimum) value of each parameter.

For this purpose, objects send a sequence of message groups simultaneously. The first group arrives at the adjuster *aCR* as a single message combining the same-name bits. Such a message transmits the highest digit of the first parameter. We represent this digit as a binary scale with the number of digits equal to the base of the numerical system selected for the parameter values. Only one bit of the scale corresponding to the digit value equals "1"; the rest are "0." For example, the scale for decimal digit 6 is 000100000. The following actions are performed to find the maximum (minimum) value of each parameter.

To find the maximum, the objects receive the digit from the adjuster *aCR* and check whether they sent the highest value of the highest digit or not. The objects that did not send the highest value will not participate in the check for the parameter. The others send a similar group of messages, but for the next digit of the parameter value. The process continues until all digits of all parameters are completed checked. As a result, the objects with the maximum value of each parameter will be identified. The minimum is found by analogy.

Note that the maximum (minimum) is determined using the same logic as the minimum sequential number in process 1; see subsection 2.1. However, this operation is significantly accelerated due to a special representation of digits (reducing the number of data exchange operations) and the process *SP*.

Analog-to-digital summation. For estimating the state of the entire system of objects, it is desirable to sum up the group of numbers sent by the objects directly in the adjuster *aCR*. Such operations were described in the paper [4]. They are organized as follows. An analog-to-digital converter (ADC) is added to the adjuster *aCR*. The digits of all summands are represented by scales, like finding the maximum (minimum).

Let us illustrate this operation on an example of summing up decimal digits simultaneously sent by a group of objects to the adjuster *aCR*. Let the 001(4)01(6)0001(7) scale be formed by superposing several digits (scales). The energy level of the signal coming to the ADC is given in parentheses for digits "7," "5," and "1," respectively. In this case, the adjuster *aCR* distributes the following result to all objects: four "7"s, six "5"s, and seven "1"s are transmitted. Each object independently collects the sum: $4 \times 7 + 6 \times 5 + 7 = 65$. For multi-digit numbers, all digits are similarly processed, and a total sum is formed. For subtraction, two sums are formed, and subtraction is performed.

Histograms estimating the state of numerous system parameters are created based on such operations. ADC operations require stable sources of optical signals. Such sources do exist. The paper [12] presented a simple LED source with output power variation below 50 ppm/ºC. The indicated operations were also performed in a time independent of the number of participants.

These operations are examples of an associative operation in which all objects participate simultaneously. The operation results are simultaneously supplied to all objects, allowing them to perform further actions considering the received data about the system state.

The examples in this section show that the objects operate as a single mobile computer cluster. It can be partitioned into small and asynchronously interacting clusters by applying the solution [13] (with an appropriate modification considering the specifics of mobile systems).

6. THE INFLUENCE OF ENVIRONMENT ON THE PROCESSES SP_0 AND SP

Changes in the environment's state, e.g., temporary deterioration of the quality of transmitted signals or the appearance of interference, affect the execution of the processes SP_0 and SP. The solutions considered in this paper are based on the results experimentally validated and applied in several fields. Therefore, to assess the implementability of the proposed approach, we address relevant publications.

The implementability of the passive adjuster. Road signs on highways reflect the vehicle signal and are clearly identified in the dark among reflections from other objects. The explanation is that road signs contain many retroreflectors. This use of retroreflectors is thoroughly studied and defined by technical requirements and standards. A passive adjuster is only required to be clearly distinguished by reflection from interference reflections. Hence, its application does not differ from the above, and it can be implemented as well.

The implementability of the active adjuster. This task is simpler. The signals coming into the adjuster and the signals returned by it are qualitatively different. Interference from them travels a longer path, lags behind the useful signal, and does not interfere with measuring the object's distance to the adjuster.

Message transmission by the active adjuster using the process *SP*. The papers [14, 15] demonstrated an interaction scheme of the optical message source and receiver as follows. The receiver sends a continuous optical signal, which is received by the retroreflector of the message source and returned by it to the receiver. There is a signal modulator on the path of the returned signal, which translates the returned continuous signal into a message. The system's operability was tested in harsh conditions: 7 km-distance between the source and receiver, sea environment, fog, and heaving. The data transfer rate was 40 Mbit/s. Here the message transmission structure is similar to that of the active adjuster. However, the latter device is in better conditions using its signal source. This example shows the implementability of message transmission on a highway section.

We give another example. In the paper [16], a source of optical non-directional digital signals at short distances (a few meters) was considered, and a transfer rate of 400 Gb/s was obtained for it. The communication structure presented therein is as follows. There is a non-directional LED source in a room, modulated by electrical signals with a frequency of 400 Gb/s. The source sends a message to a group of receivers connected to the final recipients (computers). Delayed reflections from external objects do not violate the operation of this system. Such a source is also close to the active adjuster.

Thus, we have provided examples of optical communication useful for creating active optical adjusters. Radio means are more widespread and therefore not considered here.

Interference from signals generated in neighbor highway sections. Usually, such interference is eliminated by distributing frequencies of transmitted signals: neighbors use different frequencies. As applied to highways, this approach is implemented in the following way. The k neighbor sections of the highway use different frequencies of optical and radio signals. Then the order is repeated, and each highway section will be separated from the interfering $(k - 1)$ sections. The object gets information about the chosen frequencies from its electronic map or by polling the passive adjusters. In this case, the adjuster's passive retroreflector must have additional light filters. For example, a combination of three light filters opened in different combinations corresponds to $k = 7$.

Individual correction of object signal level. Like measuring the object's distance to the adjuster, we can control the level of the object signal coming to the adjuster. With the passive adjuster, the object performs this measurement, estimating the level of the signal returned to it. Assume that the conditions for the passage of the forward and reverse signals are the same. The active adjuster sends its signal measured by the object. Alternatively, the adjuster contains an ADC to estimate the level of the signal entering the adjuster. This method corrects the influence of the environment on the system operation.

CONCLUSIONS

The autonomous collective motion adjustment methods for vehicles on a highway proposed in this paper provide new capabilities. Let us emphasize them.

The methods complement the three most commonly used approaches: radar and lidar, vision aids, and satellite navigation aids installed on the vehicle (object). The first and second approaches are proximity methods estimating the mutual location of objects only within the line of sight. However, they measure the distance with high accuracy. The third approach determines the location of vehicles on a large highway section without requiring the direct line of sight. Unfortunately, it has lower accuracy than the previous ones and is not autonomous (depends on external navigation aids). The solutions proposed in this paper autonomously determine the location of objects on a highway beyond the line of sight and have an accuracy characteristic of the first two approaches.

Moreover, the proposed methods have another capability distinguishing them from the three approaches. With simple, computer-free means of communication, objects' computers perform several *distributed* calculations (important for objects' state estimation) directly in the network. These operations are executed during the data exchange between objects, causing no additional delays. Their duration does not depend on the number of objects participating in the operation. The precise synchronization of joint operations allows combining the same-name bits of messages for a group of objects. As a result, a group of messages is replaced by a single message without increasing the number of bits.

Hopefully, the proposed solutions will be useful in the ever-expanding market of fully autonomous vehicles, complementing the known results.

REFERENCES

1. Yurtsever, E., Lambert, J., Carballo, A., and Takeda, K., A Survey of Autonomous Driving: Common Practices and Emerging Technologies, *IEEE Access*., 2020, vol. 8, pp. 58443–58469. DOI:10.1109/ACCESS.2020.2983149.
2. Badue, C., Guidolini, R., Carneiro, R.V., et al., Self-Driving Cars: A Survey, 2019. https://arxiv.org/pdf/1901.04407.pdf.
3. Stetsyura, G.G., Decentralized Autonomic Synchronization of Interaction Processes of Mobile Objects, *Control Sciences*, 2020, no. 6, pp. 47–56. DOI: http://doi.org/10.25728/pu.2020.6.5. (In Russian.)
4. Stetsyura, G.G., Network Information-Computing Support of Automatic Mobile Objects Interaction, *Automation and Remote Control*, 2019, vol. 80, no. 6, pp. 1134–1147. DOI: 10.1134/S0005117919060110.
5. *GOST* (State Standard) R 60.3.3.2-2020 (ISO/TR 13309:1995): *Robots and Robotic Devices. Industrial Manipulation Robots. Methods and Means for Estimating the Performance of Robots*. (In Russian.)
6. Katyuzhanskii, G.A., Nisnevich, L.B., and Stetsyura, G.G., Decentralized Priority Control in a One-Channel Data Exchange System, *Izv. Akad. Nauk SSSR. Tekh. Kibern.* , 1970, no. 6, pp. 115–119. (In Russian.)
7. Prangishvili, I. V., Podlazov, V. S., and Stetsyura, G. G., Local Microprocessor Computer Networks (*Lokal'nye mikroprotsessornye vychislitel'nye seti)*, Moscow: Nauka, 1984. (In Russian.)
8. Moreira, P. and Darwazeh, I., Digital Femtosecond Time Difference Circuit for CERN's Timing System, *Proceedings of the London Communications Symposium*, 2011. http://www.ee.ucl.ac.uk/lcs/previous/LCS2011/LCS1136.pdf.
9. Yu, Q., Liangfu, P., and Yongqing, H., An Ultrahigh Precision Network Time Synchronization Technology, *Frontiers in Signal Processing*, 2019, vol. 3, no. 4, pp. 82–92. https://dx.doi.org/10.22606/fsp.2019.34004.
10. Tancock, S., Arabul, E., and Dahnoun, N., A Review of New Time-to-Digital Conversion Techniques, *IEEE Transactions on Instrumentation and Measurement*, 2019, vol. 68, no. 10, pp. 3406–3417. https://doi.org/10.1109/TIM.2019.2936717.
11. Sun, J, Timurdogan, E., Yaacobi, A., et al., Large-Scale Nanophotonic Phased Array, *Nature*, 2013, vol. 493, pp.195–199. DOI:10.1038/nature11727.
12. Bosiljevac, M., Babić, D., and Sipus, Z., Temperature-Stable LED-Based Light Source without Temperature Control, *Proceedings of SPIE OPTO*, San Francisco, CA, USA, 2016, vol. 9754, pp. 1–6. DOI: 10.1117/12.2211576.
13. Stetsyura, G.G., The Computer Clusters with Fast Synchronization of Messages and with Fast Distributed Computing by the Network Hardware, *Control Sciences*, 2020, no. 4, pp. 61-69. DOI: http://doi.org/10.25728/pu.2020.4.7. (In Russian.)
14. Rabinovich, W.S., Goetz, P.G., Mahon, R., et al., 45 Mbps Cat's Eye Modulating Retroreflectors, Optical Engineering, 2007, vol. 46, no. 10, p. 104001. https://doi.org/10.1117/1.2789634.
15. Rabinovich, W., Optical Modulating Retro-Reflectors, in *Advanced Optical Wireless Communication Systems*, Cambridge University Press, 2012, pp. 328–350.
16. Gomez, A., Shi, K., Quintana, C., et al., Design and Demonstration of a 400 Gb/s Indoor Optical Wireless Communications Link, *Journal of Lightwave Technology*, 2016, vol. 34, no. 22, pp. 5332–5339.

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VULNERABILITY ANALYSIS OF COMPLEX NETWORK INFRASTRUCTURES USING A GENETIC ALGORITHM

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Abstract. This paper proposes a method for analyzing the vulnerability of network infrastructures. The method uses a genetic algorithm for finding cross-sections that block delivering resources from their sources to consumers. The well-known approaches to solving network problems based on combinatorial and evolutionary approaches are considered. A feature of the proposed method is the fitness function chosen as an algorithm for calculating the number of paths in the graph when isolating the graph vertices that make up the individual. The graph reachability matrix and simple mathematical operations are adopted to optimize the fitness function and calculate the number of paths. The efficiency of the genetic algorithm compared to combinatorial methods is shown: multiple failures are found significantly faster than using exhaustive search algorithms.

Keywords: safety, engineering networks, vulnerability, reliability, combinatorial algorithms, models for damage analysis, crucial elements of an engineering network, genetic algorithms.

INTRODUCTION

The object of this research is a network infrastructure, i.e., a complex technical system (electricity, gas, etc.) [1].

Vulnerability is an internal property that makes the object susceptible to the impact of a risk source that can lead to some consequence [2]. The presence of vulnerability contributes to the realization of a threat. For technical systems under consideration, threats are destructive effects such as natural disasters, terrorist attacks, technical failures of system components, etc. The negative consequences of realizing a threat include, first of all, the lost operability of large consumers (enterprises, research centers, etc.) due to interrupting the supply of any resource. This understanding of technical system vulnerability corresponds to the definition of supply system reliability, which is directly related to the continuous supply of resources to consumers.

