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CONTENTS

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PROBLEMS OF MANAGING THE FIRE SAFETY SYSTEM OF A FACILITY. PART I: ASSESSMENT METHODS

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Abstract. The scope and capabilities of managing the fire safety system of a facility from the position of its representative (head) are overviewed. Part I of the survey is devoted to the general problem statement and methods to assess the fire safety of a facility and the safety of people inside a building. As shown, fires and deaths of people testify to certain problems either in the facility's fire safety system or in the management of such a system. The existing methods for assessing the fire safety of a facility cannot be applied by its head: they require deep knowledge of the subject matter as well as the corresponding qualifications and tools (computer programs). In the current situation, the head (decision-maker) has no formalized objective assessment of the fire safety of his or her organization at a particular time, which significantly complicates (or even disables) rational decision-making.

Keywords: fire safety, management, facility assessment, fire safety system, fire risk.

INTRODUCTION

Each facility must have a fire safety system (FSS) to prevent fire, ensure people's safety, and protect property in the event of a fire.

Based on the purpose of this system, it is reasonable to choose the prevented fire and the absence of deaths (injuries) of people as assessment criteria. However, as shown by analysis, despite the general trend of reducing the number of fires, they still cause many deaths (at least 7000 people annually) and significant damage, exceeding 15 billion rubles annually [1]. At least 90% of people die in fires due to violations of fire safety codes and careless handling of fire. These factors generally determine the managerial aspect of the problem.

According to qualitative analysis of large fires, the main causes of fires resulting in mass deaths are violations of fire safety codes (careless handling of fire, smoking in unauthorized places, fireworks, etc.) or electrical wiring malfunction. One way or another, these problems are related to organizing fire safety at facilities.

The main causes of deaths and injuries are either the absence (disabling) of fire alarm and warning or the lack of necessary evacuation measures (mechanical blocking of evacuation exits and poorly organized evacuation). Like fire, death and injury have causes in organizing and managing the fire safety of facilities.

Thus, there are specific problems either with the fire safety system of facilities or with the management of this system.

A retrospective survey of the existing fire safety system of facilities, its operation specifics, and organizational and managerial aspects [1] showed that this system developed very slowly, and management areas were not considered for it (reduced to specifying organizational and technical measures). At the same time, there is no document regulating the management of this system, criteria for assessing the current state of the system, or its management mechanisms. Presently, the concept of a fire safety system as a controlled object is absent: this procedure is not described, there are no criteria for assessing the efficiency of fire safety systems, and the heads of organizations do not understand what they need to manage.

This paper overviews the scope and capabilities of managing a facility's fire safety system from the position of its representative (head). The survey consists of two interrelated parts. Part I is devoted to methods for assessing the fire safety of a facility and the safety of people inside it. We begin by describing the fire safety system of a facility.

1. THE FIRE SAFETY SYSTEM OF A FACILITY

Russia's current legislation[1] requirements determine the need for a fire safety system for each facility. Figure 1 presents the structure of a fire safety system.

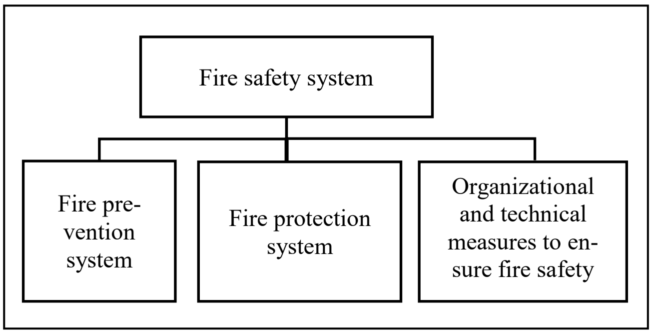

Fig. 1. Structure of the fire safety system of a facility.

An FSS consists of three main subsystems: a fire prevention system, a fire protection system, and organizational and technical measures to ensure the facility's fire safety. Let us consider them in detail.

The purpose of fire prevention systems is to exclude the conditions of fire occurrence by excluding the conditions for forming a combustible environment and (or) excluding the conditions for forming inflammation sources[1] in the combustible environment (or introducing such sources into it).

Conditions for forming a combustible environment are excluded by the following methods:

– using non-combustibles;

– limiting the mass and (or) volume of combustibles;

– using the safest methods to arrange combustibles and materials whose interaction may form a combustible environment;

– isolating a combustible environment from inflammation sources (using insulated compartments, chambers, or cabins);

– maintaining a safe concentration of oxidizers and (or) combustibles in the environment;

– reducing the concentration of oxidizers in the combustible environment in the protected volume;

– maintaining the temperature and pressure of the environment under which flame propagation is excluded;

– mechanizing and automating technological processes related to the handling of combustibles;

– installing fire-potential equipment in separate premises or open areas;

– using protective devices for production equipment that prevent combustibles from escaping into the premises or devices that prevent the formation of a combustible environment in the premises;

– removing fire-potential industrial waste, dust, and fluff from the premises, process equipment, and communications.

Conditions for forming inflammation sources in a combustible environment (or introducing such sources into it) are excluded by the following methods:

– using electrical equipment corresponding to the class of fire and (or) explosive zone and the category and group of an explosive mixture;

– using fast-response protective cutout devices for power plants or other devices that exclude the appearance of inflammable sources;

– using equipment and technological process modes with protection against static electricity;

– designing the lightning protection of buildings, facilities, and equipment;

– maintaining a safe heating temperature for substances, materials, and surfaces that come into contact with a combustible environment;

– using means and devices to limit the spark energy in the combustible environment to safe values;

– using spark foolproof tool ware when working with flammable liquids and combustible gases;

– eliminating conditions for thermal, chemical, and (or) spontaneous microbiological inflammation of circulating substances, materials, and products;

– excluding contact of pyrophoric substances with the air;

– using devices that exclude the possibility of flame propagation between adjacent volumes.

The purpose of fire protection systems is *to secure people and property against fire hazards and (or) limit the consequences of fire*. This purpose is achieved by the following methods:

– using space-planning solutions and means to limit the spread of fire beyond the hearth;

[1] Federal Law of the Russian Federation of July 22, 2008, No. 123-FZ "Technical Regulations on Fire Safety Codes."

– arranging evacuation routes that meet the requirements of the safe evacuation of people in the event of a fire;

– arranging fire detection systems (fire alarm devices and systems), warning and evacuation control in the event of a fire;

– using collective protection systems (including smoke protection) and personal protective equipment against fire hazards;

– using basic building structures with fire resistance limits and fire hazard classes corresponding to the required degree of fire resistance and structural fire hazard class of buildings and facilities; limiting the fire hazard of surface layers (finishes, facings, and fire protection means) of building structures on the evacuation routes;

– using fire retardants and building materials (facings) to increase the fire resistance limits of building structures;

– arranging the emergency drainage of flammable liquids and emergency bleeding of combustible gases from the equipment;

– arranging anti-explosion protection systems on the technological equipment;

– using primary fire extinguishing equipment;

– using automatic and (or) autonomous fire extinguishing systems;

– organizing the activities of fire protection units.

Generalizing these subsystems, we form a tree of FSS goals (Fig. 2).

According to the presented structure of the FSS, the goals of all subsystems, and methods for achieving them, the initial task is to prevent fire (assigned to a fire prevention system). If this task is not solved (a fire occurs), a fire protection system is activated. At the same time, the goals of organizational and technical measures to ensure fire safety and ways to achieve them are established (Fig. 2). However, such methods

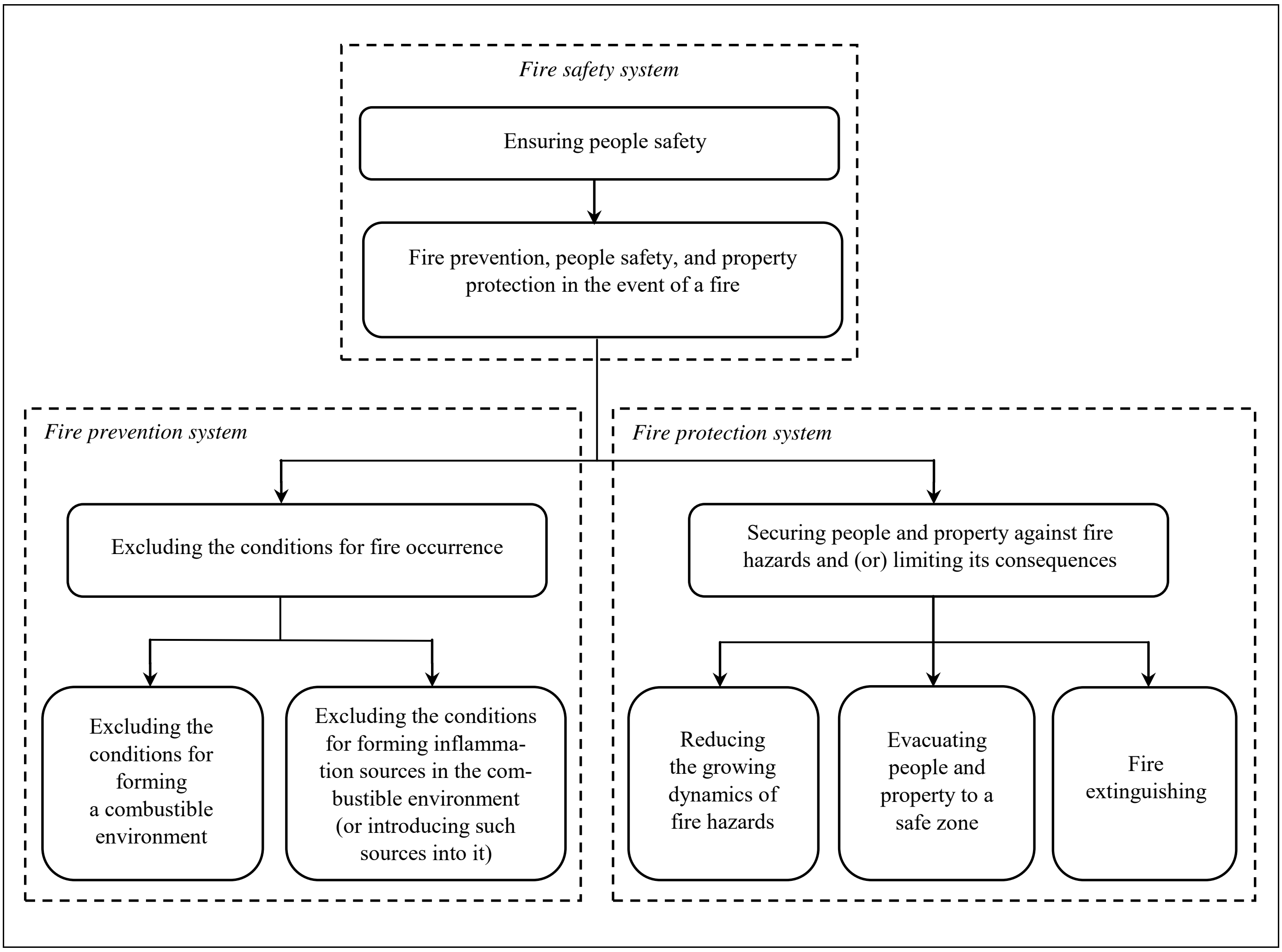

Fig. 2. The fire safety system of a facility: the tree of goals.

were previously formulated in several documents.[2,3] They included:

– developing an action plan for the administration, workers, employees, and population in the event of a fire (particularly evacuation of the people);

– making and using visual aids for fire safety;

– rationing the number of facility's employees based on safety conditions in the event of a fire;

– and others.

Note that similar requirements are now contained in the rules of the fire protection regime in the Russian Federation.[4] They are presented as a list of fire safety codes defining people's behavior, the order of production organization, and (or) maintenance of territories, buildings, facilities, premises, and other objects of organizations to ensure fire safety. Nevertheless, the goals of this set of codes are not formulated. The fire safety system was first introduced in 1977, and its structure has undergone no significant changes since then.[5] Requirements for the subsystems were refined from edition to edition. Very little attention was paid to organizational and technical activities within the system. There were changes in the criteria for assessing the system's operation, but the quantitative value of the assessment criteria was not changed and amounted to $1 \cdot 10^{-6}$. The facility's head manages the system and has the personal responsibility for observing the fire safety codes. Therefore, it is necessary to consider the entire process of state fire safety regulation and the organizational scheme of management.

Generalizing the analysis results and the facility's fire safety management system (the structural scheme in Fig. 3) [1], we make the following conclusions. In the form of requirements of federal legislation, supervisory authorities, etc., the environment obliges the facility's head to manage its fire safety system. The controlled system is the facility's fire safety system.

At the same time, in the classical statement of a control problem [2], the facility's head must carry out appropriate control actions based on the controlled system state (the facility's fire safety). From the legislative point of view, the controlled system state is characterized by obligatory fulfillment of fire safety codes and individual fire risk value or full compliance with all fire safety codes. In other words, at any time (the system is dynamic), the facility's head must monitor

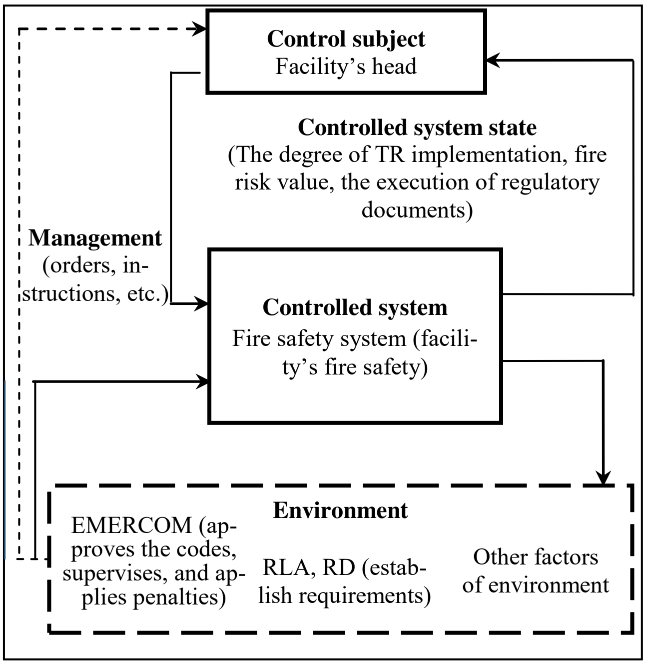

Fig. 3. The existing structure of the facility's fire safety management system: TR—technical regulations, RLA—regulatory legal acts, and RD—regulatory documentation.

the controlled system state and, if necessary, make managerial decisions to bring it to an appropriate state.

In addition, note that the facility's head has the right to appoint persons responsible for fire safety in the organization.[4] In this case, an organizational structure is formed; see an example in Fig. 4.

One way or another, being responsible for ensuring fire safety, the facility's head deals with the need to have extensive knowledge in fire safety. In Russia, the State Fire Supervision Service of EMERCOM is traditionally believed to be responsible for ensuring fire safety. However, it is not the case. The Service only controls (supervises) how the fire safety codes are observed. Thus, the facility's head is personally responsible for fire safety in the organization. Several large, high-profile fires in the last decade have shown that if people die or get injured in a fire, the facility's head bears criminal liability. As a rule, the facility's head or the person responsible for fire safety has only a superficial knowledge of fire safety, worsening the situation. However, note that qualification requirements[6] are currently being established for personnel responsible for fire safety (including a degree in fire safety), which would improve the situation.

At the same time, a profile education does not guarantee the result. The paper [3] assessed the reliability of specialists with experience from three to five years with a profile education in fire safety. Reliability

[2] GOST (State Standard) 12.1.004-85: The System of Labor Safety Standards. Fire Safety. General Requirements.

[3] GOST (State Standard) 12.1.004-91: The System of Labor Safety Standards. Fire Safety. General Requirements.

[4] Decree of the Government of the Russian Federation of September 16, 2020, No. 1479 "On Approval of the Rules of the Fire Protection Regime in the Russian Federation."

[5] GOST (State Standard) 12.1.004-76: The System of Labor Safety Standards. Fire Safety. General Requirements.

[6] Draft law No. 1188754-7 "On Amendments to Articles 24 and 37 of the Federal Law 'On Fire Safety.'"

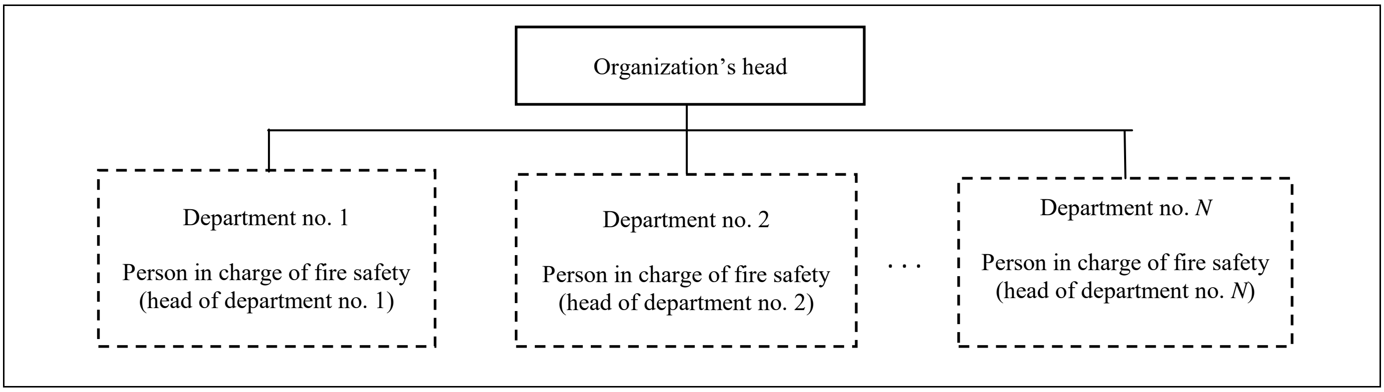

Fig. 4. The organizational structure of an enterprise in terms of fire safety (*N* is the number of departments in the organization).

was determined by the difference between the detected number of violations of fire safety codes and their total number on the example of a particular facility. The results showed that such a specialist could identify only 16–20% of the total number of violations.

In the current practice of fire safety, the situation develops as follows. After commissioning or when taking its office, the head receives a facility with an already defined set and structure of fire protection systems. Being responsible, he or she controls fire protection systems and organizes evacuation drills, periodic training, and briefings. With an irresponsible approach, he or she does not. It seems that the reason lies not even in the unwillingness of the head to manage the fire safety of the facility but rather in the impossibility of assessing the state of fire safety to make appropriate decisions. We consider the existing methods for assessing the facility's fire safety to verify this assertion.

2. METHODS FOR ASSESSING THE FIRE SAFETY OF A FACILITY AND THE SAFETY OF PEOPLE INSIDE THE BUILDING

2.1. Fire safety assessment based on Russian legislation

At present, several approaches exist to assess the compliance of facilities with fire safety codes in Russia. Strictly speaking, it is necessary to separate the fire safety of the building and the safety of people in case of a fire as its most important component. The legislation establishes forms for assessing the compliance of facilities with fire safety codes, which include:

– independent fire risk assessment (fire safety audit),

– the Federal State Fire Supervision,

– fire safety declaration,

– acceptance and commissioning of the facility and fire safety systems.

The forms mentioned above are chosen only for facilities and not products. The facility's head can initiate only independent fire risk assessment (fire safety audit). Moreover, it is a commercial service: the fire safety of a facility is confirmed by a specialized organization.

Regardless of the form of confirmation for the building, there exist two conditions for compliance with the fire safety codes:

– the fire safety codes of technical regulations are fully met, and the fire risk does not exceed the feasible value (for public facilities, $1 \cdot 10^{-6}$),

or

– the fire safety codes of technical regulations and regulatory documents on fire safety are fully met.

If one of the conditions is satisfied, the facility (in terms of legislation) is considered safe against fires. All conditions include compliance with the requirements of technical regulations, which establish general compulsory fire safety codes for fire distances, evacuation routes, evacuation and emergency exits, fire resistance of the building, etc. Usually, these are general requirements without specific statements (like, e.g., the value of a parameter *X* for a group of facilities *Y* must be at least *Z*): they just establish the need for something (e.g., buildings of class *X* must have a system of type *Y*). Thus, these conditions are prescriptive and imply that the facility is safe concerning the factor in question if they are met. This form of assessment is directive. The second part of the conditions is variable and also directive. It establishes that if all standards are met, then the facility is safe. However, an advanced tool in this area is the second part of the first condition, which establishes that only general requirements can be met, while the fire risk should not exceed $1 \cdot 10^{-6}$.

Fire risk assessment in domestic practice is carried out based on the corresponding methodology[7] for calculating the fire risk in buildings, constructions, and structures of different classes of functional fire hazard (further called the Methodology), approved by EMERCOM. As a rule, the Methodology involves computer programs.

The Methodology for calculating the fire risk is a set of procedures and their sequence, including the stages shown in Fig. 5.

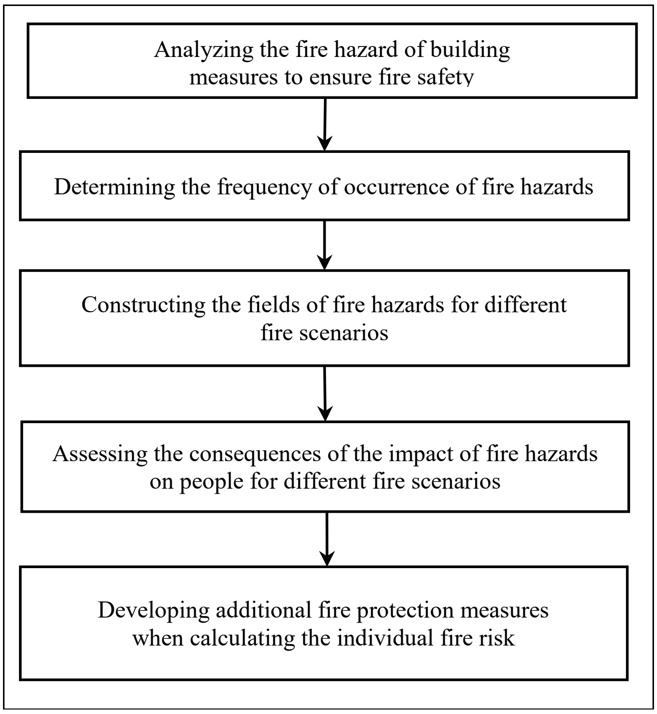

Fig. 5. Fire risk assessment stages.

The safety condition is given by the inequality

$$Q_{\text{risk}} \le Q_{\text{risk}}^{\text{norm}}, \qquad (1)$$

where Q_{risk} is the calculated individual fire risk, and $Q_{\text{risk}}^{\text{norm}}$ is the individual fire risk norm.

The risk value is calculated by the formula

$$Q_{\text{risk},i} = Q_{\text{fire},i} \cdot \left(1 - K_{\text{fire-fight},i}\right) \cdot P_{\text{people},i} \cdot \left(1 - P_{\text{evac},i}\right) \cdot \left(1 - K_{\text{fire prot},i}\right)$$

with the following notations: $Q_{\text{risk},i}$ is the individual fire risk in the ith fire scenario; $Q_{\text{fire},i}$ is the frequency of fires in the building during the year; $K_{\text{fire-fight},i}$ is the coefficient describing the compliance of automatic fire-fighting systems with the requirements of the fire safety regulations; $P_{\text{people},i}$ is the probability of people's presence in the building; $P_{\text{evac},i}$ is the probability of people's evacuation; $K_{\text{fire prot},i}$ is the coefficient describing the compliance of the fire protection system, aimed to ensure the safe evacuation of people in a fire, with the requirements of the fire safety regulations.

Then, the maximum value among all $Q_{\text{risk},i}$ is taken and compared with the norm (the expression (1)).

The Methodology provisions were discussed many times. Therefore, we consider only some of them.

According to several researchers, the Methodology requires substantial revision [4] due to the following drawbacks:

– The probability of fire is not determined for some buildings, and there are no recommendations on its choice.

– There are contradictions in the choice of the parameter P_{people}.

– There are no data and conditions for choosing the type of human clothing (this determines the projection area in the calculations) and fire load (including the criteria for choosing this load).

– It is unclear how many scenarios to consider and under what conditions to select the blocking of the evacuation exit.

– Several parameters are not considered (fire passages, fire distances, drencher curtains, outdoor water supply, personal protective equipment, etc.).

As determined, the Methodology should be improved by developing an indicator of the quality of fire protection systems and differentiated values of fire risk. According to the Law,[8] the risk value should confirm the fire safety of the facility, people, and property. However, the indicators in the calculation formula indirectly determine the risk value for people and property. The fire community (experts and supervising bodies) believes [4] that the fire risk is a tool of justification of deviations concerning the parameters of evacuation ways, and it is not an indicator of the facility's safety.

There is evidence [5] that Russia's individual fire risk norm is significantly underestimated and requires revision. The variables figuring in the expression (1) and their values and determination methods are ques-

[7] Order the Ministry of the Russian Federation for Civil Defense, Emergencies and Elimination of Consequences of Natural Disasters of June 30, 2009, No. 382 "On Approving the Methodology for Calculating Fire Risk in Buildings, Facilities, and Constructions of Different Classes of Functional Fire Hazard."

[8] Federal Law of the Russian Federation of July 22, 2008, No. 123-FZ "Technical Regulations on Fire Safety Codes."

tioned. The acceptable (normative) risk level is debatable. As mentioned earlier [6], fire risk is too abstract for life and, therefore, difficult to express understandably and acceptably for society.

Generally speaking, the Russian approach to assessing the safety of people has been repeatedly criticized (particularly in the paper [7], where proposals to improve the procedure for assessing the safety of people inside the building were presented). As described therein, the reaction time significantly affects the total evacuation time; the factors affecting the time to start evacuation were listed. According to some evidence, the facility's personnel (constantly trained) do not always behave adequately when the fire alarm system is triggered, looking for confirmation information, not starting the evacuation, etc. In addition, the person's decision to evacuate is made by perceiving incoming information about the presence of fire signs and comparing a set of factors determining the probability of fire and the truthfulness of this information. In other words, a person to decide must overcome a certain threshold ("really happened fire" vs. "false alarm"). Then the evacuation management problem reduces to bringing this threshold closer by any means as soon as possible. The following idea was proposed in the paper: correction factors can be applied to the fire risk value, e.g., if the personnel are familiar with the evacuation plan. As also argued, the methodology for determining the fire safety level overestimates the time to evacuate people in a fire.

Thus, the state-level approaches to assess the fire safety of a building consist in either fulfilling all the requirements of fire safety codes or fulfilling the compulsory codes and calculating the fire risk (assessing the individual risk of death in a fire). At the same time, the procedure of calculating this risk has been repeatedly criticized and objectively requires improvement.

2.2. Domestic approaches to fire risk assessment

Consider the approaches to assessing fire safety developed by domestic researchers.

One approach under development is express fire risk assessment [8–12]. It reduces mathematical models describing a fire hazard to a simpler form (many indicators are generalized to three or four ones). As a result, they can be calculated by any specialist of basic qualification without special software. In other words, calculations can be performed when examining the facility using simple technical devices (cell phone, calculator, etc.).

Another way to assess the facility's safety is the scenario approach: baseline scenarios of a particular emergency are studied [13]. For each baseline scenario, safety aspects associated with it are identified, and each aspect is assigned a certain number of factors affecting this scenario. Then an operator multigraph is used to construct the relationship of successive events of an emergency, and its consequences are assessed and predicted by simulation methods. This approach was tested for a station of the Moscow subway [14].

