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DESIGN OF MULTIVARIABLE TRACKING SYSTEMS VIA ENGINEERING PERFORMANCE INDICES BASED ON H_∞ APPROACH

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Abstract. This paper proposes an algorithm for designing a measured output-feedback controller with given or achievable engineering performance indices for linear multivariable systems. The plant is subjected to bounded exogenous disturbances from the class of polyharmonic functions with an infinite number of harmonics and a bounded sum of their amplitudes for each disturbance component. As a result, additional tracking errors appear in controlled variables. The problem is to design a multivariable output-feedback controller ensuring given or achievable tracking errors, the settling time determined by a given or achievable degree of stability of the closed loop system, and a set of the oscillation indices M_i for the *i*th closed loop relating the *i*th reference signal g_i to the *i*th controlled variable z_i. In addition, the controller should ensure the conditions $M_i \le \gamma$, where γ is a given number or the minimand. As shown below, H_∞ control methods are quite convenient for solving such problems. An illustrative example of designing an interconnected electric drive is presented.

Keywords: linear multivariable systems, bounded exogenous disturbances, tracking errors, settling time, degree of stability, oscillation index of the *i*th loop.

INTRODUCTION

The classical theory of automatic control of minimum-phase neutral single-input single-output (SISO) plants has shown high practical efficiency due to the physical clarity of the engineering performance indices underlying it: the tracking error, the settling time, and the oscillation index [1].

For multivariable (multi-input multi-output) plants, such an approach to controller design has not yet been formed, although the obvious first step is to solve the problem of autonomous control [2–4] going back to Voznesenskii [5].

The authors' recent works [6, 7] were devoted to single-variable plants (both minimum-phase and non-minimum-phase, stable and unstable), and controllers were constructed based on the H_∞ theory. This paper deals with multivariable plants and can be considered an extension of the approach [8] to the class of tracking systems: the robustness of a closed loop system is assessed not using the radius of stability margins but the oscillation index, a more natural and generally accepted performance index in the theory and applications of tracking systems.

Let us clarify the concept of the oscillation index of a multivariable system: it means a set of the oscillation indices M_i for the *i*th closed loop relating the *i*th reference signal g_i to the *i*th controlled variable z_i. The controller should ensure the conditions $M_i \le \gamma$, where γ is a given number or the minimand. As shown below, H_∞ control methods are quite convenient for solving such problems.

In practice, automatic systems are subjected to bounded exogenous disturbances causing additional tracking errors. This paper considers polyharmonic exogenous disturbances with an infinite number of unknown harmonics and a bounded sum of their (unknown) amplitudes for each disturbance component. The controller should ensure given (or achievable) tracking errors in the presence of such disturbances.

Note that they cover an applications-relevant class of continuous disturbances with piecewise continuous time derivatives [8, 9].

Another engineering performance index employed in control design is the settling time, characterizing the response speed of the closed loop system under nonzero initial conditions and (or) a stepwise change in the reference signal or disturbance. Below, the settling time is indirectly taken into account by ensuring a given degree of stability of the closed loop system. Although this index estimates the rate of transient processes very approximately (especially when the roots of the characteristic polynomial of the closed loop system are close to each other), it has proven itself well in applications with an initial estimate of the settling time. As demonstrated in [6, 10], an excessive increase in the degree of stability (a smaller distance from the plant's zero to the imaginary axis) catastrophically reduces the radius of stability margins (raises the oscillation index) and the phase and modulus margins even in the minimum-phase case with an output-feedback controller. Such results are unacceptable in practice due to large overshoots in the step response of the closed loop system. This phenomenon is analogous to the burst effect [11] for the output-feedback controllers. Therefore, the controller design algorithm suggested below, like the one developed in [8], provides for a gradual increase in the degree of stability.

To the authors' knowledge, Aleksandrov [12, 13] and his students are among the leading researchers stating and solving particular problems of this very difficult class within the theory of LQ control and H_∞ optimization. A detailed survey of the corresponding results was provided in the paper [8]. In the Western literature, with the appearance of the H_∞ theory in the early 1980s, stability margins were given much attention [10, 14]. At the same time, the issues of accuracy, response speed, and stability margins, combined in a unified output-feedback controller design method for multivariable tracking systems, have not received proper coverage.