As an internal property, this paper considers the object's structure. It includes sets of nodes (vertices, elements), further referred to as key ones, destructive impacts on which disrupt the continuous supply of resources to consumers.

Various methods and models are used to find key nodes. One approach is the mathematical modeling of physical processes in resource supply networks (electricity, gas, heat, water, and others). The disadvantages of this approach include the need to use information about the parameters and modes of the system's operation (which should be introduced into the model), the random occurrence of emergencies, and slow calculations of the system of high-order algebraic and differential equations for complex objects.

Topological analysis methods are preferable for systems represented by a graph [3–6]. This approach is convenient to implement since only the description of the graph structure (vertices and connections between them) is required. The disadvantages of the topological model are redundant solutions due to neglecting the system's modes and parameters. These disadvantages are partially eliminated when passing to weighted graphs, in which the edges and vertices are assigned network characteristics such as capacity, power, etc.

Topological methods yield acceptable results for small-dimension networks (tens of vertices). However, the computation time grows significantly with increasing the network scale. For example, during the exhaustive search of multiple failures, the number of possible alternatives is determined by the number of k-combinations of n elements. (Here, k is the number of damaged elements, and n denotes the total number of system elements.) As a result, the computation time has exponential growth. The well-known methods for constructing minimal cross-sections, such as Petrick's method or the method of disjoint sets [7, 8], do not eliminate this drawback.

The genetic algorithm reduces computational problems when searching key elements. It finds the minimum (or maximum) of a certain function (fitness function) characterizing the infrastructure state. The main advantages of the algorithm include the following:

– The fitness function can be represented by an algorithm.

– The implementation is simple.

– The discontinuities in the fitness function do not affect the solution search.

The main drawback of genetic algorithms is associated with the uncertainty of finding the global optimum. However, when searching key elements using the method proposed below, this disadvantage can be overcome.

The evolutionary approach has various applications in network analysis. For example, a genetic algorithm was used in [9, 10] to assess the vulnerability of a power system (and the system operator's response) when disconnecting its elements.

In the report [11], a genetic algorithm was used to analyze the vulnerability of electrical networks for two optimization levels. When optimizing the upper level, the maximum damage to the power system was determined in terms of the load-off. When optimizing the lower level, this damage was minimized by choosing the optimal operating mode of the power system. A peculiarity of the model [11] is that the system operator can change the network topology among various corrections available.

The paper [12] considered optimal solutions for the maintenance of infrastructure objects. Optimal solutions were those minimizing the network life cycle cost under the reliability and functionality requirements. A Markov chain model was used to predict the efficiency of infrastructure objects.

The report [13] presented an approach to finding the best ways of protecting infrastructure assets (adding or changing infrastructure in response to an emergency, etc.) for complex national and international network structures such as transport, telecommunications, finance, energy, etc. Also, their interconnections were investigated.

The genetic algorithm proposed in the paper [14] allows organizing the joint work of the system operator and the power system. The algorithm calculates control corrections to minimize the power system load-off. As a result, the network elements with the most severe consequences in case of failures were identified.

Potential vulnerabilities in a power system [15] can be identified by determining the power transmission lines causing maximum network disruption in case of failures. The AC power equations were adopted in the network infrastructure model. The failures were initiated by increasing the resistance of the transmission lines. As a result, the authors identified those transmission lines for which minor conductivity disturbances lead to serious network disruptions, voltage drops, and disconnection of consumers.

The genetic algorithm was applied to find vulnerable sections of power transmission lines [16]. Vulnerability assessment of lines allows identifying problematic areas of such infrastructure by modeling and, second, assessing the possibility of cascade failures.

The paper [17] proposed two approaches based on genetic algorithms to improve the system voltage stability under various operating conditions. Within the first approach, a correction is used to optimize the voltage stability index during abnormal control. The second approach involves finding an optimal arrangement of compensators and generator control to minimize the voltage stability index.

An optimal location of energy storage systems to reduce the power system's vulnerability was considered in [18]. The authors analyzed the impacts on busbars and searched the most vulnerable ones. An optimal location was chosen using a genetic algorithm.

The paper [19] was devoted to genetic algorithm-based optimal solutions for protecting and restoring infrastructure in case of accidents or disasters and identifying the assets necessary to maintain the network's operating mode.

Genetic algorithms are often used to tune neural networks [20–22].

The publications cited differ in the field of research, particular problems solved, and the fitness function chosen. Below, we consider the problem of finding key elements in engineering networks based on a specially constructed fitness function.

1. PROBLEM STATEMENT

The technical systems under consideration have a network organization that can be represented by a non-directed or partially directed graph $G = \{V, R\}$, where V and R denote the sets of vertices and edges, respectively [23].

The set of vertices V is described by a triple $\{S, C, U\}$, where S and C are the sets of power sources and consumers, respectively, and U denotes the set of network vertices in which power transformation (transformative stations), power distribution (power distribution plants, taps), and power transmission (power lines) are implemented:

$$V = S \cup C \cup U.$$

Also, we denote by C_z a given subset of the most important consumer-vertices and by V_α the subset of key vertices to be found.

The problem statement is as follows: for the graph $G = \{V, R\}$, it is required to determine the minimum subset of key vertices $V_\alpha \in U$ such that their removal from the graph G will violate the reachability of all vertices C_z from the vertices S.

Genetic algorithms involve such concepts as individuals, populations, chromosomes, and fitness functions. For the problem under consideration, these concepts are defined below.

An individual $\theta_d = \{V_\alpha\}$ is a set of key vertices V_α whose failure will disconnect important consumers. The subscript d is the individual's number in a population, and α is the number of graph vertices of an individual. The number of graph vertices contained in one individual, α, is advisable to choose according to the number of simultaneously occurring failures (single, double, triple, etc.) assessed in terms of their impact on the system: $V_\alpha \in U$.

The population is $P = (\theta_1, \theta_{2,\dots,}\theta_k)$, where k denotes the size of a subset from the set of individuals used to select the best ones. The value k is fixed and related to the graph dimension n (the total number of graph vertices). Usually, the number k is chosen in the percentage of the graph dimension n, ranging from 5% to 15% (k/n = 0.05–0.15). For high-dimensional problems, the range of k can be smaller, e.g., k/n = 0.03–0.10.

A chromosome is a numerical vector (or string) representing a particular individual as a binary string of bits (genomes). For instance, the chromosome of the individual $\theta_5 = (v_3, v_2, v_{15}, v_{50})$, containing four vertices (a quadruple failure), is shown in Fig. 1.

θ_5			
000011	000010	001111	110010
v_3	v_2	v_{15}	v_{50}

Fig. 1. An individual represented by a chromosome.

The fitness function $F(\theta_d)$ allows finding an individual with the greatest effect on the total number of routes for supplying resources to consumers. We estimate this value algorithmically [24], counting the number of limited-length paths (3–5 steps) between the graph vertices after isolating the elements that make up the individual. We calculate the number of paths by raising the adjacency matrix to a power. When raising an adjacency matrix E to the power m by ordinary arithmetic operations (instead of Boolean algebra rules), its element $e_{i,j}$ will equal the number of paths of length m from vertex i to vertex j; see [25].

A small number of steps m (hence, a small exponent) is dictated by the need to reduce the algorithm's running time when raising the matrix to a power. We make the following assumption: if isolating the vertices that make up an individual reduces the total number of paths in the graph for small exponents m, it will also decrease the number of paths for supplying resources from sources to consumers. The availability of resource supply paths is tested in three stages:

1. A genetic algorithm finds an individual (a set of vertices) whose isolation will reduce to the greatest extent the total number of paths of length m in the graph.
2. The availability of paths for supplying resources from sources to consumers is tested.
3. If there are no paths from sources to consumers, the vertices are key, and the calculations finish; otherwise, the calculations are repeated.

The flow chart of the genetic algorithm is presented in Fig. 2.

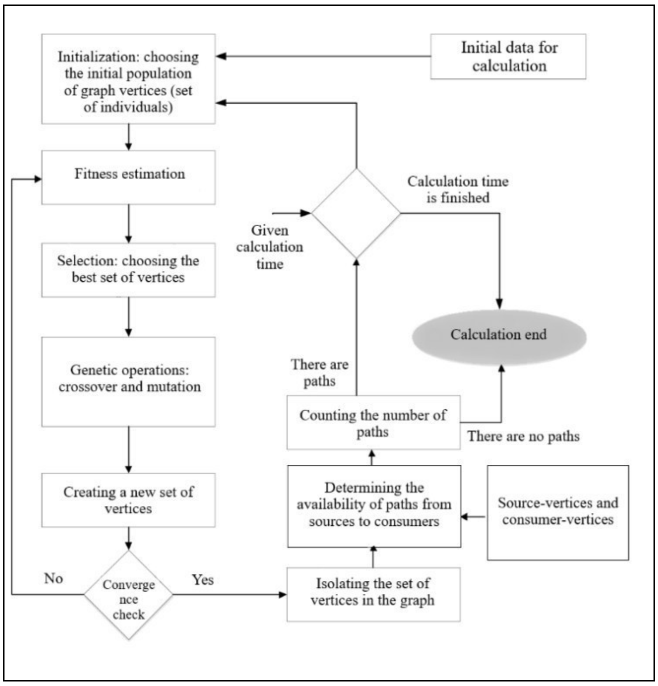

Fig. 2. Block-scheme of a genetic algorithm for finding network key vertices.

2. APPLICATION OF A GENETIC ALGORITHM

2.1. Preparing initial data for calculation

The initial information on the graphs is usually given by tables of paired relations or adjacency lists. The initial information is arranged by assigning serial numbers from 1 to n to all network vertices, $V = (v_1, v_2,\ldots, v_n)$. Then the adjacency matrix E of the graph is formed:

$$E = \left(e_{i,j}\right),$$

$$e_{i,j} = 1 \text{ if } \left(v_i, v_j\right) \in V,$$

$$e_{i,j} = 0 \text{ if } \left(v_i, v_j\right) \in \varnothing,$$

where the subscripts i and j correspond to the rows and columns of the adjacency matrix, respectively.

2.2. Initialization: choosing initial population

Populations have the following main features:

- The initial population is formed as a set of individuals with randomly chosen vertices.
- The individual's size θ is fixed. (The number of graph vertices α in an individual is constant.)
- The population size P remains invariable during the algorithm. (The number of individuals in the population $P = (\theta_1, \theta_{2,\ldots,}\theta_k)$ is constant.)
- Each individual $\theta_d = \{V_\alpha\}$ is initialized by the vertex serial numbers randomly chosen using a uniform distribution on the set of nodes V.

2.3. Fitness estimation

The network is designed to supply resources to consumers. When choosing an individual and assessing its impact on the infrastructure, it is therefore reasonable to calculate the number of graph paths after isolating certain individuals.

The total number of paths between the vertices $\left(v_i, v_j\right) \in V$ with a length not exceeding m is calculated from the reachability matrix $E^* = \left(e^*_{i,j}\right)$. (Below such paths will be called paths of length m.) This matrix is the sum of adjacency matrices E raised to powers from 1 to m:

$$E^* = E + E^2 + E^3 + \ldots + E^m.$$

We denote by S the total number of paths of length m in the graph:

$$S = \sum_{i=1}^{n}\sum_{j=1}^{n} e^*\left(i, j\right),$$

where n is the total number of graph vertices.

We test the susceptibility of the infrastructure to the impact of a particular individual θ_d by isolating its vertices. For this purpose, the elements corresponding to these vertices in the adjacency matrix are set to 0. Then a new reachability matrix is calculated, and a new number of graph paths is calculated:

$$\tilde{S} = \sum_{i=1}^{n}\sum_{j=1}^{n} e^*\left(i, j\right),$$

where $\tilde{S}$ denotes the number of paths of length m in the graph after isolating all vertices of the individual θ_d.

Dividing the number of all graph paths by the number of all graph vertices, we obtain the average number of paths per one vertex:

$$S_{\text{avg}} = \frac{\tilde{S}}{n}.$$

Then the fitness function can be written as

$$F\left(\theta_d\right) = \min\left(S_{\text{avg}}\right).$$

Therefore, using the fitness function, we determine an individual θ_d with the following property: the average number of paths S_{avg} achieves minimum after isolating its vertices.

The operation of this algorithm can be demonstrated by an example. Consider the graph G of a resource supply network containing ten vertices, two sources $\left(v_1\ v_2\right) \in G$, and one consumer v_{10} (Fig. 3). For testing the individual's impact on the number of paths, we isolated the vertices $\left(v_5\ v_8\right) \in G$. For this purpose, we set to 0 the corresponding rows and columns of the adjacency matrix (Table 1) and calculated the number of paths in the reachability matrix.