A disadvantage of this approach (in terms of fire safety) is a significant simplification of the processes occurring during fire growth. For example, the scenario approach does not consider the dynamics of fire hazards directly: calculations require preliminary modeling of fire hazards (FHs) and an array of critical values of FHs at each calculation point. The data on the evacuation of people are also taken incorrectly. For example, evacuation speed is defined as 3 persons per second (healthy people) and 2 persons per second (injured people), linearly on the entire segment [15]. In contrast, the fundamental law in the evacuation of people is a logarithmic dependence of evacuation speed on the human flow density [16, 17]; this density is determined by the geometric dimensions of evacuation routes, which are not considered within the scenario approach. Hence, this approach is appropriate for forming development scenarios and not assessing their consequences (at least in terms of fire safety).

Note the mechanisms for assessing the safety of potentially dangerous facilities [18, 19] based on the theory of active systems [20]. For its implementation, decision trees and bottom-up aggregation are used. The main stages include selecting directions that characterize the facility's state, assessing the facility by these directions, performing convolution, and obtaining a comprehensive assessment of the facility's state. In general, this approach is promising for fire state assessment due to its simplicity, efficiency, and wide approbation in various areas [21–23].

Several studies [24, 25] were devoted to index methods for fire risk assessment, particularly the Gretener method. The cited authors adapted the method to the Russian conditions (preliminary fire safety assessment, analysis of the probability of fire, etc.) However, the method was not systematically described to evaluate its efficiency. This method will be considered in detail in subsection 2.3 below.

Another fire risk assessment method involves the classical definition: the fire risk is defined as the probability of event occurrence multiplied by the expected damage [26–29]. In particular, the following typification is considered: the risk of encountering a fire, the risk of dying in a fire, the risk of destroying a structure, etc. Thus, fire risk is generalized, often at the strategic level (scale), using mathematical statistics methods. Considering such assessment directly for the facility, we obtain the expected risk of fire or damage (but not its quantitative estimate) based on the facility's characteristics.

2.3. Foreign approaches to assessing fire safety

Fire safety assessment methods in the foreign scientific literature are analyzed much wider and deeper. There are different requirements, guidelines, and approaches to assessing fire risk.

The regulation of fire safety (in general) and risk calculations (in particular) considerably varies abroad. Only the basic part of fire safety codes is established and regulated at the state level. Another feature is the approach called *performance-based design*. It is defined [30] as "an engineering approach to fire protection design based on agreed upon fire safety goals, loss objectives and performance objectives, deterministic and probabilistic evaluation of fire initiation, growth and development, the physical and chemical properties of fire and fire effluents, and quantitative assessment of design alternatives against loss and performance objectives."

Consider the regulated methods for assessing fire safety through fire risk analysis.

The Society of Fire Protection Engineers (SFPE) Guide [31] establishes general requirements for fire risk assessment. This guide is intended for professionals in construction and process design. In particular, it provides a recommended assessment procedure, hazard identification methods, data sources, and risk modeling and calculation methods.

In the US, *the National Fire Protection Association* (NFPA) developed a fire risk assessment guide [32] containing a sequence of steps to be followed by a professional. The guide provides well-developed risk assessment and analysis methods with input and output data, model assumptions and constraints, scenario selection methods, etc.

In the UK, a fire risk assessment standard was developed [33]. Like the guides discussed above, it provides a methodological framework for the professional to analyze and assess risk. In contrast to the standards mentioned above, this document establishes risk acceptance criteria and describes methods for assessing financial losses.

The International Organization for Standardization (ISO) also developed a standard conceptually describing the risk assessment procedure [34]. The document describes the risk assessment procedure, the principles of risk assessment, and uncertainty analysis methods. An essential difference of this standard is recommendations to interpret the risk assessment results.

Interestingly, in Germany, the procedure for regulating risk assessment at the legislative level is just emerging, and a corresponding standard is being developed [35].

Now we discuss methods and approaches developed for fire risk assessment.

Hazard and Operability Study (HAZOP) is a risk analysis method to identify situations entailing the failure of any elements or the entire system and assess the consequences of such failures [36]. Generally speaking, it is not intended for fire risk assessment but can be used to identify potential fire elements of a building or fire potential processes.

Failure modes and effects analysis (FMEA) and *Failure mode, effects and criticality analysis* (FMECA) are methods for analyzing failures (usually, technical elements without human factors) and the consequences of such failures and determining their criticality [36]. These methods are rarely used within the fire risk assessment procedure. Nevertheless, attempts were made [37] to adapt FMEA to fire risk assessment by synthesizing this method with fuzzy logic theory on the example of a railway tunnel. As a result, two types of risks contributing to fire and causing tragic consequences were established.

Event tree analysis (ETA) and *Fault tree analysis* (FTA) are methods to identify hazard and failure scenarios. For their application, it is necessary to construct trees with the probabilities of transition to downstream events. The probability of the resulting event is defined as the product of the probabilities preceding the given branch. The approaches mentioned above can be referred to as quantitative or qualitative risk assessment methods, depending on whether the probabilities of transition between events are assigned or not. They are often used for fire risk assessment, usually at the stage of identifying potential causes of an accident or constructing fire growth scenarios [38–42]. In addition, note that these methods are often used to assess the fire risk at production facilities.

FN-curves describe risk analysis results (frequency and consequences): a risk curve shows the probability of N or more deaths per year as a function of the event frequency F on a double logarithmic scale [43]. This value is often used to assess collective risk, mainly at production facilities [44, 45].

As low as reasonably practicable (ALARP) is a fundamental principle rather than a method to minimize risk. This principle was originally formulated in 1954 by the International Commission on Radiological Protection and was actively used to ensure the safety of nuclear power plants. Indeed, many experts [46–50] agree that achieving zero risk is not practicable due to various uncertainties, the interrelation of risks, etc. For fire risk assessment, this principle was rarely applied. Note the recent paper [49], where the ALARP principle was applied to fire risk assessment and tested on selecting an automatic water fire-fighting system and determining the size of the evacuation exit.

Fire Safety Concept Tree (FSCT) is a concept for assessing fire hazards and their consequences [51] based on two obligatory basic components, namely, fire prevention and consequence management. Like in ETA and FTA, a tree structure is used, but the probabilities of transition are not specified.

Simple Analysis Fire Risk Evaluation (SAFRE) is an approach based on the construction of failure and event trees, intended to assess the fire risk of cultural heritage buildings. It identifies possible fire growth scenarios and examines probable consequences [52].

The Gretener method [53, 54] is an index fire risk assessment method widely used in practice. It has undergone many changes and variations [55–60]. Initially, the fire risk measure was the product of the probability of an event and the degree of its hazard; the hazard was defined as the ratio of the potential hazard to the protective measures [53].

In this method, the fire risk is calculated as

$$R = A \times B,$$

where A is the probability of a fire, and B is the fire hazard (its level or the degree of consequences).

The fire hazard is given by

$$B = P / (N \times S \times F),$$

where: P is the potential fire hazard; N, S, and F are the aggregate indicators characterizing standard fire safety measures, special fire safety measures, and the fire resistance of the building, respectively.

The Carleton University model [61] is a quantitative fire risk assessment method implemented in the computer program CUrisk. This approach was developed mainly for buildings with wooden frames. The approach estimates the following parameters:

– fire growth scenarios (based on event trees),

– the dynamics of FH development (based on the *CFAST* model [62]),

– the stability of wooden structures (estimated during fire exposure based on the *WALL2D* model [63]),

– the evacuation of people (no model specified),

– the actions of fire units (no model specified),

– economic damage (no model specified).

The computer program calculates *the expected risk to life* (ERL) [64] and *the expected risk of injury* (ERI) [61] in case of deaths and disabilities.

The Edinburgh Risk Assessment Model is a matrix concept for assessing fire safety developed at the University of Edinburgh for hospitals [65]. It represents a hierarchy of matrices at the following levels:

1. the facility's fire safety policy,
2. the goals (tasks) of the facility's fire safety,
3. the strategy of ensuring the facility's fire safety,
4. the facility's fire safety components.

As stated, this approach assesses the fire safety of any facility; if necessary, additional levels of the hierarchy can be introduced to detail any component. At each level of the hierarchy, a matrix of comparisons of fire safety factors at each level is compiled. Then a relationship between the levels is established. As a result, the relationship between the components is traced at the system level. Currently, this method is developing [66, 67].

As a rule, the risk assessment approaches discussed above are particular and have a fundamental component affecting the resulting risk value. Let us consider complex approaches combining several methods.

The Australian Centre for Environmental Safety and Risk Engineering developed CESARE-Risk, a fire risk assessment method extending Australian building codes [68]. This method is intended for officials and engineers to select cost-efficient fire safety measures at an acceptable level of risk. It includes the following steps.

1. Forming an event tree for developing fire scenarios. Note that the current state of the building, the probability of a fire, the characteristics of people in the building, the availability of fire protection, and other factors are considered.

2. Fire growth modeling. An original fire growth model was used [69].

3. Evacuation modeling, including the people's reaction to a fire warning, a routing model (people can change their direction if there is smoke or open flame on their way), and the presence of people under alcohol or drugs.

4. Fire-fighting modeling: the arrival of fire crews to a building and the deployment of resources are simulated using event trees. Depending on prevailing conditions, fire extinguishing, people's search and rescue missions, and fire localization and elimination are also simulated.

5. Building stability modeling. The service time of building structures and the emergence of a limit state for fire resistance are predicted. See the publication [70] for a detailed description of this method.

FiRECAM was designed to quantify the fire risk in residential buildings and offices in accordance with Canadian fire safety codes [71]. This approach calculates the expected risk to human life and the expected fire damage for each fire scenario; based on these values, the fire safety concept is accepted or revised. It includes the following steps.

• Six fire types for each floor are considered fire growth scenarios: three types of combustion (smoldering combustion, flame combustion inside the premises, and flame combustion going outside the premises) and two door states (open or closed). The probabilities of different fire types are assigned using statistical data.

For example, according to the data presented in [72], fires in buildings have the following dynamics: 22% of fires are smoldering combustion without further growth, 54% of fires are flame combustion inside the premises, and 24% of fires are flame combustion going outside the premises.

• The probability of fire occurrence is estimated if the building is not typical and there are no data on the frequency of fires. In this case, the probability of fire occurrence is estimated based on several factors (the type and combustibility of materials, potential sources of fire, maintenance of fire protection systems, etc.).

• Fire growth is modeled to estimate the time of occurrence of fire hazards.

• The stability of building elements is modeled based on thermal effects to estimate the probability of fire spreading inside the building considering the operability of the automatic fire-fighting system.

• People's evacuation is described by the model [73], which considers the reaction time, situations when people are blocked inside the building, etc.

• The actions of the fire-fighting crews are modeled considering the movement time and response time and the possibility of rescuing people blocked inside the building.

• The fire safety costs are assessed for each fire scenario as its probability multiplied by the expected amount of damage.

• The probability of people's death is estimated by comparing the FH spread and their movement routes. The presence of a balcony nearby or a fire-safe zone is considered. See the publication [74] for a detailed description.

CRISP is a fire risk assessment approach based on simulation and the Monte Carlo method. It determines the conditions of safe evacuation by comparing the blocking time of evacuation routes and evacuation time. This approach is less comprehensive than the previous ones but advanced compared to Russia's current Methodology. In particular, much attention is paid to the stochasticity of processes (random processes and random initial conditions). In the course of multiple modeling, the probabilities of death and injury are calculated through the *fractional effective dose* (FED) [75]. This method was considered in detail in the paper [76].

Lund QRA is another risk assessment method developed at Lund University. It has two versions: standard and extended. Their principal difference consists in considering the random nature of the variables: in the extended version, the risk is calculated by the standard method with Monte Carlo simulations. The event tree is used to construct the fire growth scenarios. Each outcome in the event tree has a set of probabilities and consequences, called the Kaplan and Garrick triplet. The risk assessment results are presented in FN-diagrams or a risk profile on a logarithmic diagram. Human risk is generally defined as a "safety margin" (the conditions for safe evacuation). This method was described well in the publication [77].

We draw several conclusions based on the analysis of various methods for assessing the facility's fire safety and the safety of people inside the building in a fire.

• Under the form established by the legislation of the Russian Federation, the facility's compliance with fire safety codes (over 100 000 in total [3]) is assessed. As a result, the fire risk can be assessed, but the corresponding Methodology needs significant revision.

• As shown by the survey, foreign fire and human safety assessment approaches are strongly developed and deeply elaborated. Most likely, this is due to the absence of "tough" state regulation in fire safety.

• All approaches considered are probabilistic. Strictly speaking, the calculation results are relevant only for the conditions accepted and neglect the dynamically changing environment.

• Despite many approaches to fire safety assessment, they are difficult to implement for the facility's head: he or she needs deep knowledge of the subject matter and the availability of appropriate qualifications and tools (computer programs).

CONCLUSIONS

Concluding part I of the survey, we note the following.

• As shown by the statistical data on fires in the Russian Federation, the number of deaths is high, and the damage from fires exceeds 15 billion rubles annually. The majority (90%) of people die in fires due to violations of fire safety rules and careless handling of fire. Hence, there are problems in managing the facility's fire safety. Qualitative analysis of fires with mass death confirms this conclusion since the main causes of fire, mass death, and injuries are violations in the facility's fire safety organization and management.

• Currently, the fire safety system as a controlled object is absent at the conceptual level. The corresponding procedure is not described, there are no criteria for assessing the efficiency of the fire safety system, and the facility's head does not understand what he or she needs to manage.

• Despite numerous approaches to fire safety assessment, their application requires deep knowledge of the subject matter and appropriate qualifications and tools (computer programs). Thus, the facility's head cannot assess the security of his or her organization (facility) without qualified specialists.

Part II of the survey will consider methods to monitor the facility's fire safety and assess the state of socio-economic systems in fire safety. The existing contradictions in the fire safety management system will be shown, and some ways to resolve them will be presented.

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OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS

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Abstract. This paper considers optimal decision-making during the life cycle management of complex systems of aerospace, power, nuclear, transport, and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries as well as information systems. The system-wide peculiarities of the life cycles of complex systems are identified and analyzed. Qualitative formalisms to represent life cycles are proposed; mathematical foundations of the problem of their optimal control are described. A mathematically rigorous optimal control problem for the life cycle of complex products, objects, and systems is stated. An algorithmic solution of the optimal control problem based on the formalisms of dynamic programming is developed. A practical way of applying this algorithm based on the scenario approach is proposed; the conditions of life cycle control optimization (under which optimization is possible) are listed. The results presented below are an optimal control tool for the life cycle of complex products, objects, and systems.

Keywords: optimal control; dynamic programming; life cycles of complex products, objects, and systems.

INTRODUCTION: THE TOPICALITY OF LIFE CYCLE MANAGEMENT AND OBJECTIVE COUNTER FACTORS

Nowadays, the concept *of life cycles of complex systems (products, objects)*[1] (LCS), fundamental in systems engineering [1, 2], is widespread in the practice of managing the creation and application of products and systems of aerospace and defense industries, objects and systems of the nuclear, oil and gas, power, transport, communication, and other processing, raw material, and service industries as well as in the field of information technology.

However, despite the wide circulation of the concept of LCS, the formal bases of LCS management are still not stated: there are no strictly defined criteria to compare certain approaches, choose the best of them, and coordinate and integrate interdisciplinary decisions. Furthermore, LCS management is not even posed as a mathematical control problem. The reasons are the high complexity and heterogeneity of management processes for LCS caused by the complexity of the Systems and the uncertainty and variability of external factors for LCS.

The life cycle is a complex system and the subject of research in various knowledge domains. Within each domain, models are developed to study separate aspects of LCS with different degrees of rigor. The presence of heterogeneous models requires a reasonable choice of approaches to forming integral quantitative models of LCS.

Management of complex systems is studied primarily by cybernetics and systems theory [3, 4], whereas lifecycle management by systems engineering [1, 5–9]. However, the results obtained in these branches are, as a rule, qualitative; the models of the mathematical theory of systems [7] do not allow posing and solving optimization problems.

Another popular topic is cost estimating (costing) throughout the entire life cycle of the system or product being created; for example, see [10, 11]. In recent years, neural networks, machine learning, and other modern approaches have been used for life cycle cost-

[1] For brevity, the subject of a life cycle (products, objects, or systems) will be uniformly called the System with a capital letter.

ing; for example, see [12–16]. Cost estimating of industrial programs is investigated by leading Western firms (e.g., see [17]) and is regulated by various governmental organizations (e.g., the National Aeronautics and Space Administration [18] and United States Government Accountability Office [19]).

Much research is devoted to mathematical models of the behavior of systems consisting of many interrelated elements, an example of which is LCS: multi-agent systems [20–23]; interacting processes and systems [24–27]; systems and their properties described using systems modeling methods [8, 28]; various network structures (e.g., see [29]), particularly using graph theory (see the surveys in [29, 30]); project and program management [31–33]; stochastic networks and their applications in transport, power grids, logistics, and production [34]; firms (e.g., see the survey [35] and the references therein), organizations, and organizational structures [36–38].

Despite the significant number of models, approaches, and standards created and tested in practice, first of all, system-engineering, the absence of formal foundations considerably complicates LCS management and the coordination and integration of design, technological, economic, organizational, and other decisions. At the same time, the importance of LCS management requires defining formal bases and developing adequate models and methods of LCS management.

This paper introduces the following bases: LCS management is mathematically formalized as an optimal control problem, and approaches to solve it are proposed. Mathematical formalisms provide the maximum possible degree of rigor of the bases: mathematics has the most abstract and formal apparatus among all knowledge domains.

Representing a system, LCS requires the system-wide approach and the principle of holism. Therefore, the optimization problem is stated as a unified, holistic problem covering all LCS aspects. The multidisciplinary nature of LCS makes it difficult to form such a unified statement. The traditional practical approach is to model and optimize separate types and (or) components of LCS. This is common, for example, in operations research and related disciplines. However, the optimality of the parts does not imply the optimality of the whole. Therefore, forming a unified statement of the optimization problem becomes fundamentally important. Note that subsequent decomposition "top-to-bottom" remains correct for solving the problem by different mathematical methods at the corresponding levels of the hierarchy.

1. OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS: A QUALITATIVE DESCRIPTION OF THE PROBLEM

We consider the problem of managing the life cycle of a complex System by introducing several clarifications and definitions based on generally accepted approaches, methods, and standards.

Below, this problem will be solved for the products of aerospace, power, nuclear, transport and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries as well as information and technological systems.

Following the international and Russian standards [1, 2, 6], we understand *the life cycle* of a System as a set of repeated phenomena and processes with a period determined by the life of its standard design from conception to disposal or its particular copy from complete creation to disposal.

We use the following definitions from [5, 39]: *a project* as a set of interconnected measures to create a unique product or service under time and resource constraints; *a project program* as a set of interconnected projects and other activities to achieve a common goal under common constraints. In practice, LCS is usually implemented as a project program, i.e., a set of interconnected projects and other types of economic activities coordinated by time and resources, united by one type of System (or copy), including its updating and (or) modification, as well as by all stages of its life cycle, aimed at its development and (or) production and (or) maintenance to meet consumer requirements and obtain a positive economic result. LCS is a particular case of *complex activity*[2] (CA) [41], performed by a complex subject (an *extended enterprise*, EE) [41]. An EE is a system of autonomous but interacting firms (enterprises) united by a single structure of goals and a single technology of operation in which the parent enterprise performs the technological and business coordination.

In this case, the LCS[3] program consists of several interconnected lines of activity implemented by the extended enterprise (Fig. 1):

[2] Activity [40] is a dynamic interaction of a human with the reality in which he represents an actor (subject) purposefully influencing a subject matter (object). Complex activity [41] is an activity with a nontrivial internal structure, multiple and (or) changing goals, actor, technology, and the subject matter's role in the goal context.
[3] In this paper, the composition and sequence of lifecycle phases are given by the standards [2, 42]. Nevertheless, all results, statements, and conclusions will remain valid for other compositions of lifecycle phases as well.

- The creation and transformation of the System's information model (IM) together with the extended enterprise's IM. These activities are performed during the design (conceptual, schematic, detailed, etc.) of the System and EE. Subsequent updates again include works on System design and, if necessary, EE design.
- The creation, operation, updating, and termination (completion) of the extended enterprise itself.
- The creation, operation, updating, and disposal of the System.

Conceptualization and design (Fig. 1) consist in creating and transforming the System and extended enterprise's descriptions in the form of text documents, drawings, diagrams, and other formats, including traditional paper documents and computer data (CAD, PDM, and other engineering platforms as well as ERP, CRM, and other types of corporate management systems). In addition, various computational models are developed and used for engineering and management decision-making to assess and study the properties of the System and extended enterprise based on the current description. Computational models translate the design, engineering, logistics, and other decisions of various employees into the functional indicators of the System. The entire set of descriptions and computational models forms the information model of the System or EE, respectively; see Fig. 1.

In the early stages of the life cycle, the demand for the System, its expected characteristics, and the feasibility and effectiveness of the business idea are preliminarily analyzed; as a result, technical requirements for the System are formed. Being reflected in the operational concepts by models of the System's target application, the requirements define the desired way of operation of the future System. As the result of R&D, the IM of the elements (units, systems, and assemblies), phenomena, and processes of the System operation are formed. During detailed design and process engineering, technological and production descriptions and models of the System are formed, accompanied by the models of the extended enterprise and individual enterprises as its constituent parts. In each essential stage, the developed IMs are supplemented, updated, and detailed. They are used to verify and confirm the conformity of the System's current image to the one planned during conceptualization; see Fig. 1.

LCS management means managing the complex activity [41] of the extended enterprise: influence of the control subject on the controlled object to ensure the latter's behavior for achieving the former's goals. We define the management process of LCS as a complex activity that is:

- implemented within the LCS program throughout the entire life cycle of the System;

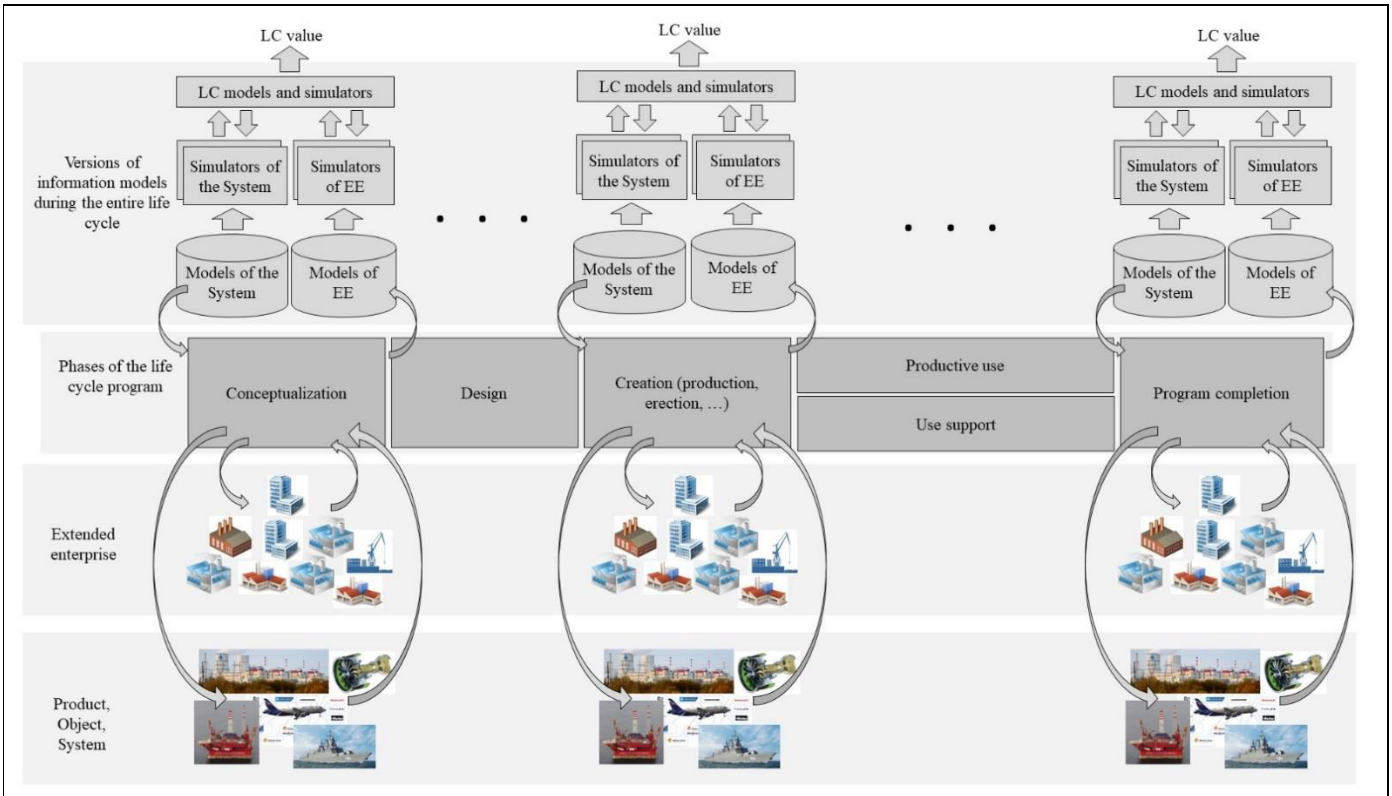

Fig. 1. Life cycle, System, extended enterprise, and their information models.

- performed by EE employees, whose design, engineering, technological, production, and other decisions affect the characteristics of the LCS program;
- coordinated by a dedicated group of specialists (the LCS program management office);
- performed to provide market or economically justified value characteristics of the program (hence, to achieve the goals of the LCS program);
- composed of:

– collecting, systematizing, and providing predictive and actual data to determine the value characteristics;

– determining and coordinating target values of the characteristics and limits on the characteristics of the System and EE, which are decomposed into limits on the components of the System and EE and groups of EE works;

– making design, engineering, technological, production, and other decisions to comply with the characteristics limits based on predicting the LCS evolution (System and EE);

– implementing responsibility for compliance with the limits by those firms, departments, and particular employees (designers, technologists, etc.) whose decisions or actions affect the corresponding component of the System and (or) EE.

