

# PRICING MECHANISMS FOR COST REDUCTION UNDER BUDGET CONSTRAINTS

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**Abstract.** The problem of evaluating the prices (cost) of individual projects of a megaproject or program is considered. The megaproject manager evaluates the cost of each project based on its planned cost reported by the project executors under the budget constraint on the total cost of the program. The executor of each project is a monopolist in the relevant area and cannot be replaced by another executor. In the deterministic case, the executors know the exact actual cost of their project; the manipulability of the mechanism for forming the cost of projects is investigated. In the stochastic case, the executors do not know the actual cost of their projects; when evaluating the planned cost, they estimate a probable value of the actual cost. For this estimate, the distribution function of the project's actual cost is used. The paper proposes a pricing mechanism for cost reduction under the budget constraint on the total cost of the program and a given probability distribution of the project's actual cost.

**Keywords:** cost price, limit price, profit, mathematical expectation, manipulability, cost reduction.

## INTRODUCTION

Expectations that a market economy would maximize production efficiency did not realize. In such an economy, competition is a prerequisite for increasing efficiency, and no competition occurs in the presence of monopolists.

According to the monograph [1], the mechanism of cost reduction is a mechanism that encourages every employee to increase production efficiency and manufacture better quality products at lower costs and lower prices. The main results on control mechanisms of cost reduction were obtained for deterministic models [1–4]. Determinism was primarily understood as the absence of random disturbances. Note that cost-reducing control mechanisms were developed within the theory of active systems [5] to fight monopoly effects such as eliminating or preventing competition [6]. (Note that they are also called counter-expensive mechanisms.) At the same time, most studies of Russian researchers were aimed at cost-reducing measures without considering the monopoly effect [7–12]. The main focus in Western literature was on applying antitrust laws [13–16], including the ones to split monopolies. In addition

to these papers, cost reduction and management were widely studied [17–19].

Russian researchers associate the solution of stochastic problems in the theory of active systems primarily with the analysis of incentive mechanisms; for details, see [20–22]. Similar problems in Western literature are considered within contract theory [23–28] and risk analysis [29, 30]. This paper deals with the following case: the project manager does not know the project cost price in advance, but he knows the cost price distribution.

## 1. PROBLEM STATEMENT

Consider a two-level system composed of a Principal (the upper level allocating budget funds for program execution) and agents (the lower level represented by program executors). The program consists of  $n$  projects executed by  $n$  organizations (agents), each being a monopolist in the corresponding area. Each agent knows the project's cost and limit prices. The Principal has budget funds in an amount  $R$ , restricting the entire program's cost, and knows the limit price (cost) of each project. Let  $c_i$  and  $l_i$  denote the cost and

limit prices of project  $i$ , respectively,  $i = 1, \dots, n$ . The problem is to determine the cost of each project.

Consider a game-theoretic statement of this problem.

1. Each agent reports an estimate of the project's cost (his strategy).
2. The Principal determines the cost (prices) of all projects based on the information received.
3. The agents and the Principal determine their payoffs. The agent's payoff is the expected profit. The Principal's payoff function can be different, but this is not important: this aims to analyze the agents' strategies.

In the sequel, we will suppose that  $C < R$ , where

$C = \sum_{i=1}^n c_i$ . The cost  $C_i$  of project  $i$ ,  $i = 1, \dots, n$ , will be determined using a pricing mechanism for cost reduction [1–4]:

$$C_i = s_i + k(l_i - s_i), \quad i = 1, \dots, n,$$

where  $s_i$  denotes the cost price estimate reported by agent  $i$ . A natural assumption is  $s_i < l_i$ : the Principal will not consider the cost price estimates higher or equal to the limit price. Another natural assumption

has the form  $\sum_{i=1}^n l_i = L > R$ .

The value  $k$  is obtained from the condition

$$R = \sum_{i=1}^n C_i = S + k(L - S), \quad (1)$$

where  $S = \sum_{i=1}^n s_i$ .