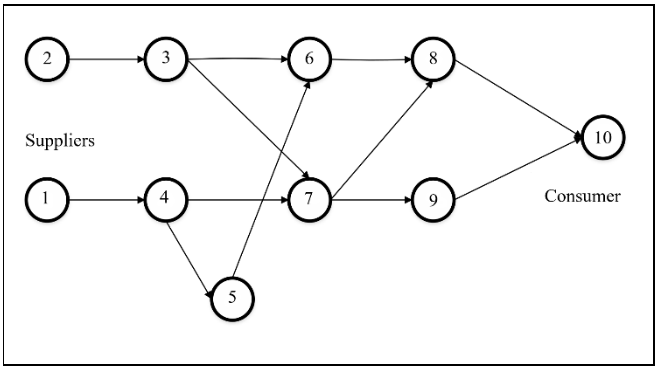

Fig. 3. Graph of a resource supply network.

The path length m was set to 3. Table 2 presents the total and average numbers of such paths (S and S_{avg}) in the original graph G and the modified graph after removing different pairs of vertices.

Clearly, the minimum average number of paths per vertex is obtained by isolating vertices v_6 and v_7. In this case, all paths between sources and the consumer are interrupted.

Table 1

Adjacency matrix after isolating vertices v_5 and v_8

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
v_1	0	0	0	1	0	0	0	0	0	0
v_2	0	0	1	0	0	0	0	0	0	0
v_3	0	0	0	0	0	1	1	0	0	0
v_4	0	0	0	0	0	0	1	0	0	0
v_5	0	0	0	0	0	0	0	0	0	0
v_6	0	0	0	0	0	1	0	0	0	0
v_7	0	0	0	0	0	0	0	0	1	0
v_8	0	0	0	0	0	0	0	0	0	0
v_9	0	0	0	0	0	0	0	0	0	1
v_{10}	0	0	0	0	0	0	0	0	0	0

Table 2

Number of paths of length 3 in graph G

Numbers of removed vertices	S	S_{avg}	Number of paths from sources to a consumer	Paths from sources to a consumer
–	45	4.5	6	1,4,7,9,10 1,4,7,8,10 1,4,5,6,8,10 2,3,6,8,10 2,3,7,8,10 2,3,7,9,10
5, 8	23	2.3	2	2,3,7,9,10 1,4,7,9,10
6, 7	11	1.1	0	–
3, 9	26	2.6	2	1,4,7,8,10 1,4,5,6,8,10

2.4. Selection, crossover, and mutation

The algorithm involves rank selection. With this method, after calculating the fitness values for crossover, $(l \times k)$ best individuals are selected, where l denotes the relative number of the best individuals in the population, and k is the population size. The parameter l describes the influence of selection on the survival of individuals in the population. In this paper, the value l ranges from 0.3 to 0.5.

Individuals selected with a given probability undergo single-point crossover (shuffling of binary strings). As a result, the offsprings receive half of the randomly determined characters from each parent. The offsprings form a new population of a given size k.

Mutation is necessary to prevent convergence to a local optimum. Since individuals are binary strings, mutation consists in inverting a randomly chosen gene for one randomly chosen individual. (Inverting means replacing 1 for 0 and vice versa.) The mutation frequency is set at the beginning of calculations, without any changes at subsequent stages.

For one generation, the search procedure stops according to the following criterion: the fitness function has the same value after selection and mutation. The resulting individual should be checked for the isolation of important consumer vertices from sources. For this purpose, the paths from sources to consumers are calculated using the Floyd–Warshall algorithm. If such paths exist, the genetic algorithm should be restarted to find a new solution for a new generation.

2.5. Application of genetic algorithm: example

The algorithm was tested on a graph of a real network segment; see Fig. 4.

The power system graph under consideration consists of 47 elements, including 15 sources and 5 consumers. We studied sextuple failures of elements.

Table 3 shows the cross-sections (sets of disconnected vertices) and the number of disconnected consumers. Clearly, the global optimum (disconnection of all consumers from the network) was achieved only in one case out of six. The time required to obtain one solution was approximately 3 s.

The sources and consumers were not analyzed (disconnected in the software implementation): such solutions are trivial and can be seen directly on the graph.

This approach can be extended to high-dimensional networks after representing in the form of graphs.

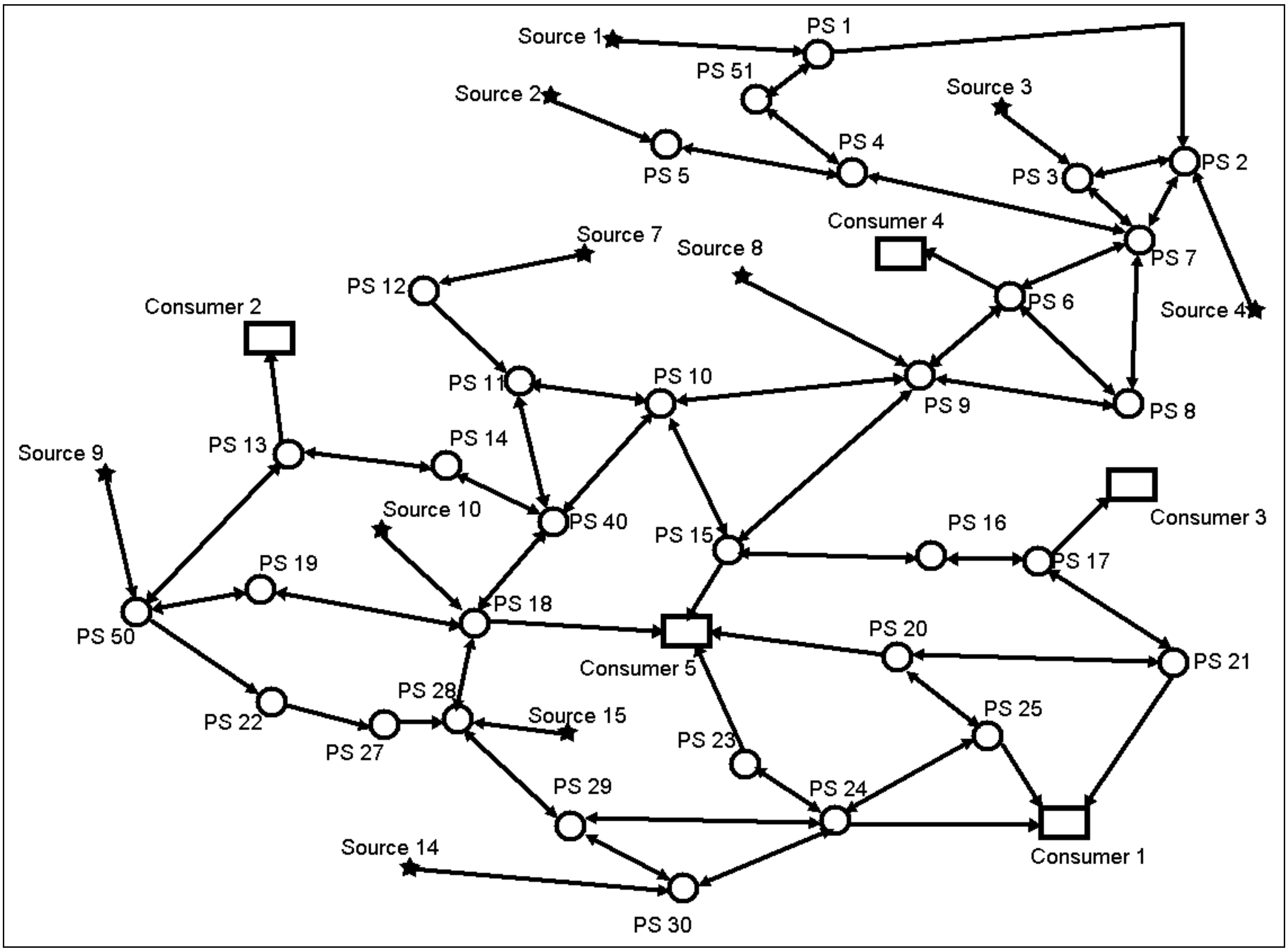

Fig. 4. Graph of a power system segment.

Table 3

Cross-sections in the graph of a power system segment: Calculation results

Disconnected vertices	PS 7, PS 10, PS 24, PS 18, PS 13, PS 9	PS 7, PS 15, PS 10, PS 13, PS 24, PS 18	PS 51, PS 22, PS 18, PS 10, PS 9, PS 24	PS 40, PS 15, PS 9, PS 7, PS 24, PS 13	PS 7, PS 9, PS 10, PS 18, PS 17, PS 24	PS 29, PS 9, PS 10, PS 18, PS 7, PS 24
Number of disconnected consumers	5	4	4	4	4	4

CONCLUSIONS

According to an analysis of publications on the subject, genetic algorithms are not applied to block the supply of resources to important consumers, despite their widespread use. At the same time, the disadvantages of traditional methods make it relevant to apply genetic algorithms for solving such problems.

The genetic algorithm proposed in this paper calculates the fitness function by estimating the average number of paths per one vertex after isolating individuals. Implementing the genetic algorithm to find cross-sections in infrastructures is not very difficult.

The proposed genetic algorithm needs significantly less computation time than the exhaustive search to determine multiple failures in high-dimensional networks.

REFERENCES

1. Bhamidipati, S., Van Der Lei, T.E., and Herder, P.M., A Layered Approach to Model Interconnected Infrastructure and Its Significance for Asset Management, *European Journal of*

Transport and Infrastructure Research, 2016, vol. 16, no. 1, pp. 254–272. DOI: 10.18757/Ejtir.2016.16.1.3130/
2. *GOST* (State Standard) R 53114-2008. *Information Protection. Ensuring Information Security in the Organization. Basic Terms and Definitions*, 2018.
3. Cagno, E., Grande, O., and Trucco, P., Towards an Integrated Vulnerability and Resilience Analysis for Underground Infrastructures, *Reliability Engineering & System Safety*, 2011, vol. 96, no. 1, pp. 139–148.
4. Valencia, V.V. and Maj, P.E., Network Interdependency Modeling for Risk Assessment on Built Infrastructure Systems, 2013. https://pdfs.semanticscholar.org/95ba/f36ae65157638a83f82084c39884b3f0fcb2.pdf?_ga=2.74081425.2003714931.1570034586-1895369083.1570034586.
5. Dorri, M.Kh., Roschin, A.A., and Sereda, L.A., Application of RDS Software Suite for Calculation and Visualization of Engineering Structures' Failure Consequences, *Automation in Industry*, 2017, no. 11, pp. 11–14. (In Russian.)
6. Grebenyuk, G.G. and Nikishov, S.M., Blocking of Energy and Resource Supply of Target Objects in Network Infrastructures, *Automation and Remote Control*, 2018, vol. 79, no. 3, pp. 535–544.
7. Ryabinin, I.A., *Nadezhnost' i bezopasnost' strukturno-slozhnykh sistem* (Reliability and Security of Structurally Complex Systems), St. Petersburg: St. Petersburg State University, 2007.
8. Haenni, R., Generating Diagnoses from Conflict Sets, *Proceedings of the 11th International FLAIRS Conference*, Sanibel Island, Florida, USA,1998. URL: www.aaai.org/Papers/FLAIRS/1998/FLAIRS98-081.pdf.
9. Arroyo, J.M. and Fernández, F.J., A Genetic Algorithm for Power System Vulnerability Analysis under Multiple Contingencies, in *Metaheuristics for Bi-level Optimization*, *Studies in Computational Intelligence*, 2013, vol. 482, pp. 41–68.
10. Arroyo, J.M. and Fernández, F.J., Application of a Genetic Algorithm to n-K Power System Security Assessment, *Electrical Power and Energy Systems*, 2013, vol. 49, pp. 114–121.
11. Genesi C., Granelli G., Marannino P., et al., A Genetic Continuation Power Flow Approach To Assess Power System Vulnerability, *Proceedings of the 43rd International Universities Power Engineering Conference*, Padua, Italy, 2008, pp. 1–5. https://ieeexplore.ieee.org/document/4651440.
12. Morcous, G. and Lounis, Z., Maintenance Optimization of Infrastructure Networks Using Genetic Algorithms, *Automation in Construction*, 2005, vol. 14, no. 1, pp. 129–142.
13. Permann, M.R., Toward Developing Genetic Algorithms to Aid in Critical Infrastructure Modeling, *Proceedings of 2007 IEEE Conference on Technologies for Homeland Security*, Woburn, MA, USA, 2007, pp. 192–197.
14. Agudelo, L., Munoz, N., and López-Lezama, J.M., Vulnerability Assessment of Power Systems to Intentional Attacks Using a Specialized Genetic Algorithm, *DYNA*, 2015, vol. 82, no. 192, pp. 78–84.
15. Kim, T., Wright, S.J., Bienstock, D., and Harnett, S., Vulnerability Analysis of Power Systems, *IEEE Transactions on Network Science and Engineering*, 2016, vol. 3, no. 3, pp. 132–146.
16. Karimi, E., Madani, S.M., and Ebrahimi, A., Power Transmission System Vulnerability Assessment Using Genetic Algorithm, *Intelligent Systems in Electrical Engineering*, 2012, vol. 3, no. 3, pp. 1–10.
17. Roselyna, J.P., Devarajb, D., and Dash, S.S. Multi-Objective Genetic Algorithm for Voltage Stability Enhancement Using Rescheduling and FACTS Devices, *Ain Shams Engineering Journal*, 2014, vol. 5, no. 3, pp. 789–801.
18. Teng, J.-H., Chen, C.-Y., and Martinez, I.C., Utilising Energy Storage Systems to Mitigate Power System Vulnerability, *IET Generation, Transmission and Distribution*, 2013, vol. 7, no. 7, pp. 790–798.
19. Permann, M.R., Genetic Algorithms for Agent-Based Infrastructure Interdependency Modeling and Analysis, *Proceedings of the 2007 Spring Simulation Multiconference* (*SpringSim* 2007), Norfolk, Virginia, USA, 2007, vol. 2, pp. 169–177.
20. URL: https://medium.com/swlh/genetic-algorithm-in-artificial-neural-network-5f5b9c9467d0.
21. URL: https://towardsdatascience.com/using-genetic-algorithms-to-train-neural-networks-b5ffe0d51321.
22. Anand, H. and Darayi, M., Power Network Component Vulnerability Analysis: A Machine Learning Approach, *Procedia Computer Science*, 2021, vol. 185, pp. 73–80.
23. Grebenuk, G.G., Dorri, M.Kh., Roshchin, A.A., and Nikishov, S.M., A Study of Influence of the Structure of Heterogeneous Engineering Networks on the Reliable Operation of Large Consumers, *Proceedings of the 11th IEEE International Conference on Application of Information and Communication Technologies* (*AICT*2017), Moscow: IEEE, 2017, vol. 2, pp. 57–61.
24. URL:https://progr-system.ru/wp-content/uploads/Math/МАПКС-15-ГрафыАтак.pdf.
25. URL:http://poivs.tsput.ru/ru/Math/DiscreteMath/GraphTheory/AccessibilityMatrix.