In essence, LCS is nothing other than implementing one or more elements of activity to obtain benefits (a business or several businesses based on creation, production, use of the subject of LCS (product, system, or object). This interpretation of LCS leads to the generalized structure of the goals of activity (business) identical to the structure of value formation. Figures 2 and 3 show such structures, correlated with the LCS

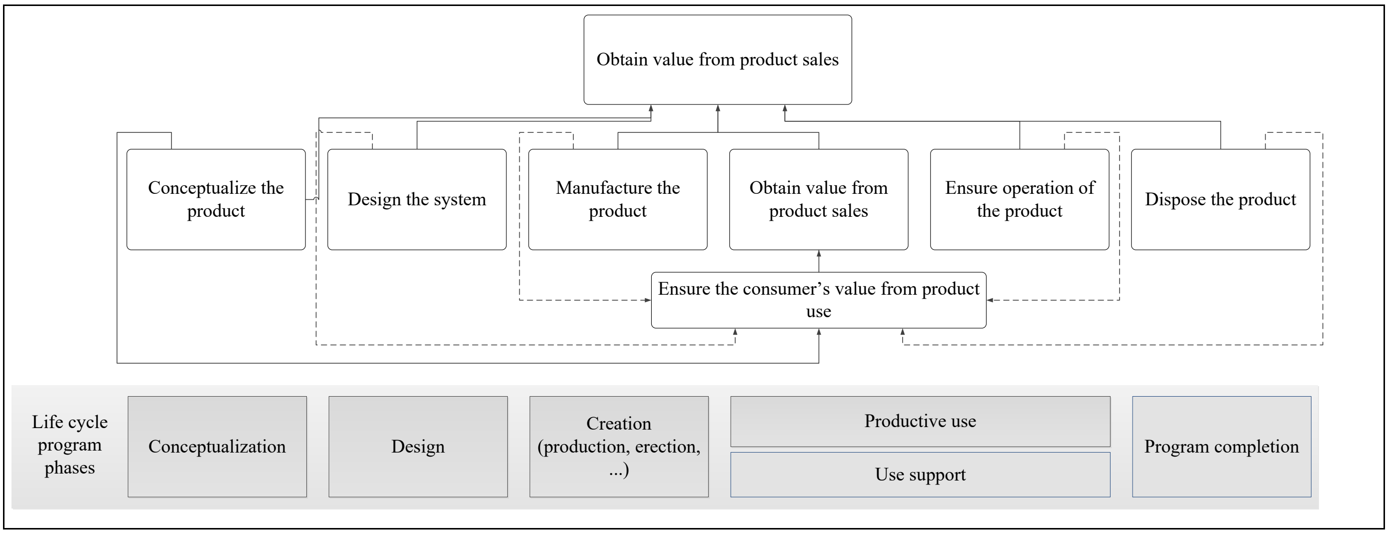

Fig. 2. The structure of forming the value or goals of complex activity performed during the life cycle of a complex system or object.

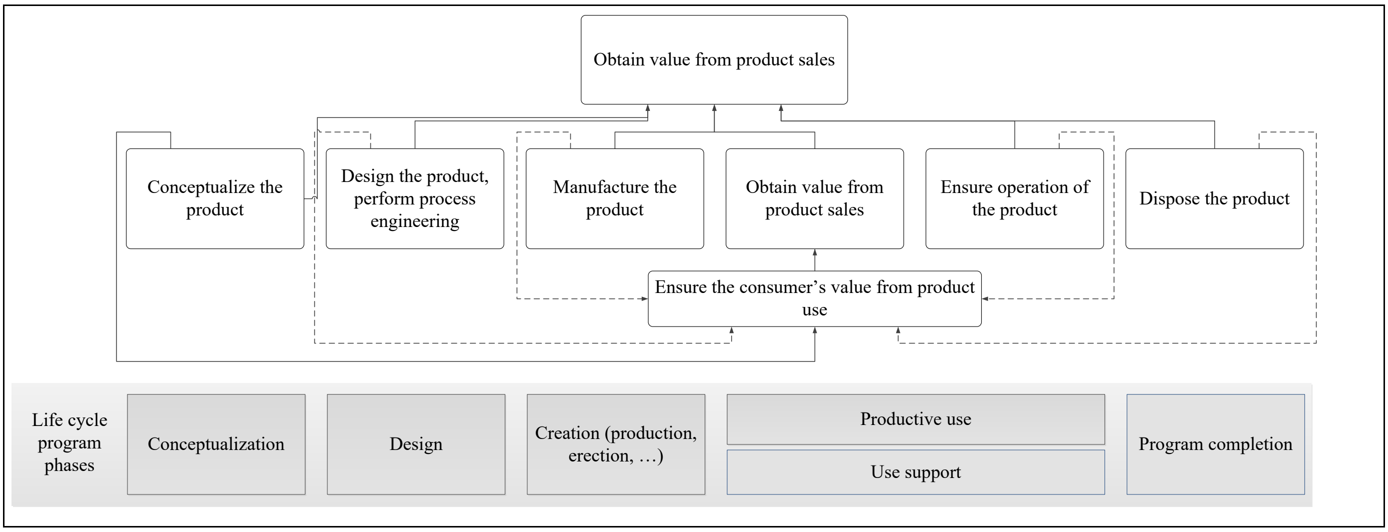

Fig. 3. The structure of forming the value or goals of complex activity performed during the life cycle of a complex product.

phases, for a complex object or system and a complex product, respectively.

The value structures (goals) are formed from the viewpoint of the subject implementing the LCS. These structures are generally self-evident and require no comment: rectangles with rounded corners represent goals and subgoals, and arrows connect the subgoals with the goals. The dashed arrows show the dependencies of the System's value on the design (including conceptual), production, operation support, and disposal works. These dependencies also have the character of "goal-subgoal" links: for the System to be useful for the consumer, the corresponding properties must be incorporated during the design, production, etc.

Note that the content of all phases of the LCS, except for productive use, includes works of various types (design, production, operation support, and disposal of the Systems). The productive use phase is primarily characterized by obtaining the target value and benefits (together with executing the corresponding works). All works during the life cycle have an auxiliary but unavoidable character and are executed only to obtain the target value and benefits during the productive use phase. At the level of the system-wide cross-industrial generalization, the LCS content can be formulated as follows.

- All LCS phases are characterized by the costs of the corresponding works directed on the creation and purposeful change of the System and EE and their information models. The achieved goals of the activity and the formed value have an auxiliary and internal character: the goals are achieved, and the value is formed in the interests of the LCS subject instead of external consumers.
- The productive use phase is characterized by obtaining the target value for the LCS subject based on the value and benefits provided to external consumers using the System.

In business practice, the control subject (or the subject implementing the LCS) is usually the management office of the LCS program, headed by a manager of the parent company, and the controlled object is the entire EE implementing the LCS. The control action is the set of decisions made by the management office of the LCS program (contracts, orders, regulations, letters, and other documents in electronic or paper form). The behavior of the controlled object is the entire set of elements of CA EE (production, engineering, technological, logistical, sales, administrative, financial, etc.), including the managerial activity of superior economic agents over subordinates.

When considering any control problem for objects with people, the key property is their ability of active choice: they act according to their internal motives and preferences. In addition, EE is a multilevel hierarchical organization (an interconnected and hierarchically subordinated set of enterprises and their subdivisions and employees). Therefore, the operation and management of EE are processes of a multilevel hierarchical nature. The presence of people in the EE structure, their property of active choice, and their key role in im0plementing LCS are the universal properties characteristic for all LCS and EE.

In such cases, it is traditional to apply game-theoretic approaches and methods of hierarchical game theory [43], active systems theory [44], organizational systems control theory [45], and contract theory [46]. Within this field of knowledge, an extended enterprise is a multilevel hierarchical dynamic active network with uncertainty and constraints on the joint activity of active elements[4] (AEs) in the form of technological networks [47–49]. In practice, an extended enterprise usually satisfies all assumptions[5] of the decomposition theorems formulated and proved in [47–49]. According to these theorems, for any feasible trajectory of LCS and irrespective of specific technological links between AEs (the LCS technology and EE organization), the control subject can construct a compensatory incentive scheme for AEs that:

– implements the trajectory of AE actions as a dominant strategy equilibrium;

– decomposes the control problem with respect to AEs, their actions, and time periods;

– ensures the minimum costs of the control subject to implement this trajectory under any possible foresight of AEs.

Such an incentive scheme reflects the principle of incentive-compatible control[6] and allows applying the enterprise control optimization scheme; see subsection 3.4.5 of the book [49]. Hence, the uncertainty of active choice of AEs can be eliminated in a mathematically correct way, and the control action can be considered the set *of action plans* of all AEs beneficial for

[4] Active elements in practice are firms, departments, divisions, work groups, and employees.

[5] The hypothesis of rational behavior of employees is the assumption that the subject chooses the actions yielding the most preferable for him results of activity considering all the information available to him; the assumption on bijective technological functions with respect to the actions of subjects and the results of their predecessors in the current period or the assumption on fully observable actions of subjects for the upper control subject (Principal); the assumption on Principal's awareness about the socially conditioned values of the cost function and reserved value of the subjects (holding for developed labor markets).

[6] Under incentive-compatible control [45], plan fulfillment is beneficial to all subjects and is an equilibrium of their game.

them. (Therefore, all AEs will seek to fulfill the action plans.) Implementing the principle of incentive-compatible control in business practice means that superior managers form tasks (plans) and incentive schemes for subordinate employees, divisions, and enterprises so that plans fulfillment be beneficial to subordinates. With such control, in particular, the strategic goals of a firm are translated to all employees down to average executives.

Now we define a class of cross-industrial methods and approaches to study LCS. For this purpose, we consider some important implementation features of LCS and, consequently, management models of LCS (Fig. 1).

When implementing the LCS, an extended enterprise plays a dual role: it acts as the subject and object of complex activity. Really, within the framework of the LCS program, there arises a need to create and change the cooperation of enterprises and create new technologies and, consequently, new enterprises.

Moreover, being complex systems, the System and EE require representations from different points of view [2] (e.g., functions, geometry, power, economics, reliability, etc.). Therefore, in practice, the System and EE are characterized by multiple descriptions and models.

The products, objects, and systems have significant industrial specifics and are often unique. In contrast, extended enterprises and life cycles have many system-wide cross-industrial similar functions and activities (financial, economic, personnel, logistics (to a considerable degree), etc.). Hence, unified descriptions and models can be used for them. As a result, all enterprises use the same best practices for organizing operational activities, the same patterns of business processes, and the same information technology platforms (ERP, CRM, MES, etc.).

Finally, in the vast majority of cases (or even always), the life cycle value is determined from the economic point of view; in turn, the economic approaches are cross-industrial and reflect the generalizing properties of the economic field of knowledge. Therefore, economic descriptions and models of the System, extended enterprise, and life cycle are typical for various industries, and unified approaches can be used. Also, the essential reflexivity of LCS is shown in the economic sphere: on the one hand, the economic parameters of the System determine the economic parameters of LCS and EE; on the other hand, the former depend on the latter. In particular, the cost price of the System considering the entire life cycle depends on the characteristics of EE, and conversely. Therefore, economic descriptions and models of the System, EE, and LCS represent an interconnected system.

Thus, we introduce the system-wide cross-industrial representation of the LCS based on the economic approaches and methods describing the processes of value formation and the associated costs.

Now we pass to the description of the quantitative model of the LCS management, formulating the four necessary components of the optimization problem:

– the state variables of the controlled system and the environment;

– constraints;

– patterns reflecting the relationships between the variables;

– the goal functions of active participants and the criterion of management effectiveness.

2. OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS: A FORMAL PROBLEM STATEMENT

The practical analysis of life cycle features allows characterizing LCS management as unfolding in time multistep decision-making process under uncertainty to achieve the best result for the control subject on the entire LCS.

Within the established economic life (business) practice, LCS are treated as assets: objects that form or should form a positive economic (business) result. Therefore, it is advisable to optimize a single quantitative indicator (the effect or value of LCS), choosing one of the widespread economic characteristics (profitability, cash flow, added value, or another).

We consider this problem in the discrete-time statement: in each period t, the LCS state is characterized by some (maybe, vector) variable $x(t)$ taking values from an admissible set X, $x(t) \in X$. Assume that the control subject selects in period t an element $u(t)$ from the set of possible decisions (and actions) U, $u(t) \in U$. In practice, under the incentive-compatibility condition (see the discussion above), the control action $u(t)$ corresponds to the set of action plans of all EE elements (firms, their subdivisions, and individual employees) formed by following the plans of the management office of the LCS program.

Regardless of the control action $u(t)$ and the LCS state $x(t)$, some value of different-nature uncertain factors is realized in each period. It is described by the vector $\omega(t) \in \Omega$, where Ω denotes the set of all possible values of the uncertain factors.

We impose no restrictions on the nature and dimension of the vectors $x(t)$, $u(t)$, and $\omega(t)$ and the sets

X, U, and Ω (except for the reachability of the corresponding maxima or minima). Also, we understand the values of LCS states and uncertain factors in an extended sense, including, if necessary, elements related to the current period t and some previous periods (possibly the entire history from the initial modeling period t_1 to the current period t inclusive). In particular, the LCS state variables $x(t)$ can be understood as a complete information model of the pair "the System (object, or system) itself; the extended enterprise implementing the LCS."

The value of the uncertain factors $\omega(t)$ is unknown to the subject when choosing the control action $u(t)$ but becomes known a posteriori. Depending on the realized values of $x(t)$, $u(t)$, and $\omega(t)$, the LCS evolves: in the next period $(t + 1)$, its state takes the value

$$x(t + 1) = F(x(t), u(t), \omega(t), t), \tag{1}$$

where $F(\cdot)$ is an *LCS dynamics function*, a given function describing the change patterns of the LCS state in the environment depending on the decisions (control actions).

The effect or value of the LCS $F(\cdot)$ during the simulation interval $[t_1, t_2]$ will be described in the traditional form:

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) = \sum_{\tau=t_1}^{t_2} \delta_{\tau,t_1,t_2} \varphi(x(\tau); u(\tau); \omega(\tau); \tau) . \tag{2}$$

Here, $\varphi(\cdot)$ is a known partial *value function of the LCS* for the control subject, δ_{τ,t_1,t_2} is the foresight function of the control subject, and the notation $\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}$ means the dependence on $x(\cdot)$, $u(\cdot)$, and $\omega(\cdot)$ on the simulation interval $[t_1, t_2]$.

For the sake of brevity, we will not write the foresight function δ_{τ,t_1,t_2} in explicit form: it can be taken into account as a factor in the partial value function $\varphi(\cdot)$.

The discrete-time representation, as well as the decisions and uncertainty reduced to a single time instant (in each period), reflects the existing practice of EE operation: planning and accounting are implemented in corresponding periods (for LCS, these are phases, stages, and more detailed periods, not necessarily of equal duration). Moreover, they do not restrict the capabilities of the proposed formalism.

The fractal hierarchy of the elements of activities, works, and, accordingly, decisions is also adequately implemented within the proposed formalism: the activities and decision-making of subordinate levels of the hierarchy are modeled in the description (2) of the LCS evolution process (the function $F(\cdot)$).

The uncertainty generated by each of the possible sources [41] (the environment, the technology and subject matter of CA, and the complex subject of activity) is fully reflected through the influence of uncertain factors (the process $\omega(\cdot)$) on the LCS evolution (the function $F(\cdot)$) and the LCS effect (value) (the function $\varphi(\cdot)$).

Then the optimal control of LCS is to maximize the LCS effect on the time interval $[t_1, t_2]$ considering the expressions (1) and (2):

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) \to \max_{\{u(\cdot)|t_1; t_2\}; u(t)\in U} . \tag{3}$$

Problem (1)–(3) is a classical dynamic programming problem with discrete time and uncertainty. Without imposing any restrictions on the nature of uncertainty, we denote by $\underset{\omega(\cdot)}{\mathrm{def}}\{\}$ the operator for eliminating the uncertainty $\omega(\cdot)$ (e.g., using the guaranteed result, expected value, or another approach).

Then the optimal control problem takes the form

$$\underset{\omega(\cdot)}{\mathrm{def}} \{\Phi(\{x(\cdot), u(\cdot)|t_1; t_2\}; \{\omega(\cdot)|t_1; t_2\})\} \to \max_{\{u(\cdot)|t_1; t_2\}; u(t)\in U} \tag{4}$$

considering the expressions (1) and (2) and the initial conditions $x(t_1 - 1) = x_0$.

3. AN ALGORITHM FOR SOLVING THE OPTIMAL CONTROL PROBLEM

We present a general algorithm for finding the optimal control (a sequence of decisions $u^*(t)$) maximizing the LCS effect.

The solution of problem (1)–(4) is based on Bellman's optimality and backward induction. We write problem (4) in the form

$$\underset{\omega(\cdot)}{\mathrm{def}}\left\{\Phi\left(\left\{x(\cdot),u(\cdot)\,|\,t_1;t_2\right\};\left\{\omega(\cdot)\,|\,t_1;t_2\right\}\right)\right\} = \underset{\omega(\cdot)}{\mathrm{def}} \sum_{\tau=t_1}^{t_2} \varphi(x(\tau); u(\tau); \omega(\tau); \tau) \to \max_{\{u(\cdot)|t_1; t_2\}; u(t)\in U} .$$

Introducing the Bellman function

$$J(x, t) = \max_{\{u(\cdot)|t; t_2\}; u(\tau)\in U(\cdot)} \underset{\omega(\cdot)}{\mathrm{def}} \sum_{\tau=t}^{t_2} \varphi(x(\tau); u(\tau); \omega(\tau); \tau),$$

we obtain a recursive formula for it starting from period t_2 backwards.

For the final period t_2,

$$J(x, t_2) = \max_{u|\in U} \{ \underset{\omega(t_2)}{\mathrm{def}} (\varphi\{x, u, \omega(t_2), t_2)\}\}. \tag{5}$$

For period $(t_2 - 1)$,

$$J(x, t_2 - 1) =$$
$$\max_{u|\in U} \{ \underset{\omega(t_2-1)}{\mathrm{def}} \{\varphi(\{x, u, \omega(t_2 - 1), t_2 - 1)\}\} +$$
$$\max_{u|\in U} \{ \underset{\omega(t_2-1)}{\mathrm{def}} \{ \underset{\omega(t_2)}{\mathrm{def}} \{\varphi(F(x, u, \omega(t_2 - 1), t_2 - 1),$$
$$u(t_2), \omega(t_2), t_2)\}\}\} =$$
$$\max_{u|\in U} \{ \underset{\omega(t_2-1)}{\mathrm{def}} \{\varphi(x, u, \omega(t_2 - 1), t_2 - 1) +$$
$$J(F(x, u, \omega(t_2 - 1), t_2 - 1), t_2)\}\}.$$

Using backward induction, we derive a general recursive formula for the Bellman function for all $t \in [t_1, t_2 - 1]$ in descending order:

$$J(x, t) = \max_{u|\in U} \left\{ \underset{\omega(t)}{\mathrm{def}} \left\{ \varphi\left(x, u, \omega(t), t\right) + J\left(F\left(x, u, \omega(t), t\right), t+1\right) \right\}\right\}. \tag{6}$$

The expressions (5) and (6) allow calculating the sequence of functions $J(x, t)$ for all $t \in [t_1, t_2]$ in descending order of the period number t. After obtaining the solution $J(x, t)$, we substitute the initial value x_0 in period $(t_1 - 1)$ to find, for all $t \in [t_1, t_2]$, the optimal control strategy (the sequence of optimal decisions $u^*(t)$) together with the optimal trajectory $x^*(t)$ of LCS implementation:

$$u^*(t) = \arg \max_{u|\in U} \{ \underset{\omega(t)}{\mathrm{def}} \{\varphi(x^*(t), u, \omega(t), t) + J(F(x^*(t), u, \omega(t), t), t + 1)\}\}; \tag{7}$$

$$x^*(t + 1) = \underset{\omega(t)}{\mathrm{def}} \{F(x^*(t), u^*(t), \omega(t), t)\}. \tag{8}$$

The relations (5)–(8) give a rigorous algorithm for making optimal decisions on LCS management. They express formal grounds for LCS management based on the following formalism:

- The state and behavior of the LCS are modeled by the vector $x(t) \in X$.
- The managerial decisions are described by the vector $u(t) \in U$.
- The uncertainty of all kinds is represented by the vector $\omega(t) \in \Omega$.
- The dynamics of the LCS and environment are described by the function $F(\cdot)$; see the relation (1).
- The LCS effect is formalized by the function $\varphi(\cdot)$ and the relation (2).

Problem (1)–(4) and the solution algorithm (5)–(8) have several fundamental properties. Let us discuss these properties and ways to apply the approach for practical LCS management and coordination and integration of heterogeneous (engineering, financial-economic, organizational, and other) decisions when implementing LCS.

We write two special cases of the optimization problem, which are important in practice.

In many cases, the partial value function is the difference between the benefits received $h(\cdot)$ and the total costs $c_i(\cdot)$ of various kinds:

$$\varphi(x(\tau), u(\tau), \omega(\tau), \tau) = h(x(\tau), u(\tau), \omega(\tau), \tau) + \sum_i c_i(x(\tau); u(\tau); \omega(\tau); \tau). \tag{9}$$

Then the effect is defined as the sum of partial values $\varphi(\cdot)$ discounted with a constant coefficient δ:

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) = \sum_{\tau=t_1}^{t_2} \delta^{(\tau-t_1)} \Big[h(x(\tau); u(\tau); \omega(\tau); \tau) - \sum_i c_i(x(\tau); u(\tau); \omega(\tau); \tau) \Big]. \tag{10}$$

The function (9) fits most (or even all) economic statements, treating benefits and costs as elements of the cash flow, profit and loss account, and accumulated value. Hence, the effect (10) can be interpreted as a *net present value* of the life cycle as an investment asset, a weighted total profit, and value added (*economic value added*, *shareholders value added*, or *market value added*).

In this case, the Bellman equations take the form

$$J(x, t_2) = \max_{u|\in U} \left\{ \underset{\omega(t_2)}{\mathrm{def}} \left\{ h(x(t_2); u(t_2); \omega(t_2); t_2) - \sum_i c_i(x(t_2); u(t_2); \omega(t_2); t_2) \right\}\right\}. \tag{11}$$

$$J(x, t) = \max_{u|\in U} \left\{ \underset{\omega(t)}{\mathrm{def}} \left\{ h(x(t); u(t); \omega(t); t) - \sum_i c_i(x(t); u(t); \omega(t); t) + \delta J\left(F\left(x, u, \omega(t), t\right), t+1\right) \right\}\right\}. \tag{12}$$

Another special case is the long life cycle with an a priori unknown completion period and stationary dynamics starting from some period:

$$x(t + 1) = F(x(t), u(t), \omega(t)). \tag{13}$$

In this case, the Bellman equations are reduced to a single equation of the form

$$J(x) = \max_{u|\in U} \left\{ \underset{\omega}{\mathrm{def}}\, h(x; u; \omega) - \sum_i c_i(x; u; \omega) + \delta J\left(F\left(x, u, \omega\right)\right) \right\}\right\}. \tag{14}$$

The solution of this equation, $J(\cdot)$, gives the optimal control

$$u^* = \arg\max_{u|\in U} \Big\{ \operatorname*{def}_{\omega} h(x;\, u;\, \omega) - \sum_i c_i(x;\, u;\, \omega) + \delta J\big(F(x, u, \omega)\big) \Big\}. \quad (15)$$

As the basic elements, the extended enterprise implementing the LCS includes the employees (individuals with the ability of active choice). In other words, the EE is an active system. This peculiarity of the problem, discussed in Section 1 above (also, see [47–49]), dictates LCS optimization conditions under the hypothesis of rational behavior of employees, the assumption on bijective technological functions, and the assumption on control subject's awareness (see footnote no. 5 and [47–49]).

Moreover, the extended enterprise is a multilevel active system with a hierarchy of technologically related firms, their subdivisions, work groups, and employees. Under such conditions, the practical formation of optimal control $u^*(t_{\text{cur}})$ consists in coordinated planning in a multilevel hierarchical dynamic active system. This problem was considered in detail in sections 2.2 and 7.1 of the books [50] and [49], respectively. The algorithmic coordinated planning models developed therein can be applied to form the optimal plan $u^*(t_{\text{cur}})$ of LCS implementation.

Another important feature of the optimal control of LCS is the need to consider the nature and characteristics of the uncertain factors $\omega(\cdot)$. Following [41], we treat all possible types of uncertainty as true uncertainty (the possibility of unique or rarely recurring events, which are not explained by the existing fundamental laws and for which there is no sufficient amount of a priori observations) or measurable uncertainty (the possibility of a priori unpredictable but repeated earlier events described by fundamental laws). In the life cycle management problem, the presence of both measurable and true uncertainty is fundamental. The reasons include the long-term duration of the life cycle, the variability of the environment (technological, political, economic, etc.), the creative nature of the life cycle processes (at least, in the early stages when designing the product, i.e., creating new knowledge about the future product, its operation in the environment, and production), and the presence of individuals with their ability of active choice within the complex subject of activity. The presence of true uncertainty specifying the behavior of $\omega(\cdot)$ and, consequently, the LCS and its effect (due to the dependencies (1) and (2)) makes it difficult to eliminate the uncertainty and solve the problem.

Traditionally, a priori knowledge about the sources and generation mechanisms of uncertainty is used to eliminate it. In this problem, due to the true uncertainty, such knowledge can never be considered objective and exhaustively complete (with respect to the described objects and phenomena) and, consequently, unchangeable. Insufficient knowledge about objective regularities compels using subjective assessments and assumptions to eliminate uncertainty (the operator $\operatorname*{def}_{\omega(t)}\{\cdot\}$ in the algorithm (5)–(8)). For dynamical phenomena, such as LCS, assessments and assumptions are formed as sets of scenarios [51] describing the evolution of phenomena under flexible control calculated depending on the realized values of uncertainty factors. The scenario approach [51] is widespread in decision-making, particularly forecasting and planning, in the areas where the true uncertainty is most significant (economics and the social and political sphere). It involves expert scenarios of the behavior of the analyzed system for calculations and forecasting. This approach is a subjective (heuristic) way to form knowledge with all its inherent disadvantages. However, this approach is applied in practice when an objective instrumental study is impossible. In the problem under consideration, the scenarios $\{x_0;\ \{\Omega_n^*(t)|\ t_1 \le t \le t_2\};\ \{U_n^*(t)|\ t_1 \le t \le t_2\}\}$ consist of the initial values x_0, the sequences of the state sets $\Omega_n^*(t)$ of uncertain factors and the sets $U_n^*(t)$ of managerial decisions (depending on the vector $\omega(t)$) ordered by the period number t.

Another important aspect of this problem and the practical application of the proposed optimization approach is the variability of environment conditions and the realization of the true uncertainty of the technology, subject matter, and subject. As a result, the optimal control strategies become irrelevant over time. Therefore, it seems reasonable to solve the problem regularly, considering all currently available information, to form the optimal control strategies. Before deciding in each current period t_{cur}, it is advisable to repeat the solution of problem (1)–(15) for the time interval $t_{\text{cur}} \le t \le t_2$ with the updated a priori knowledge (scenarios and other assumptions). Among all optimal controls $\{u^*(\cdot)|t_{\text{cur}};\ t_2\}$, only the nearest in time plan $u^*(t_{\text{cur}})$ is always used: generally speaking, the remaining controls $\{u^*(\cdot)|t_{\text{cur}} + 1;\ t_2\}$ can be not calculated, following the expressions (7)–(8). From the practical point of view, the optimal strategy should be updated when fixing each *baseline* [2] during the entire LCS.

Within the scenario approach, the interpretation of the problem solution includes a stipulation about optimal control under the accepted assumptions (realization of one scenario). On the one hand, such stipulations reduce the value of optimization; on the other, such justification is, no doubt, the best possible, especially if the set of scenarios under consideration is large, making negligibly small the possibility of realizing the LCS along a trajectory different from all such scenarios. This remark is another condition for optimizing LCS management.

The LCS state vector $x(\cdot)$, the control action $u(\cdot)$, the uncertain factors $\omega(\cdot)$, and the corresponding sets of their admissible values (X, U, and Ω) describe the complex objects and phenomena of LCS (System, EE, technology, and their evolution and operation in a complex technological, political, and economic environment). The function $F(\cdot)$ formalizes the diverse LCS evolution (all changes in the pair <System, extended enterprise>) under the managerial decisions, design, technological, production, and other works, the formation, coordination, fulfillment, and control of plans, and implementation of other activities within the EE. In turn, the function $\varphi(\cdot)$ (as well as the benefits $h(\cdot)$ and costs $c_i(\cdot)$) reflects the dependence of the LCS effect on all significant aspects of LCS implementation.