From condition (1) we find

$$k = \frac{R - S}{L - S}. \quad (2)$$

Hence,  $k < 0$  if  $S > R$ : the project's price established by the Principal is smaller than the cost price estimate reported by the agent. In the case  $S < R$ , we have  $1 > k > 0$ : the project's price exceeds its cost price estimate.

The agent's profit is given by

$$P_i = C_i - c_i = s_i + k(l_i - s_i) - c_i, \quad i = 1, \dots, n. \quad (3)$$

We write the expression (3) as

$$P_i = k(l_i - s_i) + s_i - c_i, \quad i = 1, \dots, n. \quad (4)$$

Let  $P_i^{(pl)} = k(l_i - s_i)$  be the agent's planned profit, and  $P_i^{(sp)} = s_i - c_i$  be the super-planned profit. Naturally, the matter concerns the super-planned profit if  $s_i > c_i$ . In this paper, the profit of agent  $i$  is calculated as

$$P_i = P_i^{(pl)} + qP_i^{(sp)} = k(l_i - s_i) + q(s_i - c_i), \quad (5)$$

$$i = 1, \dots, n,$$

where  $q \leq 1$ . If  $q \in (0, 1]$ , then the Principal allocates a share of the super-planned profit to the agent, and  $q$  is a normative value of the agent's super-planned profit. Finally, if  $q \leq 0$ , then  $q$  specifies a penalty coefficient for any project's cost price distortions.

In the case  $s_i < c_i$ , the agent's profit will be written in the form (4). Clearly, for  $s_i = c_i$ , the expression (4) coincides with (5).

## 2. STUDY OF MANIPULABILITY

Let the agents act under the hypothesis of weak contagion [5]. In this case, agent  $i$  neglects the effect of his estimate  $s_i$  on the value  $k$ . The agents will not benefit by overestimating the cost prices of their projects under the condition

$$\frac{\partial P_i}{\partial s_i} = -k + q < 0, \quad i = 1, \dots, n. \quad (6)$$

Inequality (6) will hold if  $q < k$ . Condition (6) being valid, each agent benefits by reporting the true estimate of the cost price  $c_i$ ,  $i = 1, \dots, n$ , and the value  $k$  is given by

$$k = \frac{R - C}{L - C}.$$

Hence, we arrive in the following conclusion: even if  $0 < q < k$  and the Principal allocates part of the super-planned profit to the agents, they will benefit not by overestimating the cost prices of their projects (to gain the super-planned profit) but by truth-telling (reporting the true estimates of the cost price).

However, a problem arises because the Principal announces the value  $q$  before the agents report their estimates of cost prices (before he calculates the value  $k$ ).

To study manipulability, we write formula (5) as

$$P_i = kl_i - qc_i - (k - q)s_i, \quad i = 1, \dots, n. \quad (7)$$

Recall that  $C < R$ .

**Case  $S < R$ .** Here, formula (2) implies  $0 < k < 1$ . Due to the expression (7), for  $q < k$ , the profit of agent  $i$  decreases with the growth of his cost price estimate. Therefore, the agent's optimal strategy has the form  $s_i^* = c_i$ ,  $i = 1, \dots, n$ . For  $q = k$ , the agent's strategy  $s_i^* = c_i$ ,  $i = 1, \dots, n$ , is also optimal. Really—see (7)—if the agent is benevolent to the Principal (i.e., the hypothesis of benevolence holds [31]), he implements an action beneficial for the Principal.

Thus, for receiving reliable information about the projects' cost prices from the agents in the case  $S < R$ , the Principal should establish  $q = \beta k$ , where  $\beta \leq 1$ .



