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MODELING THE PROCUREMENT OF SCHOOL EQUIPMENT AND COMPETITION AMONG SUPPLIERS

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Abstract. This paper is devoted to the interaction of schools and suppliers as well as the processes of competition among suppliers for public procurement. Maximizing its utility function, a school determines an optimal distribution of its budget between labor costs and the purchase of equipment. Next, different suppliers begin to compete for the equipment budget, maximizing either their profit or revenue. Depending on the market (municipal, regional, or All-Russian), the procurement processes can be described using various models, ranging from perfect competition and oligopoly to monopoly. In the case of monopoly, suppliers provide no discounts on their products; under perfect competition, suppliers reduce prices to the level of their maximum profit. New applications of several game-theoretic models to the procurement of equipment and the description of competition among suppliers are proposed.

Keywords: school education, competition, decision-making, oligopoly, monopoly, auction, game theory.

INTRODUCTION

School is an important institution for the social and economic development of any country. It lays the foundations of a personality, basic skills, abilities, and values, as well as tools of communication and emotional intelligence. A professional teaching staff and special equipment are needed for a school to implement its activities. The school equipment can be different: from common desks and blackboards to modern interactive whiteboards and robotics kits [1].

In addition to its educational function, school forms a huge market of school equipment supplies in Russia, with an annual volume of above 100 billion RUB. This market is of particular interest for analysis using game theory and mechanism design.

First of all, it is necessary to establish how a school determines the necessary list of equipment and the corresponding annual budget. At the next step, the school procures equipment by announcing an auction in accordance with regulatory and legal restrictions and other restrictions imposed on it by decision centers (municipality, regional authorities, etc.). In the Russian Federation, these key restrictions are Federal Laws Nos. 223 and 44 [2, 3] as well as the organizational and legal form of schools and the regulatory restrictions of particular regional authorities.

Procurement is carried out either directly or through special public procurement mechanisms. As a rule, several key suppliers (two to four) compete for the supply of school equipment, offering the lowest price for the purchased equipment. However, another situation is possible: given (unique) equipment can be provided by only one supplier in the market, or lots are so standard that they can be sold by numerous companies.

Suppliers decide on their participation in a procurement based on several criteria. The first criterion is the ability to supply the required equipment. The second one is the supplier's utility function, i.e., its revenue or profit. (Revenue applies to large suppliers with an annual turnover exceeding 400 million RUB provided that the project margin is positive.) The third criterion is the current market situation (the number of other suppliers) and the availability of insider and other information.

This paper considers only some models of particular real situations in the school education market. We investigate primarily the impact of school equipment and labor costs (teachers' wages) on the value of the school utility function.

1. SCHOOL UTILITY FUNCTION AND VOLUME OF EQUIPMENT PURCHASE

Consider an average Russian school, which has no entrance tests. Assume that the school operates in a stable mode (no dissatisfaction with the level of education and no funding problems). The educational outcome function [4] of schools, also representing a utility function, is the average results of graduates in the most common metric (marks of the Unified State Examination). School equipment was possibly accumulated in previous years (the variable A). In the current year, the school distributes its budget (the variable M) between teachers' wages in this year (the variable L) and the purchase of additional school equipment in this year (the variable K). All the variables are in RUB since the total school budget, the accumulated and purchased school equipment, and teachers' wages are expressed in RUB [5].

Thus, the educational result function has the general form

$$U = f(A, K, L, M).$$

The total school budget is distributed between the purchase of equipment and teachers' wages: $M = L + K$, i.e., $M = M(K, L)$. Therefore, we obtain a simplified function f of three variables instead of four:

$$U = f(A, K, L).$$

In the paper [6], various potential school utility functions were considered, including one that satisfies many requirements for such functions. (They are as follows: an increasing function in the variables K, A, and L that has a decreasing scale effect and vanishes if $A = K = L = 0$ or $L = 0$.) This function is the sum of some modifications of the Cobb–Douglas functions, which will be called the double Cobb–Douglas function [5]:

$$U = f(A, K, L) = CK^{\alpha} L^{1-\alpha} + BA^{\beta} L^{1-\beta},$$

$$0 < \alpha < 1,\ 0 < \beta < 1,\ C, B > 0.$$

The school reports on the results of the previous year and forms its budget M from the financial receipts of municipal, regional, and (or) federal authorities as well as its extra-budgetary funds. In this model, the budget M is a fixed exogenous parameter. Then the school maximizes its utility function over the set of all $A, L, K \geq 0$. In [6], different solutions of this maximization problem were considered; it was demonstrated that the problem has a solution for the given type of function, and the solution is valid for any parameter values $A, L, K \geq 0$. Thus, the school determines its optimal equipment budget in the current year (K_0, in RUB) and reports it to suppliers (Fig. 1) [5, 7].

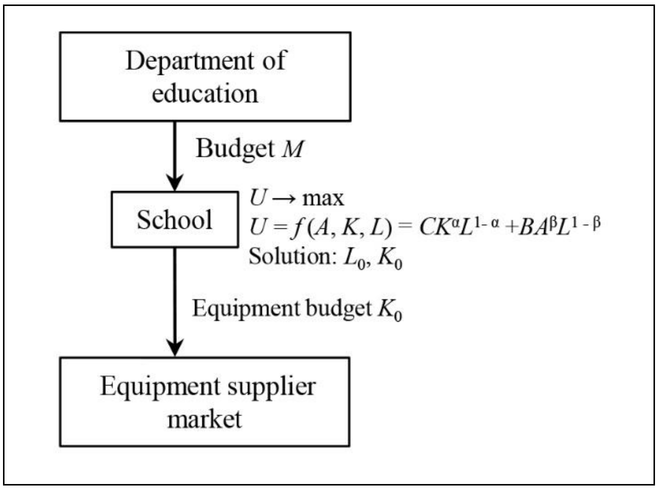

Fig. 1. The general three-level distribution scheme of school equipment budget.

For more details on the quantitative determination of a school utility function and the application of the double Cobb–Douglas function, we refer to the paper [6], where the utility function was obtained based on the quantitative data of St. Petersburg Education Committee and the Unified Information System of Public Procurement (https://zakupki.gov.ru) on the set of all $A, L, K \geq 0$. For example, the following utility function was obtained based on the real data of St. Petersburg schools:

$$U \sim K^{0.55} L^{0.45} + \frac{2}{3} A^{0.12} L^{0.88}.$$

2. MATHEMATICAL MODELING OF SUPPLIERS' COMPETITION FOR PROCUREMENT

2.1. The Basic Model

Consider a model of interaction between a school and suppliers, which will be called the basic model. This model applies to companies maximizing their profit. By assumption, companies know the utility function of each other (the double Cobb–Douglas function) but not the exact values of the variables of each other (fixed and variable costs). Also, they do not know the profit of each other, supposing that the costs of competitors are "somewhere between zero and the procurement price." In other words, the costs range from the maximum discount at the level of the supply price (if the company has almost zero costs) to the minimum (i.e., zero if the company's costs are at the level of the supply price). This model is often observed in real life when companies compete in a closed first-price auction, and there is no company that knows information about the internal processes of other companies (the public procurement process). Assume that the suppliers compete for the entire equipment budget of the school in the current year (K_0): the school spends its equipment budget in the current year once without dividing it into several procurements. As soon as the suppliers know this budget, those who can supply the equipment requested by the school begin to compete on price by offering various discounts. Let ΔM_i denote the discount offered by supplier *i* within the procurement.

We begin with the general scheme (Fig. 2): there are *n* suppliers in the market that can provide the requested equipment. Assume that all suppliers maximize their profit

$$\pi_i = K_0 - FC_i - p_i K_0 - \Delta M_i = (1 - p_i) K_0 - FC_i - \Delta M_i,$$

where p_i is the ratio of the variable costs of supplier *i* to provide school equipment in the volume K_0 to the supply amount K_0; FC_i is the fixed costs of supplier *i*. (They are individual for each supplier but independent of the volume of equipment, e.g., expenses for office rent, security, utilities, or part of labor costs). Supplier *i* maximizes its profit depending on the discount ΔM_i since the other indicators are given for each supplier.