However, generally, problem (1)–(15) cannot be solved in analytical form. Therefore, the practical implementation of the algorithm (5)–(8) and (11)–(15) requires using industry-specific models to represent the functions $F(\cdot)$, $\varphi(\cdot)$ $h(\cdot)$, and $c_i(\cdot)$ and reflect the complex relationships between the characteristics of the LCS states $x(\cdot)$, the managerial decisions $u(\cdot)$, and the uncertain factors $\omega(\cdot)$, including their impact on the LCS effect $\varphi(\cdot)$. Such models promptly yield numerical values of $F(\cdot)$, $\varphi(\cdot)$, $h(\cdot)$, and $c_i(\cdot)$ under different scenarios to apply the proposed algorithms (5)–(8) and (11)–(15). A graphic metaphor of these algorithms is shown in Fig. 4.

Finally, we again list all optimization conditions for LCS management (the conditions under which the proposed approach remains mathematically rigorous):

- The hypothesis of rational behavior of EE employees is the assumption that the subject chooses the actions yielding the most preferable for him results of activity considering all the information available to him.
- The assumption on bijective technological functions with respect to the actions of subjects and the results of their predecessors in the current period or the assumption on fully observable actions of the decision-maker (Principal).
- The assumption on Principal's awareness about the socially conditioned values of the cost function and reserved value of the subjects (holding for developed labor markets). The assumption that the set of LCS scenarios is large enough to make negligibly small the possibility of realizing an LCS trajectory is different from all such scenarios.

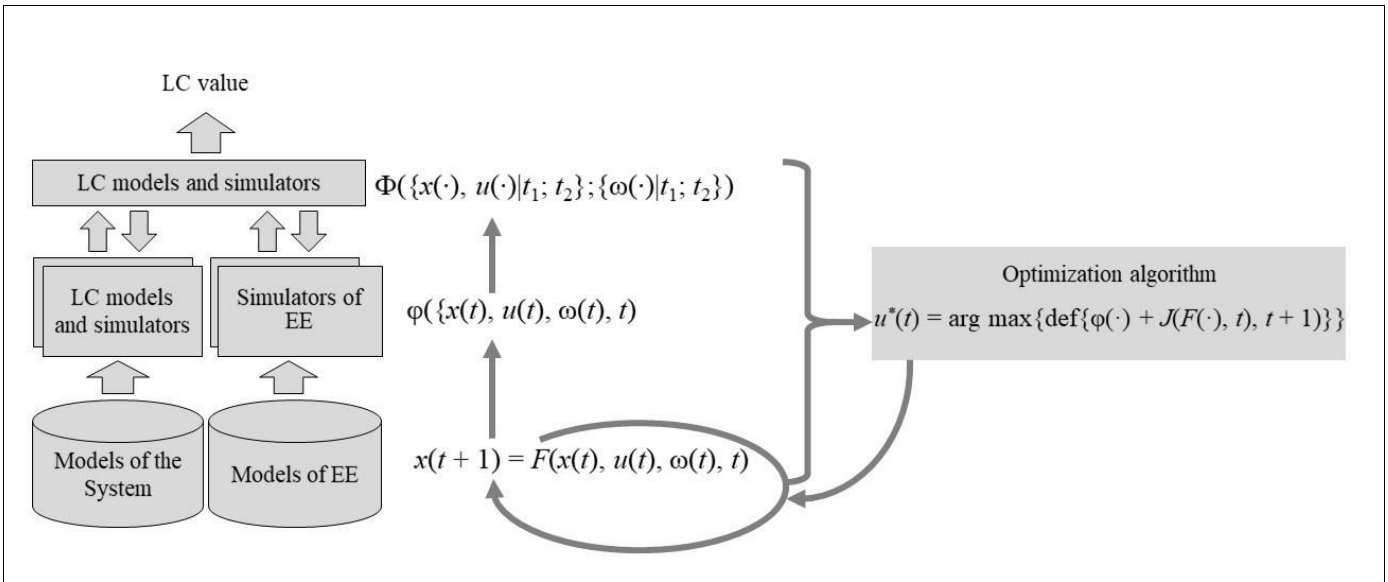

Fig. 4. Logic of the optimization algorithm.

CONCLUSIONS: AN OPTIMAL CONTROL TOOL FOR LIFE CYCLE

We summarize the results of this paper.

- A mathematically rigorous optimal control problem for the life cycle of complex products of aerospace, power, nuclear, transport, and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries, as well as information and technological systems has been stated.
- A formal algorithm for solving the corresponding optimal control problem has been presented.
- A scenario approach to apply this algorithm in practice has been proposed; optimization conditions for LCS management (the conditions under which optimization is possible) have been listed.

These results form *an optimal control tool for the LCS*.

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SOME SOCIAL AND ETHICAL NORMS OF BEHAVIOR: MATHEMATICAL MODELING USING GAME-THEORETIC APPROACHES

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Abstract. This paper overviews game-theoretic approaches to model the impact of prevailing behavioral norms (selfishness and altruism, morality on the example of Kant's imperative or the Golden Rule of ethics) on some community development. In addition, we study the effectiveness of the community depending on the prevailing worldview of its representatives. The equilibrium of the maximum cooperative income is investigated for communities whose representatives observe, to some extent, public interests rather than personal ones. The effectiveness of communities whose representatives follow Kant's imperative or the Golden Rule of ethics is considered using a game-theoretic model of social choice between two norms of behavior: one generally accepted but obsolete, and another new, not yet widespread, but advanced and progressive. The results can be used to assess the effectiveness of ongoing pedagogical work and state planning in the areas of upbringing and education.

Keywords: game theory, conflict equilibria, modeling of social and ethical norms of behavior.

INTRODUCTION

What guides each individual when choosing behavior? There are two main, essentially opposite, answers to this question.

For example, in economics, since the works of Adam Smith, there has been a tradition of believing that people are driven primarily by the interest of maximizing personal income. There is another principle: cooperating and assisting each other, individuals are willing to sacrifice personal interests to achieve a better public result. This principle can also have an economic interpretation when there is no absolute antagonism between the participants in the process under consideration, their interests overlap, and integration occurs.

However, the choice between these two models of behavior (maximizing personal income or considering public interests) may have a more general and no less important moral and ethical interpretation. How will the ethical principles and attitudes prevailing among society representatives affect social development? This question has been raised many times.

As just one example, we mention the paper [1]: T.N. Mikushina and M.L. Skuratovskaya pointed out the crisis in education, health care, ecology, and other areas due to not the economic or political situation but the declining level of ethics in society.

However, this question can also be investigated from a mathematical point of view. Below, we analyze how the ethical principles guiding individuals affect the reachability of the most favorable situations (strategy profiles). Within the traditional approach (each participant maximizes only personal income), a Nash equilibrium may not be the most beneficial strategy profile for all participants; see the well-known Prisoner's dilemma as an illustrative example.

In the model where each participant pursues personal interests considering the interests of others with some weight (cooperation and mutual assistance between participants), the most beneficial strategy profile becomes a strong equilibrium.

Also, we investigate the influence of morality (understood as Kant's imperative or the Golden Rule of ethics, close by meaning) on the decision-making process of individuals in a human community. For this purpose, we consider a game-theoretic model of choosing between two norms of behavior: one, generally accepted but less effective, and the other, new, still little known, but more beneficial for the entire community. This model demonstrates how moral and ethical norms prevailing among community members can lead the community either to progress and prosperity or, on the contrary, to decline and degradation.

First, we briefly overview the domestic and foreign research results available in the field.

1. MODELING OF SOCIAL AND ETHICAL NORMS OF BEHAVIOR: A SURVEY OF GAME-THEORETIC APPROACHES

Since A. Smith, the founder of economic theory [2], man has been supposed to be driven primarily by the individualistic motive of maximizing personal welfare. Even the term "*homo economicus*" (rational man) appeared.

However, even Smith questioned this premise. For example, in *The Theory of Moral Sentiments* [3], he introduced the concept of sympathy, which is inherent in people and sometimes causes them to act to the detriment of personal interests.

In the twentieth century, *behavioral economics* appeared. This branch of economic theory studies how psychological, moral and ethical, cognitive, and cultural factors affect decision-making. Such an analysis is in high demand because it realistically considers all aspects of human decision-making, in contrast to the classical but simplified (and often rather rough) *homo economicus* model.

Game theory is one of the mathematical tools used to analyze economic phenomena. Much has been done in this area to model processes and phenomena that previously seemed to be the subject of sociology, philosophy, and psychology.

One of the first attempts to model moral and ethical behavior using game-theoretic approaches was made in 1955 by Professor R.B. Braithwaite in a lecture delivered in Cambridge [4]. Since then, such attempts have appeared regularly in the works of various game-theory researchers.

For example, Nobel laureate J. Harsanyi [5] argued that ethical (or moral) behavior is based on the notion of collective rationality, going beyond the traditional game-theoretic concept of maximizing each participant's individual or cooperative income: "The theory of rational behavior in a social setting can be divided into game theory and ethics. Game theory deals with two or more individuals often having very different interests who try to maximize their own (selfish or unselfish) interests in a rational manner against all the other individuals who likewise try to maximize their own (selfish or unselfish) interests in a rational manner. Ethics deals with two or more individuals often having very different personal interests yet trying to promote the common interests of their society in a rational manner."

Furthermore, in the paper [6], Harsanyi used the fundamental concepts of utilitarianism to construct a more realistic model of decision-making by individuals in society. According to utilitarianism [7], the moral or ethical value of any action is determined by its aggregate *utility* or benefit for all individuals affected. In this connection, in a game-theoretic interpretation, Harsanyi introduced *a social utility function*. For each participant, its value at each point (behavioral strategy) is given by the average utility of all participants [5]. Note that utility theory was described in detail in [8].

Currently, Harsanyi's ideas have been significantly developed by many modern experts in *behavioral economics* and game theory [9–11].

Special attention should be paid to the so-called evolutionary game theory, which studies population development in biology and sociology using game-theoretic methods. Usually, this theory considers repeated games, and each strategy is assessed in terms of evolutionary stability (passing the test of time). For example, in biology, different strategies represent the genetic traits that determine the behavior of individuals and are inherited by descendants from ancestors. Evolutionary game theory substantiates the examples of gentlemanly and even altruistic behavior, i.e., behavior for the benefit of the species, often observed in nature, especially in social species. Such examples do not agree with the Darwinian assumption that natural selection occurs at the individual level [12, 13].

This survey of the foreign literature is very brief and does not claim completeness. In conclusion, we also note the book [14] by S.J. Brams, where game-theoretic approaches were applied to humanities (literature, politics, history, and even theology). Through the prism of choosing the optimal strategy of behavior, the author examined the dilemmas faced by the characters of classical literary works (e.g., Shakespeare's *Hamlet*, *Macbeth*, and *Much Ado About Nothing*) and the Biblical legends about Abraham, the Exodus of the Jews from Egypt, the Story of Samson and Delilah, and others.

Among the representatives of the domestic scientific school, we mention as an example the monograph [15] by Yu.B. Germeier and I.A. Vatel. Considering the distribution of resources between personal and public needs, the authors introduced the concept of selfishness concerning the community's needs when the participant prefers to spend all available resources on personal goals only, ignoring the community's interests.

Several ideas proposed by Germeier and Vatel' were reflected in the works devoted to social and private interests coordination engines (SPICE models) [16, 17]. These models consider a two-level community to study, like in [15], the distribution of resources between personal and public needs. In [17], all participants were divided into individualists and collectivists depending on their preferences (spending their resources on private or social purposes).

The works of V.A. Lefebvre, particularly his book *The Algebra of Conscience* [18], are also well known. The author modeled the general decision process by a person and its reflexion element (in other words, the person's analysis of himself or herself and other participants). This model is based on Boolean functions, and the set of all outcomes reduces to two ones, favorable and unfavorable. Also, the set of admissible strategies for each participant is binary and represents a choice between good and evil.

The concept of Berge equilibrium, developed by V.I. Zhukovskiy, K.S. Vaisman, and others as a counterpart to the selfish Nash equilibrium, deserves special attention. They introduced a new type of game equilibrium differing from the classical Nash equilibrium. For example, the participant's utility function (payoff) is optimized not on the set of his or her admissible strategies but the product of those of all other participants. According to the authors [19], this approach can be interpreted as follows: "...each player directs all efforts to increase the payoffs of others, forgetting about himself or herself (personal interests)."

F.L. Zak's publications [20, 21] are also of interest. The paper [21] considered the model of altruistic behavior proposed by K. Saito [22]; the paper [20], another decision-making model based on the notion *of Kantian equilibrium*. These concepts are discussed in detail in Sections 5 and 6, respectively.

In 2017, a special issue on *ethics, morality, and game theory* [10] was published in *Games*, an international peer-reviewed journal in game theory (MDPI, Switzerland). It collected papers of different contemporary authors, united by the common theme of modeling moral and ethical norms and their influence on the decision-making of game participants.

In this issue, we note the paper [11]. In addition to individualism and collectivism, it introduced the third type of behavioral strategies based on Kant's imperative ("Act only according to that maxim whereby you can at the same time will that it should become a universal law." [7]) or the Golden Rule of ethics ("Behave to others as you would like them to behave to you."). The essence of such behavior as applied to the game-theoretic model is as follows: before choosing a strategy, each participant admits that with some probability, all participants will choose the same strategy; based on this assumption, each participant makes a decision (chooses an action).

By analogy with the term "*homo economicus*" (rational man), referring to the first type of participants (individualists) guided by the personal interest of maximizing their income, the players of the third class were called *homo moralis* (moral man) [11].

This type of behavior successfully models some social, economic, and other processes. In several cases, it more realistically describes the process of human decision-making than the classical optimization model of personal payoff functions.

This paper considers a dynamic social choice model with two norms of behavior in *the coordination game*: one generally accepted but obsolete, and another new, not yet widespread, but advanced and progressive. However, applying the new norm by the overwhelming majority will yield much better results for the entire community. As shown, *homo moralis* players can serve as an example: adopt the new norm of behavior, even being in the minority and suffering losses at the initial stage, and gradually bring society to a fundamentally new qualitative level.

Under natural conditions, the transition to the new norm of behavior may not occur; see below. Hence, we consider a model of learning where the level of morality and conscientiousness in the community increases by a certain law due to educational activities. As a result, more and more individuals pass to the new norm of behavior, and gradually it becomes generally accepted in the community, leading the community to undoubted progress.

2. THE MODEL

This paper considers a game-theoretic model with N participants making choice from the same set of admissible strategies.

Assumption 1. Let Q be a metric space, and G be a compact set: $G \overset{\Delta}{=} Q^N = \underbrace{Q \times \ldots \times Q}_{N}$.

Let continuous functions (functionals) $J_i(q)$, $i = \overline{1, N}$, $q = (q_1, \ldots, q_N) \in G$, be defined on the set G; q_i denotes the strategy of player i, $q_i \in Q$, and $q^i \overset{\Delta}{=} (q_1, \ldots, q_{i-1}, q_{i+1}, \ldots, q_N)$ are the strategies of all other $(N-1)$ players under a fixed strategy q_i of player i, $q^i \in Q^{N-1}$; $J_i(q)$ is *the payoff function (functional)* of player *i*, determining the quantity of some good or resource obtained by player *i* when he or she chooses a strategy q_i and the other players choose a strategy q^i. The functions $J_i(q)$, $i=\overline{1, N}$, are supposed to be *transferable*. (In other words, the players can divide and distribute the income arbitrarily.) Note that control mechanisms for organizational systems with transferable utility functions were considered in detail in the survey [24].

Let $G(q_i)$ and $G(q^i)$ be the cutsets of the set G under a fixed strategy of player *i* (q_i) and fixed strategies of all players except *i* (opponents' strategy profile q^i), respectively.

Let $J(q) \stackrel{\Delta}{=} \sum_{k=1}^{N} J_k(q)$ be the total payoff function of all players, and $J^i(q) \stackrel{\Delta}{=} \sum_{k \neq i} J_k(q)$ be the total payoff function of all players except *i*.

Definition 1. A game satisfying Assumption 1 is called a classical game (or the game Γ) if each player *i* chooses an appropriate strategy $q_i \in Q$ to maximize his or her payoff function $J_i(q_i, q^i)$. ♦

This is a classical game formulation describing the behavior based exclusively on personal interests. To reflect that each player maximizes only his or her payoff function, distinguishing this model from the others defined below, we also call it the model of individualists or the *homo economicus* model (like in [11]).

As an alternative, we study a class of games where each player considers the interests of other participants with some weight. This fact is modeled by passing from the original game with the set of payoff functions $\left\{J_i,\ i=\overline{1, N}\right\} = \{J_i\}$ to an auxiliary one defined by the parametric family of utility functions $\{U_i(J_k, \alpha)\} = \{U_i\}$.

Definition 2. A game satisfying Assumption 1 is called the game Γ^{α} (the game of altruists) if each player seeks to maximize his or her utility function U_i defined through the payoff function $J_i(q)$ and the total payoff function $J^i(q)$ as follows:

$$U_i(q) = (1-\alpha) J_i(q) + \frac{\alpha}{N-1} J^i(q),\ \ q \in G,\ \alpha \in \mathbb{R},$$
$$\alpha \in \left[0, \frac{N-1}{N}\right],\ i = \overline{1, N}. \ ♦ \tag{1}$$

We use the term "utility function" for U_i to emphasize the transition from the original payoff functions $J_i(q)$ of the game to some new, generally speaking, artificially constructed goal functions describing a particular type of the agent's decision rationality or logic.

Altruism (from Latin *alter* "another") refers to this class of games to emphasize a special kind of decision rationality or logic: the players consider the interests of other participants with some weight. In [15], this type of rationality was called *collectivism*, and this type of behavior is commonly said to be *prosocial.*

We introduce the change of variables $\beta \stackrel{\Delta}{=} \alpha \frac{N}{N-1}$. Since $\alpha \in \left[0, \frac{N-1}{N}\right]$, we have $\beta \in [0, 1]$, and the utility function $U_i(q)$ can be written as

$$U_i(q) = (1-\beta) J_i(q) + \frac{\beta}{N} J(q),\ \ \beta \in [0, 1]\,. \tag{2}$$

The model defined by the utility functions (2) can be treated as *a public goods game*: the functions $\beta J_i(q)$ determine the contribution of participant *i* to some socially important needs. The term $(1-\beta) J_i(q)$ is the resource share he or she leaves for personal needs, and the term $\frac{\beta}{N} J(q)$ is what he or she obtains from society.

Different representations of the function U_i are convenient in different games. For example, in two-player games ($N=2$), formula (1) reduces to $U_i(q) = (1-\alpha) J_i(q) + \alpha J^i(q)$, which is very convenient to use. The expression (2) is more suitable in multiplayer games, and the main results below are obtained for it.

Note that the original goal functions of participants are replaced by some new functions (a linear combination of the original goal functions of other participants) in the theory of active systems to solve *criterion control* problems; in particular, see the papers [25, 26] by V.N. Burkov, D.A. Novikov, and colleagues. Individual rationality (benefit) may differ from the collective one for the active elements composing an active system. If there is a third party (Principal) interested in the best collective results and endowed with appropriate powers, the original individual payoff functions of the players are replaced by linear combinations of the payoff functions of other players. This transformation reflects the requirement imposed on the active elements: work together for a common result. The Principal determines the effectiveness criterion of the result, and the linear combination coefficients can be assigned accordingly.

As concluded by the authors [24], the efficient operation of socio-economic systems cannot be ensured without coordinating the interests of all system participants.

3. THE SYSTEM OF CONFLICT EQUILIBRIA

First, we define the classical Nash equilibrium in the notations introduced above.

Definition 3. A strategy profile $q^* \in G$ is called a Nash equilibrium (a $\bar{C}^N$-optimal strategy profile) if

$$\max_{q_i \in G\left(q^{i*}\right)} J_i\left(q^{i*}, q_i\right) = J_i\left(q^*\right),\ i = \overline{1, N}, \tag{3}$$

where $q^i = \left(q_1, \ldots, q_{i-1},\ q_{i+1}, \ldots, q_N\right)$. ♦

The maximum in (3) is taken over all admissible strategies q_i of participant *i* from the cutset of the set G under a fixed opponents' strategy profile q^{i*} in the equilibrium q^*.

However, this equilibrium has several drawbacks: it does not always exist, and even when it does, it may not determine the most advantageous strategy profile for all participants (see an example below). Therefore, in addition to this classical equilibrium, this paper considers the system of conflict equilibria developed by E.R. Smol'yakov [27, 28]. This system is a set of strengthening equilibria in which the weakest one exists in any game satisfying Assumption 1. Thus, it is possible to find the strongest equilibrium (solution) in any game among the existing ones.

Some basic equilibria of this system are as follows.

Definition 4. A strategy profile (point) $q^* \in G$ is said to be A_i-optimal if either $G\left(q^{i*}\right) = q_i^*$, or with each strategy $q_i \in G\left(q^{i*}\right) \backslash q_i^*$ of player *i* we can associate at least one response strategy $\hat{q}^i = \hat{q}^i \left\langle q_i \right\rangle$ of the other ($N-1$) players such that

$$J_i\left(\hat{q}^i,\ q_i\right) \le J_i\left(q^i\right).$$

Denoting by A_i the set of all A_i-optimal strategy profiles, we call a strategy profile (point) $q^* \in G$ a symmetric weak active equilibrium (in short, *A*-equilibrium) if $q^* \in A_1 \cap \ldots \cap A_N \overset{\Delta}{=} A$. ♦

The notation $\hat{q}^i \left\langle q_i \right\rangle$ indicates that the other participants choose their strategy $\hat{q}^i$ responding to the strategy q_i of participant *i* if he or she decides to deviate from the equilibrium strategy $q_i^* \left(q_i \in G\left(q^{i*}\right) \backslash q_i^*\right)$.

Simply put, the equilibrium given by Definition 4 has the following meaning. If participant *i* wishes to deviate from the equilibrium strategy q_i^* to another acceptable strategy q_i in a given strategy profile (seeking to increase his or her payoff function J_i), then the other participants can punish the renegade with the counterstrategy $\hat{q}^i \left\langle q_i \right\rangle$. As a result, participant *i* will not receive more than he or she would receive in the equilibrium q^*. Therefore, the strategy profile q^* is called an equilibrium: none of the participants will benefit from deviating from it. (Otherwise, they risk being punished by the other players.)

The symmetric *A-equilibrium* is weakest in the system of conflict equilibria under consideration. As proved in [27], this equilibrium exists at least in any ε-approximation, $\forall \varepsilon > 0$, in any games satisfying (rather general) Assumption 1. In the numerical solution of real problems, equilibria are approximated. Hence, it does not matter for applications whether the strategy profile q^* turns out to be an exact *A-equilibrium* or an equilibrium with an admissible accuracy ε, where ε is an arbitrarily small number. Thus, introducing this equilibrium resolves the existence problem of game solutions.

However, as a rule, *A-equilibria* are not the only ones. Therefore, the following concepts naturally strengthen (contract) the set *of A-equilibria*.

Definition 5. A strategy profile (point) $q^* \in A_i$ is said to be B_i-optimal if

$$\max_{q^i \in A_i\left(q_i^*\right)} J^i\left(q_i^*,\ q^i\right) = J^i\left(q^*\right). \tag{4}$$

A strategy profile q^* is called an $\in$-equilibrium if $q^* \in \bigcap_{i=1}^{N} B_i \overset{\Delta}{=} B$, where B_i denotes the set of all B_i-optimal strategy profiles.

B-equilibrium is based on the following logic: each participant selects a circle of strategy profiles from which he or she finds it disadvantageous to deviate due to threats to decrease the payoff (the set of A_i-equilibria); then he or she suggests the other participants choose the best strategy profiles for them on this set. Thus, the equilibrium becomes more stable to any deviations of the participants.

Therefore, the expression (4) extracts the best strategy profile for the other participants in the cutset of the set A_i under the fixed equilibrium strategy q_i^* of participant *i*.

The next equilibrium is a possible strengthening of *B*-equilibrium.

Definition 6. A strategy profile (point) $q^* \in A_i$ is said to be C_i-optimal if

$$\max_{q^i \in G\left(q_i^*\right)} J^i\left(q_i^*,\ q^i\right) = J^i\left(q^*\right). \tag{5}$$

A strategy profile $q^* \in G$ is called a C-equilibrium if $q^* \in \bigcap_{i=1}^{N} C_i \overset{\Delta}{=} C$, where C_i denotes the set of all C_i-optimal strategy profiles. ♦

The difference between B- and C-equilibria is as follows. When searching for B-equilibrium, participant i suggests the others choose their best strategy profile on the set of A_i-optimal ones (the maximum in (4) is taken over the cutset $A_i\left(q_i^*\right)$). When searching for C-equilibrium, the other participants are suggested in (5) to choose the best strategy profile on the entire cutset of the game set $G\left(q_i^*\right)$. Hence, C-equilibrium is more stable to the deviations of all participants than B-equilibrium.

In two-player games, C-equilibrium and Nash equilibrium coincide.

Let us further strengthen B- and C-equilibria.

Definition 7. A strategy profile $q^* \in B_i$ is said to be $\bar{D}_i$-optimal if

$$\max_{q \in B_i} J_i(q) = J_i\left(q^*\right) \tag{6}$$

or (in an equivalent expanded form),

$$\max_{q_i \in Pr_{Q_i} A_i} J_i\left(\operatorname{Arg} \max_{q^i \in A_i(q_i)} J^i\left(q_i, q^i\right)\right) = J_i\left(q^*\right).$$

It is called a $\bar{D}$-equilibrium if $q^* \in \bigcap_{i=1}^{N} \bar{D}_i \overset{\Delta}{=} \bar{D}$. ♦

This equilibrium strengthens the concept of *C-equilibrium* introduced above.

It has the following interpretation. First, all participants except i choose their most beneficial strategy profiles in the cutsets of the set $A_i\left(q_i\right)$ for each admissible strategy of participant i (the set of B_i-optimal strategy profiles; see the variable of J_i in (6)). Then participant i chooses a strategy from the projection of the set A_i into the set of his or her admissible strategies $Q_i - Pr_{Q_i} A_i$ that maximize the goal functional J_i.

The concept of D-optimal strategy profiles (see below) has a similar meaning. The difference is that player i chooses not on the set B_i but the set C_i.

Definition 8. A strategy profile $q^* \in C_i$ is said to be D_i-optimal if

$$\max_{q \in C_i} J_i(q) = J_i\left(q^*\right).$$

It is called a D-equilibrium if $q^* \in \bigcap_{i=1}^{N} D_i \overset{\Delta}{=} D$. ♦

As an example, consider these equilibria for both classes of players, Γ and Γ^{α}.

4. COMPARING DIFFERENT SYSTEMS OF CONFLICT EQUILIBRIA

Note that the principle of threats and counterthreats (see below) is also used in different variations in other systems of conflict equilibria, particularly the concept *of equilibrium in safe strategies* (ESS) proposed and developed by M.B. Iskakov et al. in [29, 30] and other publications.

A key notion in this concept is a threat to player i by player j in a strategy profile $q \in G$.

A strategy profile $\left(q_j^{'}, q^j\right)$ is threatening if $J_j\left(q_j^{'}, q^j\right) \geq J_j(q)$ and $J_i\left(q_j^{'}, q^j\right) < J_i(q)$ [29].

We make two remarks as follows.

