DOI: http://doi.org/10.25728/cs.2022.1.2

OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS

M.V. Belov

Skolkovo Institute of Science and Technology, Moscow, Russia

✉ mbelov59@mail.ru

Abstract. This paper considers optimal decision-making during the life cycle management of complex systems of aerospace, power, nuclear, transport, and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries as well as information systems. The system-wide peculiarities of the life cycles of complex systems are identified and analyzed. Qualitative formalisms to represent life cycles are proposed; mathematical foundations of the problem of their optimal control are described. A mathematically rigorous optimal control problem for the life cycle of complex products, objects, and systems is stated. An algorithmic solution of the optimal control problem based on the formalisms of dynamic programming is developed. A practical way of applying this algorithm based on the scenario approach is proposed; the conditions of life cycle control optimization (under which optimization is possible) are listed. The results presented below are an optimal control tool for the life cycle of complex products, objects, and systems.

Keywords: optimal control; dynamic programming; life cycles of complex products, objects, and systems.

INTRODUCTION: THE TOPICALITY OF LIFE CYCLE MANAGEMENT AND OBJECTIVE COUNTER FACTORS

Nowadays, the concept *of life cycles of complex systems (products, objects)*[1] (LCS), fundamental in systems engineering [1, 2], is widespread in the practice of managing the creation and application of products and systems of aerospace and defense industries, objects and systems of the nuclear, oil and gas, power, transport, communication, and other processing, raw material, and service industries as well as in the field of information technology.

However, despite the wide circulation of the concept of LCS, the formal bases of LCS management are still not stated: there are no strictly defined criteria to compare certain approaches, choose the best of them, and coordinate and integrate interdisciplinary decisions. Furthermore, LCS management is not even posed as a mathematical control problem. The reasons are the high complexity and heterogeneity of management processes for LCS caused by the complexity of the Systems and the uncertainty and variability of external factors for LCS.

The life cycle is a complex system and the subject of research in various knowledge domains. Within each domain, models are developed to study separate aspects of LCS with different degrees of rigor. The presence of heterogeneous models requires a reasonable choice of approaches to forming integral quantitative models of LCS.

Management of complex systems is studied primarily by cybernetics and systems theory [3, 4], whereas lifecycle management by systems engineering [1, 5–9]. However, the results obtained in these branches are, as a rule, qualitative; the models of the mathematical theory of systems [7] do not allow posing and solving optimization problems.

Another popular topic is cost estimating (costing) throughout the entire life cycle of the system or product being created; for example, see [10, 11]. In recent years, neural networks, machine learning, and other modern approaches have been used for life cycle cost-

---

[1] For brevity, the subject of a life cycle (products, objects, or systems) will be uniformly called the System with a capital letter.

ing; for example, see [12–16]. Cost estimating of industrial programs is investigated by leading Western firms (e.g., see [17]) and is regulated by various governmental organizations (e.g., the National Aeronautics and Space Administration [18] and United States Government Accountability Office [19]).

Much research is devoted to mathematical models of the behavior of systems consisting of many interrelated elements, an example of which is LCS: multiagent systems [20–23]; interacting processes and systems [24–27]; systems and their properties described using systems modeling methods [8, 28]; various network structures (e.g., see [29]), particularly using graph theory (see the surveys in [29, 30]); project and program management [31–33]; stochastic networks and their applications in transport, power grids, logistics, and production [34]; firms (e.g., see the survey [35] and the references therein), organizations, and organizational structures [36–38].

Despite the significant number of models, approaches, and standards created and tested in practice, first of all, system-engineering, the absence of formal foundations considerably complicates LCS management and the coordination and integration of design, technological, economic, organizational, and other decisions. At the same time, the importance of LCS management requires defining formal bases and developing adequate models and methods of LCS management.

This paper introduces the following bases: LCS management is mathematically formalized as an optimal control problem, and approaches to solve it are proposed. Mathematical formalisms provide the maximum possible degree of rigor of the bases: mathematics has the most abstract and formal apparatus among all knowledge domains.