**Case  $S > R$ .** Here, formula (2) implies  $k < 0$ . Due to the expression (7), for  $k - q > 0$ , the profit of agent  $i$  becomes negative (a loss occurs). For reducing this loss, the agent benefits by underestimating the cost price. Therefore, the agent's optimal strategy has the form  $s_i^* = c_i, i = 1, \dots, n$ . Note that  $q < k$  corresponds to  $q < 0$ : the agent is penalized for distorting the true information. For  $q = k$ , the agent's strategy  $s_i^* = c_i, i = 1, \dots, n$ , is also optimal. This fact follows from the considerations in the case  $S < R$ .

Thus, if  $q = \beta k$ , where  $\beta \leq 1$ , the mechanism will be cost-reducing in both cases.

**Case  $S = R$ .** Here,  $k = 0$ , and the profit of agent  $i$  is given by

$$P_i = q(s_i - c_i), i = 1, \dots, n.$$

According to this expression, for  $q < 0$ , the agent benefits by underestimating the cost price. Therefore, the agent's optimal strategy has the form  $s_i^* = c_i, i = 1, \dots, n$ . For  $q > 0$ , the agents should be interested in overestimating the cost prices. However, they cannot realize this scenario: any increase in the cost price estimate will immediately cause transition from  $S = R$  to  $S > R$ , where the constraint  $q \leq k$  should be satisfied.

Note that the mechanism remains cost-reducing for one agent. Indeed, for one agent,

$$k^{(1)} = \frac{R - s_1}{l_1 - s_1} \tag{8}$$

and  $P_1 = R - qc_1 - (1 - q)s_1$ . Since  $q \in (0, 1]$ , the optimal strategy has the form  $s_1^* = c_1$ .

### 3. STOCHASTIC CASE

Suppose that when planning the project's cost, each agent cannot accurately determine its cost price but knows the cost price distribution function  $F(x_i), F(l_i) = 1$ , and the density function  $f(x_i) = F'(x_i)$ . As mentioned above,  $s_i \leq l_i$ . In the sequel, all cost price estimates reported by the agents to the Principal satisfy the conditions  $s_i \in [d_i, l_i], i = 1, \dots, n$ , and  $c_i \in [d_i, l_i], i = 1, \dots, n$ . Therefore, the Principal and agents know that the project's cost cannot be smaller than  $d_i, i = 1, \dots, n$ . Recall that in the deterministic case, the mechanism is cost-reducing if the agent's optimal strategy is reporting the true cost price  $s_i^* = c_i, i = 1, \dots, n$ . In the stochastic case, in contrast, the mechanism is cost-reducing if the agent's optimal strategy is reporting a planned cost price less than the limit price.

First, consider the problem with one agent ( $n = 1$ ).

Since the value  $k^{(1)}$  is given by (8), the agent's profit can be written as

$$P_1(s_1) = R - s_1 + \begin{cases} q(s_1 - x_1) & \text{for } x_1 \leq s_1, \\ s_1 - x_1 & \text{for } x_1 > s_1. \end{cases}$$

We calculate the expected profit:

$$M[P_1(s_1)] = R - s_1 + q \int_{d_1}^{s_1} (s_1 - x_1) f(x_1) dx_1 + \int_{s_1}^{l_1} (s_1 - x_1) f(x_1) dx_1. \tag{9}$$

From the expression (9) it follows that

$$\frac{dM[P_1(s_1)]}{ds_1} = -(1 - q)F(s_1).$$

Hence,  $\frac{dM[P_1(s_1)]}{ds_1} < 0$ . In this case, the expected

profit achieves maximum at  $s_1 = d_1$ , which corresponds to the that that the mechanism is cost-reducing:

$$M[P_1(d_1)] = R - \int_{d_1}^{l_1} x_1 f(x_1) dx_1.$$

For example, let the random value  $x_i, i = 1$ , obey the uniform distribution on the interval  $[d_i, l_i]$ . Then the density function  $f(x_i)$  has the form

$$f(x_i) = \frac{1}{l_i - d_i}. \tag{10}$$

The expected profit achieves the maximum value

$$M[P_1(d_1)] = R - \frac{l_1 + d_1}{2}.$$

Consider the case of  $n$  agents. In view of the expressions (4) and (5), the profit of agent  $i$  can be written as

$$P_i = k(L_i - s_i) + \begin{cases} q(s_i - x_i) & \text{for } x_i \leq s_i, \\ s_i - x_i & \text{for } x_i \geq s_i, \end{cases} i = 1, \dots, n.$$