In fact, the suppliers play a Bayesian game [7] (also called a game with incomplete information). This game corresponds to the situation when at least one player does not know minimally one utility function of other players. Within the basic model, the utility function of each supplier is unknown to all other suppliers.

We reduce the supplier's profit function to

$$\pi_i = (1 - p_i)\, K_0 - FC_i - \Delta M_i = g(p_i, FC_i, K_0) - \Delta M_i = U_i - \Delta M_i.$$

Assume that:

(a) Suppliers do not know the prices offered by each other.

(b) The utility functions of all suppliers are mutually independent.

According to the legislation of the Russian Federation, the procurement is actually carried out as a first-price auction [8, 9], i.e., the company offering the lowest price (the highest discount on the required equipment) gains the right to supply.

Since the utilities U_i of different suppliers are independent of each other, player *i* treats π_j as a value with the uniform distribution from zero to K_0 $\forall j,\ j \in 1\ldots n,\ j \neq i$ (Fig. 3).

The payoff function of supplier *i* takes the following form:

$$\pi_i(\Delta M_1, \Delta M_2, \ldots, \Delta M_n) = \begin{cases} U_i - \Delta M_i \text{ if } \Delta M_i = \max\limits_j (\Delta M_1, \Delta M_2, \ldots, \Delta M_n) \\ 0 \text{ if } \Delta M_i < \max\limits_j (\Delta M_1, \Delta M_2, \ldots, \Delta M_n). \end{cases}$$

We begin with a special case of oligopoly where three suppliers compete in the market. This situation will be analyzed from the viewpoint of the conditional first supplier (*i* = 1).

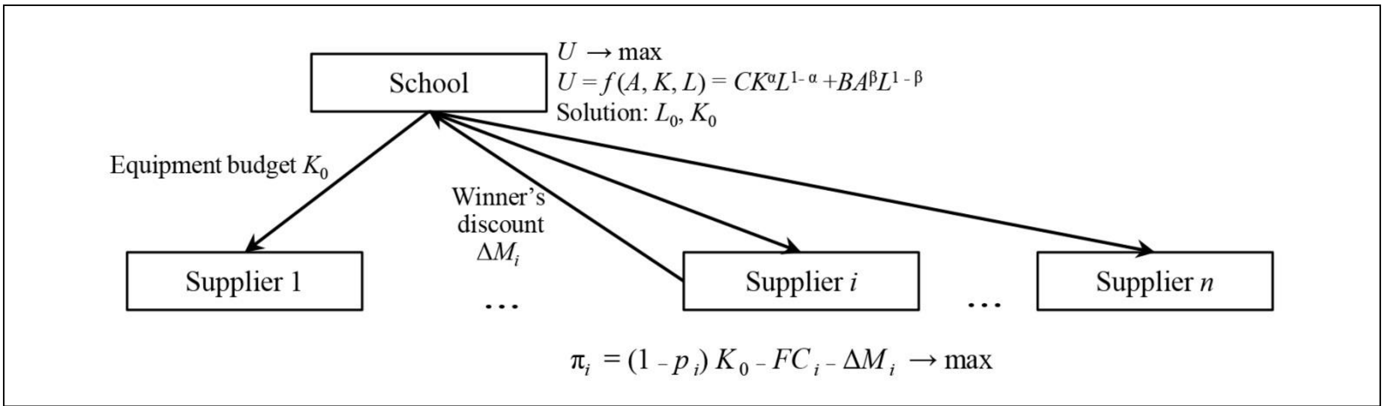

Fig. 2. The two-level generalized competition scheme of suppliers.