Representing a system, LCS requires the system-wide approach and the principle of holism. Therefore, the optimization problem is stated as a unified, holistic problem covering all LCS aspects. The multidisciplinary nature of LCS makes it difficult to form such a unified statement. The traditional practical approach is to model and optimize separate types and (or) components of LCS. This is common, for example, in operations research and related disciplines. However, the optimality of the parts does not imply the optimality of the whole. Therefore, forming a unified statement of the optimization problem becomes fundamentally important. Note that subsequent decomposition "top-to-bottom" remains correct for solving the problem by different mathematical methods at the corresponding levels of the hierarchy.

1. OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS: A QUALITATIVE DESCRIPTION OF THE PROBLEM

We consider the problem of managing the life cycle of a complex System by introducing several clarifications and definitions based on generally accepted approaches, methods, and standards.

Below, this problem will be solved for the products of aerospace, power, nuclear, transport and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries as well as information and technological systems.

Following the international and Russian standards [1, 2, 6], we understand *the life cycle* of a System as a set of repeated phenomena and processes with a period determined by the life of its standard design from conception to disposal or its particular copy from complete creation to disposal.

We use the following definitions from [5, 39]: *a project* as a set of interconnected measures to create a unique product or service under time and resource constraints; *a project program* as a set of interconnected projects and other activities to achieve a common goal under common constraints. In practice, LCS is usually implemented as a project program, i.e., a set of interconnected projects and other types of economic activities coordinated by time and resources, united by one type of System (or copy), including its updating and (or) modification, as well as by all stages of its life cycle, aimed at its development and (or) production and (or) maintenance to meet consumer requirements and obtain a positive economic result. LCS is a particular case of *complex activity*[2] (CA) [41], performed by a complex subject (an *extended enterprise,* EE) [41]. An EE is a system of autonomous but interacting firms (enterprises) united by a single structure of goals and a single technology of operation in which the parent enterprise performs the technological and business coordination.

In this case, the LCS[3] program consists of several interconnected lines of activity implemented by the extended enterprise (Fig. 1):

---

[2] Activity [40] is a dynamic interaction of a human with the reality in which he represents an actor (subject) purposefully influencing a subject matter (object). Complex activity [41] is an activity with a nontrivial internal structure, multiple and (or) changing goals, actor, technology, and the subject matter's role in the goal context.

[3] In this paper, the composition and sequence of lifecycle phases are given by the standards [2, 42]. Nevertheless, all results, statements, and conclusions will remain valid for other compositions of lifecycle phases as well.

- The creation and transformation of the System's information model (IM) together with the extended enterprise's IM. These activities are performed during the design (conceptual, schematic, detailed, etc.) of the System and EE. Subsequent updates again include works on System design and, if necessary, EE design.
- The creation, operation, updating, and termination (completion) of the extended enterprise itself.
- The creation, operation, updating, and disposal of the System.

Conceptualization and design (Fig. 1) consist in creating and transforming the System and extended enterprise's descriptions in the form of text documents, drawings, diagrams, and other formats, including traditional paper documents and computer data (CAD, PDM, and other engineering platforms as well as ERP, CRM, and other types of corporate management systems). In addition, various computational models are developed and used for engineering and management decision-making to assess and study the properties of the System and extended enterprise based on the current description. Computational models translate the design, engineering, logistics, and other decisions of various employees into the functional indicators of the System. The entire set of descriptions and computational models forms the information model of the System or EE, respectively; see Fig. 1.

In the early stages of the life cycle, the demand for the System, its expected characteristics, and the feasibility and effectiveness of the business idea are preliminarily analyzed; as a result, technical requirements for the System are formed. Being reflected in the operational concepts by models of the System's target application, the requirements define the desired way of operation of the future System. As the result of R&D, the IM of the elements (units, systems, and assemblies), phenomena, and processes of the System operation are formed. During detailed design and process engineering, technological and production descriptions and models of the System are formed, accompanied by the models of the extended enterprise and individual enterprises as its constituent parts. In each essential stage, the developed IMs are supplemented, updated, and detailed. They are used to verify and confirm the conformity of the System's current image to the one planned during conceptualization; see Fig. 1.