- The concept of ESS assumes that players threaten each other separately. For example, the possibility of two players jointly choosing a strategy profile to punish a third player is not considered a threat. As noted in [30], designing a construction similar to ESS for coalition interaction seems difficult so far. According to the definition of A-equilibrium, the other players can create threats collectively (via a common strategy $\hat{q}^i = \hat{q}^i \langle q_i \rangle$ of other participants in response to the strategy q_i of participant i).
- A-equilibrium is based on the assumption that players can threaten each other even to their detriment. In other words, the requirement $J_j\left(q_j^{'}, q^j\right) \geq J_j(q)$ is not applied. (It means that in a threatening strategy profile $\left(q_j^{'}, q^j\right)$, the value of the payoff function of threatening players must be greater than in a strategy profile $q \in G$ containing a threat.)

The absence of this requirement indicates that, generally speaking, there may be some agreements and hidden coalitions between the players: someone may sacrifice himself or herself (jump on the embrasure) to reduce his or her payoff function, letting his or her team win in the end. However, as emphasized above, collective interaction is not considered in ESS.

Due to these features, the two systems of equilibria are difficult to compare because, e.g., some A-equilibrium may contain threats from other players and, therefore, will not be a simple (zero-order) ESS [29, 30].

ESS of higher orders can be contained in the set of A-equilibria. For example, consider the following bimatrix game [30]:

$$J_1 = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix},\ J_2 = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix};$$

$$A_1=\begin{bmatrix}+ & +\\ + & .\end{bmatrix},\ A_2=\begin{bmatrix}+ & .\\ + & +\end{bmatrix},\ A=A_1\bigcap A_2=\begin{bmatrix}+ & .\\ + & .\end{bmatrix};$$

$$B_1=\left(a_{11},\ a_{21}\right),\ B_2=\left(a_{21},\ a_{22}\right),\ B=\left\{a_{21}\right\};$$

$$C_1=\left\{a_{11}\right\},\ C_2=\left\{a_{21}\right\},\ C=\varnothing;$$

$$\bar{D}_1=\left\{a_{21}\right\},\ \bar{D}_2=\left\{a_{22}\right\},\ \bar{D}=\varnothing.$$

Here, the strongest game equilibrium is $B=\left\{a_{21}\right\}$. By the way, this strategy profile corresponds to the maximum cooperative income. The simple ESS is a_{11}, not the most beneficial strategy profile for both participants; the first-order ESS is a_{21}. For details, see [30].

Note that the compared systems have a common iterative design scheme. For example, suppose that the initial (zero) iteration yielded no equilibrium stronger than A- equilibrium. Then we would consider the next (first) iteration, replacing the entire set G of admissible strategy profiles with its subset of A-equilibria. For the new game, we would find the corresponding equilibria $A^1,\ B^1, C^1,$ and D^1. This process can be continued until a next iteration yields a sufficiently strong equilibrium. Similarly, in the concept of ESS, we search for equilibria of orders 0, 1, and so on.

Another example of an iterative equilibrium design scheme is the concept of the *double best response* proposed by N.I. Bazenkov [31]. However, the equilibrium search algorithm described in this paper stops after the second iteration.

Finally, note that Iskakov [30] considered Smol'yakov's system of conflict equilibria. As he suggested, ESS can be included in this system as one of the equilibria $A,\ B,\ C, D$, etc.

5. THE EXISTENCE OF CONFLICT EQUILIBRIA IN THE CLASS Γ^α

Recall the notations adopted in Assumption 1:

– $J\left(q\right)\overset{\Delta}{=}\sum_{i=1}^{N}J_i\left(q\right),\ q\in G$, is the total payoff function of the players in the original game G.

– $J^i\left(q\right)\overset{\Delta}{=}\sum_{k\neq i}J_k\left(q\right)$ is the total payoff function of all players except *i*.

– $U_i\left(q\right)\overset{\Delta}{=}\left(1-\alpha\right)J_i\left(q\right)+\dfrac{\alpha}{N-1}\sum_{\substack{k=1,\\ k\neq i}}^{N}J_k\left(q\right),\ q\in G,$ is the utility function of player *i* in the game Γ^α.

Definition 9. The total payoff function J is said to achieve maximum in a strategy profile $q^*\in G$ if $\forall q\in G, q\neq q^*:\ J\left(q^*\right)\geq J\left(q\right)$.♦

Under Assumption 1, $J_i\left(q\right)$ are continuous functions defined on G (a compact set defined in the product $Q_1\times\ldots\times Q_N$ of metric spaces Q_i, $i=\overline{1,\ N}$). Hence, their sum (the total payoff function $J\left(q\right)=\sum_{i=1}^{N}J_i\left(q\right)$) is also a continuous function on the set G.

A continuous function achieves its supremum and infimum on a compact set. Therefore, the function J achieves maximum on the set G.

Let us formulate an existence theorem for Nash equilibrium in games satisfying Assumption 1. We denote by $\Gamma^{\alpha_{NE}}$ the games of the type Γ^α with $\alpha=\alpha_{NE}$.

Theorem. *Assume that in a game satisfying Assumption* 1, *the total payoff function achieves maximum in a strategy profile* q^*. *Then there exists* $\alpha_{NE}\in\mathbb{R},\ \alpha_{NE}\in\left[0,\ \dfrac{N-1}{N}\right]$, *such that:*

– The strategy profile q^* *is a Nash equilibrium (* $\bar{C}^N$*-optimal) in the game* $\Gamma^{\alpha_{NE}}$.

– The strategy profile q^* *is a Nash equilibrium in the game* Γ^α $\forall\alpha\in\left[\alpha_{NE},\ \dfrac{N-1}{N}\right]$.

Theorem 1 is proved in the Appendix. Similar existence theorems for other equilibria ($B, C,$ and D) can be formulated and proved by analogy.

This theorem leads to the following conclusion. In the game Γ^α with cooperation and mutual integration between the participants, the most beneficial strategy profile (the maximum cooperative income) becomes a Nash equilibrium for some $\alpha=\alpha_{NE}$. For all α exceeding this value, the strategy profile remains an equilibrium.

In non-antagonistic games with possible cooperation between the players, the parties can agree on appropriate strategies maximizing the cooperative income, even if it is not an equilibrium. This is true under one condition: the parties can agree on a fair and stable distribution (imputation) of the cooperative income (i.e., no player will deviate from it). Unfortunately, the classical theory of cooperative games does not provide such a solution.

Nevertheless, fair cooperative income distribution methods can be applied to determine the coefficients α. Smol'yakov [32] proposed a method for distributing the cooperative income of a coalition P_N of N participants in a strategy profile q^0 proportionally to the payoffs of the coalition participants in the strongest equilibrium q^* (by assumption, unique). The basic equilibria to find the strongest one have been defined in the previous section. Their various modifications were introduced in [27]. The paper [32] also presented a scheme of relations between existing equilibria to identify the strongest one in the game.

The imputation of the cooperative income $J_{P_N}(q^0)=\sum_{k\in P_N} J_k\left(q^0\right)$ of a coalition P_N at the coalition's maximum income point q^0 is given by $x_1+x_2+x_3+\cdots+x_N=J_{P_N}(q^0)$, where x_i denotes the income of participant *i*.

In addition, $x_i=\gamma_i\cdot J_{P_N}(q^0)$, where

$$\gamma_i=\frac{J_i\left(q^*\right)}{J_{P_N}\left(q^*\right)}=\frac{J_i\left(q^*\right)}{\sum_{k\in P_N} J_k\left(q^*\right)},\ i=1,\ 2,\ldots,\ N. \quad (7)$$

The expression (7) implies $\sum_{k\in P_N}\gamma_k=1$.

For heterogeneous communities (different coefficients α for different participants), the utility functions $U_i\left(q\right)$ (2) can be represented as

$$U_i(q)=(1-\gamma_i)J_i(q)+\frac{1}{N}\sum_{k=1}^{N}J_k(q)\,\gamma_k,\ i=1,2,\ldots,N. \quad (8)$$

For each participant *i*, the coefficient γ_i (7) is the value of his or her goal function J_i divided by the cooperative income $J\left(q^*\right)=\sum_{i=1}^{N}J_i\left(q^*\right)$: $\gamma_i=\frac{J_i\left(q^*\right)}{J\left(q^*\right)}$ at the point q^* of the strongest equilibrium.

Note that the utility functions (2) and (8) establish a rule to distribute financial resources in the community. For example, in F.L. Zak's paper [33], a function like (8) was used to construct a taxation model. With $J_i\left(q\right)$ and γ_i being interpreted as the income and tax rate of participant *i*, respectively, the first term in (8) describes the residual financial resources of participant *i* after paying taxes.

The second term in (8) expresses the amount of public goods received by each individual (in this case, equally distributed among all community members). As emphasized by Zak, this resource distribution scheme in the community has a disadvantage: the Nash equilibrium turns out to be inefficient (far from the strategy profile with the maximum cooperative income for all participants). The reasons are the following: as each agent believes, his or her efforts have little effect on the amount of public goods received.

Of course, the fair distribution of resources in a community is an issue going beyond pure mathematics into the realm of sociology, economics, and even philosophy. For example, some thinkers, particularly T.N. Mikushina and E.Yu Il'ina [34], claim that the amount of public goods due to an individual should be proportional to the individual's contribution to these goods. In this case, the function $U_i\left(q\right)$ can be written as

$$U_i(q)=(1-\gamma_i)J_i(q)+\gamma_i\sum_{k=1}^{N}J_k(q)\,\gamma_k,\ i=1,2,\ldots,N.$$

The difference with (8) is distributing the total amount of public goods $\sum_{i=1}^{N}J_i\left(q\right)\gamma_i$ among the community members not equally but proportionally to γ_i (the shares of their contributions to the cooperative income $J\left(q^*\right)$ in the strongest equilibrium).

As a way out of the quandary, Zak suggested another mechanism based on the so-called *Kantian equilibrium*, which differs from Nash equilibrium. We consider a similar model in the next section.

6. MODELING MORALITY IN THE SENSE OF FOLLOWING THE GOLDEN RULE OF ETHICS OR KANT'S IMPERATIVE

Passing to the third model, we note that the types of behavior and decision-making by Definitions 1 and 2 have something in common. Both individualists and collectivists (or altruists, as they are called in some works) are somewhat indiscriminate in their means: the former pursue personal interest only, whereas the latter also care about the public welfare with some weight. According to Assumption 1, however, the set of admissible strategies (actions) of all participants is the same (*Q*), and players of both classes neglect the consequences when the other participants choose the same strategy. Such analysis is performed by players of the third class, *homo moralis* (moral man) [11, 35].

This type of behavior is based on a well-known ethical principle, Kant's categorical imperative: "act only according to that maxim whereby you can at the same time will that it should become a universal law." A close ethical principle is called the Golden Rule of ethics: "*behave to others as you would like them to behave to you*" [37].

In [11, 35], this principle was modeled as follows. Participant *i* assumes that each of the other players will choose the same strategy as he or she does with a probability $k_i\in[0,1]$ and a different strategy with the probability $(1-k_i)$. (The value k_i can be treated as the morality level of participant *i*.) Choosing a strategy $q_i\in Q$, each participant *i* obtains the well-known Bernoulli scheme with $(N-1)$ trials (corresponding to the other players) and two outcomes for trial j, $j=\overline{1, N-1}$: participant j chooses the strategy $q_j = q_i$ or another strategy $(q_j \neq q_i)$. Instead of the original payoff functions, the game is played on the utility functions representing the mathematical expectations of the binomial distribution for each player.

Definition 10. A game satisfying Assumption 1 is called the game Γ^{hm} (in the class of moralists) if each player seeks to maximize not his or her original payoff function J_i but the utility function

$$W_i(q_i, q^i) = \mathbb{E}_{k_i}[J_i(q_i, \tilde{q}^{\,i})],\ q_i \in Q,$$

$$k_i \in \mathbb{R},\ k_i \in [0, 1],\ i = \overline{1, N}, \qquad (9)$$

where $\tilde{q}^i$ is a random $(N-1)$-dimensional vector taking values from the set Q^{N-1} with the following distribution: $m \in \{0, \ldots, N-1\}$ components are equal to q_i with the probability $k_i^m (1-k_i)^{N-m-1}$, and the other components preserve their original values. ♦

For every m, there are $\binom{N-1}{m} \overset{\Delta}{=} C_{N-1}^m$ ways to choose m from $(N-1)$ components of q^i.

In addition, for $k_i = 0$, only one value of the random vector $\tilde{q}^i = q^i$ will have non-zero (unitary, full) probability. That is, the random vector takes a single value, the one in the variable of W_i. In this case, $W_i(q_i, q^i) \equiv J_i(q_i, q^i)$: *homo moralis* players with $k_i =$ 0 are, in fact, individualists of the first class Γ. This is illustrated in the social choice model below.

For example, consider a three-player game. The utility function (9) takes the form

$$W_i(q_i, q_j, q_k) = (1-k_i)^2 J_i(q_i, q_j, q_k) +$$
$$k_i(1-k_i) J_i(q_i, q_i, q_k) +$$
$$k_i(1-k_i) J_i(q_i, q_j, q_i) + k_i^2 J_i(q_i, q_i, q_i).$$

On the contrary, for $k_i = 1$, this decision-making model becomes identical to the *Kantian equilibrium* model pioneered by J.-J. Laffont [38] and further developed by J. Roemer [39, 40]. In Kantian optimization, players ask themselves: "If I deviate from my strategy and all other participants similarly deviate from their strategies, would I prefer the new state?" [33].

7. THE SOCIAL CHOICE MODEL WITH TWO BEHAVIORAL NORMS

We consider the following problem as a simple example of a coordination game. A married couple decides on spending the evening: paying a visit or staying at home. In addition, the payoff matrices have greater values on the diagonal (the couple spends the evening together, away or at home). The table below presents the payoff matrix of this game. (The payoffs of the parties are given in parentheses.) Clearly, this game has two Nash equilibria when the spouses decide to spend the evening together: away or at home. Moreover, the strategy profile "paying a visit together" is Pareto-efficient with respect to the strategy profile "staying at home together."

The Family Choice Game

	Paying a visit	Staying at home
Paying a visit	(10, 10)	(0, 0)
Staying at home	(0, 0)	(5, 5)

Note that coordination games have many economic applications [5].

Now we consider a coordination game described in [2]. It represents a social choice model with many participants. Let *N* participants of some community independently choose between two norms of behavior (strategies) *A* and *B*. Norm *A* is more efficient than norm *B*: if all individuals pass to *A*, the welfare of each participant (in the broad sense) will be higher than in the case when everyone chooses *B*. However, norm *B* is generally accepted. In the beginning, all participants in the model choose norm *B*, and norm *A* is new for them.

For example, young people, going through the socialization process and finding themselves in new social groups (classmates, friends, etc.), adopt bad habits from some group members. However, there are also opposite examples. Let us imagine a group of acquaintances addicted to some bad habit. If someone from this company manages to give up the habit, he or she first feels discomfort as a kind of black sheep. Gradually, however, the example of this person is followed by others; after a critical share of those given up, those still addicted to the habit receive dubious glances of the former. Gradually, society begins to change its attitude to the bad habit: its advertising in the media is banned, and its sales to minors are prohibited. Thus, changing attitudes in society and increasing restrictions make it more and more difficult to follow bad habits until, finally, a healthy lifestyle becomes the norm. In turn, society obtains fewer illnesses, healthier children, and a stronger gene pool. In short, the transition to a new norm of behavior positively affects the development of the entire community. There are many similar examples.

Under what conditions will the community pass from the less efficient (obsolete) norm *B* to the more efficient (progressive) norm *A*? To answer this question, we formulate the model as a game. First, we consider the static case and then investigate the model in dynamics.

Let $q_i \in Q = \{0, 1\}$ be the choice of participant *i*, where $q_i = 1$ corresponds to norm *A* and $q_i = 0$ to norm *B*. If participant *i* chooses norm *A*, and n_A other participants choose the same norm, the payoff function of participant *i* will take the value $a\, n_A$. If he or she chooses norm *B*, and n_B other participants do the same, the value of his or her utility function will be $b\, n_B$. Assume that $0 < b < a$.

In the individualist model Γ, the payoff functions have the form

$$J_i(q_i, q^i) = aq_i \sum_{\substack{j=1,\\ j\neq i}}^{N} q_j + b(1-q_j)\sum_{\substack{j=1,\\ j\neq i}}^{N}(1-q_i), \qquad (10)$$
$$q_i \in Q,\ q^i \in Q^{N-1}.$$

For collectivists (the game Γ^α), the utility functions have the form

$$U_i(q_i, q^i) = (1-\alpha)J_i(q_i, q^i) + \frac{\alpha}{N-1}\sum_{\substack{k=1,\\ k\neq i}}^{N} J_k(q_k, q^k),\ q_i \in Q,\ q^i \in Q^{N-1}, \qquad (11)$$

where J_i and J_k are given by (10), and the parameter $\alpha \in \left[0, \frac{N-1}{N}\right]$ determines the extent to which each individual prefers the public interests. For $\alpha = 0$, the functions (10) and (11) become equivalent: $J_i \equiv U_i$. Clearly, for the first and second classes of players, the game has two Nash equilibria regardless of the coefficient α: all participants choose norm *A* ($q = (1,\ldots, 1)$) or norm B ($q = (0,\ldots,0)$).

Thus, if norm *B* is widespread, each player believes that the others will choose it, there are many players, and direct agreement (cooperation) between them is impossible, then norm *B* will continue to be an equilibrium for the participants pursuing exclusively personal interests. Really, each player will gain nothing from deciding to choose norm *A* singly.

A similar situation occurs in the class Γ^α (players considering the interests of others). Even for the highest coefficient α, when $U_i(q) = \frac{1}{N}J(q) = \frac{1}{N}\sum_{k=1}^{N} J_k$ (the utility function maximized by each player is directly proportional to the total payoff function), none of the players will wish to deviate from the less efficient norm B: the entire community will gain less from single participant's pass to norm A.

Thus, norm *B* will remain an equilibrium in both classes: the entire community will not pass to the new norm.

However, the situation fundamentally changes for the third class of players (*homo moralis*). According to Definition 3, these players maximize the utility functions

$$W_i(q) = \mathbb{E}_{k_i}\,[J_i(q_i,\ \tilde{q}^i)],$$

where $\tilde{q}^i$ is a random $(N-1)$-dimensional vector with the following distribution: $m \in \{0,\ldots, N-1\}$ components are equal to q_i with the probability $k_i^m(1-k_i)^{N-m-1}$, and the other components preserve their original values. This distribution is similar to the well-known binomial distribution $B_{k_i}^{N-1}$ but has the following difference: $(N-m-1)$ components must preserve their original values (those at the point $q \in G$ where the function $W_i(q)$ is determined).

Thus, the function $W_i(q_i, q^i)$ is given by

$$W_i(q_i, q^i) = \sum_{m=0}^{N-1}\binom{N-1}{m} k_i^m (1-k_i)^{N-m-1} \times$$
$$\overbrace{[aq_i\ (mq_i + \frac{N-1-m}{N-1}\sum_{\substack{j=1,\\ j\neq i}}^{N} q_j) +}^{\mathrm{I}}$$
$$\underbrace{b(1-q_i)(m(1-q_j) + \frac{N-1-m}{N-1}\sum_{\substack{j=1,\\ j\neq i}}^{N}(1-q_j))]}_{\mathrm{II}}, \qquad (12)$$

where expressions I and II correspond to the cases $q_i = 1$ and $q_i = 0$, respectively. The terms with the coefficient $\frac{N-1-m}{N-1}$ reflect that the other players (except those m with the strategies q_i) preserve their original strategies.

Clearly, for $k_i = 0$, formulas (10) and (12) become identical, demonstrating again the property $W_i(q_i, q^i) \equiv J_I(q_i, q^i)$ for $k_i = 0$. (*Homo moralis* participants with k_i=0 become individualists.)

If all players choose strategy *A*, participant *i* will obtain $(n-1)a$ by choosing *A*. With strategy *B*, his or her utility function takes the form

$$W_i(0, q^i = (1,\ldots, 1)) = \sum_{m=0}^{N-1}\binom{N-1}{m} k_i^m (1-k_i)^{N-m-1} m. \qquad (13)$$

Now we simplify the expression (13). For $m = 0$, the corresponding term of the series is 0. Hence, the summation can be performed starting from $m = 1$.

The expression (13) can be written as

$$W_i(0,\ q^i = (1,\ldots,1)) = b(n-1)k_i.$$

If all players choose strategy *B*, then participant *i* will gain following them. Choosing strategy *A* singly, he or she obtains

$$W_i(1,\ q^i = (0,\ldots,0)) = \sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m \times$$
$$(1-k_i)^{N-m-1}m = a(n-1)k_i.$$

Thus, $W_i(1,\ q^i = (0,\ldots,0)) > W_i(0,\ldots,0)$ for $k_i > \frac{b}{a}$: the player with a sufficiently high coefficient k_i is willing to pass to the more efficient norm *A* even singly. In a homogeneous community with all $k_i > \frac{b}{a}$, the strategy profile $q = (1,\ldots,1)$ (all participants choose norm A) is a unique Nash equilibrium.

However, the heterogeneous case is more realistic: generally speaking, the coefficients k_i may differ for different community members.

8. THRESHOLDS IN THE HETEROGENEOUS COMMUNITY MODEL

To investigate such heterogeneous communities, we introduce the notion of *a threshold*. The threshold θ_i of participant i is the minimum share of other community members passing to norm A under which he or she will do the same. For example, participants i and j will pass to norm A when expecting half and one-third of the community to choose it, respectively. Then $\theta_i = \frac{1}{2}$ and $\theta_j = \frac{1}{3}$.

The concept of thresholds to describe decision-making dynamics in large communities first appeared in the research of American sociologist M. Granovetter [41] and T. Schelling [3]. However, it found many supporters and was substantially developed by modern researchers. In particular, we mention V.V. Breer's publications [42–44], devoted to studying the so-called *conformity behavior* (behavior based on following established social norms).

The threshold for each participant can be defined as follows. Let participant $i \in \{1, N\}$ expect that $\tilde{n} \in \{0,\ldots, N-1\}$ other participants will choose norm A. In the case of choosing norm B, his or her utility function is given by

$$W_i(0,q^i) = b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-1-m} \times$$
$$[\frac{N-1-m}{N-1}(N-\tilde{n}-1)+m] =$$
$$b\,[(N-\tilde{n}-1)+\tilde{n}k_i].$$

Choosing norm *A* under the same conditions, participant *i* obtains

$$W_i(1,\ q^i) =$$
$$b\sum_{m=0}^{N-1}\binom{N-1}{m}k_i^m(1-k_i)^{N-1-m}[\frac{N-1-m}{N-1}\,\tilde{n}+m] =$$
$$a\,[(1-k_i)\tilde{n}+(N-1)k_i] = a\,[\tilde{n}+(N-\tilde{n}-1)k_i].$$

Thus, participant i will choose norm A if $W_i(1,\ q^i) > W_i(0,\ q^i)$, i.e., $a\,[\tilde{n}+(N-\tilde{n}-1)k_i] \geq b\,[(N-\tilde{n}-1)+\tilde{n}k_i]$. This condition is equivalent to

$$\frac{\tilde{n}}{N-1} \geq \frac{b-k_ia}{(a+b)(1-k_i)} \overset{\Delta}{=} \theta_i$$

where θ_i denotes the threshold of participant *i* (the minimum share of participants choosing norm *A* under which he or she will do the same). Interestingly, under the condition $k_i > \frac{b}{a}$, the threshold θ_i is negative. This result can be interpreted as follows. With a sufficiently large coefficient k_i (the level of morality [2]), participant *i* is willing to pass to a new norm even singly.

Note that the players with the lowest coefficient $k_i = 0$ have the threshold $\theta_i = \frac{b}{a+b}$. In other words, if the share of the community's representatives passing to norm A exceeds this value, even the individualists will pass to norm *A*.

We model a heterogeneous community with different coefficients k_i and thresholds θ_i using a distribution function $F(x): \mathbb{R} \to [0,1]$. It shows the share of community members whose threshold θ_i does not exceed x.

Let the threshold θ of a community member be represented as a random variable on the interval $(-\infty, \frac{b}{a+b}]$. In this case, $F(x)$ can be treated as a distribution function of this random variable: $F(x) = P(\theta < x)$, where P denotes the corresponding probability (the share of community members whose threshold θ does not exceed x).

Moral statistics can be useful for finding the parameters of the threshold distribution function describing the member's willingness to pass to a new norm.

Moral statistics is a branch of statistics covering a wide range of problems related to negative phenomena in society (crimes and violations of public order and moral and ethical norms) and the positive ones characterizing the moral character of people. They include the participation of citizens in public organizations for environmental protection, selfless donation, participation in various rescue services, etc. [45].

For example, consider a company or university organizing a regular voluntary donor blood campaign. Then every employee (student or lecturer) has two strategies: donating blood (norm A) or not (norm B). The moral satisfaction from participating in such events is difficult to formalize. Hence, it is impossible to determine the coefficients a and b. However, the thresholds of passing from norm A to norm B can be determined.

For this purpose, a sociological survey of blood donors is conducted. The participants should be asked the following question: how many familiar people had donated blood before you decided to do the same? As a result, the threshold for each participant will be found.

Of course, distribution functions will differ from problem to problem. The social model under consideration assumes a sufficiently large number of participants. Therefore, we can choose the Gaussian distribution with a mean μ and a variance σ^2 as the parameters characterizing the community: $F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{x} e^{-(u-\mu)^2/(2\sigma^2)} du$ (Fig. 1).

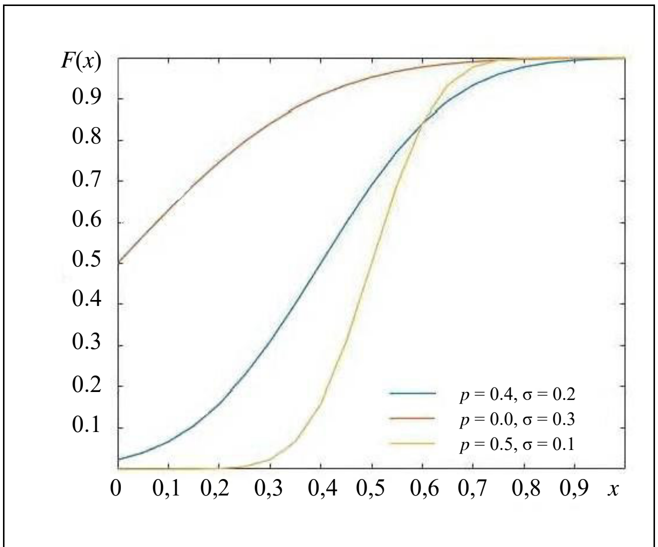

Fig. 1. The graph of the distribution function $F(x)$ of thresholds θ_i.

However, there are examples of other threshold distribution functions in the literature. In particular, the β-distribution was used in [44].

In the blood donation example, the average value of the thresholds of all surveyed participants yields the mean, and the mean square of the deviations from the mean yields the variance.

Figure 2 shows the graphs of the distributions under different values of the parameters μ and σ. Note that $F(x) = 1$ for $x \geq \frac{b}{a+b}$.

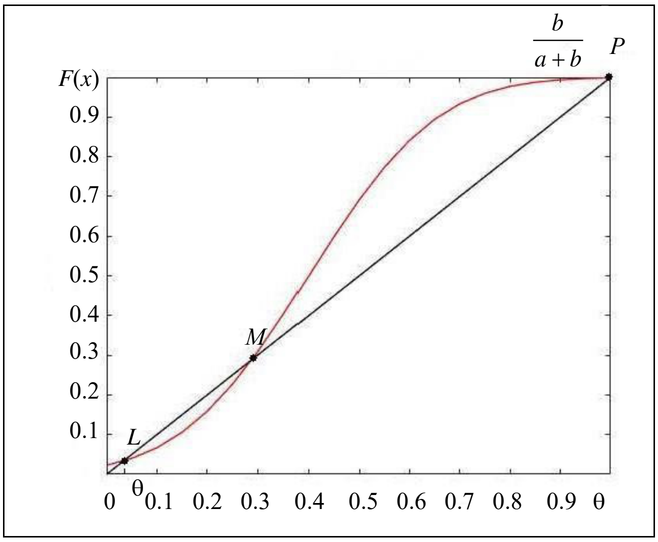

Fig. 2. Graph of the threshold distribution function with marked equilibria.