We calculate the expected profit:

$$M[P_i] = k(L_i - s_i) + s_i - (1 - q) \times \left( s_i F(s_i) + \int_{s_i}^{l_i} x_i f(x_i) dx_i \right) - q \int_{d_i}^{s_i} x_i f(x_i) dx_i.$$

First, assume that the hypothesis of weak contagion [5] holds: the agent's estimate  $s_i$  has negligible effect on the value  $k$ , i.e.,  $\frac{\partial k}{\partial s_i} = 0$ . Then

$$\frac{\partial M[P_i]}{\partial s_i} = (1 - k) - (1 - q)F(s_i), i = 1, \dots, n. \tag{11}$$

The inequality  $\frac{\partial M[P_i]}{\partial s_i} > 0$  holds if

$(1 - k) > (1 - q)$ , or equivalently,  $S > \frac{R - qL}{1 - q}$ . The latter inequality is valid under  $D > \frac{R - qL}{1 - q}$ , yielding

$$q > 1 - \frac{L-R}{L-D}. \quad (12)$$

If the value  $q$  satisfies (12), the expected profit will tend to the maximum value as  $s_i \rightarrow l_i$ ,  $i = 1, \dots, n$ . In other words, the mechanism is not cost-reducing.

Now consider the case  $q < 1 - \frac{L-R}{L-D}$ . To find the agents' estimates  $s_i$ ,  $i = 1, \dots, n$ , maximizing the expected profit, we solve the system of equations

$$(1-k) - (1-q)F(s_i) = 0, \quad i = 1, \dots, n.$$

If the random value  $x_i$ ,  $i = 1, \dots, n$ , obeys the uniform distribution on the interval  $[d_i, l_i]$ , then this system (see formula (10)) can be written as

$$\frac{s_i - d_i}{l_i - d_i} = \frac{1-k}{1-q}, \quad i = 1, \dots, n. \quad (13)$$

The solution of (13) is given by

$$s_i^{(1)} = \frac{l_i + d_i}{2} + \frac{l_i - d_i}{2} \sqrt{1-V}, \quad i = 1, \dots, n,$$

and

$$s_i^{(2)} = \frac{l_i + d_i}{2} - \frac{l_i - d_i}{2} \sqrt{1-V}, \quad i = 1, \dots, n,$$

where  $V = \frac{4}{1-q} \frac{L-R}{L-D}$  and  $D = \sum_{i=1}^n d_i$ .

Hence, the system (13) is solvable if

$$q \leq 1 - 4 \frac{L-R}{L-D}. \quad (14)$$

As noted, for  $q \in (0, 1]$ , the Principal allocates part of the super-planned profit to the agent. From inequality (14) it follows that part of the super-planned profit is at the agent's disposal if  $q > 0$ , or  $R > \frac{1}{4}(3L+D)$ .

Since

$$\frac{\partial^2 M[P_i]}{\partial s_i^2} = -(1-q)f(s_i), \quad i = 1, \dots, n,$$

the expected profit has two local maxima at the points  $\{s_i^{(1)}\}$  and  $\{s_i^{(2)}\}$ ,  $i = 1, \dots, n$ .