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20TH IFAC CONFERENCE ON TECHNOLOGY, CULTURE AND INTERNATIONAL STABILITY (TECIS'2021)

INTRODUCTION

The 20th International *Conference on Technology, Culture and International Stability* (TECIS'2021) was held on September 14–17, 2021, in the virtual format at Trapeznikov Institute of Control Sciences, Russian Academy of Sciences (ICS RAS, also referred to as the Institute). TECIS conferences are traditionally held under the auspices of the International Federation of Automatic Control (IFAC). The organizer of the anniversary 20th conference was ICS RAS, together with the National Committee on Automatic Control and IFAC Technical Committee (TC) 9.5. Technology, Culture and International Stability. Also, the conference was supported by IFAC TC 5.4. Large Scale Complex Systems and IFAC TC 9.1. Economic, Business and Financial Systems.

In his opening speech, *D.A. Novikov*, Director of ICS RAS, welcomed the participants of TECIS'2021. He reminded that the IFAC has been a scientific community in the field of control since 1957. In 1960, the first IFAC Congress was held in Moscow, particularly at the Institute. Nowadays, the IFAC is a worldwide federation of researchers and experts in the theory and practice of control from fifty countries from all continents.[1] The IFAC holds dozens of scientific conferences and symposia annually. Russia's scientific organizations resumed holding conferences under the auspices of the IFAC in 2009. Only five IFAC scientific events were held in the USSR and Russia, and four of them were organized by the Institute.

1. TRADITIONS AND FUNDAMENTAL PRINCIPLES OF TECIS CONFERENCES

TC 9.5 is one of the most important IFAC committees, traditionally dealing with global technological challenges to humanity that affect the technical, social, and political spheres. The forerunner of TC 9.5 was the IFAC Committee on "Supplemental Ways for Improving International Stability – SWIIS," founded in 1983 *by H. Chestnut* and *P. Kopacek*.[2] Its main direction was to study and develop the possibilities of applying control theory tools to improve international stability at the end of the 20th century. As a result, international mechanisms for managing dual-purpose technologies[3] were developed and implemented at the beginning of the 21 century.

Accelerating changes lead to new technological challenges with technical, social, and political implications. Consequently, the work of control and management professionals is changing accordingly. Who will conduct research and development in these spheres? According to The 2019 World Economic Forum in Davos, engineers are actively entering politics. As noted in a feature article in *The Guardian*, engineers are going into the core of politics... Is that not a way of adapting to rapid change?

Traditionally, the fundamental principle of SWIIS and TC 9.5 is *engineering outside the box*. Guided by this principle, TECIS conferences have been held for 25 years to discuss global aspects of technology development and its impact on technical, social, and political trends.

2. PAPERS, SESSIONS, AND DIRECTIONS

Initially, it was supposed to hold seventeen regular TECIS sessions. In addition, groups of researchers from different countries applied for seven invited sessions. The International Programme Committee (IPC) and the National Organizing Committee (NOC) supported six invited sessions for TECIS'2021.

A total of 177 papers were submitted to the conference. Their distribution is illustrated in Fig. 1. Note that 138 papers (78%) were accepted after peer review. The conference was attended by 262 researchers from 25 countries: Russia, Austria,

[1] https://www.ifac-control.org/about/overview-of-ifac

[2] https://tc.ifac-control.org/9/5

[3] Dual-purpose technologies can be used both for peaceful tasks and the creation of mass destruction weapons.

Bulgaria, Ireland, Colombia, Norway, Slovenia, the USA, and others. The participants represented all continents (Fig. 2). The proceedings of TECIS'2021 were published in *IFAC-PapersOnLine*, 2021, vol. 54, iss. 13, pp. 1–774, by Elsevier.[4]

The TECIS'2021 program included four plenary sessions, a panel discussion, seventeen regular sessions, and six invited sessions initiated by the speakers; see Sections 4 and 5 of this paper.

The main keywords of the conference papers are "intelligent systems and applications," "modelling and simulation," "control and automation for improved stability," "artificial intelligence," and others (Fig. 3).

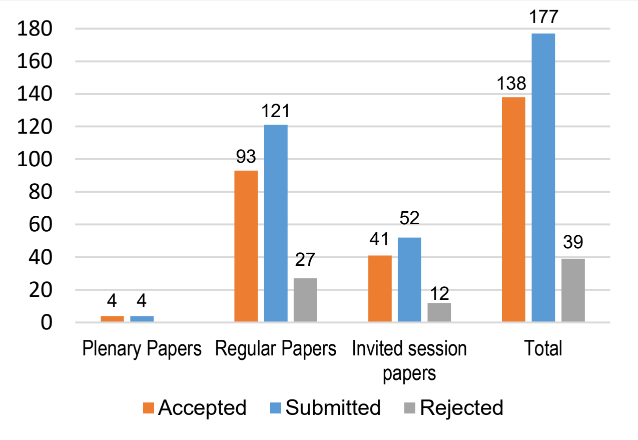

Fig. 1. Distribution of papers submitted to TECIS'2021.

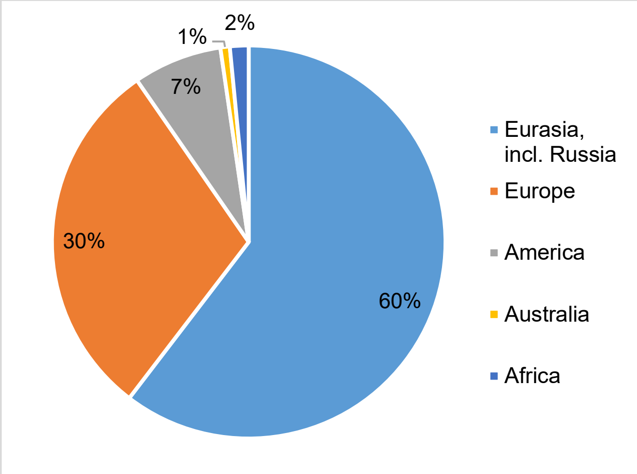

Fig. 2. Distribution of TECIS'2021 participants by continent.

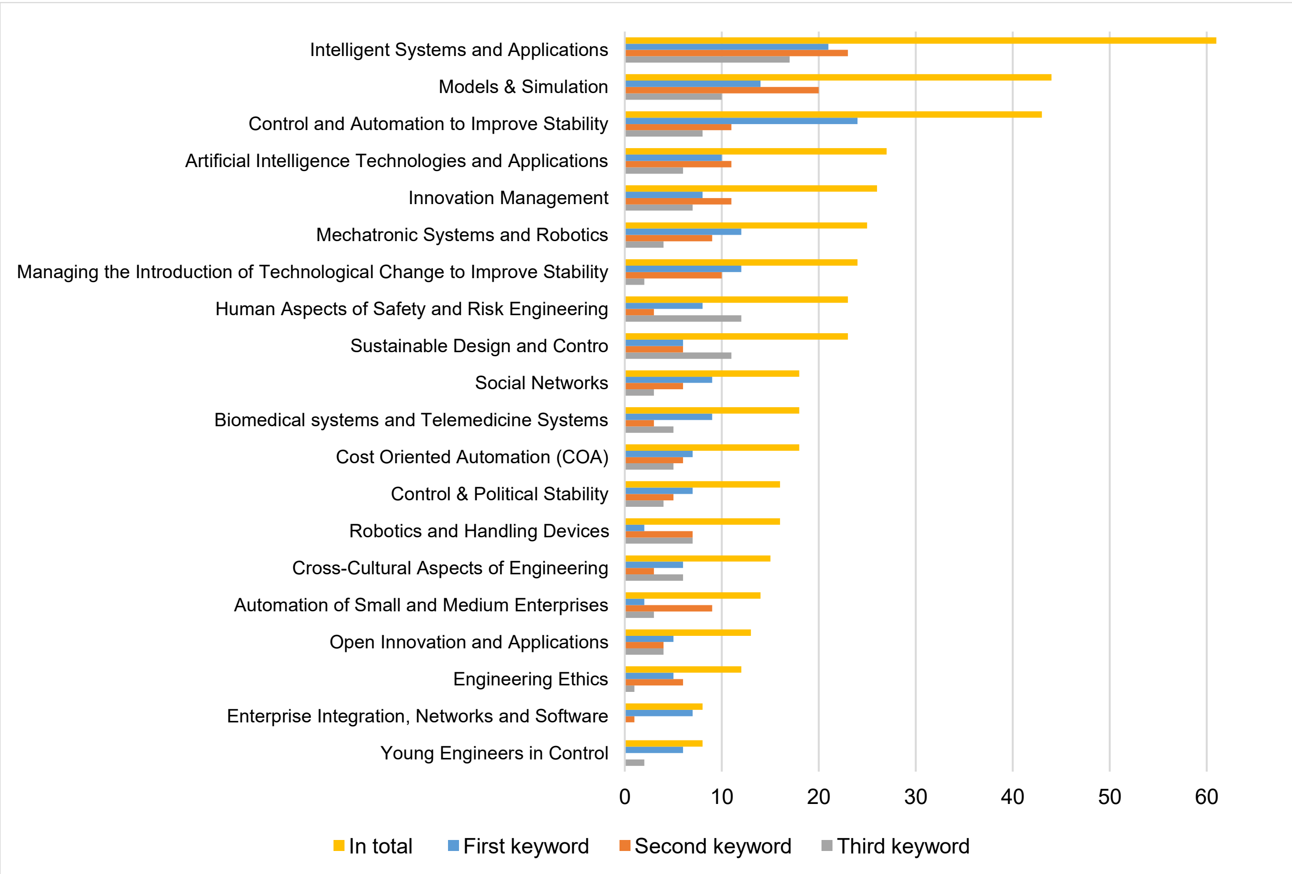

Fig. 3. Keywords in TECIS'2021 papers: distribution by occurrence.

[4]https://www.sciencedirect.com/journal/ifac-papersonline/vol/54/issue/13

3. PROGRAM AND MAIN DIRECTIONS OF TECIS'2021

Based on these quantitative data (Fig. 3) and the conceptual analysis of the papers, three main directions of TECIS'2021 can be identified:

- Control and Automation to Improve Stability;
- Intelligent Technologies and Their Applications to Manufacturing, Robotics, Mechatronics, Industrial, Medical, and Other Systems;
- Social Aspects of Automation.