Let us mention some important publications on different aspects of this range of problems. For example, the issues of nonsmooth H_∞ optimization were considered in [15], and the results were later used in [16] to design controllers of a given structure and order (particularly PID controllers). The matter is that controllers based on modern design techniques have a high order. Hence, they are "fragile": lose stability under small deviations of their parameters from the calculated ones [17]. This property is usually expressed in small phase and modulus margins of control loops. Also, a still-unsolved problem is choosing weight functions in the design of multivariable H_∞ controllers, noted in [18]. (For scalar systems, some rules were suggested in the monograph [14].) Below, we introduce a strict mathematical rule for choosing a weight for a given tracking accuracy.

Besides a different measure of robustness for the closed loop system, this paper involves a fundamentally novel approach to accuracy compared to [8]: a new vector of weighted controlled variables is not introduced, but the vector of exogenous disturbances is weighted. With this approach, the degree of sufficiency of the results is considerably decreased.

As shown below, the problem to ensure the engineering performance indices reduces to a special H_∞ optimization problem [19, 20]. A numerical solution of such a degenerate problem can be conveniently obtained using the technique of Linear Matrix Inequalities (LMIs) [21, 22], e.g., in MatLab's Robust Control Toolbox [23]. Finally, an illustrative example of designing an interconnected electric drive [8] is presented.

1. PROBLEM STATEMENT

Consider a plant described by the state-space equations

$$\dot{x} = Ax + B_1 f + B_2 u, \quad z = Cx, \tag{1}$$

where $x \in R^n$, $u \in R^m$, and $z \in R^{m_1}$ denote the plant's state vector, the control vector, and the vector of controlled variables, respectively; $f \in R^{m_3}$ is the vector of unmeasured exogenous disturbances.

Let the plant (1) be looped by a stabilizing output-feedback controller

$$\dot{x}_c = A_c x_c + B_c \varepsilon, \quad u = C_c x_c + D_c \varepsilon, \quad \varepsilon = g - z, \tag{2}$$

where $x_c \in R^{n_c}$ $(n_c \le n)$, $g \in R^{m_1}$, and $\varepsilon \in R^{m_1}$ denote the controller's state vector, the vector of reference signals, and the vector of measured tracking errors; A_c, B_c, C_c, and D_c are numerical matrices.

The exogenous disturbance vector has bounded components of the form

$$f_i(t) = \sum_{k=1}^{\infty} f_{ik} \sin(\omega_k t + \psi_{ik}), \quad i = \overline{1, m_3}, \tag{3}$$

where the amplitudes $f_{ik} \ge 0$, the initial phases ψ_{ik}, and the frequencies ω_k $(i = \overline{1, m_3},\ k = \overline{1, \infty})$ are unknown, and the number of harmonics is infinite.

Assume that the exogenous disturbance is bounded:

$$\sum_{k=1}^{\infty} f_{ik} \le f_i^*, \quad i = \overline{1, m_3}, \tag{4}$$

where $f_i^* > 0$, $i = \overline{1, m_3}$, are given numbers.

Conditions (3) and (4) mean the inequalities $|f_i(t)| \le f_i^*$, $(i = \overline{1, m_3})$. The model (3) and (4) covers a wide applications-relevant class of continuous exogenous disturbances with piecewise continuous time derivatives [8]. Therefore, such disturbances can be expanded into an absolutely convergent Fourier series [9], representing a special case of (3) with multiple frequencies. In addition, the series (3) is not necessarily a periodic function of time. For example, choosing all frequencies in (3) equal to 0 and the initial phases equal to $(2k+1)\pi/2$, where $k=\overline{0,\infty}$, we arrive at a step function.

The tracking errors caused by the exogenous disturbance (3), (4) are defined as

$$\varepsilon_{i,st} = \sup_{t \geq t_{\text{set}}} |\varepsilon_i(t)|, \quad i=\overline{1,m_1},$$

where t_{set} denotes the settling time. A requirement common in practice is

$$\varepsilon_{i,st} \leq \varepsilon_i^*, \quad i=\overline{1,m_1}, \tag{5}$$

where $\varepsilon_i^* > 0$ are given numbers (the desired tracking errors).

The settling time in the closed loop system (1) and (2) can be approximately estimated as $t_{\text{set}} \approx 3/\beta$, where β specifies the system's degree of stability (the minimum distance from the eigenvalues of the closed loop system matrix

$$A_{cl} = \begin{bmatrix} A - B_2 D_c C & B_2 C_c \\ -B_c C & A_c \end{bmatrix}$$

to the imaginary axis).