The Gaussian distribution is taken here as some approximation: in reality, the human factor should be considered when analyzing social processes due to the self-organization and memory of people.

Several classical works (e.g., F. Blackman [46] and L. von Bertalanffy and J. Woodger [47]) involved the logistic model and, accordingly, the sigmoid (S-shaped) function to describe the probability distribution function of states in social systems. Such an approach is suitable for the considerations below.

Several modern authors (D.O. Zhukov, T.Yu. Khvatova, and others [48, 49]) investigated the stochastic dynamics in social systems based on a cellular automaton with the memory effect of system participants. (The state of each individual depends on his or her states at the previous instants.) Given some initial parameters of the system (e.g., the number of contacts between community members), this model yields a threshold distribution function for the community's transition between states.

The next section is devoted to the transition process between norms *A* and *B* in dynamics.

9. THE SOCIAL MODEL IN DYNAMICS

Consider the transition dynamics of community members between norms *A* and *B* on some time interval $[t_0, T]$. We begin with the model with a discrete step Δt, then letting $\Delta t \to 0$. We denote by $N_A(t)$ the number of community members choosing norm *A* at an instant t. The initial condition is $N_A(t_0) = 0$.

Then $\frac{N_A(t)}{N-1}$ defines the share of those passing to norm *A* at the instant t. According to the definition of the function $F(x)$, the value $F\left(\frac{N_A(t)}{N-1}\right)$ is the share of individuals with a threshold not exceeding $\frac{N_A(t)}{N-1}$. Therefore, the number of those passing to norm A at a next instant is given by $N_A(t+\Delta t) = F\left(\frac{N_A(t)}{N-1}\right)N$. If the community is supposed large enough, $N-1 \approx N$. Denoting by $x(t) \overset{\Delta}{=} \frac{N_A(t)}{N}$ the share of participants passing to norm A at the instant t, we obtain the relation

$$x(t+\Delta t) = F(x(t)) \tag{14}$$

or

$$x(t+\Delta t) - x(t) = F(x(t)) - x(t).$$

If $F(x) > x$, then $x(t)$ and $N_A(t)$ are increasing functions of time; if $F(x) < x$, they are decreasing function of time. Letting $\Delta t \to 0$ in (14) yields the equilibrium condition $x(t) = F(x(t))$. Under this condition, the number of individuals passing to norm *A* stabilizes. Fixed points of the mapping *F* correspond to equilibria. However, these states can be stable and unstable. To illustrate this fact, we turn to an example.

10. STABILITY OF EQUILIBRIA

Consider a community with the distribution function $F(x)$ of the thresholds θ shown in Fig. 2. First, we study the discrete-time model. The initial condition is $N_A(t_0) = 0$. All individuals with a negative threshold will be the first to pass to norm A. Therefore, $N_A(\Delta t) = F(0) \cdot N$. At the next instant, those with a threshold not exceeding the share of participants choosing *A* at the previous instant will pass. That is, $N_A(2 \cdot \Delta t) = F\left(\frac{N_A(\Delta t)}{N}\right) = F(F(0))$, and so on. Letting $\Delta t \to 0$, we obtain a continuous-time process.

The function *F* in Fig. 2 has three fixed points, and the corresponding equilibria are the points L (near the origin), M, and P (near the unity).

The equilibria L and P are stable: if the share of individuals passing to norm A approaches the threshold θ_L or θ_P, it will fluctuate near them. Really, $F(x) > x$ for $x < \theta_L$ (see above); therefore, the number $N_A(t)$ will increase. Conversely, $N_A(t)$ will decrease for $x > \theta_L$.

The equilibrium *M* is unstable: if the share of those passing to norm A exceeds θ_M by an arbitrarily small number, then $F(x) > x$, and $N_A(t)$ will increase. This process will continue until the share of those choosing A stabilizes, reaching the nearest stable equilibrium $\theta_P = 1$. (In other words, the entire community will pass to norm *A*.) Conversely, if the share $x(t) = \frac{N_A(t)}{N}$ is smaller than the threshold θ_M by an arbitrarily small number, it will continue decreasing until stabilizing near the threshold θ_L. (The community will slide into the inefficient norm B.) The stability theory of fixed point as applied to economic, social, and biological processes was considered in detail in [50].

Note that the fixed points of a continuous distribution function where an equilibrium is reached will be inflection points. If the function is concave (convex) on the left of a fixed point, the point will be stable (unstable, respectively).

Since $F(x) = 1$ for $x > \frac{b}{a+b}$, the function $F(x)$ will be convex as $x \to 1-0$. Therefore, the point $x = 1$ (the entire community passes to the new norm *A*) is always stable. Suppose that a fixed point of the distribution function *F* represents a stable equilibrium with a value below 1. In this case, the community will never completely pass to the more efficient norm, being “stuck” in the neighborhood of the stable equilibrium closest to the origin. However, if some enlightenment (educational) activity is conducted to raise the level of morality in society, it can completely pass to the more efficient norm. The presence of educational elements in the social choice model and the resulting changes in the graph of the probability distribution function (particularly, the arrangement of equilibria) were considered in [53, pp. 78–80].

CONCLUSIONS

The behavioral model of individuals following the principle of morality (Kant's imperative) considered above was developed by I. Alger and J. Weibull; e.g., see [11, 51]. This model shows an essential difference between the behavior of individuals called *homo moralis* and those traditionally considered in game theory (*homo economicus*).

As another model encountered in the literature, we mention the model of collectivism or altruism. In this model, each participant considers the interests of other participants with some weight. In several studies (e.g., [9, 11]), collectivism was modeled by introducing special utility functions. For example, in a two-player game, each player seeks to maximize not his or her original payoff function $J_i(q)$ but the utility function $U_i(q) = (1-\alpha)J_i(q) + aJ^i(q),\ \alpha \in [0, 1]$. In [52], it was generalized to an arbitrary number N of participants: $U_i(q) = (1-\alpha)J_i(q) + \frac{\alpha}{N}\sum_{k=1}^{N} J_k(q),\ \alpha \in [0, 1]$. The function $U_i(q) = \frac{1}{N}\sum_{k=1}^{N} J_k(q)$ proposed by Harsanyi [5] is its particular case for $\alpha = 1$.

The essential difference between *homo moralis* and the so-called individualists (*homo economicus*) and even altruists consists in the following. While assessing the potential benefits of adopting a new norm of behavior by all community members, *homo moralis* can become a catalyst for the process, pioneers. Neither individualists nor altruists can do it.

This peculiarity indicates some evolutionary stability for such a behavioral model. Interestingly, its stability can be implicitly confirmed by evolutionary game theory methods. As already mentioned, this theory operates repeated games, and each behavioral strategy is investigated for success not in a particular strategy profile but the long run (a lengthy series of strategy profiles).

For example, the authors [54] studied the evolutionary stability of a special model of altruistic behavior using the infinite repeated prisoner's dilemma.

We recall the essence of this dilemma. Two players choose between cooperating and betraying the partner. If both players cooperate, they benefit. But each player is tempted to deception: in the case of success, the deceiver gets even more than both cooperating players. However, the deceived player suffers a loss. If both players, having yielded to temptation, deceive each other, they will be punished by their greed with the least favorable strategy profile in the game.

A nontrivial result was established in the paper [54]: altruism will be evolutionarily more stable (the total value of the utility function will be higher after many repetitions of the dilemma) only if community members have at least an approximate idea of each other's preferences. Otherwise, an egoist accidentally entering an altruistic community will have significant advantages (being in favor of others) until the nature of his or her behavior is revealed. According to the authors, this circumstance explains why altruism (prosocial behavior) tends to occur between relatives, friends, and colleagues (in short, people who know something about each other).

Nevertheless, we draw the following conclusion based on the arguments about the stability of equilibria in heterogeneous communities (see the discussion above). The new (progressive) model of behavior may never become a widespread norm in natural conditions. In this case, society may be forever "stuck" on the obsolete (less efficient) model of behavior unless some additional measures are taken to raise the level of morality (the coefficient k_i of the model). Such measures may include, in particular, enlightenment and educational work.

However, the model has an obvious disadvantage: the parameters (e.g., the coefficients k_i) are weakly formalizable and difficult to determine in applications.

At the same time, statistical methods [45] can be used to solve this problem. Therefore, the theoretical grounds presented above will serve for assessing the efficiency of state policy in the areas of upbringing and education.

APPENDIX

P r o o f of the theorem. According to Definition 3, a strategy profile q^* is a Nash equilibrium if

$$U_i\left(q^*\right) \ge U_i\left(q^{i*}, q_i\right),\ \forall q_i \in G\left(q^{i*}\right),\ i = \overline{1, N}, \quad \text{(A1)}$$

where $q^{i*} \overset{\Delta}{=} \left(q_1^*, \ldots, q_{i-1}^*, q_{i+1}^*, \ldots, q_N^*\right)$ denotes the opponents' strategy profile for player *i*. Using the payoff function (2), we write inequality (A1) as

$$\begin{gathered}(1-\alpha)J_i\left(q^*\right) + \frac{\alpha}{N-1}J^i\left(q^*\right) \ge (1-\alpha)J_i\left(q^{i*}, q_i\right) + \\ \frac{\alpha}{N-1}J^i\left(q^{i*}, q_i\right),\ \forall q_i \in G\left(q^{i*}\right),\ i = \overline{1, N}.\end{gathered} \quad \text{(A2)}$$

With the change of variables $\beta \overset{\Delta}{=} \alpha\frac{N}{N-1}$, the payoff function $U_i(q)$ is expressed in the equivalent form $U_i(q) = (1-\beta)J_i(q) + \frac{\beta}{N}J(q), \beta \in [0,1]$.

Then inequality (A2) reduces to

$$(1-\beta)\left(J_i\left(q^*\right)-J_i\left(q^{*i}, q_i\right)\right)+\frac{\beta}{N}\left(J\left(q^*\right)-J\left(q^{*i}, q_i\right)\right)\geq 0, \quad \forall q_i \in G\left(q^{i*}\right),\ i=\overline{1, N}. \tag{A3}$$

For $\beta \in [0, 1]$, the expression on the left-hand side of (A3) defines a real-line segment between the points

$$Q(q_i) \overset{\Delta}{=} J_i\left(q^*\right)-J_i\left(q^{*i}, q_i\right), \quad P(q_i) \overset{\Delta}{=} \frac{1}{N}\left(J\left(q^*\right)-J\left(q^{*i}, q_i\right)\right). \tag{A4}$$

We have $P(q_i)\geq 0$ since $J(q)$, $q \in G$, achieves maximum at the point q^*. Therefore, one of the following cases is true: $\forall q_i \in G(qi^*)$ the segment between the points $Q(q_i)$ and $P(q_i)$ lies on the right of zero if $Q(q_i)>0$; zero lies inside the segment $\left[Q(q_i), P(q_i)\right]$ if $Q(q_i)<0$; zero coincides with one of its bounds if $Q(q_i)=0$ or $P(q_i)=0$; zero coincides with both bounds if $P(q_i)\equiv Q(q_i)=0$. (In the latter case, the segment becomes a point.) In other words, there exists a value $\beta^i_{NE}\langle q_i\rangle \in \mathbb{R}, \beta^i_{NE} \in [0, 1]$, such that inequality (A4) holds $\forall \beta \in \left[\beta^i_{NE}, 1\right]$.

Assigning the value $\beta^i_{NE}\langle q_i\rangle \in [0,1]$ to each $q_i \in G(qi^*)$, we define a bounded function $\beta^i_{NE}(q_i)\leq 1, \forall q_i \in G(qi^*)$:

$$\beta^i_{NE}(q_i) \overset{\Delta}{=} \begin{cases} \dfrac{-Q(q_i)}{P(q_i)-Q(q_i)} \text{ if } Q(q_i)<0, \\ 0 \text{ if } Q(q_i)\geq 0. \end{cases}$$

Let $\beta^i_{NE} \overset{\Delta}{=} \sup\limits_{q_i \in G\left(q^{*i}\right)} \beta^i_{NE}(q_i)$. Due to $\beta^i_{NE}(q_i)\leq 1$, we have $\beta^i_{NE}\leq 1$.

Now we introduce $\beta_{NE} \overset{\Delta}{=} \max\limits_{i=\overline{1,N}} \beta^i_{NE}$. Since $\beta^i_{NE}\leq 1$, it follows that $\beta_{NE}\leq 1$, and $\forall \beta \in [\beta_{NE}, 1]$ the point q^* will be a Nash equilibrium in the game with the utility functions $U_i(q)$ (2), where $\beta=\beta_{NE}$.

Returning to the original parameters $\alpha_{NE}=\dfrac{N-1}{N}\beta_{NE}$, we finally obtain a value α_{NE} such that $\forall \alpha \in \left[\alpha_{NE}, \dfrac{N-1}{N}\right]$ the strategy profile q^* will be a Nash equilibrium in the game $\Gamma^{\alpha_{NE}}$.

The proof of the theorem is complete. ♦

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A CONCEPTUAL APPLIED GEOGRAPHIC INFORMATION SYSTEM FOR MODELING SEARCH AUTONOMOUS CORRELATION-EXTREME NAVIGATION SYSTEMS

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Abstract. This paper presents a conceptual applied geographic information system (AGIS) for modeling search correlation-extreme navigation systems (CENSs) to control moving objects. As demonstrated below, the development and mass implementation of autonomous navigation systems of this type as the only alternative to satellite navigation systems can currently be based on subject-oriented information technology. The AGIS can be used to assemble models of a wide range of CENSs and models of technologies for adjusting their operation in specified areas with necessary computational experiments. The required software components, storage structure, and interface features are determined by constructing a general mathematical model. While preserving all specifics of the search algorithms of CENSs, this model covers the well-known image combining algorithms and, moreover, includes a synthesis scheme for search algorithms of new-type CENSs using pattern recognition and scene analysis, clustering, neural network training, and cloud data processing. Stress testing is the most important type of computational experiments with CENS models. A mathematical model of stress effects is constructed for a particular case. It describes various operating conditions for CENSs, including fatal deviations from normal operation.

Keywords: applied geographic information system, correlation-extreme navigation system, shooting system, pattern recognition, scene analysis, learning machines, neural network, parallel computing, cloud computing, mathematical modeling, stress testing of the system.

INTRODUCTION

Correlation-extreme navigation systems (CENSs) serve to refine off-line information about the location, orientation, and other parameters of a moving object coming from the main navigation system. A control system uses this information to compensate the deviations in the object's motion parameters to follow a given route. Search CENSs check hypotheses about the values of motion parameters by matching the current terrain sector image received by the airborne shooting system with fragments of a reference image of the application area. The reference images are prepared in advance and stored in the memory of the airborne computer. When searching for a reference image fragment close by content to the current image (in the sense of a closeness function in the onboard algorithm), a regular shift grid of the frame selecting the next fragment of the reference image is used. The hypotheses that the sought parameters have values equal to those at the grid nodes are checked. The hypothesis for which the closeness function achieves maximum is accepted. Global search schemes, gradient methods from the arsenal of numerical optimization methods, and their combinations are often used [1].

Until the late 1990s, R&D works on various aspects of CENSs were carried out intensively. At different stages, the solution of motion control and navigation problems was associated with general synthesis principles that would yield control parameters for moving objects in the automatic mode under specific circumstances. Directions for the further development of CENSs were defined:

• new design principles for onboard algorithms, their intellectualization and self-organization;

• application of new types of shooting systems and their combination;

• development of parallel processors, including specialized processors for implementing algorithms with a single parallel structure.

At that time, the level of information technology and the achievable characteristics of airborne computers restrained the practical implementation of the directions mentioned above. Satellite navigation systems were developed, and CENSs moved to the background.

However, the situation has changed dramatically to date. The accelerated development of CENSs in the above directions has become topical due to the following factors: the intensive development of various-purpose unmanned vehicles and the appearance of modern shooting systems, large-memory processors, and processors with parallel computing schemes in their control systems; mass distribution and development of programming tools for artificial intelligence systems; training neural networks on big data in cloud computing environments. Hence, the potential of CENSs as the only alternative to satellite navigation systems needs to be tapped more than in the existing solutions [2].

As it turned out, satellite control systems for moving objects are vulnerable in today's environment. We are increasingly aware that satellite control systems need to be protected, strengthened, and expanded. Orbital stations can be disabled or simply destroyed.

Under these conditions, the accelerated development of this problem domain can be provided by expanding R&D works using a subject-oriented computational complex. Such a complex can provide an engineer with all the necessary tools to assemble models of a wide range of CENSs and draft technologies to adjust their operation in given application areas from ready-made software components through a special interface to the component storage and conduct necessary computational experiments with them.

Note that besides the specific functionality focused on modeling search CENSs and draft technologies to adjust their operation in given application areas, such a subject-oriented complex should provide user access to all universal means of handling geospatial information (particularly in the form of *application programming* interface (API) for external programs). In other words, the complex should provide user access to the general-purpose functionality of modern geographic information systems (GISs). Thus, it should be created as an applied geographic information system based on the extended functionality of general-purpose GISs [3].

Therefore, developing a subject-oriented complex for modeling search CENSs in the form of an applied geographic information system (AGIS CENSs) is a topical problem. First of all, it is necessary to consider new design principles of onboard algorithms (particularly, their intellectualization and self-organization), modern processors oriented at the parallel implementation of algorithms, new types of shooting systems and their integration, and other recent achievements of information technology. Such analysis is needed to justify the composition of the software components of AGIS CENSs, identify the ones with functionality implemented in related problem domains, and determine a CENS-specific assembly scheme for different onboard algorithms and draft technologies of data preparation. New variants of search CENSs will improve the dynamics of autonomous control systems using the principles of reconfigurable structures.

This paper considers two-dimensional sensing CENSs: the current and reference terrain images are compared for active and passive airborne shooting systems in the electromagnetic radiation wavelength ranges for which means of stress exposure are available or can be developed.

In Section 1, we present a mathematical model of search algorithms and their adjustment procedures to perform their task in a given application area, which is rather general but, at the same time, preserves all specifics of search CENS algorithms. This model includes the model of image matching as a particular case and describes the scheme of assembling a significantly wider range of CENSs from the software components discussed above.

Since tough requirements are applied to the reliability of CENSs, stress testing is the most important type of computational experiments with CENS models. In Section 2, we construct a general mathematical model of stress exposures causing fatal deviations from the normal operating conditions of CENSs. Also, we justify requirements to their modeling tools within AGIS CENSs.

1. MATHEMATICAL MODEL OF SEARCH CENS ALGORITHMS

Consider CENSs in which the shooting system captures a scene image S on a terrain section, and the onboard algorithm refines the planned coordinates of the carrier at the shooting instant. In other words, the parameter to be refined is $d=(X,Y)$. These limitations are adopted just to illustrate the main features of the mathematical model and facilitate their perception. In the final analysis, they will affect neither the set of CENS variants (and the procedures of their adjustment to perform a particular task in a given application area) covered by the model, nor the generality of the analysis results and their practical importance for the conceptual AGIS CENSs.

Let an application area be defined if the set D of all possible values of the refined parameter of the carrier motion at the shooting instant is defined, i.e., $d\in D$. We denote by M the set of all possible images S coming from the shooting system to the input of the onboard algorithm of the CENS in a given application area (under the condition $d\in D$).

Then the CENS is prepared to perform its task in the application area if for any $S\in M$, the onboard algorithm is ready to output a correct approximation $\hat{d}=(\hat{X},\hat{Y})\in\hat{D}$ to the true value $d\in D$ at the instant of receiving the image S, where $\hat{D}$ is the set of all possible outputs of the onboard algorithm. For known search algorithms, the set $\hat{D}$ is finite and coincides with the set of coordinates for the shift grid nodes of the frame selecting the next reference image fragment when matching the current and reference images in the application area: $\hat{D}=\{\hat{d}_{j_1 j_2}\}$, $j_1=1,2,\ldots,N_1$; $j_2=1,2,\ldots,N_2$. Since $D\subset R^2$, the approximation is correct if $\rho(\hat{d},d)\le\varepsilon$, where ρ is the distance function between two points on the plane R^2, and ε is an admissible error (e.g., in meters). Note that this interpretation of the CENS preparation to perform its task in a given region can be treated as the first iteration towards a general mathematical statement of the problem. In traditional terms, this problem consists in preparing the actual reference image for a given application area.

Obviously, the CENS ability to perform a task in a given application area is determined by the relationship between the images obtained from the shooting system and the values of the refined parameter of the moving object at the shooting instant. We describe this relationship by a function $f(S): M\to D$. Note that in the general case, this function is multi-valued: the images getting into the shot under different values of the refined parameter of the moving object do not necessarily differ. Furthermore, the image content depends on other factors and parameters. For example, the moving object's height affects the image and should be considered when refining the planned coordinates. Such factors and parameters will be called disturbances. Hence, the function $f^{-1}(d)$, inverse to $f(S)$, is also multi-valued in the general case.

Other disturbances include the time of the year, weather conditions in the application area, etc. Stress exposures on the shooting systems of CENSs should be considered separately. (They can be purposeful.) Stress exposures may cause fatal deviations from the normal operating conditions of CENSs: the system will be unable to perform the task. Stress modeling in AGIS CENSs is described in Section 2.

Thus, the navigational properties of a CENS in the application area and the conditions of its orientation therein can be studied by analyzing the function $f(S): M\to D$.

We specify the form of initial information about the approximated function $I_0\{f(S): M\to D\}$ when adjusting the CENS to operate in a given application area. Assume that this information is described by a computer simulation model of the shooting system:

$$I_0\{f(S): M\to D\} = \hat{f}^{-1}(d,p), d\in D,\ p\in P,$$

where the vector $p\in P$ consists of the disturbing parameters included in the shooting system model. The vector p has the admissible domain P. The computer simulation model of the shooting system should approximate the function $f^{-1}(d), d\in D$, inverse to $f(S): M\to D$.

Now we pass to the onboard algorithms of CENSs.

If the CENS is already prepared to perform its task in a given application area, for any $S\in M$ the onboard algorithm will output some value $\hat{d} \in \hat{D}$. In other words, the algorithm is ready to calculate the value of a one-valued function $\hat{f}: M \to \hat{D}$ for any $S \in M$. Hence, we may suppose the following: during the preparation process, the value of some generalized parameter $\alpha^* \in A$ was calculated and saved in the onboard memory, adjusting the CENS to calculate the values of this particular function by separating it from a parametric family. Therefore, the CENS can be treated as a technical implementation of the parametric family of one-valued functions $\left\{\hat{f}(\alpha; S)\right\}_{\alpha \in A}$, where $\hat{f}(\alpha; S): M \to \hat{D}$ is a particular function from this family uniquely defined by the generalized parameter value $\alpha \in A$. In the traditional interpretation, this parameter is the reference image of the application area. Under the most general assumptions, the problem of preparing the CENS for operation in a given application area turns out to be that of function approximation: as the result of preparation, a function $\hat{f}\left(\alpha^*; S\right)$ is chosen to approximate the function $f(S)$ in an exact sense dictated by practical requirements to the CENS. Assume that the criterion of closeness of the two functions has the form $\rho_M\left(\hat{f}, f\right) \le \varepsilon$, where ρ_M is a metric in the space of such functions, and ε is a positive number.

We choose the simplest parametric approximating family of classical numerical functions, generalizing it to functions with image matrix variables S that take values in the plane R^2 (the elementary case). The simplest family consists of step functions of one numerical variable. Their definitional domain on the numerical axis is divided into disjoint segments, where the function has a constant value.

We write the definitional domain of the function $\hat{f}$ as a union of l disjoint sets (called classes):

$$M = \bigcup_{j=1}^{l} K_j, \text{ where } K_j \bigcap K_t = \varnothing \text{ for } j \ne t. \tag{1}$$

The function $\hat{f}(S)$ will be called a generalized step function if

$$\hat{f}(S) = \sum_{j=1}^{l} \chi_j(S) \hat{d}_j, \tag{2}$$

where $\chi_j(S) = 1$ if $S \in K_j$, and $\chi_j(S) = 0$ otherwise, and $\hat{D} = \left\{\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_l\right\}$. (In other words, $\chi_j(S)$ are the characteristic functions of classes K_j.) We introduce the notations

$$\mathbf{d} = (\hat{d}_1, \hat{d}_2, \ldots, \hat{d}_l), \quad \mathbf{X}(S) = \left(\chi_1(S), \chi_2(S), \ldots, \chi_l(S)\right).$$

Then the expression (2) takes the vector form

$$\hat{f}(S) = \left\langle \mathbf{X}(S), \mathbf{d} \right\rangle. \tag{3}$$

Due to (3), calculating the value $\mathbf{X}(S)$ is a common problem of assigning an object to one of l disjoint classes $K_1, K_2, \ldots, K_l$. Any algorithm for solving this problem is by definition a pattern recognition algorithm [4]. Obviously, the known search algorithms solve a particular case of the pattern recognition problem assuming that the set of images M is a union of $l = N_1 \times N_2$ disjoint classes K_{ij}. An image S belongs to the class $K_{j_1 j_2}$ if it is obtained in a small neighborhood $\hat{D}_{j_1 j_2}$ of a node $\hat{d}_{j_1 j_2}$ of the shift grid $\hat{D} = \left\{\hat{d}_{j_1 j_2}\right\}$, where $j_1 = 1, 2, \ldots, N_1$ and $j_2 = 1, 2, \ldots, N_2$. Any image $S \in K_{ij}$ coincides with the reference image fragment corresponding to the node $\hat{d}_{j_1 j_2}$ with an accuracy to a random "term" obeying a known distribution (as a rule, Gaussian). Such assumptions apply to the original image spaces obtained from the shooting system and the image spaces preliminarily transformed by image quality improvement methods or compiled from the scene descriptions of terrain sectors got into the shot [5, 6]. Thus, we have a special case of comparison with reference objects when each class is described by a single reference object.

In the context of pattern recognition, preliminary transformations of images extract working features of recognition and must be included in the model of onboard algorithms. Under a preliminary image transformation $\pi(S)$, the expressions (1), (2), and (3) are written as follows:

$$\pi M = \bigcup_{j=1}^{l} \pi K_j, \tag{4}$$

where $\pi K_j \bigcap \pi K_t = \varnothing$ for $j \ne t$;

$$\hat{f}(S)=\sum_{j=1}^{l}\chi_j\left(\pi(S)\right)\hat{d}_j, \tag{5}$$

where $\chi_j\left(\pi(S)\right)=1$ if $\pi(S)\in\pi K_j$, and $\chi_j\left(\pi(S)\right)=0$ otherwise. Hence, they are characteristic functions of the classes πK_j.