The expected profit takes the following values:

– at the point  $\{s_i^{(1)}\}$ , the value

$$M\left[P_i\left(s_i^{(1)}\right)\right] = \frac{l_i - d_i}{2} \times \left[ \frac{1+q}{2} - \frac{1-q}{2} \left( \sqrt{1-V} + \frac{1}{2}V \right) \right], \quad i = 1, \dots, n; \quad (15)$$

– at the point  $\{s_i^{(2)}\}$ , the value

$$M\left[P_i\left(s_i^{(2)}\right)\right] = \frac{l_i - d_i}{2} \times \left[ \frac{1+q}{2} + \frac{1-q}{2} \left( \sqrt{1-V} - \frac{1}{2}V \right) \right], \quad i = 1, \dots, n. \quad (16)$$

Comparing the expressions (15) and (16), we observe the following: the agents gain the maximum expected profit at the point  $\{s_i^{(2)}\}$ . In other words, the mechanism is cost-reducing.

The paper [5] introduced the concept of a reliable estimate of the element's plan when studying the interaction between the Principal and one stochastic element. By analogy with reliability, let us define the probability that the project's random cost price estimate  $\{s_i\}$  will take a value not exceeding  $\{s_i^{(2)}\}$ . Due to the distribution function formula, the probability  $p(s_i \leq s_i^{(2)})$  is given by

$$p(s_i \leq s_i^{(2)}) = \frac{1 - \sqrt{1-V}}{2}.$$

This probability (the reliability of the estimate  $\{s_i\}$ ) will be not smaller than  $u$  under the condition

$$\frac{1 - \sqrt{1-V}}{2} \geq u.$$

Hence, the maximum value of  $u$  never exceeds 0.5. This reliability can be ensured by an appropriate choice of  $q$ . Indeed, it suffices to choose  $q$  so that inequality (14) turns into equality. Note that the choice of  $q$  determines the value  $\{s_i^{(2)}\}$ . For  $q = 1 - 4 \frac{L-R}{L-D}$ , we

obtain  $s_i^{(2)} = \frac{l_i + d_i}{2}$ , and moreover,

$$p\left(s_i \leq \frac{l_i + d_i}{2}\right) = \frac{1}{2}.$$

Next, consider the case  $\frac{\partial k}{\partial s_i} \neq 0$ . Here, formula

(11) can be written as

$$\frac{\partial M[P_i]}{\partial s_i} = (1-k) \left( 1 - \frac{l_i - s_i}{L-S} \right) - (1-q)F(s_i), \quad (17)$$

$$i = 1, \dots, n.$$

Assuming that

$$\frac{l_i - s_i}{L-S} = \frac{l_i - s_i}{\sum_{j=1}^n (l_j - s_j)} \approx \frac{1}{n}$$



for sufficiently great  $n$ , we arrive at

$$\frac{\partial M [P_i]}{\partial s_i} = \frac{(n-1)(1-k)}{n} - (1-q)F(s_i), i = 1, \dots, n.$$

Clearly, the inequality  $\frac{\partial M [P_i]}{\partial s_i} > 0$  will hold if  $\frac{(n-1)(1-k)}{n} > (1-q)$ , or equivalently,  $S > \frac{(n-1)R - (nq-1)L}{n(1-q)}$ . The latter inequality is the

case under  $D > \frac{(n-1)R - (nq-1)L}{n(1-q)}$ , yielding

$$q > 1 - \frac{n-1}{n} \frac{L-R}{L-D}. \tag{18}$$

Well, if the value  $q$  satisfies (18), the expected profit will tend to the maximum as  $s_i \rightarrow l_i, i = 1, \dots, n$ . In other words, the mechanism is not cost-reducing.