Problem. Find a stabilizing controller (2) under which:

The system's accuracy requirements

$$\varepsilon_{i,st} \leq \gamma_1 \varepsilon_i^*, \quad i=\overline{1,m_1}, \tag{6}$$

hold, where γ_1 is a given number or the minimand.

The oscillation indices do not exceed a given number (or the minimand) γ_2,

$$M_i = \|t_i\|_\infty \leq \gamma_2, \quad i=\overline{1,m_1}, \tag{7}$$

where $t_i(s)$ is the transfer function of the closed loop system relating the ith reference signal g_i to the ith controlled variable z_i, and $\|t_i\|_\infty$ denotes its H_∞ norm.

The eigenvalues of the matrix A_{cl} of the closed loop system (1) and (2) satisfy the condition

$$\operatorname{Re} \lambda_i(A_{cl}) \leq -\beta, \quad i=\overline{1,n+n_c}, \tag{8}$$

where $\beta \geq 0$ is a given number.

Let us comment on this problem, further referred to as the original problem.

If the plant (1) is non-minimum-phase in the control variable (has zeros in the right half-plane), then the initial accuracy requirement (5) cannot be satisfied for any ε_i^*; therefore, the requirements (6) should be considered instead. Moreover, if the plant is also unstable [14], then the value γ_2 on the right-hand side of inequality (7) always exceeds 1, and it has a lower bound on M_i that cannot be overcome by any linear controller. And finally, the degree of stability β cannot be made greater than the plant's zero closest to the imaginary axis, which sharply decreases the stability margin: the high accuracy requirement contradicts the requirement of low oscillation indices, and high performance (a large value of β) contradicts the requirement for stability margins (small values of M_i).

This paper seeks a reasonable compromise between the mutually contradictory engineering performance indices based on the H_∞ optimization technique, which became a very convenient tool for designing applications-relevant controllers.

2. SOLUTION BASED ON H_∞ APPROACH

For solving the original problem, we first establish a connection between the oscillation indices M_i, $i=\overline{1,m_1}$, and the H_∞ norm of the transfer function $T(s)$ of the closed loop system relating the reference signal vector g to the controlled variable vector z. The following result is true.

Lemma. If

$$\|T\|_\infty \leq \gamma, \tag{9}$$

then

$$M_i \leq \gamma, \quad i=\overline{1,m_1}. \tag{10}$$

Note that due to (9), a similar inequality will hold for any element of the matrix $T(s)$ [24], particularly for any diagonal element $t_i(s)$: $\|t_i\|_\infty \leq \gamma$. Since the transfer function $t_i(s)$ relates the ith reference signal g_i to the ith controlled variable z_i, by definition we obtain $M_i = \|t_i\|_\infty$ and consequently (10).

At its input, the closed loop system (1) and (2) receives two exogenous signals, g and f. We form the augmented vector $w^{\mathrm{T}} = \begin{bmatrix} g^{\mathrm{T}} & f^{\mathrm{T}} \end{bmatrix}$ and choose the vector z as the controlled output. In the closed loop system, these vectors are connected via the transfer function $T_{zw}(s)$:

$$z = T_{zw}(s) w = \begin{bmatrix} T(s) & T_f(s) \end{bmatrix} w, \tag{11}$$

where $T(s)$ denotes the transfer function of the closed loop system relating the vector g to the vector z; $T_f(s)$ denotes the transfer function of the closed loop system relating the vector f to the vector z.

Consider an auxiliary H_∞ optimization problem of the form

$$\|T_{zw}\|_\infty \le \gamma\,, \tag{12}$$

where γ is a given number or the minimand.

Due to the transfer function structure (11), condition (12) can be written in the equivalent frequency representation

$$T^{\mathrm T}(-j\omega)T(j\omega)+T_f^{\mathrm T}(-j\omega)T_f(j\omega)\le\gamma^2 I,\quad \omega\in[0,\infty), \tag{13}$$

where I denotes an identity matrix of compatible dimensions. Hence,

$$T^{\mathrm T}(-j\omega)T(j\omega)\le\gamma^2 I \qquad T_f^{\mathrm T}(-j\omega)T_f(j\omega)\le\gamma^2 I,\quad \omega\in[0,\infty), \tag{14}$$

where the former condition is equivalent to (9), and the latter one means that $\|T_f\|_\infty \le \gamma$.