LCS management means managing the complex activity [41] of the extended enterprise: influence of the control subject on the controlled object to ensure the latter's behavior for achieving the former's goals. We define the management process of LCS as a complex activity that is:

- implemented within the LCS program throughout the entire life cycle of the System;

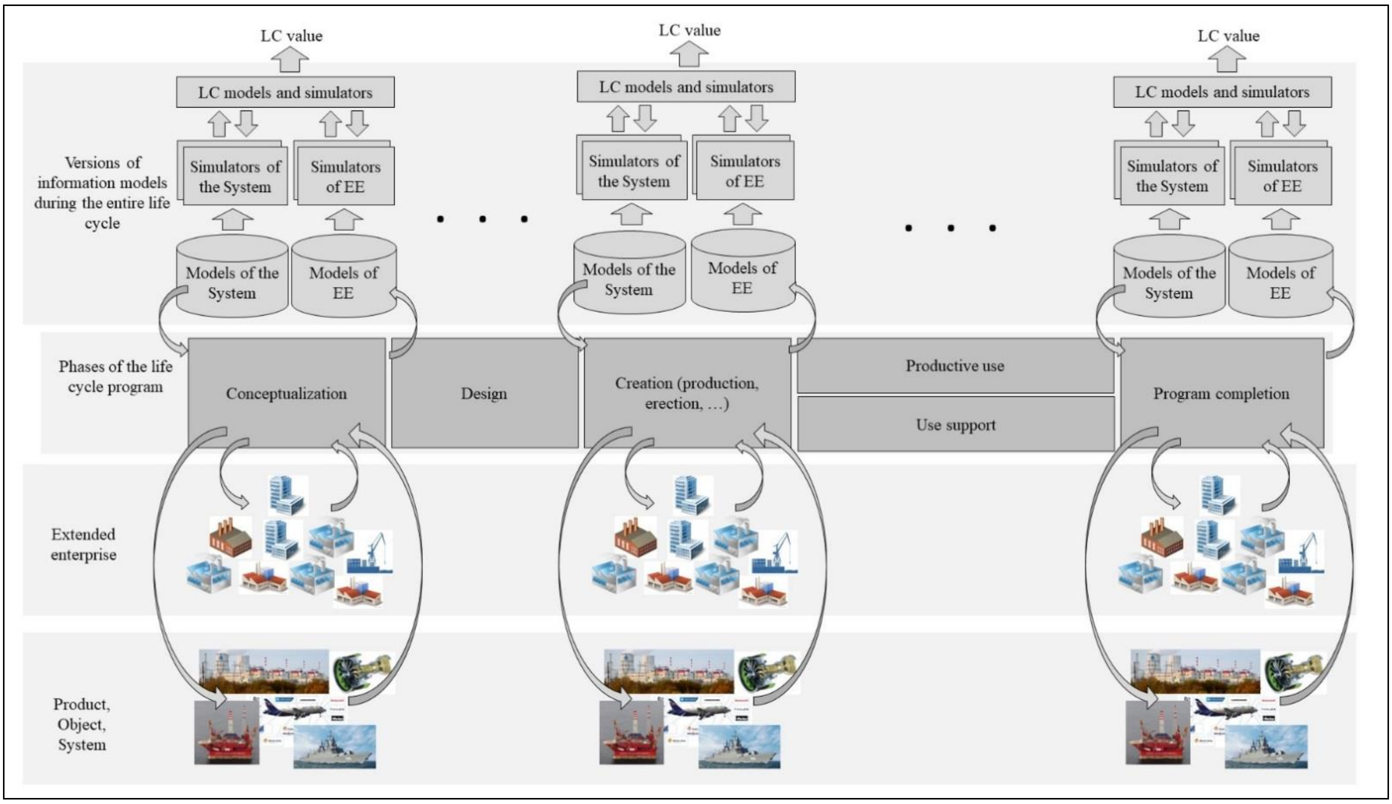

Fig. 1. Life cycle, System, extended enterprise, and their information models.