Denoting

$$\mathbf{X}\left(\pi(S)\right)=\left(\chi_1\left(\pi(S)\right),\chi_2\left(\pi(S)\right),\ldots,\chi_l\left(\pi(S)\right)\right),$$

we write the expression (5) in the vector form

$$\hat{f}(S)=\left\langle \mathbf{X}\left(\pi(S)\right),\mathbf{d}\right\rangle. \tag{6}$$

In certain conditions, known search algorithms face the problem of large search zones: global schemes lead to indiscriminate enumeration, and local gradient methods may not reach the global optimum of the comparison function. At the same time, in the presence of powerful disturbances, the enumeration procedure has to be performed in the space of disturbing parameters as well. (Computational resources have to be consumed on extraneous problems.) In these conditions, a possible approach is to use hierarchical partitions of the set M into classes. Let such partitions have $r = 2$ levels.

For the generalized step function defined on a two-level hierarchical partition, the expression (5) takes the form

$$\hat{f}(S)=\sum_{j_1=1}^{l}\chi_{j_1}\left(\pi(S)\right)\sum_{j_2=1}^{l_1}\chi_{j_1 j_2}\left(\pi_{j_1}(S)\right)\hat{d}_{j_1 j_2}. \tag{7}$$

In the general case (an arbitrary number r of levels), we have

$$\begin{aligned}\hat{f}(S)=&\sum_{j_1=1}^{l}\chi_{j_1}\left(\pi(S)\right)\sum_{j_2=1}^{l_1}\chi_{j_1 j_2}\left(\pi_{j_1}(S)\right)\ldots\\ &\ldots\sum_{j_r=1}^{l_r}\chi_{j_1 j_2\ldots j_r}\left(\pi_{j_1 j_2\ldots j_{r-1}}(S)\right)\hat{d}_{j_1 j_2\ldots j_r}.\end{aligned} \tag{8}$$

Thus, the CENS can be adjusted to operate in a given application area by solving a function approximation problem:

– The function $f(S): M\to D$ is given by the algorithm for calculating the values of the function $\hat{f}^{-1}(d,p), d\in D, p\in P$, that approximates the inverse $f^{-1}(d): D\to M$ of the function f.

– In the parametric family of all generalized step functions $\left\{\hat{f}(\alpha;S): M\to\hat{D}\right\}_{\alpha\in A}$ (8), it is required to find a value $\alpha^*\in A$ such that the function $\hat{f}(\alpha^*;S)$ approximates the function $f(S)$.

– One example of the approximation criterion is $\rho_M\left(\hat{f},f\right)\le\varepsilon$, where ρ_M is the metric in the space of such functions, and ε is a positive number.

At this stage of the study, the criteria for approximating the inverse function and the approximation criterion are not required to clarify. It suffices to assume that the computer simulation model of the shooting system (the algorithm for calculating the inverse function $\hat{f}^{-1}(d,p)$) yields representative sets of samples $\left((d,p);S\right)$ describing the behavior of the approximated function on the set M.

Consider an illustrative example: implementing a family of generalized step functions (7) in the airborne computer. The term “reference terrain map” is common in the literature devoted to CENSs. The current example involves a cartographic interpretation of the concepts of the proposed mathematical model, which is justified by history: the compilation and application of maps by humans for terrain orientation underlie the development of CENSs [5, 7–9].

Figure 1 shows the operating principle of a traditional CENS supplemented with a module for the approximate preliminary estimation of the object’s coordinates. This module narrows the search area for the exact module by approximately three times. Note that the main purpose of the approximate estimation modules is not just to narrow the search area for exact algorithms in the coordinate space. Such modules are universal aggregators, bringing the uncertainty to a form for which the aggregated exact modules are effective. For example, let the terrain in the correction area be such that the function used in the exact coordinate determination module to compare the current image with fragments of the reference image has several close-value optima. Then the module will not solve the problem in the zone E. If the zoning procedure gives a unique optimum in each subzone $E1$, $E2$, and $E3$, and the aggregator correctly determines the one with the shot, then the problem will be solved. These changes in the onboard algorithms of CENSs generate new requirements to reference images for them. Traditional reference images are analytical maps of geophysical fields. They are compiled for areas covering the boundaries of all landscape images possibly obtained by the shooting system during the next session of refinement and correction of aircraft motion parameters considering their deviations from the planned values. In contrast, reference images for additional hierarchical search modules of next-generation CENSs must be compiled for areas of probable locations of the aircraft (search zones) rather than the boundaries of images; they must be special synthetic maps describing the typology of landscapes in the terrain sectors that can get into the shot in the search area with the sensor trigger. Such maps are search area zoning maps on which one area contains

the aircraft locations to which all shots correspond (survey areas) with the same-type landscape detected by the landscape characteristics automatically extracted from the image data received by the CENS sensors during a correction session. The information content is determined by the type of sensors and onboard preliminary image and pattern recognition algorithms.

Changes in the reference image content are shown in Figs. 2 and 3 on the example of a two-level CENS (see formula (7)). In this system, a simple description of the scene on the current image is formed during the

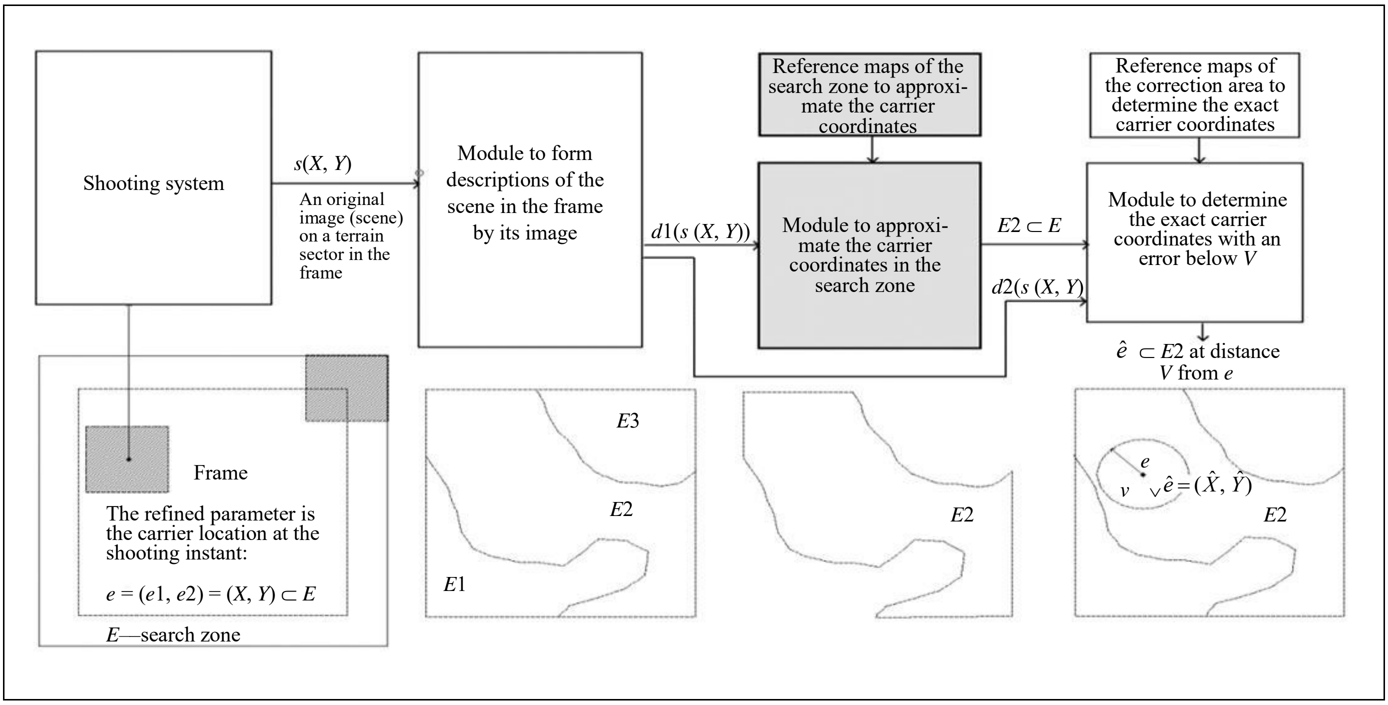

Fig. 1. Two-level onboard algorithm for CENS (r = 2).

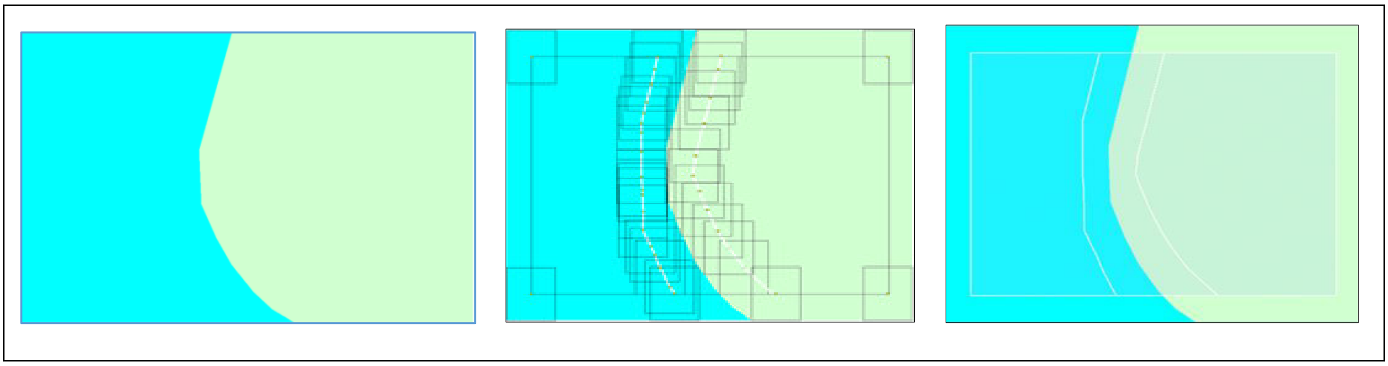

Fig. 2. Correction area, shooting sectors (shots), and search area.

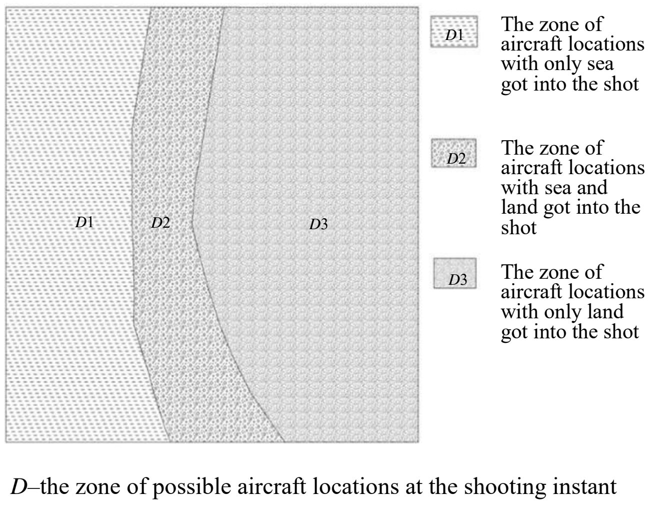

Fig. 3. A synthetic reference map of search area zoning.

preliminary transformation and extraction of two binary features, $P1$ and $P2$. The feature $P1$ takes value 1 if a hydrographic object (a morphological terrain element) is found on the image, and 0 otherwise. The feature $P2$ takes value 1 if a land object is found on the image, and 0 otherwise.

Such scene descriptions are called morphological landmarks [4]. Synthetic reference maps with the boundaries of the zones $D1$–$D3$ and three references (0,1), (1,0), (1,1) have to be prepared in advance [4]. We emphasize that this elementary example illustrates the operating principle of preliminary estimation modules: an advance controlled clustering of the set of possible images, consistent with the zoning of the set of possible locations, and application of pattern recognition algorithms (attributing the input image to one of the pre-fixed classes according to their description in the language of features).

A feature space is any of those tested in a large recognition practice. A particular case is structural (morphological, syntactic) feature spaces, e.g., considered above, or mixed ones (a vector of the areas occupied by morphological terrain elements in the frame, in percentage); see [4, 5, 9].

In the general case, morphological descriptions can be built considering the set of terrain objects. The set of morphological landmark objects is limited, on the one hand, by the possibility of obtaining actual information about the boundaries of their distribution, and on the other hand, by the possibility of automatic detection of these objects in an acceptable time by the pre-processing module of the onboard algorithm during the activation period.

Another direction of detailing morphological descriptions is to study relations and connections between objects in the scene. In this case, it is possible to consider the terrain specifics more fully when solving the CENS task. Besides the information about the boundaries of objects distribution, the information about the relations between them should be taken into account; the scene description module should be adjusted to recognize these relations and connections automatically during the activation session of the CENS. Some examples are the nesting relation (one object is located inside another object) or the order relation (the sequence of the objects when viewing the scene by given rules) [10–12].

If disturbing parameters significantly affect the scenes and their images, a series of reference maps of zoning by distribution areas of morphological landmarks may be required for certain values of these parameters. For example, let the disturbing parameter be the shooting altitude. Then the series corresponds to a specially selected set of altitudes: a multiscale zoning map by distribution areas of morphological landmarks is needed.

2. MATHEMATICAL MODEL OF STRESS EXPOSURES

Problem statement. The physical field brightness distribution a_{ij} in the test area of a CENS is known. There are N means of stress exposure. Let these means be arranged as follows: means k is placed in a point with pixel coordinates (x_k, y_k), $k = 1, \ldots, N$. It is required to calculate automatically the optimal power A_k of each means of stress exposure minimizing the correlation between the current image and the stress-induced one. In the elementary case, the correlation is calculated on the window $1 \le i \le Mx$, $1 \le j \le My$.

Problem solution. With this arrangement of the means of stress exposure, the field brightness at a point (i, j) is given by

$$a_{ij} + \sum_{k=1}^{N} A_k \delta_k (i - x_k,\ j - y_k)\,. \qquad (9)$$

The correlation between these images has the form

$$C=\frac{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}\ \sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2}}. \tag{10}$$

To solve the problem, we find the vector of amplitudes (A_1, ..., A_N) minimizing the correlation value C. According to the first-order optimality condition, all partial derivatives of C with respect to A_k must be equal to 0. We calculate the partial derivatives:

$$\frac{\partial C}{\partial A_s}=\frac{1}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}}\left\{\frac{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2}}-\right.$$

$$\left.\frac{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\right)}{\sqrt{\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2\right)^3}}\right\}=$$

$$\frac{1}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}\ \sqrt{\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2\right)^3}}\times$$

$$\left\{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2-\right.$$

$$\left.\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\right)\right\}=$$

$$\frac{1}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}\ \sqrt{\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2\right)^3}}\times$$

$$\left\{\underline{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}+2\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)+\right.$$

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2-$$

$$\underline{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)}-\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)-$$

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)-$$

$$\left.\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\right\}=$$

(Author's note: *the underlined terms are mutually reduced; for* $A_k = 0$*, all derivatives vanish, which corresponds to the correlation maximum*)

$$\frac{1}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}\ \sqrt{\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2\right)^3}}\times$$

$$\left\{2\sum_{k=1}^{N}A_k\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_k(i-x_k,\ j-y_k)\right)+\right.$$

$$\sum_{k=1}^{N}A_k\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2\sum_{i=1}^{Mx}\sum_{j=1}^{My}\delta_s(i-x_s,\ j-y_s)\delta_k(i-x_k,\ j-y_k)\right)-$$

$$\sum_{k=1}^{N}A_k\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_k(i-x_k,\ j-y_k)\right)+$$

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2-$$

$$\left.\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\right\}=$$

$$\frac{1}{\sqrt{\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2}\ \sqrt{\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}+\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2\right)^3}}\times$$

$$\left\{\sum_{k=1}^{N}A_k\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_k(i-x_k,\ j-y_k)\right)+\right.$$

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)^2-$$

$$\sum_{k=1}^{N}A_k\left(\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}^2\sum_{i=1}^{Mx}\sum_{j=1}^{My}\delta_s(i-x_s,\ j-y_s)\delta_k(i-x_k,\ j-y_k)\right)-$$

$$\left.\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(a_{ij}\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\sum_{k=1}^{N}A_k\delta_k(i-x_k,\ j-y_k)\right)\right\}.$$

Thus, the optimal amplitudes satisfy a system of *N* quadratic equations with *N* unknowns. The factor at A_k^2 is

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_s(i-x_s,\ j-y_s)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\delta_k(i-x_k,\ j-y_k)^2-$$

$$\sum_{i=1}^{Mx}\sum_{j=1}^{My}a_{ij}\delta_k(i-x_k,\ j-y_k)\sum_{i=1}^{Mx}\sum_{j=1}^{My}\left(\delta_s(i-x_s,\ j-y_s)\,\delta_k(i-x_k,\ j-y_k)\right). \tag{11}$$

For $s = k$, we obtain $A_s = 0$. Thus, the derivative $\frac{\partial C}{\partial A_s}$ depends on A_s linearly; for fixed values A_k, $k \neq s$, the optimal value A_s is unique (11).

Due to the linear dependence, we can control the correlation C; its minimum admissible value can be set in advance according to the frequency-metric properties of the testing field. Arranging the means of stress exposure with the amplitudes A_k at specific field points, we construct the vector of their optimal amplitudes minimizing the correlation C. The sufficient condition of the minimum correlation between the current image and the stress-induced one is determined by the vector A_k and its dimension corresponding to the number N of the means of stress exposure, depending on the field properties of the testing area.

The algorithm for finding the optimal parameters of the means of stress exposure is as follows:

1. Calculating the quadratic functions in the numerators of the partial derivatives.

2. Organizing a loop on convergence. In each loop:

2.1. Organizing a loop on s from 1 to N. In each cycle:

2.1.1 Calculating the unique values A_s for fixed values A_k, $k \neq s$.

Each iteration minimizes the correlation by selecting one amplitude (the correlation decreases with each iteration). Thus, the monotonicity of the objective function ensures the convergence of this algorithm.

The points (i, j) are selected sequentially where the field brightness in a given wavelength range λ significantly exceeds the average field brightness of the test area. In each iteration cycle, the value A_k changes until reaching the minimum correlation between the current image and the stress-induced image on the testing area in a window with dimensions $1 \leq i \leq Mx$, $1 \leq j \leq My$. A specific range of A_k is set for the selected image type to control loop calculations.

This problem is solved using an iterative process of generalized coordinate-wise descent. Each iteration involves:

- Minimizing the correlation by selecting the optimal amplitudes under the current arrangement of the means of stress exposure. This problem is now solved.
- Minimizing the correlation by choosing the optimal locations of the means of stress exposure under their current amplitudes. This problem is also solved. However, its solution is omitted here.

The values A_k for particular means of stress exposure under given wavelength ranges of electromagnetic radiation are presented in special sources. As a rule, the means of stress exposure are calibrated by the shooting system of CENSs before the testing procedure.

When implementing this algorithm, each means of stress exposure should be placed according to the calculated coordinates. A necessary means of stress exposure for the environment of CENSs is selected depending on the amplitude at the point with the given coordinates. Due to the considerable volume of such studies, their results are not included in this paper. However, control of the means of stress exposure for CENSs is a significant problem that needs to be solved [13, 14].

The means of stress exposure for the shooting systems of CENSs should change the parameters of the Earth's physical fields in different wavelength ranges of electromagnetic radiation. As a rule, these are optical, thermal, radio-thermal, radar, geomagnetic, and other geospatial fields of the Earth. The mathematical model of stress exposures on CENSs should be considered when designing new navigation systems to create modern and high-efficient CENSs.

CONCLUSIONS

A mathematical model of search algorithms—a parametric family of algorithms—for calculating generalized step functions has been obtained. This model includes an image matching model as a special case and describes a scheme to assemble a wider range of onboard algorithms of CENSs from the algorithms of feature extraction on images, scene description, and classification.

The problem of adjusting the onboard algorithm of a CENS to perform its task in a given area has been reduced to that of approximating a given function of images by generalized step functions in the space of the object's refined parameters. This problem is solved by training and self-training of the hierarchical classification of images.

An applied GIS for the developers of CENSs and technologies of their adjustment to perform their tasks in given areas (the AGIS CENS) accelerates the development of control systems for moving objects equipped with CENSs. As shown by the analysis of the obtained mathematical model, the existing image matching modules (which estimate the motion parameters with the required accuracy by correlation methods) should be supplemented with preliminary estimation modules for motion parameters using hierarchical pattern recognition methods. At each iteration, such modules attribute images to one of several classes by solving nondegenerate pattern recognition problems. Image matching modules are applied at the last step. The additional modules are intended to reduce the level of uncertainty so that the image matching modules can effectively solve their problem with the required accuracy (which is impossible under the initial level of uncertainty). Hence, the software components of the AGIS CENS should include components implementing known algorithms for pattern recognition, feature extraction on images, and scene description.

A CENS cannot be adjusted to perform its task in a given area without computer simulation models of the shooting systems. Such models should synthesize images similar to shooting systems for all possible areas and conditions of their application. The software components implementing these models, geospatial data, and terrain models must be included in the AGIS CENS.

The adjustment problem of CENSs is reduced to nondegenerate pattern recognition problems. It involves hierarchical partitioning into classes and tuning of recognition algorithms consistent with the zoning of the application area using the entire arsenal of modern training and self-training tools of recognition systems, including neural networks. Therefore, the software components of the AGIS CENS should include libraries of components from the related R & D areas.

The set of views on synthesizing the conditions of application of CENSs in a variable environment with stress exposures forms an interconnected system. Without due consideration of this system, it is impossible to achieve the necessary effect of CENS application.

Further research can be focused on the following issues:

- Refining the approximation problem statement in terms of approximation criteria, requirements to the simulation models of shooting systems and their complexes, and in-depth mathematical study of this problem.
- Constructing a general scheme to assemble onboard algorithms of CENSs and procedures for their adjustment to perform their tasks in given areas based on the above expressions and schemes for training and self-training in pattern recognition and classification.
- Developing methods to construct hierarchical partitions of images into classes consistent with the zoning of the application area of CENSs.
- Developing the mathematical model of generalized step functions to clarify the parallel structure specifics of the onboard algorithms of CENSs and design special processors for airborne computers.
- Solving the problem of stress exposures for different types of CENSs using the physical fields of the Earth in different wavelength spectra of electromagnetic radiation.

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GROUP CONTROL OF UNMANNED AERIAL VEHICLES: A GENERALIZED PROBLEM STATEMENT OF APPLYING ARTIFICIAL INTELLIGENCE TECHNOLOGIES

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Abstract. This paper considers elements of a group of unmanned aerial vehicles (UAVs) to form various tasks of the group and within a group of aerial systems. Different phases to execute control actions for a group of aerial systems of UAVs are proposed. These phases are shown by an example of selecting different targets for group elements (UAVs). The phases are elements of the large-scale behavior of the group and in the group of UAVs and can be included in the cycle when using artificial intelligence technologies. The approach is formalized for single-function UAVs (choosing a set of end actions) and multifunction UAVs (performing one or more impact functions within the group). A group control problem for applying artificial intelligence technologies is stated. The main elements of the system of relations and conditions for effectively performing tasks by a group of UAVs and executing actions within the group as a large-scale system are formulated. This system reflects the problem statement for applying artificial intelligence technologies. As noted, using homogeneous and heterogeneous groups of UAVs is a promising approach to interpret the formal behavior of robotic systems.

Keywords: UAV, robotics, unmanned aerial systems, artificial intelligence.

INTRODUCTION

Prospects for the development of unmanned technology, especially unmanned aerial systems, are associated with group application, the creation of large groups of heterogeneous unmanned aerial vehicles (UAVs), deeply informationally interconnected and acting together in the interests of a common task. When creating such unmanned systems (large-scale objects), a significant place should be assigned to the methods and technologies of artificial intelligence (AI) being intensively developed nowadays. This paper discusses approaches to UAV group control and directions for using AI technologies in the collective application of UAVs. In contrast to control of an individual UAV, group control of UAVs has a distinctive feature (see below): the latter is a sequence of decision-making subproblems and, to a lesser extent, dynamic subproblems for implementing these decisions.

Currently, the unmanned technology is dominated by remotely piloted means with the "natural intelligence" of the operator. The unreliability or overload of communication lines and the predicted mass application of UAVs, coming into conflict with the acceptable number of operators, lead to a transition to partially or fully autonomous control. Hence, there is the need to intensify research towards the autonomous actions of UAVs and their groups [1].

Artificial intelligence technologies are currently focused primarily on applications in UAV control systems and UAV payloads. Two single UAVs, one with artificial intelligence elements and the other without them, were compared in [2]. As demonstrated, perception, decision-making, behavior, and learning determine the efficiency of detection evaluated by errors of the first and second kind. Higher assessments were given to UAVs with artificial intelligence technology. Of course, there are wide-spread solutions of *simulta-*

neous localization and mapping (SLAM) problems for UAVs using AI technology [3]; breakthrough results would hardly appear soon.

Several publications considered particular problems of applying AI technologies in the operation of UAVs. For example, the paper [4] presented an onboard detection, tracking, and target evasion system for low-cost UAV flight controllers using artificial intelligence technologies. Data processing methods are commonly used to handle information from vision systems for navigation [5], reconnaissance (search and recognition of objects of interest [6, 7]), and control [8]. Frameworks for embedding AI technologies in UAVs are actively developing [9]. Any control system needs cybersecurity [10–14]. The original results in this area are methods of controlling control channels and counteracting attacks on them, particularly implementing *the smart city concept*.

As we believe, the most promising and effective application of AI should be considered in the field of unmanned systems for various purposes (even more promising, in the creation of large-scale multifunction unmanned systems [15]). Such systems can perform different tasks in agriculture [16, 17], forest protection and bioprotection [18], power line maintenance [19], motion control, natural and man-induced disasters [20, 21] (delivery, evacuation, transportation, and other tasks following current requests). Perhaps, such solutions will appear in the defense industry [22, 23]. Modern swarm control and MANET (*Mobile Ad hoc NETwork*) communication technologies are of interest to combine efficient organization in a group [13, 24]. However, they do not provide conditions for developing a full-fledged heterogeneous system.

In the near future, it is necessary to consider the use of large-scale information-executive aerial systems of UAVs [15]. They consist of informationally interconnected heterogeneous UAVs. It seems reasonable that these systems should consist of single-function UAVs for different information and executive purposes.

When organizing and controlling large groups of UAVs, much attention should be paid to the following aspects: a new set of problems associated with the deep group interaction of vehicles that form a diverse group when executing complex (often ill-defined) actions; new approaches and methods to solve the problems of group control.

In the concept of autonomous behavior of UAVs (particularly aerial systems of UAVs), the key role is assigned to control intellectualization for the behavior of individual vehicles in the group to coordinate their actions when performing a common task. Here, a separate place is occupied by decision-making (implementing methods and techniques of actions and distributing functions in the collective actions of UAVs within a complex target of the aerial system of UAVs. In the case of elementary targets, it is possible to use traditional methods of creating appropriate intelligent control algorithms [23]. However, with the complication of these targets and a significant increase in the group size, the reasonability and even need to apply artificial intelligence (AI) technologies become apparent.

Therefore, it is required to formalize intellectualization problems for aerial systems of UAVs: structure them, identify separate subproblems, and determine adequate methods to solve them using AI technologies.

1. PHASES TO SOLVE CONTROL PROBLEMS FOR AERIAL SYSTEMS OF UAVS

To concretize the AI problem statements, we deploy the actions of the aerial systems of UAVs into phases under a given target. Each phase can be assigned a scientific problem on control of group actions and decision-making (choosing an appropriate action method). Let us define separate control problems for aerial systems of UAVs necessary to implement in these phases.