Thus, with the controller (2) obtained by solving problem (12) numerically, we satisfy the target requirement (7) for $\gamma_2=\gamma$, where γ is the value realized during the calculations.

Next, consider the accuracy requirements (6). To account for them when solving problem (12), we replace the matrix B_1 of the plant (1) by $B_1\cdot Q^{1/2}$, where $Q^{1/2}$ is a scalar weight specified below. Then the second condition of (14) takes the form

$$T_f^{\mathrm T}(-j\omega)QT_f(j\omega)\le\gamma^2 I,\quad \omega\in[0,\infty).$$

Using the lemma on the working process from [8], for the steady-state values of the controlled variables $z_{i,st}=\sup\limits_{t\ge t_p}|z_i(t)|,\ \ i=\overline{1,m_1}$, we get

$$Qz_{i,st}^2\le\gamma^2\left(\sum_{j=1}^{m_3} f_j^*\right)^2,\ \ i=\overline{1,m_1}\,, \tag{15}$$

where f_j^* are known bounds on the exogenous disturbance components from (4). In contrast to the paper [8], formula (15) involves the common weight Q for all variables $z_{i,st}$. Therefore, we choose it based on the least error $z_{\min}=\varepsilon_{\min}=\min\left\{\varepsilon_1^*,\varepsilon_2^*,\ldots,\varepsilon_{m_1}^*\right\}$:

$$Q=\left(\sum_{j=1}^{m_3} f_j^*\right)^2\Big/\left(\varepsilon_{\min}^*\right)^2. \tag{16}$$

In this case, the tracking error due to the exogenous disturbance f satisfies the relation $\varepsilon_{i,st}=z_{i,st}$. It follows from inequality (15) that

$$\varepsilon_{i,st}\le\gamma\varepsilon_{\min}^*,\ \ i=\overline{1,m_1}\,,$$

and the accuracy requirements (6) are satisfied.

Now, we account for the stability requirements (8) to the closed loop system, which determine the settling time. Following the paper [8], when solving problem (12), we replace the matrix A of the plant (1) by $\tilde A=A+\beta I$, where β is the desired degree of stability. Then the solution of the shifted problem (12) with $\tilde A$ yields a shifted controller with matrices $\tilde A_c$, B_c, C_c, and D_c. According to [8], the desired controller (2) solving the original problem has the matrices

$$A_c=\tilde A_c-\beta I,\ B_c,\ C_c,\ D_c\,. \tag{17}$$

Summarizing the considerations above, we formulate the following result.

Theorem. The controller (2) and (17) solves the original problem if the weight Q in the shifted H_∞ problem (12) satisfies condition (16). In this case, the values of γ_1 and γ_2 (see the target requirements (6) and (7)) coincide with the value of γ realized when solving problem (12) and (13) numerically.

Note that transition from inequality (12) to inequalities (14) makes this result sufficient.

3. NUMERICAL SOLUTION

Since the controlled variable vector of the system (11) contains no controls, problem (12) is singular and cannot be solved numerically using the 2-Riccati method [19]. A preferable approach is based on the LMI technique [21, 22] and calculations in MatLab [23]; see the details below. As noted earlier, an excessive increase in the degree of stability β sharply raises the oscillation indices, causing large overshoots in the step response of the closed loop system. Therefore, the design algorithm presented below involves the principle of gradually increasing the response speed or the value of β. This algorithm includes the following steps.

1. Replace the plant's matrix A by the matrix $\tilde A=A+\beta I$, first letting $\beta=0$.

2. Choose a weight from equality (16) and construct the four matrices A_{gen}, B_{gen}, C_{gen}, and D_{gen} of the state-space equations of the generalized plant: $A_{\mathrm{gen}}=\tilde A$, $B_{\mathrm{gen}}=\begin{bmatrix}0 & B_1\cdot Q^{1/2} & B_2\end{bmatrix}$, $C_{\mathrm{gen}}=\begin{bmatrix}C\\ -C\end{bmatrix}$, and $D_{\mathrm{gen}}=\begin{bmatrix}D_{11} & D_{12}\\ D_{21} & D_{22}\end{bmatrix}$, where $D_{11}=\begin{bmatrix}0 & 0\end{bmatrix}$, $D_{21}=\begin{bmatrix}I & 0\end{bmatrix}$, $D_{22}=0$, $D_{12}=0$, and all matrices have compatible dimensions.