Now consider the case  $q < 1 - \frac{n-1}{n} \frac{L-R}{L-D}$ . To find the agents' estimates  $s_i, i = 1, \dots, n$ , maximizing the expected profit, we solve the system of equations

$$\frac{(n-1)(1-k)}{n} - (1-q)F(s_i) = 0, i = 1, \dots, n.$$

If the random value  $x_i, i = 1, \dots, n$ , obeys the uniform distribution on the interval  $[d_i, l_i]$ , then its density function has the form (10). Therefore, this equation can be written as

$$\frac{s_i - d_i}{l_i - d_i} = \frac{n-1}{n} \frac{L-R}{L-S}, i = 1, \dots, n. \tag{19}$$

The solution of (19) is given by

$$\hat{s}_i^{(1)} = \frac{l_i + d_i}{2} - \frac{l_i - d_i}{2} \sqrt{1 - \frac{n-1}{n} V}, i = 1, \dots, n,$$

and

$$\hat{s}_i^{(2)} = \frac{l_i + d_i}{2} + \frac{l_i - d_i}{2} \sqrt{1 - \frac{n-1}{n} V}, i = 1, \dots, n.$$

Hence, the system (19) is solvable if

$$q \leq 1 - 4 \frac{n-1}{n} \frac{L-R}{L-D}. \tag{20}$$

Recall that the Principal allocates part of the super-planned profit when  $q > 0$ . From inequality (20) it follows that part of the super-planned profit is at the agent's disposal if  $R > \frac{(3n-4)L + nD}{4(n-1)}$ .

Since

$$\frac{\partial^2 M [P_i]}{\partial s_i^2} = -\frac{n-1}{n} \frac{\partial k}{\partial s_i} - (1-q)f(s_i),$$

we have

$$\begin{aligned} \frac{\partial^2 M [P_i]}{\partial s_i^2} \Big|_{s_i = \hat{s}_i^{(1)}} &= \frac{1-q}{l_i - d_i} \times \\ &\times \left( \frac{n}{n-1} \frac{l_i - d_i}{(L-D)V} \left( 1 - \sqrt{1 - \frac{n-1}{n} V} \right)^2 - 1 \right) < 0. \end{aligned}$$

Consequently, the expected profit achieves maximum at the point  $\{\hat{s}_i^{(1)}\}, i = 1, \dots, n$ . This maximum is equal to

$$\begin{aligned} M [P_i] &= \frac{l_i - d_i}{2} \times \\ &\times \left[ \frac{1+q}{2} + \frac{1-q}{2} \left( \sqrt{1 - \frac{n-1}{n} V} - \frac{n+1}{2n} V \right) \right], \tag{21} \\ &i = 1, \dots, n. \end{aligned}$$

In other words, the mechanism is cost-reducing.

For the case under consideration, we also define the probability that the project's random cost price estimate  $\{s_i\}$  will take a value not exceeding  $\{\hat{s}_i^{(1)}\}$ . As is easily shown, in this case, the maximum value of  $u$  will not exceed 0.5. To achieve this value, it suffices to choose  $q$  so that inequality (20) turns into equality.

Due to (15), (16), and (21), the maximum expected profits of agents diverge from each other only under unequal differences between the limit price  $l_i$  and the minimum cost estimate  $d_i$ .

Let the cost prices of agents' projects vary insignificantly; in this case, assume that the difference between the limit price  $l_i$  and the minimum cost estimate  $d_i$  is the same for all agents:  $l_i - d_i = w, i = 1, \dots, n$ . Under the uniform distribution of the random value  $x_i, i = 1, \dots, n$ , on the interval  $[d_i, l_i]$ , the derivative (17) can be written as

$$\begin{aligned} \frac{\partial M [P_i]}{\partial s_i} &= (1-k) \left( 1 - \frac{l_i - s_i}{L-S} \right) - (1-q) \frac{s_i - d_i}{w}, \\ &i = 1, \dots, n. \end{aligned}$$

To find the agents' estimates  $s_i, i = 1, \dots, n$ , maximizing the expected profit, we solve the system of equations

$$(1-k) \left( 1 - \frac{l_i - s_i}{L-S} \right) - (1-q) \frac{s_i - d_i}{w} = 0, \quad i = 1, \dots, n. \quad (22)$$