- performed by EE employees, whose design, engineering, technological, production, and other decisions affect the characteristics of the LCS program;
- coordinated by a dedicated group of specialists (the LCS program management office);
- performed to provide market or economically justified value characteristics of the program (hence, to achieve the goals of the LCS program);
- composed of:

– collecting, systematizing, and providing predictive and actual data to determine the value characteristics;

– determining and coordinating target values of the characteristics and limits on the characteristics of the System and EE, which are decomposed into limits on the components of the System and EE and groups of EE works;

– making design, engineering, technological, production, and other decisions to comply with the characteristics limits based on predicting the LCS evolution (System and EE);

– implementing responsibility for compliance with the limits by those firms, departments, and particular employees (designers, technologists, etc.) whose decisions or actions affect the corresponding component of the System and (or) EE.

In essence, LCS is nothing other than implementing one or more elements of activity to obtain benefits (a business or several businesses based on creation, production, use of the subject of LCS (product, system, or object). This interpretation of LCS leads to the generalized structure of the goals of activity (business) identical to the structure of value formation. Figures 2 and 3 show such structures, correlated with the LCS

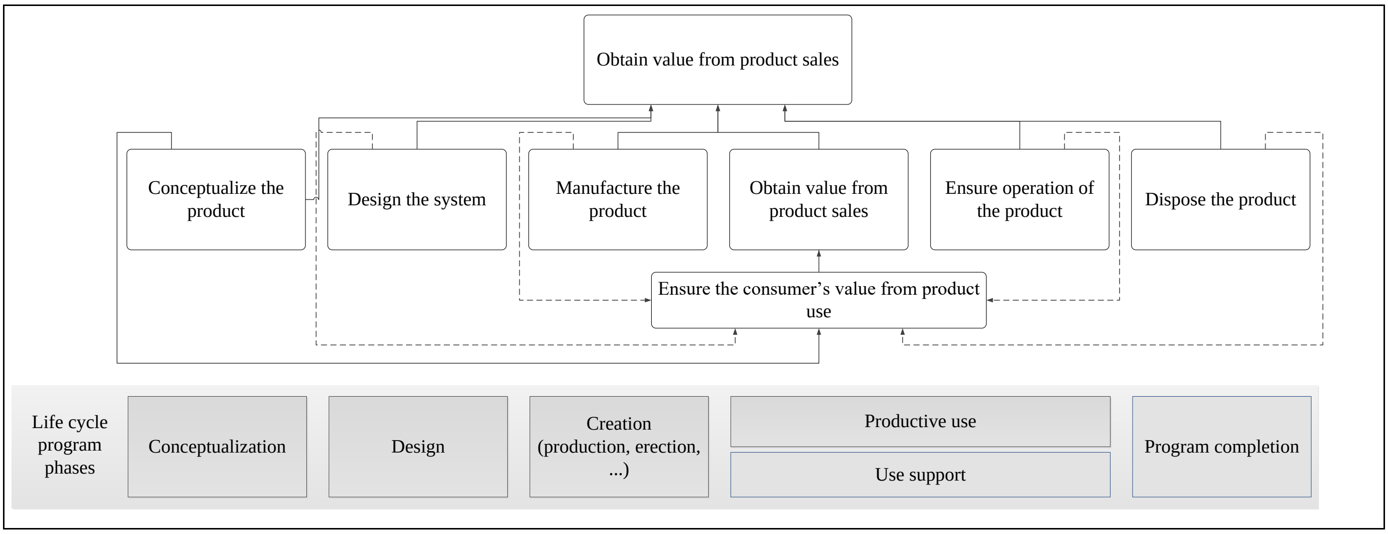

Fig. 2. The structure of forming the value or goals of complex activity performed during the life cycle of a complex system or object.