3. Form the generalized plant's system matrix using the procedure

$$P=\mathrm{ltisys}(A_{\mathrm{gen}},\,B_{\mathrm{gen}},\,C_{\mathrm{gen}},\,D_{\mathrm{gen}})\,.$$

4. Find the optimal value $\gamma_0 = \text{hinflmi}(P, [m_2, m])$ in problem (12), where m_2 and m are the numbers of the controller's inputs and outputs.

5. Choose $\gamma > \gamma_0$ and construct a controller's system matrix K that solves problem (12) using the procedure $[\gamma, K] = \text{hinflmi}(P, [m_2, m], \gamma, \varepsilon)$, where ε is the accuracy of calculating γ.

6. Using the procedure $[\tilde{A}_c, B_c, C_c, D_c] = \text{ltiss}(K)$, extract the state-space matrices of the shifted controller from the system matrix K.

7. Find the matrices $A_c = \tilde{A}_c - \beta I$, B_c, C_c, and D_c of the desired controller (17).

8. Construct the step response of the closed loop system under the exogenous disturbances (3) and (4), and find the tracking errors, the settling time, and the oscillation indices M_i. If the accuracy requirements (6) and (or) the oscillation requirements (7) do not hold, the problem is unsolvable by the proposed approach. Otherwise, proceed to Step 9.

9. If the response speed requirements do not hold, increase the value of β and get back to Step 1. Otherwise, the problem is solved.

For the first iteration of the algorithm, a natural choice is $\gamma = \gamma_0$. If the requirements (6) and (or) (7) do not hold, consider separate controller design problems with the oscillation index ($T_{zw}(s) = T(s)$) or with a given accuracy ($T_{zw}(s) = T_f(s)$ with the weight from formula (16)). These problems have a necessary and sufficient character, yielding γ_0 that determines the achievable accuracy (6) or oscillation indices $M_i = \gamma_0$.

4. CONTROLLER DESIGN FOR AN INTERCONNECTED ELECTRIC DRIVE

Consider an interconnected electric drive model described in the paper [8]. In [25], it was classified as a parallel system. The structural diagram of the model is shown in Fig. 1.

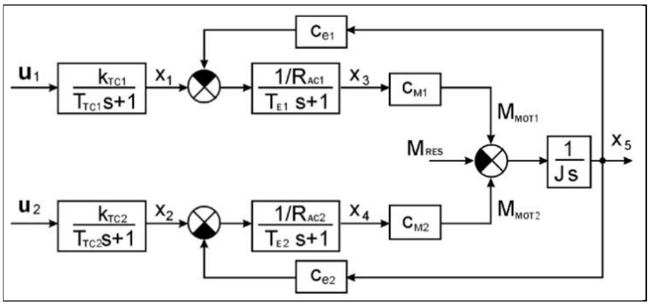

Fig. 1. The structural diagram of plant.

This diagram has the following notations: x_1 and x_2 are the deviations of the output voltages of the thyristor converters from the rated ones supplied to the armature circuits of the motors; x_3 and x_4 are the deviations of the armature currents of the drive motors; x_5 is the deviation of the angular rate of rotation of the motor shaft; u_1 and u_2 are the deviations of the control voltages supplied to the thyristor converters from the drive control system; M_{MOT1} and M_{MOT2} are the deviations of the electromagnetic moments developed by the motors from the rated values; M_{RES} is the deviation of the moment of resistance (load); T_{TC1} and T_{TC2} are the time constants of the thyristor converters; k_{TC1} and k_{TC2} are the gains of the thyristor converters; c_{M1}, c_{M2}, c_{e1}, and c_{e2} are the design constants of the motors; R_{AC1} and R_{AC2} are the active resistances of the armature circuits of the motors; T_{E1} and T_{E2} are the electromagnetic constants of the armature circuits of the motors; J is the total moment of inertia reduced to one of the motor shafts.