The solution of (22) is given by

$$\tilde{s}_i^{(1)} = l_i - \frac{w}{2} \left( 1 - \sqrt{1 - \frac{n-1}{n} V} \right), \quad i = 1, \dots, n,$$

and

$$\tilde{s}_i^{(2)} = l_i - \frac{w}{2} \left( 1 + \sqrt{1 - \frac{n-1}{n} V} \right), \quad i = 1, \dots, n. \quad (23)$$

Clearly, the system (22) is solvable under inequality (20). For the case  $l_i - d_i = w$ ,  $i = 1, \dots, n$ , inequality

$$(20) \text{ reduces to } q \leq 1 - \frac{4}{n-1} \frac{L-R}{w}.$$

Due to

$$\frac{\partial^2 M [P_i]}{\partial s_i^2} = 2 \left( 1 - \frac{l_i - s_i}{L-S} \right) \frac{(1-k)}{L-S} - (1-q) f(s_i), \quad (24)$$

$$i = 1, \dots, n,$$

and formulas (2), (10), and (23), the expression (24) can be written as

$$\left. \frac{\partial^2 M [P_i]}{\partial s_i^2} \right|_{s_i = \tilde{s}_i^{(2)}} = 2 \frac{1-q}{w} \left[ \frac{1}{(n-1)V} \left( 1 - \sqrt{1 - \frac{n-1}{n} V} \right)^2 - \frac{1}{2} \right],$$

$$i = 1, \dots, n.$$

Obviously,  $\left. \frac{\partial^2 M [P_i]}{\partial s_i^2} \right|_{s_i = \tilde{s}_i^{(2)}} < 0$ , and the expected

profit achieves maximum at the point  $\tilde{s}_i^{(2)}$ . Therefore, the mechanism is cost-reducing.

The expected profit takes the value

$$M \left[ P_i \left( \tilde{s}_i^{(2)} \right) \right] =$$

$$= \frac{w}{4} \left[ 1 + q - (1-q) \left( \frac{n+1}{2n} V - \sqrt{1 - V \frac{n-1}{n}} \right) \right], \quad (25)$$

$$i = 1, \dots, n.$$

Hence, if the difference between the limit price  $l_i$  and the minimum cost estimate  $d_i$  is the same for all agents, the maximum expected profits of agents do not diverge from each other.

The expression (16) with  $l_i - d_i = w$ ,  $i = 1, \dots, n$ , can be written as

$$M \left[ P_i \left( s_i^{(2)} \right) \right] = \frac{w}{4} \left[ 1 + q - (1-q) \left( \frac{V}{2} - \sqrt{1-V} \right) \right]. \quad (26)$$

Comparing formulas (25) and (26), we establish that

$$M \left[ P_i \left( \tilde{s}_i^{(2)} \right) \right] > M \left[ P_i \left( s_i^{(2)} \right) \right].$$

Thus, under the hypothesis of weak contagion, the expected profit is smaller compared to the case when the agents disregard it.

## CONCLUSIONS

The problem of determining the prices of individual projects within a single program has been considered in the deterministic and stochastic statements. In the deterministic case, the cost reduction property of the pricing mechanism is ensured by choosing the super-planned profit  $q$  allocated to the agent ( $q \leq k$ ). As for the stochastic case, the cost reduction conditions, for known reasons, can no longer ensure the coincidence of the planned cost of the project with the actual cost but encourage agents to report the planned cost prices below the limit prices. An appropriate choice of  $q$  yields the cost price estimates below the limit prices and, moreover, the conditions to calculate the expected profits. In addition, note that the parameters of the pricing mechanisms for projects with budget constraints can be expressed analytically under the hypothesis of weak contagion (when the actions of one agent negligibly affect the performance of the entire system). Weak contagion holds under very many agents. At the same time, with an increase in the number of projects (agents), the agents can obtain part of the super-planned profit under more stringent constraints on the value  $q$ .

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