4. PLENARY PAPERS AND PANEL DISCUSSION

The conference sessions were opened with "Selected Development Trends of TECIS," the plenary paper by *P. Kopacek* (TU Wien, Austria) and *M. Doyle-Kent* (Waterford Institute of Technology, Ireland). The authors noted that automation technologies and related information systems are important components of all kinds of objects, systems, and social relations of developed countries. Developing countries mainly need access to these technologies and systems to improve the living standards of their population. In response to these demands of the times, the IFAC is moving towards better management of the human-machine interface to enhance social and international stability.

The authors employed a socio-technical approach to creating Industry 4.0 technologies. Its essence is that new technologies should create favorable conditions for improving workers' level and quality of life. The authors highlighted the development trends of environmentally friendly technologies, mechatronic systems and robotics, smart factories, and products within cost oriented automation (COA). The main aspects of automation using robot systems were also considered. The ability of robots to learn will allow considering the characteristics of the environment and its living beings (inhabitants). In addition, robots have to assess their behavior through a reflexive process and learn their own experience, replicating the natural evolutionary processes of living beings' minds. The development of legislation, standards, and norms that adequately address the requirements of automation and robotics is crucial to the creation of such a technology. With increased technical complexity and competitive pressures, a new generation of managers is faced with the challenges of managing complex objects of an interdisciplinary nature. To manage effectively in a dynamic and often unstructured environment, managers must understand the technical, organizational, and behavioral aspects of these challenges.

P. Groumpos (The University of Patras, Greece) presented the paper "A Critical Historical and Scientific Overview of All Industrial Revolutions." He showed that the TECIS conferences are focused on the theories and technologies of control and automation to make the world a more stable place for all Earth inhabitants. The traditional vision of the community of scientists united around the TECIS conferences is a civilization in which technologies, systems, and processes serve all of humanity.

N. Jesse (QuinScape GmbH, Germany) presented the paper "Data Strategy and Data Trust – Drivers for Business Development." Insufficient data competence is one of the reasons why companies failed in the escalating process of creative destruction. The author addressed three competence dimensions: data architecture, data preparation and the interchange of data. The competence in these fields is a precondition for a company's survival and the ability to make profitable business decisions.

COVID-19 pandemic has created a difficult situation for the planet. This topic was considered by *F. Aleskerov* (ICS RAS) in the paper "COVID-19 – Spread of Morbidity, Quarantine Measures, Their Efficiency." The following panel discussion, "Covid, Technology and Ethics," was also attended by *M. Hersch* (The University of Glasgow, the UK), *D. Brandt* (Aachen University, Germany), *M. Doyle-Kent* (Waterford Institute of Technology, Ireland), *E. Bula* (The University of Business and Technology, Pristina, Kosovo), and other scientists. The participants discussed the issues of pandemic forecasting and the use of automation and robotics technologies to control the spread of diseases. The discussion showed that the pandemic had raised several ethical questions for individual citizens and society. Due to the pandemic, it was impossible to welcome TECIS'2021 participants from all continents in Moscow. The IFAC Conference App, a web application provided by the IFAC Secretariat, was widely used to create their virtual communication environment.

5. REGULAR AND INVITED SESSION PAPERS

The TECIS'2021 sessions dealt with a wide range of topical scientific and applied issues; see Fig. 3. It seems bold and reckless to describe all the scientific and applied results presented at TECIS'2021. Therefore, let us characterize the papers that generated the most interest among the participants, judging by expert appraisals and user activity in the IFAC Conference App. These papers are grouped below in 3 subsections corresponding to the TECIS'2021 main directions (Section 3 of this paper). The titles of the corresponding sessions are italicized.

5.1. Control and Automation to Improve Stability

Control and Automation to Improve Stability is a traditional session of TECIS conferences going back to the SWIIS committee. At TECIS'2021, *D. Novikov* and *A. Enaleev* (ICS RAS) presented two papers, "Sustainable Control of Active Systems: Decentralization and Incentive Compatibility" (*jointly*) and "Incentive Mechanisms for the Implementation of Management Automation to Improve Stability" (*A. Enaleev* only). The papers made a significant contribution to the theory of control in organizational and technical systems. Control mechanisms with optimal planning and incentive procedures for agents were proposed and studied. They ensure decentralized and strategy-proof (non-manipulable) control in systems with a network structure of connections between agents, and therefore, their sustainable operation. The authors introduced an approach to the organization and incentives to improve automation efficiency in man-machine complexes under asymmetric uncertainty in the "human-automated system" loop. Incentive-compatible and strategy-proof mechanisms were developed to solve Principal-agent problems with side payments. These mechanisms decompose the interaction of agents, allowing the Principal to consider incentive and planning problems independently by solving general optimization problems. With the proposed mechanisms, the agents provide reliable data (truth-telling) and fulfill their plans.

In the papers "Mechanism of Citizen Evaluation of Policy Using Machine Self-Learning" and "Machine Learning of Citizens with a Teacher and Political Stability," *V. Tsyganov* (ICS RAS) considered the stability problem of a social system that depends on the supply of a vital commodity (such as a COVID-19 vaccine). According to the author, political stability is achieved if society approves the authorities' actions to increase the supply of this commodity. However, the supply opportunities depend on random factors unknown to citizens. Consequently, they must learn to recognize and adequately evaluate the authorities' actions under uncertainty. The problem was considered for a society model in which citizens are either self-taught or trained by a teacher (media, social networks, etc.). Social stability is guaranteed if every citizen regularly evaluates the activity of a politician positively. Mechanisms were developed to quantify the evaluation of a politician by citizens using artificial intelligence procedures such as machine self-learning and teacher-assisted learning. Sufficient conditions were found to design progressive mechanisms in which the politician is interested in using all available opportunities for the benefit of citizens. The operation of these mechanisms was illustrated by an example of machine learning and evaluation of COVID-19 vaccination policies in Northern Ireland and the entire UK. Any citizen can use the mechanisms developed to evaluate policies using machine learning procedures continuously. For this purpose, he or she needs to install a corresponding application on a smartphone. In this way, artificial intelligence will contribute to social and political stability.

R. Bertelsen (UiT The Arctic University of Norway) addressed practical aspects of international stability management in his paper "Space Science & Technology in the Arctic: Promises of Cooperation and Development amid New Security Challenges." The Arctic houses key infrastructure for nuclear strategic stability, weapon systems, early warning and ballistic missile defense. Arctic high latitudes provide significant advantages to space science and technology (S&T) with the potential for dual-use. Space, which now underpins all societies, may become militarized. Strategic stability safeguards humanity. During the Cold War, there was an effective decision-making process mediated by game theory and modelling. However, new technologies, a far more complex multi-player, multi-nation environment and new space S&T reduce the effectiveness of traditional approaches. The author proposed to address the current intellectual vacuum by creating a clearer understanding of the parameters, players, technologies and their interactions and develop a new and robust theoretical basis that will contribute in the longer term towards appropriate and balanced Arctic governance for achieving strategic stability and space security. New theory, empirical overview, and modelling will provide a new intellectual basis for governance of Arctic space S&T for strategic stability and space security in the new era.

N. Kereselidze (Sokhumi State University, Georgia) presented the paper "Models of Epidemiological Security Management in the Spread of the

SARS-CoV-2 Virus." The author constructed a mathematical model of the spread of this virus without and with vaccination based on a system of differential equations. Applying solutions to the Cauchy problem, he verified the model on the epidemiological control protocol adopted in Georgia. The national epidemiological safety management problem was posed: maximize an objective function considering the financial consequences of introducing a lockdown in the country and the cost of treating the infected. A computational experiment confirmed the effectiveness of the model's computer implementation to predict the spread of the SARS-CoV-2 virus and showed that the analytical solution of the national epidemiological security management problem is applicable to justify appropriate decisions.

In the paper "Towards Strategic Reengineering the Global Computer Environment for Control of Sustainable Development of Social Systems," *Yu. Zatuliveter* and *E. Fishchenko* (ICS RAS) considered the trends of global computer environment (GCE) formation as a tool of global digitalization and its impact on social systems. The causes of intra-system imbalances in GCE development and their destructive impact on the sustainability of social systems were identified and analyzed. Ways to eliminate combinatorial barriers of the complexity of network resources integration by eliminating the fundamental causes of GCE heterogeneity were proposed. Also, ways to form a universal algorithmic space of distributed computing in the GCE based on computers with non-microprocessor architecture were developed.

L. Stapleton (Waterford Institute of Technology, Ireland) and *F. Janisch* (TU Wien, Austria) presented the paper "Digital Currencies and Community Empowerment in Austria: Gesell's Concept of Effective Demand as a Basis for Local Digital Currencies." The authors described the digital infrastructure of national cryptocurrencies and their acceptance by Austrian society. As it turned out, local cryptocurrencies have high potential and are more cost-effective than global crypto currencies. An experiment showed that local cryptocurrencies have a favorable effect when exacerbating economic crises caused by lockdowns in the pandemic process.

Z. Avdeeva, *S. Kovriga*, and *H. Grebenuk* (ICS RAS) presented the paper "Cognitive Modelling-Driven Time Series Forecasting for Predicting Target Indicators in Non-Stationary Processes." The approach developed by the authors allows improving the quality of forecasts by building and correcting competing models based on time series with subsequent activation of dominant models through correcting signals. These signals are formed (in monitoring mode) as a result of the analysis of qualitative information (judgments and opinions of the decision-makers and experts) using a fuzzy cognitive map of the situation – a model for representing causal influences between system-forming factors in such processes.

Large-Scale Systems Stability. In the paper "Structural Analysis of Large-Scale Socio-Technical Systems Based on the Concept of Influence," *O. Dranko* (ICS RAS), *Y. Rykov* (Keldysh Institute of Applied Mathematics RAS), and *A. Karandeev* (Plekhanov Russian University of Economics) represented such a system as a fuzzy cognitive map (influence digraph). *A. Pashchenko* (ICS RAS) presented the paper "Smart Management for Smart Cities – Synchronized Solutions." He proposed a management system and an integrated operation center to improve efficiency in the management and operation of a city.

5.2. Intelligent Systems and Applications

The main focus of TECIS'2021 was intellectualization technologies and their applications in robotics, mechatronics, industrial, social, medical, and other systems.

Robotics. *F. d'Apolito* and *C. Sulzbachner* (Austrian Institute of Technology) presented the paper "Flight Control of a Multicopter Using Reinforcement Learning." Machine Learning, and in particular Reinforcement Learning, is a persistent trend in automation and robotics in recent years. Many researchers worldwide are developing intelligent controllers using Reinforcement Learning techniques. The authors presented a proof-of-concept Reinforcement Learning flight controller for a multicopter. The agent was trained in the Airsim simulation environment to achieve stable flight conditions by controlling its roll, pitch, yaw, and throttle. After training, the agent was tested on the same environment to prove its ability to maintain stable flight conditions while following a determined route.

The paper "An Incentive Mechanism for UAVs Crowdsensing Markets, a Negotiation Approach" *by L. Jaimes* and *J. Kahr* (Florida Polytechnic University, the USA) and *J. Calderon* (Bethune Cookman University, the USA) proposed a solution to the problem of sensing coverage of lower regions of the atmosphere where a set of UAVs transverse it as part of their daily activities. Through sensors, UAVs acquire data while following their regular trajectories. In this model, participants use negotiation to compete and cooperate with each other while participating in data collection campaigns. Using the Virtual Robotics Environment (VRep) and extensive simulations, the authors showed that the algorithm performs well in terms of sensing coverage and participants retention while using a limited budget.

In the paper "COVID-Bot: UV-C Based Autonomous Sanitizing Robotic Platform for COVID-19," *E. Camacho* (Universidad Santo Tomas, Colombia), *N. Ospina* (Universidad Nacional De Colombia), and *J. Calderon* (Bethune Cookman University, the USA) described the design and implementation of an open-source robotic platform for sanitizing single plant environments such as offices, houses, apartments. The proposed solution—a low-cost and easy-to-replicate robot—disinfects surfaces through type C ultraviolet radiation. According to the tests, the system is adequate to autonomously cover a one-floor apartment, based on the theoretical radiation distance of the used lamps.

N. Unanyan and *A. Belov* (ICS RAS) presented the paper "Anthropomorphic Arm Control System with Remote Gesture Tracking." The authors theoretically justified and developed an automatic control algorithm for a five-grip robotic arm. Such an arm can be applied in rehabilitation systems, cosmonautics, industry, and aggressive inaccessible environments. The authors described the design of an anthropomorphic robotic arm with an embedded solution for human gesture tracking using a radio-transmitter sensor glove with an experimentally validated feedback rate provided by optimization techniques.