The model parameters in this diagram have the following numerical values: $c_{M1} = 8.1\frac{\text{N}\cdot\text{m}}{\text{A}}$, $c_{M2} = 8.262\frac{\text{N}\cdot\text{m}}{\text{A}}$, $c_{e1} = 8.15\frac{\text{V}\cdot\text{s}}{\text{rad}}$, $c_{e2} = 8.313\frac{\text{V}\cdot\text{s}}{\text{rad}}$, $T_{E1} = 0.0886\text{s}$, $T_{E2} = 0.090372\text{s}$, $T_{TC1} = 0.01$ s, $T_{TC2} = 0.012$ s, $R_{AC1} = 0.0819$ Ohm, $R_{AC2} = 0.08358$ Ohm, $k_{TC1} = 161.2$, $k_{TC2} = 164.424$, and $J = 32.5\ \text{kg}\cdot\text{m}^2$.

The exogenous disturbance $f = M_c$ is the deviation of the moment of resistance (load) from the rated value. It does not exceed $f^* = 600$ Nm (20% of the rated motor moment). The measured variables for this plant are related to the physical variables: $y_1 = x_3$, $y_2 = x_4$, and $y_3 = x_5$. The exogenous disturbance f and the controls u_1 and u_2 are applied at different points. The main controlled variable of the plant is the angular rate of rotation of the motors: $z_3 = y_3 = x_5$. In addition, an important practical requirement to parallel systems is an equal load of the motors (close values of their armature currents) when operating on a common load. This requirement is often not satisfied when using standard PI controllers [25]. According to experimental evidence, if the angular rate of rotation of the motors is chosen as the only controlled variable ($z_3 = x_5$), then the equal load requirement may not hold, and most importantly, the stability margins for the measured variables $y_1 = x_3$ and $y_2 = x_4$ (the motor currents) at the plant's output may be very small, which is unacceptable in applications. Therefore, we will consider all measured variables of the plant as the controlled variables: $z_1 = y_1 = x_3$, $z_2 = y_2 = x_4$, and $z_3 = y_3 = x_5$ (the currents and the angular rate of rotation of the motors). Thus, the reference signals g_1 and g_2 will be fictitious and used for controller design only. Note that the behavior of the motor currents in the rated mode is completely determined by the variations in the load moment (disturbance).

The plant's matrices (1) have the following form [8]:

$$A = \begin{bmatrix} -100 & 0 & 0 & 0 & 0 \\ 0 & -83.333 & 0 & 0 & 0 \\ 137.811 & 0 & -11.287 & 0 & -1123.155 \\ 0 & 132.459 & 0 & -11.065 & -1101.133 \\ 0 & 0 & 0.2487 & 0.254 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -0.031 \end{bmatrix}, \; B_2 = \begin{bmatrix} 16120 & 0 \\ 0 & 13702 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The system requirements are:

• The tracking error in the angular rate of rotation should be $\varepsilon_{3,st} = z_{3,st} \le z_3^* = 1$ rad/s, and the current deviations in the transient modes should be $|z_1| \le 375$ A and $|z_2| \le 375$ A.

• The oscillation indices for the plant's measured outputs (y_1, y_2, , and y_3) should not exceed 1.

• The settling time should be $t_{set} = 0.25$ s.

We will design an appropriate controller using the algorithm from Section 3. For this purpose, we find the weight $Q^{1/2} = f^* / z_3^* = 600/1 = 600$ from formula (16) and let $\beta = 0$. The resulting controller matrices (2) and the realized value of γ are:

$$A_c = \begin{bmatrix} -387.822 & -91.669 & 421.791 & 902.372 & -4003.386 \\ -18.692 & -371.046 & -82.211 & 1772.776 & -7073.098 \\ -74.743 & -12.302 & -65.282 & 170.106 & -613.257 \\ -6.386 & -24.3 & -3.326 & -10.924 & 135.406 \\ 0.0525 & 0.0697 & 0.066 & -484.352 & -484.698 \end{bmatrix},$$

$$B_c = \begin{bmatrix} -0.00119 & 0.00132 & 0.000937 \\ 0.000693 & -0.000701 & 0.00003 \\ -0.00223 & 0.00252 & 0.000215 \\ 0.552 & 0.579 & 4.817 \\ 63.954 & 62.713 & -13.418 \end{bmatrix},$$