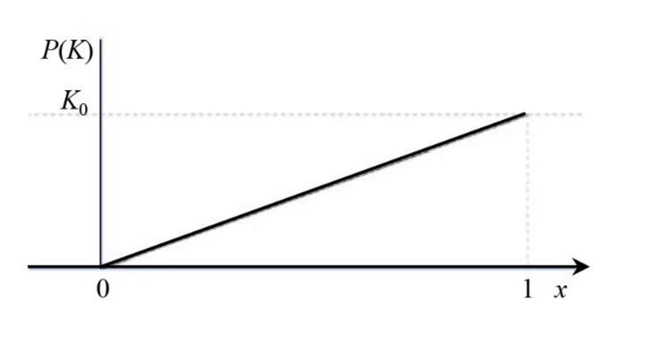

Fig. 3. The probability density of other suppliers' discounts considered by supplier *i*.

Let us calculate the probability that $\Delta M_1 > \Delta M_2$ and $\Delta M_1 > \Delta M_3$ from the viewpoint of the first supplier. If supplier 1 gives the highest discount, it wins the procurement:

$$p\left(\Delta M_1 > \Delta M_{j,\, j=2,3}\right)$$
$$= p(\Delta M_1 > \Delta M_2 \cup \Delta M_1 > \Delta M_3)$$
$$= p\left(\Delta M_1 > \Delta M_2\right)\, p\left(\Delta M_1 > \Delta M_3 \,|\, \Delta M_1 > \Delta M_2\right).$$
$$= \frac{\Delta M_1}{K_0} \times \frac{\Delta M_1}{K_0} = \left(\frac{\Delta M_1}{K_0}\right)^2.$$

The same result can be obtained in a more general case with the known probability distribution function

$$h_j(x) = \begin{cases} \dfrac{1}{K_0} \text{ if } \Delta M_i \in [0,\, K_0] \\ 0 \text{ if } \Delta M_i \notin [0,\, K_0], \end{cases}$$

$$\forall j \to p\left(\Delta M_i > \Delta M_j\right) = \int_0^{\Delta M_i} \frac{1}{K_0}\, dx = \frac{\Delta M_i}{K_0}.$$

In fact, supplier i maximizes its mathematical expectation: $(U_i - \Delta M_i) \times (\Delta M_i / K_0)^2 \to \max$. Solving this problem, we establish that the greatest mathematical expectation is achieved at $\Delta M_i = 2U_i / 3$. Thus, all suppliers will offer such discounts, and the supplier with the highest discount will be the winner.

By analogy, solving the problem with $n = 2$ yields $\Delta M_i = U_i / 2$ and $p\left(\Delta M_1 > \Delta M_2\right) = \Delta M_1 / K_0$. This result will be the base of mathematical induction for proving the general statement.

Let $p\left(\Delta M_i > \forall \Delta M_j,\, i \neq j\right) = \Delta M_i^{k-1} / K_0^{k-1}$ for some $n = k$. We fix the random numbering of suppliers. Then, for $n = k + 1$ suppliers, it follows that

$$p\left(\Delta M_1 > \forall \Delta M_{j,\, j=2,3,\ldots,k+1}\right)$$
$$= p\big(\Delta M_1 > \Delta M_2 \cup \Delta M_1 > \Delta M_3 \cup \Delta M_1$$
$$> \Delta M_4 \ldots \cup \Delta M_1 > \Delta M_{k+1}\big)$$
$$= p\big(\Delta M_1 > \Delta M_2 \cup \Delta M_1$$
$$> \Delta M_3 \cup \Delta M_1 > \Delta M_4 \ldots \cup \Delta M_1 > \Delta M_k\big)$$
$$\times p\big(\Delta M_1 > \Delta M_{n+1} | \Delta M_1 > \Delta M_2 \cup \Delta M_1$$
$$> \Delta M_3 \cup \Delta M_1 > \Delta M_4 \ldots \cup \Delta M_1 > \Delta M_k\big)$$
$$= \frac{\Delta M_1^{k-1}}{K_0^{k-1}} \times \frac{\Delta M_1}{K_0} = \frac{\Delta M_1^k}{K_0^k}.$$

By the principle of mathematical induction, for any $n \geq 2$, we therefore have $p\left(\Delta M_i > \forall \Delta M_j,\, i \neq j\right) = \Delta M_i^{n-1} / K_0^{n-1}$.