A. Stepanov and *M. Stepanov* (Saratov Technical University, Russia) presented the paper "Self-Organizing Control Systems for Autonomous Educational Robotic Complexes." To adapt the robot's functionality to changes in the environment, the authors proposed automatically synthesizing a control law adequate to the current situation. *Artificial neural planning networks* (ANPN) were used for this purpose. The results were applied in a correctional and rehabilitation educational center for schoolchildren with disabilities.

The paper "Design and Construction of a Cost-Oriented Mobile Robot for Domestic Assistance" by *B. Pallares Olivares*, *T. Rozo*, *E. Camacho*, and *J. Guarnizo* (Universidad Santo Tomas, Colombia) was awarded a diploma for the best work of young researchers. The authors described the design of the mechanics, electronics and software necessary for the operation of the robot and carried out tests to ensure its correct operation. The robot is intended for many home applications. It can be easily equipped with multiple actuators for different social tasks (elderly supervision, medicine supply, support of dependent people, etc.).

Intelligent Control for Integrated Enterprises, Safety and Risk Engineering. This session was organized by *R. Meshcheryakov*, *E. Jharko*, *A. Poletykin*, and *A. Iskhakov* (ICS RAS). It was devoted to intelligent data analysis methods in control and information processing and promising information security methods for objects of different nature. Components of the modern world, including industries and enterprises, tend to be more interdependent and form large-scale complex systems. Their control systems rest on the top of a complex and distributed architecture, implementing multiple functions and integrating multiple digital components. Deeper automation of large-scale systems with intelligent support and control elements creates the potential for increased production, lower costs, and improved safety.

A research group led by *F. Paschenko* (ICS RAS) presented several papers on solving applied problems by modern methods of analysis, modeling, management, and decision-making based on machine learning, big data processing, and artificial intelligence. The paper "Convolutional Neural Network for Convolution of Aerial Survey Images" by *Van Trong Nguyen* (Moscow Institute of Physics and Technology (MIPT)), *F. Paschenko* (ICS RAS), *Duc Tip Le* (MIPT), and *Chien Cong Vu* (MIPT) was devoted to artificial intelligence and machine learning in aviation and geoinformation systems. The papers by *F. Paschenko* (ICS RAS), *E. Arakelyan*, *A. Andryushin*, *S. Mezin*, *A. Kosoy*, and *J. Yagupova* (Moscow Power Engineering Institute) were devoted to the problem of optimal control of the CHP operation modes with a complex composition of equipment, including traditional heating units and steam-gas installations. The possibility of applying various mathematical methods of multi-criteria optimization to solve the problem in the presence of many internal and external constraints and conditions in the form of inequalities and balance equations was considered. Also, the possibility of integrating station-level optimization algorithms into the application software of modern program and technical complexes (PTC) to increase the intelligence of the automatic control systems (ACS TP) of thermal power plants was considered.

In the paper "Decision Support Systems for Stable Development of Agricultural SMEs," *V. Akinfiev* and *A. Tsvirkun* (ICS RAS) considered the problems of agricultural development for small and medium enterprises (SMEs). The features of modeling business processes in agriculture were analyzed. A financial decision support system was proposed to increase sustainability and reduce risks in the development of agricultural SMEs. The software modules are based on TEO-INVEST. This system considers the specifics of business processes in agriculture: duration of the production cycle exceeding the planning period, accounting for complex processing technology, the use of financial leasing for the purchase of agricultural machinery, etc.

E. Vergini and *P. Groumpus* (The University of Patras, Greece) presented the paper "Advanced State Fuzzy Cognitive Maps Applied on Nearly Zero Energy Building Model." Experimental application of the maps confirmed the possibility of minimizing the energy consumption of a smart building under weather conditions.

In addition to these papers, the session included substantive discussions of cybersecurity risks generated by the active implementation of machine learning technologies in protection systems. The participants' experience in building secure cyber-physical systems, designing a risk management system for the transport network of a smart city, and applying fingerprint technologies for personal identification and authentication was considered. Of particular interest were the papers on the problems of applying the visual and cognitive approach in systems engineering.

Socio-Informational Aspects of Managing Complex Systems under Conditions of Uncertainty and Risk. A research group from ICS RAS, led by *V. Kulba*, presented several papers on strategic planning and management problems solved using the methodology of scenario analysis and modeling. This methodology is intended to study the development processes of complex systems and multidisciplinary and multidimensional problems as well as construct and comprehensively analyze rational development scenarios (within the given objectives) for such systems. Therefore, it provides effective support when preparing, adopting, and implementing managerial decisions of different levels. *V. Schultz*, *V. Kulba*, *A. Shelkov*, *L. Bogatyreva* presented the paper "Scenario Analysis of Improving the Effectiveness of Cybercrime Investigation Management Problems," devoted to transforming the law enforcement system to counteract and investigate cybercrime. The current state and trends in the development of cybercrime analysis were described. The creation of a national cybercrime investigative agency was proposed based on the results of multigraph model research. In the paper "Scenario Analysis of the Impact of Rocket and Space Activities on the State of the Environment," I. Chernov (ICS RAS) introduced a basic graph model and performed a scenario study of the impact of rocket and space activities (RSA) on the ecological burden on the environment in the surrounding regions. Forecast scenarios of socio-economic development of areas with rocket and space technology intensive operation under various RSA development strategies were created. *V. Kulba*, *A. Shelkov*, and *Z. Avdeeva* (ICS RAS) presented the paper "Analysis of Anti-Corruption Management Effectiveness Based on the Scenario Approach (on the Example of the Construction Industry)." The authors presented the results of a scenario study of the corresponding multigraph models and showed the possibility of anti-corruption analysis of management decisions at the stages of their preparation, adoption, and implementation control. The paper "Methods of Socio-Economic Systems Analysis in Order to Diagnose the Problems of Transformation of Law in the Context of Digitalization" by *L. Bogatyreva*, *O. Shepeleva*, and *V. Gruzman* (ICS RAS) proposed using the scenario-based predictive examination of draft laws to anticipate their quality assessment in conditions of uncertainty.

When addressing a wide range of problems to increase organizational control performance, the methodology proposed in these papers can be applied: to build simulation models describing alternative directions of development of the studied complex systems and their components; study problems with fuzzy factors and relationships, considering the set of current and possible changes in the environment; assess the current situation by analyzing the mutual influences of describing its factors; identify positive and negative trends in the development of situations; predict possible ways of development of the emerging situations and their analysis by key objectives; adapt the management system dynamically to the arising changes and enable its operation ahead of the anticipated complex problems within tough temporal constraints on strategic, tactical, and operative decision-making.

Biomedical Systems. *A. Ivanov*, *N. Chivarov*, *K. Hrisafov* (Institute of Information and Communication Technologies, Bulgarian Academy of Sciences), *I. Budinska* (Institute of Informatics, Slovak Academy of Sciences), and *S. Chivarov* (TU Wien, Austria) presented the paper "Tele-Medical System for Remote Monitoring of Patients with Covid 19 and Other Infectious Diseases." The authors showed how an effective telemedicine platform could be created by combining free and open-source software products with commercial hardware.

Cost Oriented Automation (COA). *V. Borodin*, *A. Borodin*, *D. Frantsev*, and *M. Yudin* (Experimental Factory of Scientific Engineering (EZAN), Russia) presented the paper "Adaptive Automated Control Systems for Growing Single Crystals by the Methods of Czochralski, Stepanov (efg) and Kyropoulos Using a Weight Sensor." The authors described models and methods of economic crystal growth for industrial and scientific needs.

5.3. Social Aspects of Automation

In terms of theory, the social aspect of automation is a complex subject of interdisciplinary research that requires the consistent formalization of subject areas at the intersection of humanities, natural and technical sciences. In this sphere, let us mention the papers "Notes about the Attitude Control Problem" by *V. Korepanov* (ICS RAS), "Psychological Antecedents and Opportunities for Correcting Negative Attitudes towards COVID-19 Prevention Measures" by *V. Latynov* and *A. Vanin* (Institute of Psychology RAS), and "Identification of Integrated Rating Mechanisms as an Approach to Discrete Data Analysis" by *V. Sergeev* and *N. Korgin* (ICS RAS).

A research group from Florida Polytechnic and Bethune Cookman Universities (the USA) and Universidad Nacional De Colombia and Universidad Santo Tomas (Colombia), led *by J. Calderon*, presented several papers on intelligent systems and their applications in the social sphere. For example, the paper "Automation System Based on NLP for Legal Clinic Assistance" presented an algorithm for classifying natural language clinic requests and identifying problems for automated free consultation. As a result, the efficiency of identifying these problems reached 95%.

Development of Integrated Care Systems for Provision of Health and Social Care Services in Ageing Regions. The papers within this session considered the development and introduction of innovative technologies in the social sphere of ageing regions, where rapid changes in the population's age structure towards older people (including those with decreasing functional capacity) were observed. These changes shape the demand structure for adapted social infrastructure and health and care services. There is a need to expand the range and quality of services for older people. New services, technologies, and systems, such as *Ambient Assisted Living* and *Ambient Intelligence*, are being developed and implemented. The models and methods used in these systems focus on older people and are integrated into their living environment to support their independence and autonomy.

V. Rogelj (Institute INRISK, Slovenia), *D. Bogataj* (Institute INRISK and The University of Ljubljana, Slovenia), and *S. Temeljotov* (Norwegian University of Science and Technology) presented the paper "Digital Transformation of Community Health and Social Services for Ageing Cohorts." The authors examined the challenges and opportunities for social infrastructure development and financing concerning the situation in Slovenia. They showed how technological innovations (such as home automation and robotics, Internet of Things, and supply chain optimization) and organizational innovations (self-managed communities) create new services, businesses, and enterprises to deliver goods and services to older people, providing new employment opportunities for young people.

J. Peterlin and *V. Dimovski* (The University of Ljubljana, Slovenia) and *M. Bogataj* (Institute INRISK, Slovenia) presented the paper "Engineering Technology-Based Social Innovations Accommodating Functional Decline of Older Adults." As shown by the authors, the development of digital technologies such as cyber-physical systems, big data, machine learning, blockchain, artificial intelligence and Internet-of-Things, presents new opportunities for technology-based social innovations. The paper proposed the integration of social innovation methodology in the development of products or services for older people. The development of social innovations supporting multiple intelligences of older people was reviewed, and examples of good practice were provided to develop a model and propose a future research agenda. Cybersecurity and the ethical dimension within technically based social innovation were highlighted.

S. Colnar and *V. Dimovski* (The University of Ljubljana, Slovenia) and *D. Bogataj* (The University of Ljubljana, Slovenia) presented the paper "Review of Telecare in Smart Age-Friendly Cities." They considered the digital transformation of social infrastructure as a direction of smart city development to support the autonomy of older people.

CONCLUSIONS

The closing ceremony highlighted the sessions and papers that generated the most interest among the conference participants. Their evaluations were based on user activity in the IFAC Conference App. When awarding young researchers and delivering the closing speeches, *P. Kopacek* (the IPC Chair) and *L. Stapleton* (the IPC Co-Chair) noted the high scientific and practical level of TECIS'2021. *Z. Avdeeva*, an IPC and NOC member, thanked the NOC members, *P. Kopacek*, *L. Stapleton*, and the IFAC Secretariat staff for their assistance in organizing the conference.

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24TH INTERNATIONAL CONFERENCE ON DISTRIBUTED COMPUTER AND COMMUNICATION NETWORKS: CONTROL, COMPUTATION, COMMUNICATIONS (DCCN-2021)

The 24th International Conference on *Distributed Computer and Communication Networks: Control, Computation, and Communications* (DCCN-2021) was held on September 20–24, 2021, at Trapeznikov Institute of Control Sciences, Russian Academy of Sciences. This annual conference is devoted to discussing topical problems and innovative tasks in the information and telecommunication industry.

Like in 2020, the conference was held online due to the challenging global pandemic.

The traditional organizers of the conference were Trapeznikov Institute of Control Sciences, Russian Academy of Sciences (ICS RAS), Peoples' Friendship University of Russia (RUDN University), National Research Tomsk State University (TSU), and Institute of Information and Communication Technologies, Bulgarian Academy of Sciences. As in the previous five years, the organization of the conference together with RUDN University and TSU significantly expanded the geography of Russian participants and strengthened the integration of academic institutions and research schools of universities. Dr. Sci. (Eng.), Prof. *V.M. Vishnevsky* (ICS RAS) acted as the Chair of the Program Committee, and Dr. Sci. (Eng.), Prof. *K.E. Samouylov* (RUDN University) as the Vice-Chair of the Program Committee.