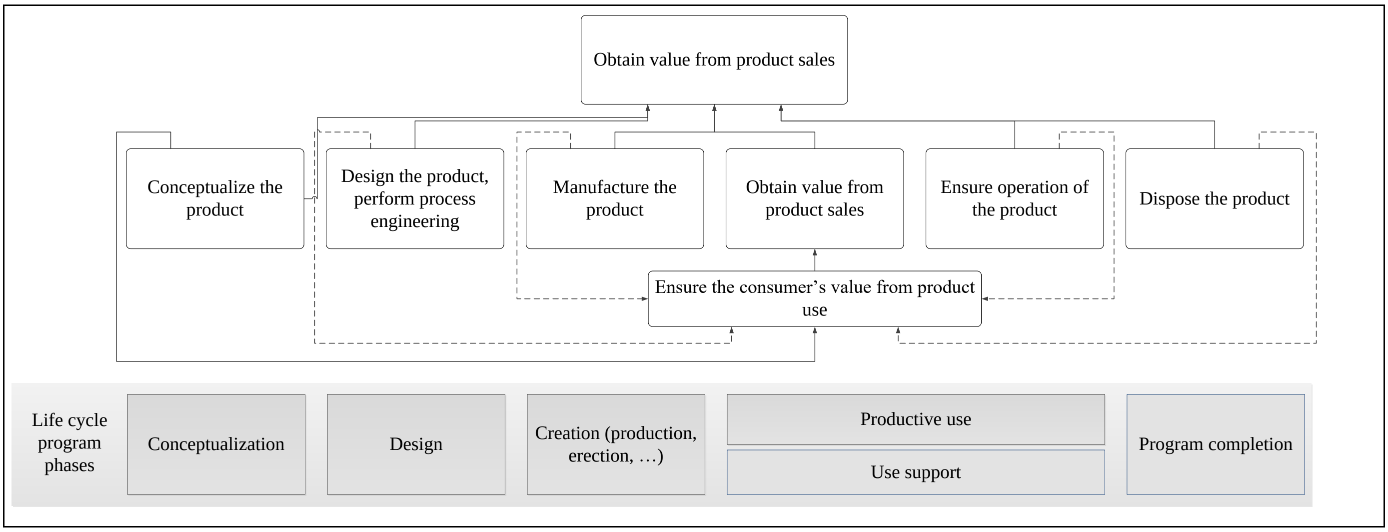

Fig. 3. The structure of forming the value or goals of complex activity performed during the life cycle of a complex product.

phases, for a complex object or system and a complex product, respectively.

The value structures (goals) are formed from the viewpoint of the subject implementing the LCS. These structures are generally self-evident and require no comment: rectangles with rounded corners represent goals and subgoals, and arrows connect the subgoals with the goals. The dashed arrows show the dependencies of the System's value on the design (including conceptual), production, operation support, and disposal works. These dependencies also have the character of "goal-subgoal" links: for the System to be useful for the consumer, the corresponding properties must be incorporated during the design, production, etc.

Note that the content of all phases of the LCS, except for productive use, includes works of various types (design, production, operation support, and disposal of the Systems). The productive use phase is primarily characterized by obtaining the target value and benefits (together with executing the corresponding works). All works during the life cycle have an auxiliary but unavoidable character and are executed only to obtain the target value and benefits during the productive use phase. At the level of the system-wide cross-industrial generalization, the LCS content can be formulated as follows.

- All LCS phases are characterized by the costs of the corresponding works directed on the creation and purposeful change of the System and EE and their information models. The achieved goals of the activity and the formed value have an auxiliary and internal character: the goals are achieved, and the value is formed in the interests of the LCS subject instead of external consumers.
- The productive use phase is characterized by obtaining the target value for the LCS subject based on the value and benefits provided to external consumers using the System.

In business practice, the control subject (or the subject implementing the LCS) is usually the management office of the LCS program, headed by a manager of the parent company, and the controlled object is the entire EE implementing the LCS. The control action is the set of decisions made by the management office of the LCS program (contracts, orders, regulations, letters, and other documents in electronic or paper form). The behavior of the controlled object is the entire set of elements of CA EE (production, engineering, technological, logistical, sales, administrative, financial, etc.), including the managerial activity of superior economic agents over subordinates.

When considering any control problem for objects with people, the key property is their ability of active choice: they act according to their internal motives and preferences. In addition, EE is a multilevel hierarchical organization (an interconnected and hierarchically subordinated set of enterprises and their subdivisions and employees). Therefore, the operation and management of EE are processes of a multilevel hierarchical nature. The presence of people in the EE structure, their property of active choice, and their key role in im0plementing LCS are the universal properties characteristic for all LCS and EE.