Consider maximization of the expected profit of the procurement by one supplier. In the general case, this problem has the form $(U_i - \Delta M_i) \times (\Delta M_i / K_0)^{n-1} \to \max$. As a result, the supplier's discount is

$$\Delta M_i = \frac{U_i (n-1)}{n}.$$

This model describes the reality. Indeed, in the extreme case of monopoly, the equipment supplier will offer no discounts $(\Delta M_i = 0)$. The solution is logical; in particular, $\Delta M_i \to U_i$ as $n \to \infty$, i.e., in the case of perfect competition, suppliers will offer to supply equipment with zero profit for them.

In also reflects the real market of school products. One striking example is *Prosveshchenie*, a monopolistic group of companies in several segments of the market. For some products (such as textbooks), the supplier refuses to discount, which also affects the market price and its growth in recent years. On the other hand, the stationery market represents an almost perfect competition. This market has low margins due to the presence of many suppliers.

2.2. The Additional Model

Consider a model involving two and, then, several suppliers, some maximizing their profit and some their revenue. As in subsection 2.1, companies know the utility functions of each other but not the exact values of the variables of each other (fixed and variable costs). Also, they do not know the profit of each other, supposing that the costs of competitors are "somewhere between zero and the procurement price." In other words, the costs range from the maximum discount at the level of the supply price (if the company has almost zero costs) to the minimum (i.e., zero if the company's costs are at the level of the supply price). This case describes a situation where large suppliers (with an annual revenue exceeding 400 million RUB) intervene in the procurement. The basic assumption in subsection 2.1 is that all suppliers of school equipment maximize their profit, which represents a function of the supplier's utility. However, this is not always the case. Large suppliers competing for procurement with small ones often maximize their revenue.

Thus, their utility function is revenue, and a large supplier solves the following problem:

$$TR_i \to \max,$$
$$TR_i = K_0 - \Delta M_i\,,\ \pi_i = (1 - p_i)K_0 - FC_i - \Delta M_i$$
$$= g(p_i\,,\ FC_i,\ K_0) - \Delta M_i = U_i - \Delta M_i \geq 0.$$

Hence, the supplier's maximum discount is $\Delta M_i = (1 - p_i)K_0 - FC_i$.

Consider the case of two competing suppliers, one maximizing profit and the other maximizing revenue, provided that both know their types (Fig. 4).

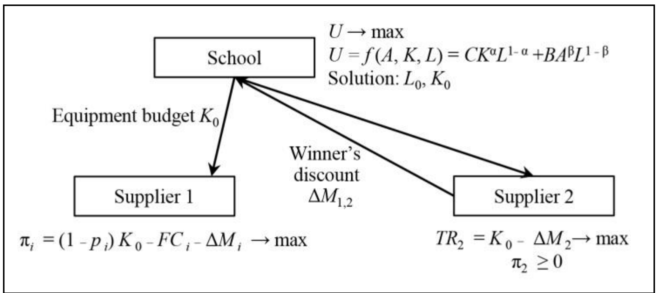

Fig. 4. The two-level competition scheme of two suppliers: suppliers 1 and 2 maximize profit and revenue, respectively.

Suppliers do not know the probability that their discount is greater than the competitor's one. Therefore, they believe that the discount of each other is uniformly distributed between zero and K_0. Similar to the previous case, the profit-maximizing supplier (supplier 1) will offer the discount $\Delta M_1 = U_1 / 2$. Supplier 2 (the revenue-maximizing one) will maximize the expected payoff $(K_0 - \Delta M_2) \times (\Delta M_2 / K_0)$, thus offering the discount $\Delta M_2 = K_0 / 2$ if the profit remains positive. Otherwise, the discount will be $\Delta M_2 = U_2$. The auction's winner will be determined by the higher discount.