Traditionally, the conference was supported by IEEE Russia Section. Information support for DCCN-2021 was provided by Springer and MDPI.

The event united researchers from universities and research centers in the field of theory and practice of building computer and telecommunication networks, mathematical modeling, control and optimization methods of distributed systems and continued the series of conferences that have been held in Russia, Bulgaria, and Israel in the recent 25 years.

DCCN-2021 was held in the format of plenary and sectional sessions with a wide range of issues covering the most relevant areas of research in the field of information and telecommunication technologies:

– Communication networks algorithms and protocols.

– Computer and telecommunication networks control and management.

– Performance analysis, QoS/QoE evaluation and network efficiency.

– Analytical modeling and simulation of communication systems.

– Evolution of wireless networks toward 5G/6G.

– Centimeter- and millimeter-wave radio technologies.

– RFID technologies and their applications.

– Internet of Things and Fog Computing.

– Cloud computing, distributed and parallel systems.

– Machine learning, big data, artificial intelligence.

– Probabilistic and statistical models in information systems.

– Queuing theory and reliability theory applications.

– High-altitude telecommunications platforms.

Despite pandemic limitations, the conference received 150 papers from 240 participants representing 26 countries. In particular, the breadth of the conference geography is emphasized by the list of plenary speakers, which included leading experts in the theory and practice of communication networks from the USA, Israel, Hungary, Portugal, Italy, India, and Russia.

The following events took place within DCCN-2021.

- Conference opening (September 20, 2021), including the opening address and information message about the event by *V.M. Vishnevsky*, the Chair of the Program Committee; welcome speech by *D.A. Novikov*, Director of ICS RAS, RAS Corresponding Member, and *K.E. Samouylov*, Head of the Department of Applied Computer Science and Probability Theory at RUDN University.

- Plenary session (September 20–21, 2021) with the papers on actual problems presented by leading Russian and foreign researchers:

– *D. Selvamuthu* (India) "Performance Analysis of DRX Mechanism in LTE-A Networks using Markov Modeling." The problem of prolonging the life cycle of smartphone batteries was considered. As noted, modern devices use *Discontinuous Reception* (DRX) mechanism for energy saving. Recommendations were provided to achieve the minimum power consumption of the device using the mechanism.

– *E. Levner* (Israel) and *V. Vishnevsky* (Russia) "Recent Advances in Scheduling Theory and Applications in Robotics and Communications." The survey's main focus was on the latest achievements in scheduling theory and a wide range of its new applications: from cloud computing to robots and communication networks. The authors' view on current trends and acute problems and limitations inherent in this promising area of research was presented.

– *L. Correia* (Portugal) "Bridging 5G to 6G Networks: Problems and Challenges." The paper discussed the problems of mobile and wireless networks that have not been solved when implementing 5G networks. These problems should be properly considered when designing 6G networks. The need for further network virtualization and continued research on cloud computing was emphasized. The existing physical constraints (network bandwidth) were considered to address these issues and limitations of service devices (latency).

– *J. Sztrik* (Hungary) "Recent Results in Performance Modelling of Finite-Source Retrial Queues with Collisions and Their Applications." The author surveyed modern achievements in the models of finite-source retrial queues with collisions. Some examples were provided to illustrate the accuracy and area of applicability of the asymptotic method for studying the probability distribution of retrials.

– *K. Trivedi* (USA) "Software Fault Tolerance via Environmental Diversity." The paper discussed the fault-tolerance problem of software systems in the light of ensuring their high reliability. Software errors were classified; methods to reduce the damage from software errors were described; examples of existing systems involving these methods were provided.

– *G. Araniti* (Italy) "Towards 6G Non-Terrestrial Networks." The concept of *non-terrestrial networks* (*NTN*) was presented, and their properties allowing future generations of telecommunication networks to meet users' expectations better were considered. The latest developments and ongoing research in this field were described, and the still open problems were discussed. The importance of using non-terrestrial networks for building next-generation wireless communication networks was highlighted.

- Sectional Sessions (September 21–23, 2021) with over 150 papers presented by researchers from Russian and foreign universities, academia and industry, and research centers. Sectional sessions were grouped into three main thematic areas (tracks):

Track A: Computer and Communication Networks: Architecture, Protocols and Technologies.

Track B: Modeling of Distributed Systems and Networks.

Track C: Distributed Systems Applications.

Please visit the conference website https://2021.dccn.ru/ for detailed information about the participants and paper abstracts.

- Conference closing (September 24, 2021). In his closing speech, the Chair of the Organizing Committee, *V.M. Vishnevsky*, summarized the event, noting the high level and versatility of the conference papers and the originality of approaches to the problems posed. The Organizing Committee made several official statements:

– The high level of organization and conduct of the conference was noted.

– The conference topics were considered important and applicable to a wide range of problems covering the most relevant research areas in information and telecommunication technologies and the development of science in general.

– The papers presented at the conference were recognized for their high level and diverse nature. The authors were noted for a deep analysis of the state-of-the-art research in the theory and practice of building computer and telecommunication networks, mathematical modeling, information-telecommunication technologies, methods of management and optimization of distributed systems, and forecast of their development for the coming years.

– The conference Organizing Committee was decided to promote further expansion of scientific contacts with representatives of universities, academia and industry, and research centers, in Russia and neighbor and faraway countries, to cooperate in the field of information and telecommunication technologies, information exchange, development of new research methods, etc.

– Sincere gratitude was expressed to the co-organizers of the conference: Peoples' Friendship University of Russia (RUDN), National Research Tomsk State University, and Institute of Information and Communication Technologies, Bulgarian Academy of Sciences.

– The next, 25th, International Conference on Distributed Computer and Communication Networks: Control, Computation, and Communications (DCCN-2022) was scheduled for September 2022.

The papers of the DCCN-2021 participants were published in the Conference Proceedings[1]. Based on the results of the sectional sessions, 65 papers in English were recommended by the Section Chairs and selected by the Program Committee for publication in *Communications in Computer and Information Science* (CCIS) and *Lecture Notes in Computer Science* (LNCS) series by Springer. In addition, extended versions of the conference papers recommended by the Program Committee will be submitted to special issues of two MDPI journals, *Mathematics* and *Sensors*.

Chair of the Organizing Committee
V. M. Vishnevsky

Secretary of the Organizing Committee
D. V. Kozyrev

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[1]https://dccn.ru/downloads/DCCN-2021_Proceedings.pdf

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14TH INTERNATIONAL CONFERENCE ON MANAGEMENT OF LARGE-SCALE SYSTEM DEVELOPMENT (MLSD'2021)

The 14th International Conference on Management of Large-Scale System Development (MLSD'2021) was held on September 27–29, 2021. This conference is organized annually by Trapeznikov Institute of Control Sciences, Russian Academy of Sciences (ICS RAS), with the support of the IEEE Russia Section. The conference aims to promote international R&D cooperation on various managerial aspects of large-scale system development at sectoral, regional, national, and transnational levels.

Due to the ongoing restrictions imposed by the COVID-19 pandemic, the last two conferences, MLSD'2020 and MLSD'2021, were held online.

Leading scientists from academia, research institutes, universities, governmental and commercial organizations, professionally involved in the theory and practice of management in the modern era of the information society, took part in the MLSD'2021 conference.

The conference program included original research results in the following sections:

Section 1. Problems of managing large-scale system development, including multinational corporations, state holdings, and state corporations.

Section 2. Methods and tools for managing investment projects and programs.

Section 3. Managing the development of a digital economy. Design offices, situational and prediction and analytical centers, institutes of large-scale system development.

Section 4. Simulation and optimization in problems of managing large-scale system development.

Section 5. Nonlinear processes and computing methods in problems of managing large-scale systems.

Section 6. Managing the development of banking and financial systems.

Section 7. Management of fuel, power, infrastructure, and other systems.

Section 8. Management of transport systems.

Section 9. Managing the development of aerospace and other large-scale organizational-technical complexes.

Section 10. Managing the development of regional, urban, and municipal systems.

Section 11. Management of nuclear power objects and other objects of increased danger.

Section 12. Infoware and software for management systems of large-scale production.

Section 13. Methodology, methods, software, and knoware for big data processing and intelligent analysis.

Section 14. Monitoring in managing the development of large-scale systems.

Section 15. Managing the development of large-scale health systems, biomedical systems, and technologies.

Section 16. Managing the development of social systems.

According to the schedule, the conference was held over three days. On the first day, there was a plenary session; on the next two days, sectional sessions.

In total, 255 papers were presented at the conference. Amongst them, 153 papers were selected, extended, and published electronically in IEEE *Xplore*; please visit https://ieeexplore.ieee.org/xpl/conhome/9600061/proceeding. Also, several papers were recommended for publication in *Automation and Remote Control*, *Control Sciences*, and other scientific journals.

The central theme of MLSD'2021 was elaborating a model-oriented approach to system analysis of large-scale systems based on the meta-model of the developing system of developing systems (DSDS). The DSDS meta-model generalizes the classical concept of "the system of systems," bringing to the forefront the life cycle management of each system (development, application, modernization, and disposal). The new view on systems analysis forms the requirements to the information sources involved, extends the key indices and indicators of development, initiates the development of software engineering of network situation modeling, forecasting, and goal-setting. The role and scope of DSDS-class modeling and meta-modeling are increasing significantly against the background of total digitalization and interdisciplinary globalization of management. Ideas of this circle permeate the papers presented in the plenary session and were further detailed in different directions during the sectional sessions.

The **plenary sessions** with all conference participants usually feature guest speakers who address the most pressing issues and present new solutions for managing large-scale systems development. This year's plenary session was very rich and heated. The plenary program of MLSD'2021 was almost entirely devoted to promoting a model-centric strategic planning style in the era of the digital revolution and digital transformation of the economy. Two groups of the most striking plenary papers aroused great interest and active discussions. In the first group, five plenary papers were devoted to generally significant problems of digital strategic planning; in seven papers of the second group, these problems were considered from the sectoral perspective.

The first group included the following plenary papers:

- "Management of large-scale systems development under new conditions" by Dr. Sci. (Eng.), Prof. *A.D. Tsvirkun* (ICS RAS). The paper considered the digital transformation problem of large-scale systems, listed by the Government of the Russian Federation among strategic activities. Based on system model-oriented research, the author presented a holistic and interlinked strategic planning methodology for large-scale systems development. The methodology involves the classical definitions and concepts necessary to digitize algorithms, technologies, and methods of forecasting the development of systems characterized by complex (intersectoral, interregional) interaction of elements, distributed over a large territory, and requiring substantial resources and time. The methodology takes into account several distinctive features of such systems: dynamism (incoming raw materials, continuous technological operations, transport flows, funding with material and financial resources, depreciation, etc.); the presence of uncertain and uncontrollable factors, the need to manage risks (insurance, loss of competitiveness, reduced financial and economic potential, etc.); the complexity of balanced multilevel and multicriteria goal-setting and an appropriate strategy of goal-achievement; the use of sectoral, intersectoral, regional, national, and transnational classifiers and standards, etc. The author described TEO-INVEST, a software package for the feasibility study of investment strategies built on the prospective principles of project-program and aggregate-decomposition approaches. This theme was further revealed in the papers of Sections 1 and 2, devoted to the key areas of development and their solutions through investment projects and programs.
- "Large-scale projects in the strategic planning system of the Russian Federation" by Dr. Sci. (Econ.), Prof. *V.G. Varnavskii* (ICS RAS). The paper considered the fundamental problems of coordinating different incentive strategic development mechanisms (resource, technological, and institutional). The advantages and disadvantages of the modern normative base for strategic planning in the Russian Federation were analyzed in detail. Particular attention was paid to the transport sector. Priority large-scale investment projects in this sector for the period up to 2030 were highlighted. A strategic planning management system with the public-private partnership was proposed.
- "Oil market: modeling problems" by Dr. Sci. (Eng.) *V.K. Akinfiev* (ICS RAS). The paper is an important supplement to the strategic planning methodology of large-scale systems development based on the model-oriented approach. The problem of oil market modeling was considered, and mathematical models of oil market forecasting were surveyed. The competition between conventional and shale oil producers was modeled to assess the effects of the OPEC+ agreement. The prospects of the oil market development after 2020 were considered under possible scenarios for implementing the energy transition policy.
- "Management problems in large-scale projects of the mixed economy" by Dr. Sci. (Eng.), Prof. *F.I. Ereshko* (Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" RAS) and his co-authors *A.Yu. Mushkov* (All-Russia Scientific and Research Institute "Center"), *N.I. Turko* (State Corporation Rostec, Academy of Military Sciences), and Dr. Sci. (Eng.), Prof. *A.D. Tsvirkun* (ICS RAS). The paper presented the authors' research on the management of large industrial infrastructure systems within the global digitalization trend of the economy. The initial foundations of the research and development were described, and domestic experience in using mathematical models, information and communication technologies, and large volumes of information in management systems was surveyed. The issues of centralization and decentralization of management in complex systems were considered. Theoretical decision-making for assessing the prospects of state-business partnership development within the existing legal norms was given. A block of conceptual models corresponding to the planning level of large-scale organizational systems was presented. The issues of data preparation, algorithmic software development, and the combination of macro- and micro-descriptions of economic systems were considered.
- "Breakthrough development toolkit and its applicability in the first phase of implementing the unified national development plan" by Dr. Sci. (Eng.), Prof. *V.A. Irikov* (Moscow Institute of Physics and Technology) and his co-author, Cand. Sci. (Phys.–Math.) *N.S. Biryukov*. The paper noted the special role of the breakthrough development toolkit as a non-standard mechanism for implementing national pro-

jects and achieving ambitious goals, such as a multiply increased growth rate. A scheme was proposed to identify bottlenecks and eliminate them at minimum cost using a trajectory model and system optimization algorithms. Measures to eliminate bottlenecks hindering the required growth were elaborated and ranked as follows: (1) the creation of a program and an annual, quarterly comprehensive action plan for breakthrough development; (2) the development of a unified and goal-oriented multilevel automated system for preparation, adoption, and execution of decisions; (3) the provision of timely and complete managerial feedback information on the implementation of local development programs and actual results; (4) management of the policy of training qualified personnel and teams capable of developing and implementing effective and efficient tools of targeted management of breakthrough innovation development; (5) management of deficit-free federal and regional development budgets, etc.