In such cases, it is traditional to apply game-theoretic approaches and methods of hierarchical game theory [43], active systems theory [44], organizational systems control theory [45], and contract theory [46]. Within this field of knowledge, an extended enterprise is a multilevel hierarchical dynamic active network with uncertainty and constraints on the joint activity of active elements[4] (AEs) in the form of technological networks [47–49]. In practice, an extended enterprise usually satisfies all assumptions[5] of the decomposition theorems formulated and proved in [47–49]. According to these theorems, for any feasible trajectory of LCS and irrespective of specific technological links between AEs (the LCS technology and EE organization), the control subject can construct a compensatory incentive scheme for AEs that:

– implements the trajectory of AE actions as a dominant strategy equilibrium;

– decomposes the control problem with respect to AEs, their actions, and time periods;

– ensures the minimum costs of the control subject to implement this trajectory under any possible foresight of AEs.

Such an incentive scheme reflects the principle of incentive-compatible control[6] and allows applying the enterprise control optimization scheme; see subsection 3.4.5 of the book [49]. Hence, the uncertainty of active choice of AEs can be eliminated in a mathematically correct way, and the control action can be considered the set *of action plans* of all AEs beneficial for

---

[4] Active elements in practice are firms, departments, divisions, work groups, and employees.

[5] The hypothesis of rational behavior of employees is the assumption that the subject chooses the actions yielding the most preferable for him results of activity considering all the information available to him; the assumption on bijective technological functions with respect to the actions of subjects and the results of their predecessors in the current period or the assumption on fully observable actions of subjects for the upper control subject (Principal); the assumption on Principal's awareness about the socially conditioned values of the cost function and reserved value of the subjects (holding for developed labor markets).

[6] Under incentive-compatible control [45], plan fulfillment is beneficial to all subjects and is an equilibrium of their game.

them. (Therefore, all AEs will seek to fulfill the action plans.) Implementing the principle of incentive-compatible control in business practice means that superior managers form tasks (plans) and incentive schemes for subordinate employees, divisions, and enterprises so that plans fulfillment be beneficial to subordinates. With such control, in particular, the strategic goals of a firm are translated to all employees down to average executives.

Now we define a class of cross-industrial methods and approaches to study LCS. For this purpose, we consider some important implementation features of LCS and, consequently, management models of LCS (Fig. 1).

When implementing the LCS, an extended enterprise plays a dual role: it acts as the subject and object of complex activity. Really, within the framework of the LCS program, there arises a need to create and change the cooperation of enterprises and create new technologies and, consequently, new enterprises.

Moreover, being complex systems, the System and EE require representations from different points of view [2] (e.g., functions, geometry, power, economics, reliability, etc.). Therefore, in practice, the System and EE are characterized by multiple descriptions and models.

The products, objects, and systems have significant industrial specifics and are often unique. In contrast, extended enterprises and life cycles have many system-wide cross-industrial similar functions and activities (financial, economic, personnel, logistics (to a considerable degree), etc.). Hence, unified descriptions and models can be used for them. As a result, all enterprises use the same best practices for organizing operational activities, the same patterns of business processes, and the same information technology platforms (ERP, CRM, MES, etc.).

Finally, in the vast majority of cases (or even always), the life cycle value is determined from the economic point of view; in turn, the economic approaches are cross-industrial and reflect the generalizing properties of the economic field of knowledge. Therefore, economic descriptions and models of the System, extended enterprise, and life cycle are typical for various industries, and unified approaches can be used. Also, the essential reflexivity of LCS is shown in the economic sphere: on the one hand, the economic parameters of the System determine the economic parameters of LCS and EE; on the other hand, the former depend on the latter. In particular, the cost price of the System considering the entire life cycle depends on the characteristics of EE, and conversely. Therefore, economic descriptions and models of the System, EE, and LCS represent an interconnected system.

Thus, we introduce the system-wide cross-industrial representation of the LCS based on the economic approaches and methods describing the processes of value formation and the associated costs.