Consider the general case of n suppliers, $n = k + m$, where k is the number of profit-maximizing suppliers (type I) and m is the number of revenue-maximizing suppliers (type II). Then the companies will offer the following discounts:

- $\Delta M_{i(k)} = \dfrac{U_i\,(n-1)}{n}$ (type I);
- $\begin{cases} \Delta M_{i(m)} = \dfrac{K_0(n-1)}{n} \text{ if } U_i - \dfrac{K_0(n-1)}{n} \geq 0 \\ \Delta M_{i(m)} = U_i \text{ otherwise} \end{cases}$

(type II).

Hence, we arrive at rather a logical outcome as follows. Consider a well-managed company of type II with small internal costs (*FC*, *p*). This company has a very high discount ceiling and ample opportunities to win the procurement against a profit-maximizing company. In the realities of the Russian school equipment market, a small (profit-maximizing) supplier facing a large (revenue-maximizing) one in the procurement with absolute competition and zero impact of non-market mechanisms always wins (e.g., *Prosveshchenie*, *Shkol'nyi Mir*, and other large companies).

CONCLUSIONS

This paper has proposed the models of public procurement of school equipment from suppliers and the models of competition among suppliers. In accordance with the legislation applicable to schools [2, 3], a significant part of procurement in this sphere is carried out within a Bayesian game: suppliers have no information about the utility functions of each other, and the procurement represents a first-price auction.

In the first case, several profit-maximizing suppliers compete for the procurement (the basic model). An individual supplier in the basic model treats the profit of another supplier as a uniformly distributed value between zero and the procurement price. Quite logically, a monopolistic supplier will not decrease its supply price by offering equipment discounts to the school. However, under perfect competition, market participants will offer discounts at the level of their maximum profit.

The second important case is competition among suppliers maximizing either profit or revenue (the additional model). In this case, the revenue-maximizing company with rather efficient internal processes will win the procurement.

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REFERENCES

1. Kayan-Fadlelmula, F., Sellami, A., Abdelkader, N., and Umer, S., A Systematic Review of STEM Education Research in the GCC Countries: Trends, Gaps, and Barriers, *International Journal of STEM Education*, 2022, vol. 9, art. no. 2. DOI: https://doi.org/10.1186/s40594-021-00319-7.
2. Federal Law of the Russian Federation of July 18, 2011, No. 223-FZ "On Procurement of Goods, Works, and Services by Certain Types of Legal Entities." URL: http://pravo.gov.ru/proxy/ips/?docbody=&nd=102149420. (Accessed April 10, 2023.) (In Russian.)
3. Federal Law of the Russian Federation of April 5, 2013, No. 44-FZ "On Contract System in the Sphere of Procurement of Goods, Works, and Services for State and Municipal Needs." URL: https://www.consultant.ru/document/cons_doc_LAW_144624/. (Accessed April 10, 2023.) (In Russian.)
4. Goubko, M., Burkov, V., Kondrat'ev, V., Korgin, N., and Novikov, D., *Mechanism Design and Management: Mathematical Methods for Smart Organizations*, Nova Science, 2013.
5. Bogdanov, A., Shchepkin, A., and Kolobov, D., Behavior Model of General Secondary Schools and Factors Influencing Their Results, *Proceedings of 2022 15th International Conference on Management of Large-Scale System Development (MLSD'2022)*, Moscow, 2022, pp. 1-4. DOI: 10.1109/MLSD55143.2022.9934285.
6. Bogdanov, A.D., Kolobov, D.V., and Shchepkin, A.V., Quantitative Validation of the Utility Function of Secondary Schools Depending on the Costs Incurred and Determination of Its Key Parameters, *Trudy Mosk. Fiz.-Tekhn. Inst.*, 2022, vol. 14, no. 4 (56), pp. 42–51. (In Russian.)
7. Burkov, V.N., Burkova, I.V., Goubko, M.V., et al., *Mekhanizmy upravleniya* (Control Mechanisms), Novikov, D.A., Ed., Moscow: LENAND, 2013. (In Russian.)
8. Shagin, V.L., *Teoriya igr* (Game Theory), Moscow: Yurait, 2023. (In Russian.)
9. Kremlev, A.G., *Osnovnye ponyatiya teorii igr* (Basic Notions of Game Theory), Yekaterinburg: Ural State University, 2016. (In Russian.)

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