The second group included the following plenary papers:

- "Transformation of electric power systems: directions and problems" by RAS Corresponding Member *N.I. Voropai* (Melentiev Energy Systems Institute, Siberian Branch RAS). The paper presented a methodology for justifying the development of the electric power sector and electric power systems and companies. The methodological basis of the approach was stated. Models and methods for forming and studying electric power sector development conditions and models and methods for justifying electric power systems development are given. Power consumption management features and models and methods for studying the influence of liberalization relations on the development of the power sector were considered. The basic knoware and software tools for solving these problems were described.
- "A conceptual design of a simulation complex to manage the operation and development of the hydrogen energy sector" by RAS Corresponding Member A.F. Rezchikov (ICS RAS) and his co-authors, Drs. Sci. (Eng.), Profs. A.D. Tsvirkun and O.I. Dranko (ICS RAS), Dr. Sci. (Eng.), Prof. V.A. Kushnikov (Institute for Problems of Precision Mechanics and Control RAS), Dr. Sci. (Eng.) A.S. Bogomolov (Saratov State University), and Cand. Sci. (Eng.) I.A. Stepanovskaya (ICS RAS). The paper covered information and analytical support for developing a new digital economy sector (hydrogen energy), a topical problem attracting the scientific community's attention worldwide. The national hydrogen programs aimed at decarbonization of sectors by 2050 were surveyed. As a result, the authors proposed the concept of systematic model-oriented analysis tools for national roadmaps integrated into the hydrogen energy security lifecycle (production, storage, transportation, trade, consumption). The approach to strategic forecasting, planning, and control is based on the advanced principles of neural mapping from GIScience. This theme was further reflected in the sectional papers of management system dynamics, a scientific school developed in ICS RAS jointly with IPPMC RAS, Saratov State University, and Belarusian State University. The range of papers under consideration described computational methods for solving several large-scale strategic analysis and management problems: minimization of losses from harmful effects of industrial and vehicular pollutants; the structural design of network management for hydrogen fuel transportation; network management for hydrogen fuel supply to vehicles; control of possible deviations in the main national economic indicators under the influence of hydrogen energy development, etc.
- "A complex of strategic management models for large-scale transport infrastructure" by Dr. Sci. (Eng.), Prof. *V.V. Tsyganov* (ICS RAS). The paper considered a topical problem: the globalization of transport infrastructure management to increase the efficiency of the economy's real sector and Russia's socio-economic and spatial development. The author hypothesized that the strategic development of macroregions is impossible without the advanced development of transport infrastructure in the changing environment. This hypothesis characterizes theoretical and methodological foundations, methods, technologies, and components of the complex of strategic management models for large-scale transport infrastructure development. Within the proposed complex, five functional subcomplexes of models were developed to simulate and maintain practical processes of transport infrastructure development support: managing strategic development; selecting and examinating large-scale development projects; training and adapting; forming transport corridors; ensuring safety. As an illustrative example, the complex was applied to the strategic management of transport infrastructure development in Siberia, the Far East, and the Russian Arctic.
- "Strategic management of low-carbon development of the electric power sector in Russia: problems and opportunities" by Cand. Sci. (Econ.) *F.V. Veselov* (Energy Research Institute RAS). The paper revealed multidimensional modeling in the strategic energy planning loop, covering energy systems and energy markets, perspective development of the power sector and energy complex, and market mechanisms of development management in the energy sector. The author identified power sector development problems solved using the models: long-term forecasting of power sector development; ensuring technical and economic competitiveness of various energy technolo-

gies and modeling changes in the technological structure of the power sector; assessing investment strategies, performing financial and economic analysis, and predicting conditions for the sustainable investment and development of energy-related sectors and companies; modeling and assessing the effects of liberalization and restructuring in the power sector; elaborating mechanisms to manage development in a competitive environment; studying integration effects for electric power markets; optimizing conditions for the operation and development of the electric power sector within the single market; simulating greenhouse gas emission scenarios in the energy sector; analyzing technological possibilities, mechanisms, and consequences of the economic regulation of greenhouse gas emissions; examining the prospects, trends, and effectiveness of the development of intelligent energy, active consumers, and distributed power generation in the Unified Energy System of Russia.

- "Studies on forming an unmanned aerial vehicle complex as a large-scale system" by Academician of the Russian Academy of Rocket and Artillery Sciences *V.P. Kutakhov* (National Research Center "Zhukovsky Institute") and his co-authors, Dr. Sci. (Eng.), Prof. *R.V. Meshcheryakov* (ICS RAS) and *A.L. Smolin.* As noted by the authors, unmanned aerial vehicles have the prospects of continuous improvement and modernization due to their high science intensity and expanding range of applications. This determines the relevance of developing a digital model-oriented strategic design platform. The proposed approach is based on a canonical ontological model of an unmanned aerial vehicle complex that implements the concept of a distributed hybrid working environment with wireless communication between stationary and mobile network agents. The advantage of such a model is focusing on software engineering of digital tests of system breakthrough solutions covering materials, propulsion systems, onboard equipment, and communication and information exchange systems in ill-structured collaborative structures. The topics raised by the authors were developed quite extensively in the sectional papers, including sharing the experience of participation in the Aerobot 2020 competition, monitoring concepts, and proposals on the configuration management of unmanned aircraft systems, etc.

- "Considering regional peculiarities in the strategic management of agro-industrial integrated formations in the single digital interaction space" by Dr. Sci. (Eng.), Prof. *V.V. Kulba* (ICS RAS) and Dr. Sci. (Eng.), Prof. *V.I. Medennikov V.I.* (Dorodnicyn Computing Center, Federal Research Center "Computer Science and Control" RAS). The paper considered a mathematical strategic planning model of agro-industrial integrated formations under the digital transformation of enterprises within the requirements of complementarity theory (long-term partnership). As shown by the authors, formation planning strategies on a single digital platform lead to a flexible management system for the relations between farm producers and processing, servicing, marketing, and trading enterprises: everyone "sees" all the participants in the chain, up to the end consumer. The growth of agro-industrial integrated formations, including agricultural holdings, poses the problems of effectively integrating material, labor, financial, and information resources. Examples are the complete absorption of enterprises or the preservation of production and social integrity with possible independent actions in the economic and legal space. The proposed digital platform incorporates significant regional factors in the model: investment in production, human capital, and the management system considering the territorial logistics component; regional competitiveness strategies. The authors demonstrated that the global agriculture digitalization trends are shifting towards the concept of a single digital platform of integrated formations; under certain conditions, they will become a single platform of the entire agro-industrial complex.

- "Control of the dynamics of multidimensional opinions in social networks" by Cand. Sci. (Eng.) *D.A. Gubanov* and Dr. Sci. (Phys.–Math.) *A.G. Chkhartishvili* (ICS RAS). The paper was concerned with developing a strategic technology of digital society, represented by the concept of control in social networks (e.g., for designing new strategies with maximum public support). The paper considered the following situation: a control subject (Principal) applies an informational impact on agents. The Principal's strategy is to choose the interval of this informational impact (the initial and terminal instants). The Principal's goal (payoff) is to minimize the distance between the average opinion of all agents and its position. A model of opinion dynamics in social networks was presented. The model has two interrelated information processes: the spread of activity and the formation of opinions. The following problem was formulated and solved for this model: choose a Principal's impact strategy that eventually gains the greatest support in the social network. According to the authors, promising lines of further research are the analysis of optimal control actions for different values of the structure and dynamics parameters and consideration of informational confrontation. This new direction of large-scale management was considered in detail by several papers of Section 16. In particular, the spread of coronavirus information, the increased effectiveness of advertising activity, street protests, and other issues were studied therein.

Sectional sessions of the conference traditionally serve as a platform for discussing management problems of large-scale systems development in the sectoral and instrumental-methodological context. The sectional program of MLSD'2021 continued studies of the model-oriented approach to the strategic design of large-scale systems towards developing digital twins of enterprises, products, technological processes, production, etc. Conceptually, a digital twin is a multiconnected set of end-to-end technologies with a formal description of real-world objects to predict the twin's properties, functions and behavior, response to perturbations, and interaction with other digital twins. The papers presented on this topic can be divided into projects, models, and methods.

The development of digital twins at the project level was the subject of the following papers:

- "A distributed software development technology with virtualization and digital twins for process control systems of nuclear power plants" by Dr. Sci. (Eng.) *A.G. Poletykin*, Cand. Sci. (Phys.–Math.) *V.G. Promyslov*, Cand. Sci. (Phys.–Math.) *K.V. Semenkov*, *N.E. Mengazetdinov*, and Cands. Sci. (Eng.) *M.E. Byvaikov* and *V.N. Stepanov* (ICS RAS);
- "A study of the digital twin of an enterprise" by *A.N. Sytov*, *A.V. Vakhranev*, and Dr. Sci. (Eng.), Prof. *F.I. Ereshko*;
- "Models and methods of technological infrastructure management based on digital twins" by Dr. Sci. (Eng.) *G.G. Grebenyuk*, Dr. Sci. (Eng.), Prof. *G.N. Kalyanov,* Dr. Sci. (Phys.–Math.), Prof. *S.P. Kovalev*, Cand. Sci. (Eng.) *A.A. Krygin*, Dr. Sci. (Eng.) *O.V. Lukinova*, and Cand. Sci. (Eng.) *S.M. Nikishov*;
- "A module of analytical methods for minimizing repair cost on pipeline networks with forming an infrastructure digital twin" by Cand. Sci. (Eng.) *A.A. Krygin*;
- "Some peculiarities in the development and application of adaptive digital twins for managing large-scale high-tech production" by *M.V. Zenkovich,* Dr. Sci. (Eng.), Prof. *Yu.G. Drevs*, and *V.S. Inozemtseva.*

A large place in the development of digital twins is given to classical analysis techniques such as simulation and correlation models, discussed in detail on Section 4.

Services supporting digital twins in the strategic planning loop may include intangible assets accounting. Interesting new approaches in this area were presented in the papers of Section 3 (in particular, the ontologization of scientific discoveries, a unique digital platform for intangible assets, ratings as a digital benchmarking tool, and others).

The application of business intelligence methods to digital twins also seems promising. The methods of banking and financial systems presented in the papers of Section 6 can be a suitable tool here. They model inflation, optimize microfinance and lending, etc.

Nonlinear dynamic models of physical media and mechatronic structures under external factors are also significant for using digital twins effectively. These extremely important issues were considered on Section 5. The papers presented therein contribute to creating adequate simulation and optimization methods.

The data mining and monitoring methods described in the papers of Sections 13 and 14 can be used to manage the big data flows of digital twins.

In conclusion, let us emphasize the following: the papers presented at MLSD'2021 show a scientific groundwork for a uniform strategic planning scheme for developing large-scale systems with different applications. This approach enhances the integrated innovation processes envisaged by the state science and technology policy. Therefore, it seems appropriate to conduct further research to standardize and compile promising samples of digital models and twins.

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