$$C_c = \begin{bmatrix} -0.00639 & 0.0118 & 0.0162 & -0.0697 & 0.265 \\ 0.0197 & 0.0161 & -0.0237 & -0.12 & 0.506 \end{bmatrix},$$

$$D_c = 0_{2x3}, \; \gamma = 51.86.$$

Note that the response of the closed loop system (1) and (2) ($g_3 \to z_3$) to the reference signal $g_3 \to z_3$ gives a large static error in the angular rate of rotation. To eliminate it, we rescale the reference signal by a value inverse to this error (the closed loop system gain, easily found from the open loop Nyquist contour for the angular rate of rotation; see below). After such scaling, the amplitude-frequency response of the closed loop system for the reference signal is presented in Fig. 2*b*, and the system's response in the angular rate-of-rotation channel is shown in Fig. 3*b*. The amplitude-frequency response of the closed loop system in the disturbance channel ($f \to z_3$) scaled by 600 Nm is demonstrated in Fig. 2*a*. This response is monotonically decreasing: for the closed loop system, the worst disturbance from the class (3) and (4) is the step function. In the angular rate-of-rotation channel, the corresponding response to such a disturbance of 600 Nm is shown in Fig. 3*a*. Clearly, the accuracy requirements are satisfied, like the response speed requirements to the closed loop system ($t_{set} \le 0.25$ s). The motor currents in the transient modes are very close

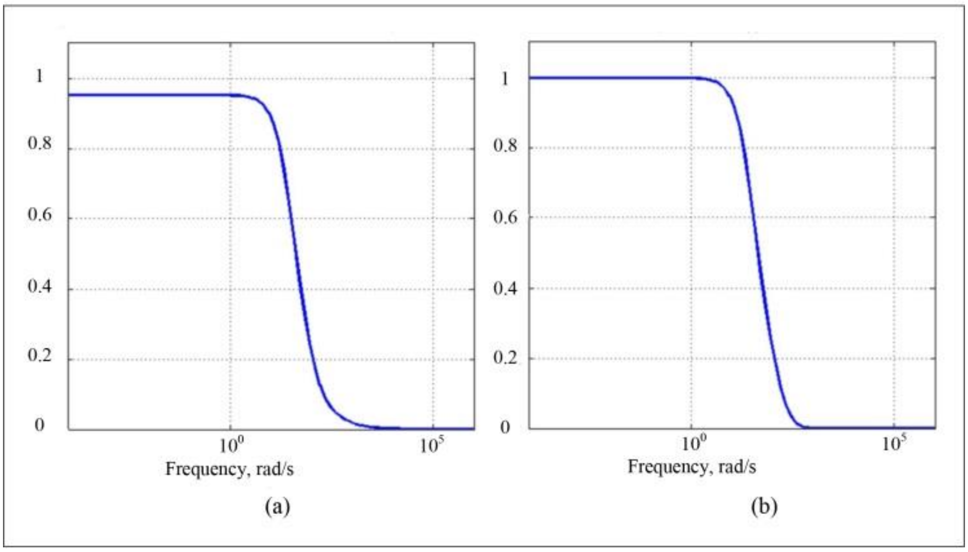

Fig. 2. The amplitude-frequency response of the closed loop system:
(a) disturbance and (b) reference signal

(the motors are equally loaded), and their deviations from the rated value are much less than the admissible value of 375 A.

Figure 4 provides the Nyquist contour of the open loop system in the corresponding measured variables: the motor currents (on the left) and the angular rate of rotation (on the right). Obviously, the system has an infinite phase margin for the measured variables - since the curves $w_i(j\omega)$ are entirely inside the unit circle. The modulus margins for the first and second measured variables (the currents of the first and second motors) are 1.6 and 1.4, respectively. The modulus margin for the main controlled variable (the angular rate of rotation of the motors) is 833. Thus, the system has significant stability margins for the measured variables. Moreover, the contours do not encircle the critical point $(-1, j0)$: the open loop systems are stable, which is important from a practical viewpoint. Checking the stability margins at the plant's physical input, we establish that the stability margins for the control variables u_1 and u_2 have radii 0.46 and 0.42, respectively, and open loop systems are stable in the control variable u_1 or u_2.