Now we pass to the description of the quantitative model of the LCS management, formulating the four necessary components of the optimization problem:

– the state variables of the controlled system and the environment;

– constraints;

– patterns reflecting the relationships between the variables;

– the goal functions of active participants and the criterion of management effectiveness.

2. OPTIMAL CONTROL OF THE LIFE CYCLE OF COMPLEX SYSTEMS: A FORMAL PROBLEM STATEMENT

The practical analysis of life cycle features allows characterizing LCS management as unfolding in time multistep decision-making process under uncertainty to achieve the best result for the control subject on the entire LCS.

Within the established economic life (business) practice, LCS are treated as assets: objects that form or should form a positive economic (business) result. Therefore, it is advisable to optimize a single quantitative indicator (the effect or value of LCS), choosing one of the widespread economic characteristics (profitability, cash flow, added value, or another).

We consider this problem in the discrete-time statement: in each period t, the LCS state is characterized by some (maybe, vector) variable $x(t)$ taking values from an admissible set X, $x(t) \in X$. Assume that the control subject selects in period t an element $u(t)$ from the set of possible decisions (and actions) U, $u(t) \in U$. In practice, under the incentive-compatibility condition (see the discussion above), the control action $u(t)$ corresponds to the set of action plans of all EE elements (firms, their subdivisions, and individual employees) formed by following the plans of the management office of the LCS program.

Regardless of the control action $u(t)$ and the LCS state $x(t)$, some value of different-nature uncertain factors is realized in each period. It is described by the vector $\omega(t) \in \Omega$, where Ω denotes the set of all possible values of the uncertain factors.

We impose no restrictions on the nature and dimension of the vectors $x(t)$, $u(t)$, and $\omega(t)$ and the sets

X, U, and Ω (except for the reachability of the corresponding maxima or minima). Also, we understand the values of LCS states and uncertain factors in an extended sense, including, if necessary, elements related to the current period t and some previous periods (possibly the entire history from the initial modeling period t_1 to the current period t inclusive). In particular, the LCS state variables $x(t)$ can be understood as a complete information model of the pair "the System (object, or system) itself; the extended enterprise implementing the LCS."

The value of the uncertain factors $\omega(t)$ is unknown to the subject when choosing the control action $u(t)$ but becomes known a posteriori. Depending on the realized values of $x(t)$, $u(t)$, and $\omega(t)$, the LCS evolves: in the next period $(t + 1)$, its state takes the value

$$x(t+1) = F(x(t), u(t), \omega(t), t), \qquad (1)$$

where $F(\cdot)$ is an *LCS dynamics function*, a given function describing the change patterns of the LCS state in the environment depending on the decisions (control actions).

The effect or value of the LCS $F(\cdot)$ during the simulation interval $[t_1, t_2]$ will be described in the traditional form:

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) = \sum_{\tau=t_1}^{t_2} \delta_{\tau,t_1,t_2} \varphi(x(\tau); u(\tau); \omega(\tau); \tau)\,. \qquad (2)$$

Here, $\varphi(\cdot)$ is a known partial *value function of the LCS* for the control subject, δ_{τ,t_1,t_2} is the foresight function of the control subject, and the notation $\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}$ means the dependence on $x(\cdot)$, $u(\cdot)$, and $\omega(\cdot)$ on the simulation interval $[t_1, t_2]$.

For the sake of brevity, we will not write the foresight function δ_{τ,t_1,t_2} in explicit form: it can be taken into account as a factor in the partial value function $\varphi(\cdot)$.

The discrete-time representation, as well as the decisions and uncertainty reduced to a single time instant (in each period), reflects the existing practice of EE operation: planning and accounting are implemented in corresponding periods (for LCS, these are phases, stages, and more detailed periods, not necessarily of equal duration). Moreover, they do not restrict the capabilities of the proposed formalism.

The fractal hierarchy of the elements of activities, works, and, accordingly, decisions is also adequately implemented within the proposed formalism: the activities and decision-making of subordinate levels of the hierarchy are modeled in the description (2) of the LCS evolution process (the function $F(\cdot)$).

The uncertainty generated by each of the possible sources [41] (the environment, the technology and subject matter of CA, and the complex subject of activity) is fully reflected through the influence of uncertain factors (the process $\omega(\cdot)$) on the LCS evolution (the function $F(\cdot)$) and the LCS effect (value) (the function $\varphi(\cdot)$).