Phase 1 is to determine the group's composition based on the target. The problem here is to form a rational composition of a group of heterogeneous, single-function UAVs based on the planned spatial range of actions (the area of operation) and a priori data about the objectives, conditions of actions, and available resources. This is a decision problem under constraints. The problem is complicated by uncertain a priori data and the blurred prediction of the a priori data for the period of the group's approach. Moreover, which is very important for the large-scale application of UAVs, the problem is complicated by the uncertain target formulation (e.g., carrying out a set of measures in the disaster zone with maximum efficiency).

Phase 2 is to manage group formation in the area of operation. This is building an appropriate spatial configuration of the group based on the functional capabilities of its elements. Nowadays, this problem is solved by an expert, even for small groups of executors. This problem should also be related to decision-making.

Phase 3 is to monitor the area of responsibility by various information means of different spatially distributed UAVs, detect target objects using combined information from heterogeneous information systems, and assess the situation in the area of operation. Let situation assessment be defined as a decision problem.

Phase 4 is to allocate the targets (i.e., distribute particular actions over particular objects among the UAVs of the group). This decision problem will be considered below.

Phase 5 is to designate the targets according to the allocation results considering the spatial location of UAVs and objects, the impact of the environment, and the differences and peculiarities of the information features of objects.

Phase 6 is to assess the effectiveness of actions and the technical state of the group elements.

Phase 7 is to reconfigure the aerial system of UAVs (perform Phase 3 again under new current conditions).

This phase deployment is conditional and illustrative. The phases may be combined in time. (In this case, the listed subproblems become significantly complicated.) For example, the reconfiguration process may occur in any phase depending on the variable conditions of the target.

In addition, this list of subproblems can be supplemented with those of managing information interaction in the group, selecting the hierarchical structure of group control in each stage, and some others. (Their analysis goes beyond the scope of this paper.) Moreover, it is often impossible to separate them into a phase.

Generally speaking, there are many subproblems and technologies to solve them within the scientific problems on UAV group control and their information interaction discussed above. Many of them partially overlap. We repeatedly emphasize that if the groups are small, these subproblems can be solved by traditional ways and methods; when the group scale increases, AI methods and technologies should be applied. And the most difficult problem (the megaproblem for AI) is to form control by solving all these subproblems.

2. CONTROL OF AERIAL SYSTEMS OF UAVS: PROBLEM STATEMENT

Let us return to the scientific problems on group control of UAVs (aerial systems of UAVs). As declared above, each phase can be assigned a scientific problem, and a rational solution should be obtained for it. We formalize the problem statement corresponding to Phase 4 (target allocation); see the figure below.

Consider a group of N UAVs formed in the previous phases. Each UAV i has a particular function. The set of UAVs is represented by the N-dimensional vectors of their coordinates r_i and properties q_i. The group includes UAVs equipped with information systems to detect and recognize objects (optoelectronic, radar, and electronic reconnaissance means). The group also includes UAVs with impact means, transport UAVs, etc. A special UAV (leader) may be assigned to control the group and organize interaction. The entire group is united in a single information space.

In addition, consider a group of objects detected and identified during the previous phases: M objects with the numbers $j=1, ..., M$, coordinates r_j, and properties q_j. The elements of groups N and M are distributed in space and are moving: their current coordinates depend on time t. All UAVs from group N have some relations with each object from group M; see the arrows in the figure. (Only some of them are shown in the figure for compact presentation.)

First, we present a system of relations: a matrix describing the geometric distances between elements i and j:

$$\rho(t)=\begin{pmatrix}\rho_{11}(t) & \cdots & \rho_{1M}(t)\\ \vdots & \rho_{ij}(t) & \vdots\\ \rho_{N1}(t) & \cdots & \rho_{NM}(t)\end{pmatrix}.$$

These distances, to some extent, determine the potential capabilities of elements i to search and detect elements j and impact the latter by appropriate means.

Group N contains information elements: single-function UAVs equipped with various information means (radar, optoelectronic, and electronic reconnaissance) described by the properties q_i.

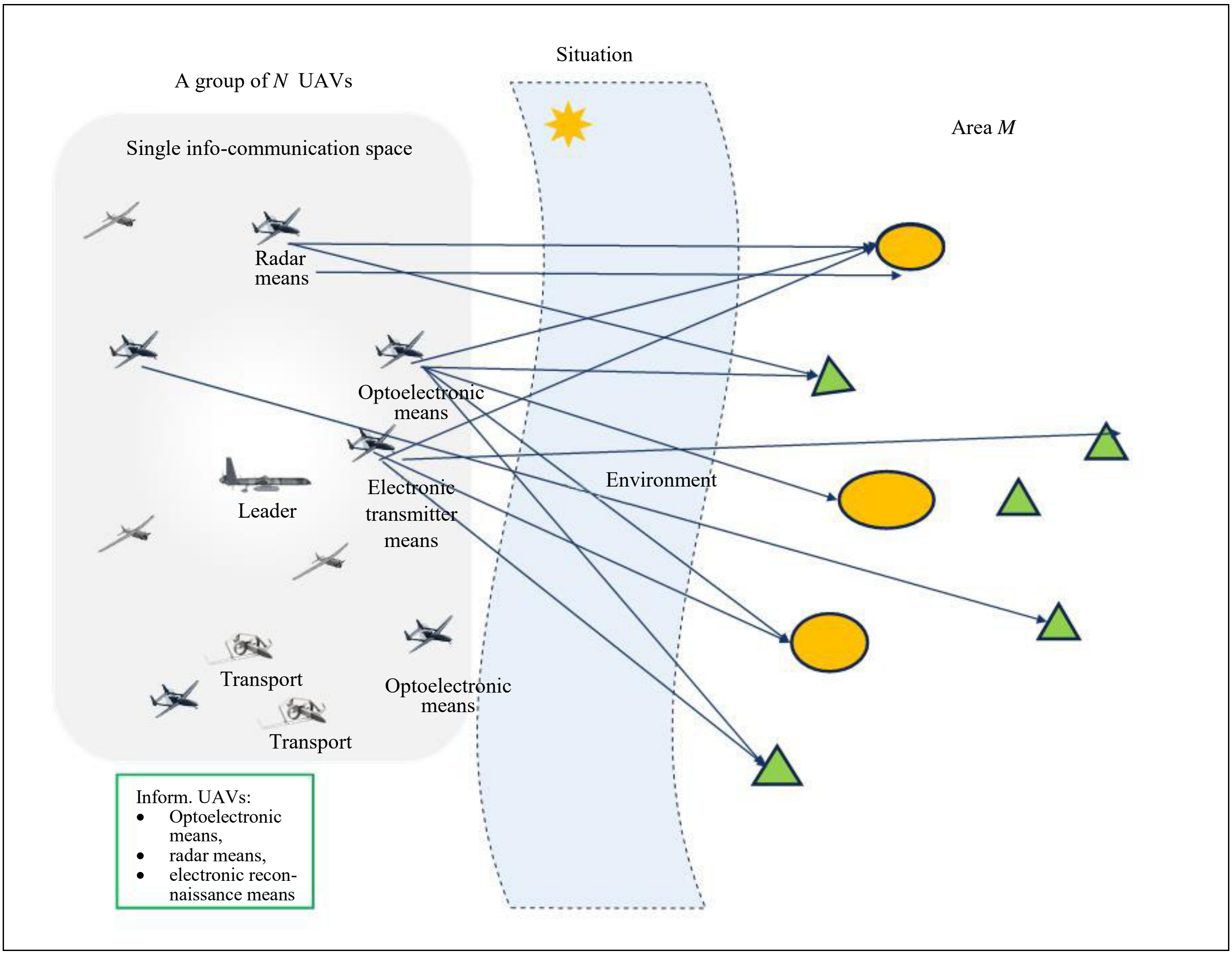
Group M includes objects with different information attributes and different reliability of detection and identification using information means with the properties q_i. Therefore, the information capabilities of each UAV with respect to objects can be represented by a system of information relations of the form

$$S(t)=\begin{pmatrix}S_{11}(t) & \cdots & S_{1M}(t)\\ \vdots & S_{ij}(t) & \vdots\\ S_{N1}(t) & \cdots & S_{NM}(t)\end{pmatrix}.$$

The elements S_{ij}, of course, depend on many factors (particularly the distances, the properties of objects and the environment, and others).

Elements from group M can be impacted by different means from group N to different degrees. The degrees depend on the impact means mounted on UAVs and the properties of the objects with respect to these means. The system of impact relations can be written as

$$B(t)=\begin{pmatrix}B_{11}(t) & \cdots & B_{1M}(t)\\ \vdots & B_{ij}(t) & \vdots\\ B_{N1}(t) & \cdots & B_{NM}(t)\end{pmatrix}.$$



The interaction scheme of a group of UAVs with objects in the target allocation phase: inform. UAVs—information UAVs; leader—the UAV deciding on the group application scenario.

Of course, these matrices contain zero elements for the impact UAVs (the system of information relations S) and the information support UAVs without impact means. (The expressions above are only matrix representations, and their elements are much more complex.)

An environment in the space between elements i and j affects the elements of the information and impact relation matrices:

$$E(t)=\begin{pmatrix} E_{11}(t) & \cdots & E_{1M}(t) \\ \vdots & E_{ij}(t) & \vdots \\ E_{N1}(t) & \cdots & E_{NM}(t) \end{pmatrix}.$$

Note that all elements of the matrices mentioned depend on time t in the operation process.

Next, the target allocation procedure needs an optimization criterion for the problem solution. As we believe, such a criterion can be the predicted efficiency of the impact of all elements i on all elements j considering the information support of their actions. Hence, the criterion can be written as

$$\mathrm{Eff}_{\max} = Y\big(B(\rho,\ S,\ E,\ t)\big),$$

where Eff is some function (functional) determining the generalized efficiency of the UAV group actions on the objects. It depends on Q (the distribution of tasks between the UAVs). The formation of this function is beyond the scope of this paper and will be considered separately. In the limit case, the optimization problem should be solved in real time using the onboard computers of UAVs.

The target allocation problem is to find a matrix Q consisting of values 0 or 1 (whether element i is assigned to interact with object j or not):

$$Q(t)^* = \begin{pmatrix} & \cdots & \\ \vdots & Q_{ij} & \vdots \\ & \cdots & \end{pmatrix}.$$

It seems that under sufficiently large dimensions of the matrices, these procedures can be obtained only using AI methods and technologies. Here, we do not analyze AI methods and technologies as applied to the listed subproblems. This paper considers a set of individual subproblems, mainly decision ones, constituting the general control problem for an intelligent autonomous group of objects.

The formal procedure for solving a particular target designation problem in the group control of UAVs has been described. It is suitable for organizing the actions of robotic groups, and the proposed approach to solve it is universal and has wide application [15].

Other components of the control problem for groups of UAVs (large-scale aerial systems of UAVs) can be described by analogy, including:

– organizing the structure of information interactions between group elements based on the current and predicted configuration and forming and reconfiguring the local information field;

– reconfiguring the group depending on changes in the targets' state, the detection of new objects, and the technical state dynamics of the group elements;

– combining information from spatially distributed sources via sensors with different signal structures and assessing the situation based on this information;

– forming a hierarchy for solving information-control subproblems in the group;

– forming a hierarchy for solving control subproblems in the group.

A significant part of these control subproblems is decision-making or discrete ones requiring specific solution principles. Of course, besides the listed subproblems, it is necessary to create specialized UAVs for group work and collective behavior to perform complex aerial tasks.

CONCLUSIONS

This paper has considered the range of subproblems and ways to solve them when creating control means and systems for heterogeneous multicomponent groups of UAVs. Such vehicles are designed to carry out complex activities for different, often ill-defined targets. Of course, each subproblem can be structured into several smaller-scale scientific problems. The subproblems discussed above would require using AI technologies along with traditional approaches. One of such subproblems—target allocation in a group—has been formalized in detail. The subproblems were previously considered in a superficial way without a generalized statement of applying artificial intelligence technologies in a group of UAVs; see [2, 8, 13, 14].

Despite the seeming long-term character of the general problem, research on the set of subproblems listed above should be deployed even now.

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29TH INTERNATIONAL CONFERENCE ON PROBLEMS OF COMPLEX SYSTEMS SECURITY CONTROL

In December 2020, the 29th International Conference on Problems of Complex Systems Security Control took place at Trapeznikov Institute of Control Sciences, Russian Academy of Sciences (RAS), Moscow. The conference was organized by the Ministry of Science and Higher Education of the Russian Federation, Trapeznikov Institute of Control Sciences RAS, Keldysh Institute of Applied Mathematics RAS, the RAS Scientific Council on the Theory of Controlled Processes and Automation, and the Ministry of the Russian Federation for Civil Defense, Emergencies and Elimination of Consequences of Natural Disasters.

The conference was attended by 123 authors from 49 organizations (Russia and several foreign countries). The conference program included 84 papers in eight sections:

1. Theoretical and methodological questions of security support;
2. Problems of economic and sociopolitical security support;
3. Problems of information security support;
4. Cybersecurity. Security aspects in social networks;
5. Ecological and technogenic security;
6. Modeling and decision-making for complex systems security control;
7. Automatic systems and means of complex systems security support;
8. Legal aspects of complex systems security support.

This annual conference traditionally takes place in the second half of December. The past 2021 was extremely tense and eventful. First of all, note the growing complexity of counteraction to the COVID-19 pandemic due to the emergence of new virus mutations. Despite the need to consolidate the efforts of the global community for the survival of humanity in the face of the pandemic, the international situation continued to deteriorate, and the crisis in relations between Russia and Western countries deepened, taking the form of intense military-political and economic confrontation, as well as an open information war.

The COVID-19 pandemic and the resulting global financial crisis affected foreign economic relations, world trade, and the domestic commodity and financial markets of even relatively prosperous countries. These negative processes resulted in large-scale bankruptcies of small businesses, job cuts, employment problems, and the need to transfer employees of many organizations to the remote working mode, causing a significant increase in cybercrimes using remote attacks, phishing, social engineering technologies, etc. In addition, new types of cybercrimes emerged, including those exploiting the COVID-19 topic in various ways.

The year 2021 will also be remembered for natural and climatic anomalies: abnormal heat in Russia, the USA, Canada, and several European countries; large-scale forest fires in Yakutia (Russia), Turkey, Greece, and the USA; floods in the Crimea and Krasnodar Krai (Russia), the Far East, China, India, Austria, Czech Republic, Germany, and other countries; typhoons and downpours in the Far East; an unprecedentedly destructive tornado in the USA; frosts in Africa and South America, etc. There is still no consensus in the scientific community about their causes. Also, there were man-induced accidents and disasters at industrial and transport facilities.

The current situation and explicit negative trends of further development (possible or even quite probable in some areas) call for comprehensive measures and systemic mechanisms to improve the effectiveness of countering various external and internal threats to the security of individuals, society, and the state. These factors increase the relevance and importance of comprehensive interdisciplinary (fundamental and applied) research of methods, tools, and mechanisms to improve the effectiveness of security control (in the broadest sense). No doubt, it affected the topics of conference papers.

According to an established tradition, the conference was opened with a detailed paper, “How not to be in the sixteenth century,” by *G.G. Malinetskii, V.V. Kul’ba, T.S. Akhromeeva, S.A. Toropygina, and S.A. Posashkov*. The authors analyzed the impact of the ongoing global changes and growing contradictions in world development and the associated risks and threats. The paper considered key strategic objectives for developing Russian society and the state in the long term. Much attention was paid to the problems in culture, science, high technologies, and demography aggravated by the pandemic and other priority tasks of progressive economic development of the country. Based on the analysis of risks and global threats to the Russian state and society, the authors focused on the ways out of the current difficult situation to achieve the basic national development goals of the Russian Federation in the short and long run.

For the second year, conference participants dealt with improving the efficiency of counteraction to the COVID-19 pandemic in their papers. Among them, note the paper "A technology to create monitoring and forecasting systems for the state of dangerous phenomena and objects (on the example of COVID-19 pandemic)" by *A.V. Sokolov, G.V. Roizenzon, and N.P. Komendantova*. It was devoted to developing a methodology to assess the effectiveness of restrictive measures as a tool to combat the spread of COVID-19. Three basic groups of effectiveness criteria were identified: available resources of different types (bedspace, medical staff, equipment, medicines, etc.); the rates of spending and replenishing necessary resources to combat the pandemic; the degree of achieving the set goals. To assess the effectiveness of restrictive measures, the authors proposed applying multicriteria order classification and verbal analysis methods for decisions made. Also, the paper generalized the experience accumulated by the authors in monitoring and forecasting new COVID-19 cases in Moscow in 2020 and 2021.

The problems of counteraction to the pandemic were also addressed in the following papers: "On a peculiarity of modeling the first stage of COVID-19 infection" by *M.E. Stepantsov*; "New models of SARS-CoV-2 virus spread and security control problems" by *N.G. Kereselidze*; "The impact of COVID-19 restrictions on economic systems security" by *T.Kh. Usmanova and N.N. Volodina*.

A distinctive feature of the conference is numerous papers on various topics, presenting research results on a wide range of methodological and applied problems of improving the effectiveness of security control processes in the context of digitalization, the rapid development of information and communication technologies, and the threats and risks associated with these processes.

A research group headed by *V.L. Shultz*, Corresponding Member of the RAS, presented the paper "Analysis of the uncertainty factor in preparing managerial decisions." The authors considered the problems of increasing the efficiency of organizational management under risk. As stated in the paper, such uncertainty has two main sources: the subjective (epistemological) source due to insufficient knowledge necessary for making managerial decisions, and the objective (aleatoric, ontological) source due to the stochastic nature of the control object or its environment. A separate class is a linguistic (subjective) uncertainty due to several objective properties of natural language. According to the authors, the methodology for evaluating uncertainty factors is being developed mainly towards methods for applied problems within the studied segments of subject areas due to the multifaceted character of uncertainty. Attempts to develop universal methods for assessing the impact of uncertainty on the effectiveness of managerial decisions face significant difficulties. They can be partially overcome using scenario analysis.

In the paper "Cryptocurrencies as a threat to the national security of Russia: legal counteraction mechanisms," *A.A. Timoshenko* considered problems caused by the lack of legal regulation of many aspects of cryptocurrency circulation in the Russian Federation. With the worldwide recognition of cryptocurrencies as an instrument of forming alternative financial relations, the author paid special attention to the analysis of threats to the national security of Russia, particularly in the situation when virtual currencies are used for illegal purposes. Threats to uncontrolled circulation of cryptocurrencies were analyzed in terms of goals, objectives, and functions of law enforcement agencies. As a result, detailed proposals to improve the Russian system of legislative regulation were formulated, including the introduction of appropriate changes in the current legislation and vesting the Government and relevant agencies with the power to regulate and control the circulation of digital financial assets. Fully agreeing with the conclusions of the author, we emphasize the growing relevance of the problems considered in the paper: the mass use of cryptocurrencies in the national payment cycle and numerous issuers uncontrolled by the state may eventually lead to a critical disorder of the financial system and, most importantly, to the inability to effectively plan and implement a uniform monetary policy of the state with all the ensuing negative consequences.

Traditionally, conference participants are interested in the problems of information and cybersecurity control. *R.V. Meshcheryakov* presented the paper "An approach to secure intelligent control of robots and their coalitions using human-robot(s) and robot-robot(s) interfaces." He considered the problems of forming secure mechanisms of inter-machine data exchange. The topicality of such research is currently increasing due to the development of the Internet of Things and the absence of security standards for robotic control systems using human-machine interfaces. As shown in the paper, the developed models and security mechanisms of the systems should be based on different interfaces for redundant communication channels when giving commands and receiving feedback from control objects. In addition, these models and mechanisms should consider such factors as noise immunity of measuring channels, fault tolerance of the entire system, the reproducibility of the reference signal, and the presence of a single transmission format for measurement information.

This broad topic was also treated in the following papers: “A two-factor authentication algorithm as a tool to reduce FRR for a proactive attack detection filter” by *A.M. Smirnov and A.Yu. Iskhakov*; “Situational awareness for the safe and effective operation of agrorobots” by *V.K. Abrosimov and A.N. Raikov*; “Some issues in the verification and validation process of cybersecurity control” by *E.F. Jharko*; “A concept of information security for a swarm of cyber-physical systems” by *D.I. Pravikov*; “Determining the success of intruder’s actions in a homogeneous environment” by *K.A. Bugaiskii*; “Using SSL/TLS technology to create secure network channels in distributed systems” by *R.E. Asratyan*; “Methods to counteract tracing user’s browser fingerprints” by *A.A. Salomatina*; “Service-browser and Man-in-the-middle attacks” by *V.L. Orlov and E.A. Kurako*; “The problem of optimizing the reconstruction scheme of destroyed operating data reserve in distributed systems” by *S.K. Somov*; “Information reliability as an element of information security and assessment of its level” by *A.D. Kozlov and N.L. Noga*; “Goals, tasks, and principles of security of digital intellectual property management systems” by *V.O. Sirotyuk*; “Ensuring the continuous development of software products certified by security requirements” by *A.A. Melikhov*.

Several interesting papers were devoted to topical security problems in social networks. Among them, we mention the following: “Features of mathematical tools used to build security systems in social networks” by *L.V. Zhukovskaya*; “Systematization of psychological factors to change beliefs and attitudes as a result of communicative influences in the form of causal influence model” by *Z.K. Avdeeva and S.V. Kovriga*; “Assessment of risks of destructive content in social networks” by *M.V. Mamchenko and A.S. Rey*; “Ethical aspects of applying artificial intelligence tools to ensure the space of trust in electronic media” by *G.K. Boreskov*; “Developing a dynamic system for the operation of social network communities” by *E.P. Okhapkina*; “Digitalization: threats and risks” by *V.V. Muromtsev and A.V. Muromtseva.*

Conference participants presented many interesting papers on problems of managing economic, environmental, energy, and technogenic security in the context of high-tech development, associated recently with the international “green” or “climate” agenda. However, the declared and real goals of this agenda are a separate subject of detailed analysis widely discussed by scientific and expert communities.

In the paper “Safety of the aquatic ecosystem of Azov–Black Sea region: cognitive study,” *G.V. Gorelova, E.V. Melnik, M.V. Orda-Zhigulina, and D.V. Orda-Zhigulina* performed the cognitive analysis and simulation of processes in the regional aquatic ecosystem to predict environmental threats and ensure the safety of the population and coastal infrastructure. The authors presented the functional structure of an original monitoring system for hazardous phenomena in natural systems, designed to observe the corresponding processes continuously.

The approach proposed by the authors is promising: a single monitoring system integrates diverse and multi-temporal data from various sources to reveal (particularly implicit) cause-effect relations between the parameters of the hydro-ecosystem studied by traditional methods. Thus, the system determines the patterns of ecosystem processes and provides intellectual support for decision-making processes to counteract environmental threats based on cognitive modeling.

Among the contributions on this broad topic, note the following papers: “Problems of managing the development of large-scale socio-economic systems” by *N.N. Volodina, N.I. Komkov, and V.V. Sutyagin*; “Formalization of institutions, adverse selection, and control of agents’ corrupt behavior” by *R.M. Nizhegorodtsev*; “Preparation of Russian public administration system for supercritical situations of natural and man-induced character” by *E.P. Grabchak and E.L. Loginov*; “On an approach to critical infrastructure risk management” by *E.A. Abdulova*; “Structural stability of the Arctic as a territorial economic ecosystem” by *N.N. Lanter*; “Digital transformation and import substitution related to nuclear facility security” by *T.A. Piskureva and A.N. Makhov*; “A system approach to the application of artificial intelligence to resolve environmental safety problems in the digital transformation of agriculture” by *V.I. Medennikov*; “Fundamentals of information support of electricity supply under the destructive impact of hydrometeorological factors” by *M.A. Polyukhovich*; “Integrated geo-ecological monitoring of forest geo-ecosystems of the Moscow metropolitan region” by *R.E. Torgashev.*

Traditionally, conference participants show great interest in the problems of technogenic and industrial security. In the paper “Management of cybersecurity risk at the design stage of industrial systems,” *V.G. Promyslov and K.V. Semenkov* described a cybersecurity risk assessment technology for the design process of critical industrial facilities. The proposed technology consists of two basic stages. The first stage includes a general risk assessment for the system to be designed under uncertainty in understanding the details of system implementation and, in part, the requirements imposed on it. In this stage, the basic technical solutions for cybersecurity are laid down. Moreover, the generalized assessments of risks can be used to prioritize their detailed elaboration when designing the security architecture of critical industrial facilities (e.g., for division into security zones or classification of assets). The second (optional) stage includes a detailed risk assessment considering the specifics of the system architecture and the threat model.

The proposed technology has several advantages: the ability to prevent critical errors in the system design process due to the under- or overestimation of cybersecurity requirements; the ability to reduce the volume (the cost and time) of design work by eliminating detailed risk assessment procedures for individual subsystems if the integral assessment for the entire system does not exceed a given threshold.

Several interesting papers were devoted to the problems of prevention and elimination of man-induced and natural emergencies and the safety and reliability of technological complexes and transport systems: "Increasing the safety of managing complex objects under implicit variations of technological process parameters" by *V.O. Chinakal*; "Safe dispatch control in intelligent unmanned traffic control systems" by *L.A. Baranov, E.P. Balakina, and V.G. Sidorenko;* "Mathematical modeling of seismic stress waves in a half-plane by a vertical cavity of rubber: a width-to-height ration of 1:10" by *V.K. Musaev*; "Stochastic modeling of cascade scenarios of accidents and disasters" by *M.Yu. Prus*; "Fire security analysis of a thermal power plant based on the study of fire hazards" by *A.V. Evdokimova*; "On the justification of industrial safety audit" by *E.V. Klovach and V.A. Tkachenko*; "Vibration fatigue of mechanisms and machines: standardization and rate setting" by *O.B. Skvortsov*.

In addition, note several conference papers of different topics united by the topicality of the problems considered and the demand for their solutions: "Tools for digitalizing the personnel security management of a regional production cluster" by *V.V. Bystrov, A.V. Masloboev, and I.O. Datiev*; "Natural computing in risk management of complex systems: models and methods" by *A.A. Shiroky*; "Risk management of a complex computer network based on a general arbitration scheme" by *E. V. Anikina*; "Fundamentals of modeling information security measures to ensure the conflict stability of socio-economic organizations" by *L.E. Mistrov and E. Golovchenko*; "A methodology for calculating economic damage from drug addiction" by *A.N. Fomichev*; "Migration policy and city security" by *V.V. Kafidov*; "National security in the sphere of intellectual property in Russia" by *V.V. Leshchenko*; "Information decision support means for assessing the capabilities of technical intelligence" by *I.A. Sidorenko, O.N. Dudarikov, and N.E. Khodyreva*; "Analysis of applied ways to increase the metrological reliability of measuring transducers" by *A.M. Anokhin*; "Peculiarities of estimating vibration influences in electromechanical systems with pulse control" by *V.I. Stashenko, O.B. Skvortsov, and O.A. Troitsky*; "Computational load balancing under the parallel solution of a minimax scheduling problem by the branch-and-bound method" by *D.R. Gonchar.*

The papers can be found in the conference proceedings[1] or on the official conference website: URL:https://iccss2021.ipu.ru/prcdngs.

In his closing remarks, the Conference Chair, Dr. Sci. (Eng.), Prof. *V.V. Kul'ba* announced plans to hold the 30th Anniversary International Conference on Problems of Complex Systems Security Control, according to the established tradition, in December 2022 at Trapeznikov Institute of Control Sciences RAS. Please contact the Organizing Committee via phone + 7 495 198-17-20 (ext. 1407) or e-mail iccss@ipu.ru. The Technical Secretary of the conference is *Alla Farissovna Ibragimova*.

Academic Secretary of the Organizing Committee
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