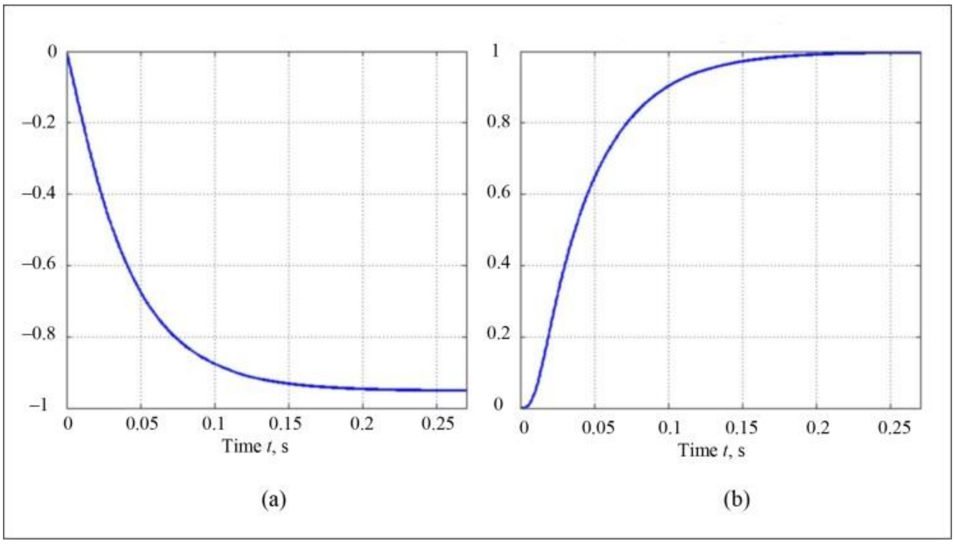

Fig. 3. The system's response to step disturbance and reference signal:
(a) output under $f(t) = 600$ and (b) output under $g_3(t) = 85.47$.

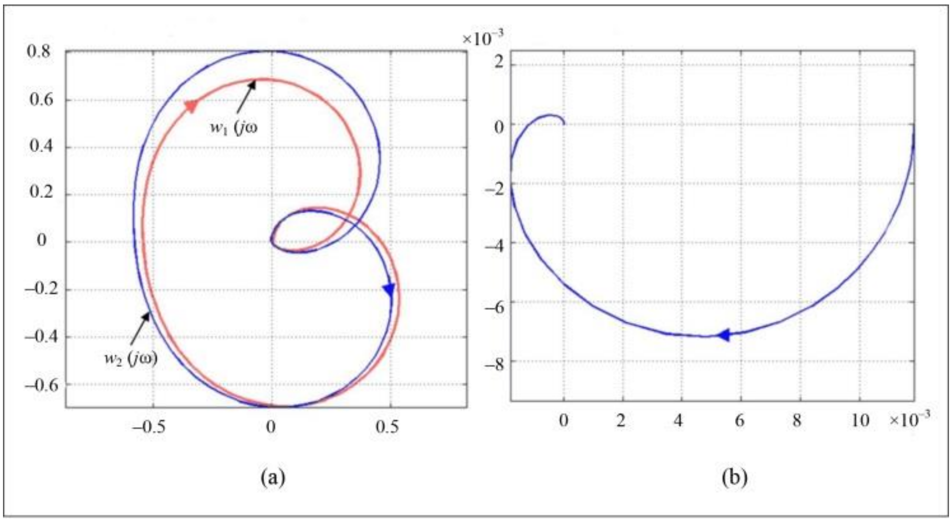

Fig. 4. The Nyquist contour of the closed loop system for different outputs:
(a) $w_1(j\omega)$ and $w_2(j\omega)$, (b) $w_3(j\omega)$.

CONCLUSIONS

This paper has presented an approach to design multivariable tracking systems based on engineering performance indices: the tracking errors due to unmeasured exogenous disturbances, a set of oscillation indices M_i for the ith closed loop relating the ith reference signal g_i to the ith controlled variable z_i, and the settling time. Note some features of the approach that are attractive from an engineering viewpoint:

- Clear engineering performance indices are applied.
- The controller design procedure reduces to the standard H_∞ optimization problem, solved by powerful software tools.
- The controller's order does not exceed that of the original physical object.
- An applications-relevant class of continuous disturbances with piecewise continuous time derivatives is considered.

If the oscillation indices should be provided at the physical input of a plant, then extra (fictitious) exogenous disturbances have to be introduced additively with the controls, and the controls have to be treated as the controlled variables.

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