Then the optimal control of LCS is to maximize the LCS effect on the time interval $[t_1, t_2]$ considering the expressions (1) and (2):

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) \to \max_{\{u(\cdot)|t_1;t_2\};u(t)\in U}\,. \qquad (3)$$

Problem (1)–(3) is a classical dynamic programming problem with discrete time and uncertainty. Without imposing any restrictions on the nature of uncertainty, we denote by $\underset{\omega(\cdot)}{\mathrm{def}}\{\cdot\}$ the operator for eliminating the uncertainty $\omega(\cdot)$ (e.g., using the guaranteed result, expected value, or another approach).

Then the optimal control problem takes the form

$$\underset{\omega(\cdot)}{\mathrm{def}}\ \{\Phi(\{x(\cdot), u(\cdot)|t_1; t_2\};\{\omega(\cdot)|t_1; t_2\})\} \to \max_{\{u(\cdot)|t_1;t_2\};u(t)\in U} \qquad (4)$$

considering the expressions (1) and (2) and the initial conditions $x(t_1 - 1) = x_0$.

3. AN ALGORITHM FOR SOLVING THE OPTIMAL CONTROL PROBLEM

We present a general algorithm for finding the optimal control (a sequence of decisions $u^*(t)$) maximizing the LCS effect.

The solution of problem (1)–(4) is based on Bellman's optimality and backward induction. We write problem (4) in the form

$$\underset{\omega(\cdot)}{\mathrm{def}}\left\{\Phi\left(\left\{x(\cdot),u(\cdot)\,|\,t_1;t_2\right\};\{\omega(\cdot)\,|\,t_1;t_2\}\right)\right\}=$$

$$\underset{\omega(\cdot)}{\mathrm{def}}\sum_{\tau=t_1}^{t_2}\varphi(x(\tau); u(\tau); \omega(\tau); \tau) \to \max_{\{u(\cdot)|t_1;t_2\};u(t)\in U}\,.$$

Introducing the Bellman function

$$J(x, t) = \max_{\{u(\cdot)|t;t_2\};u(\tau)\in U(\cdot)}\ \underset{\omega(\cdot)}{\mathrm{def}}\sum_{\tau=t}^{t_2}\varphi(x(\tau); u(\tau); \omega(\tau); \tau)\,,$$

we obtain a recursive formula for it starting from period t_2 backwards.

For the final period t_2,

$$J(x, t_2) = \max_{u|\in U}\ \{\underset{\omega(t_2)}{\mathrm{def}}\ (\varphi\{x, u, \omega(t_2), t_2)\}\}. \qquad (5)$$

For period $(t_2 - 1)$,

$$J(x, t_2 - 1) =$$

$$\max_{u|\in U}\{\underset{\omega(t_2-1)}{\mathrm{def}}\{\varphi(\{x, u, \omega(t_2 - 1), t_2 - 1)\}\} +$$

$$\max_{u|\in U}\{\underset{\omega(t_2-1)}{\mathrm{def}}\{\underset{\omega(t_2)}{\mathrm{def}}\{\varphi(F(x, u, \omega(t_2 - 1), t_2 - 1),$$

$$u(t_2), \omega(t_2), t_2)\}\}\} =$$

$$\max_{u|\in U}\{\underset{\omega(t_2-1)}{\mathrm{def}}\{\varphi(x, u, \omega(t_2 - 1), t_2 - 1) +$$

$$J(F(x, u, \omega(t_2 - 1), t_2 - 1), t_2)\}\}.$$

Using backward induction, we derive a general recursive formula for the Bellman function for all $t \in [t_1, t_2 - 1]$ in descending order:

$$J(x, t) = \max_{u|\in U}\left\{\underset{\omega(t)}{\mathrm{def}}\left\{\varphi\left(x, u, \omega(t), t\right) + J\left(F\left(x, u, \omega(t), t\right), t+1\right)\right\}\right\}. \tag{6}$$

The expressions (5) and (6) allow calculating the sequence of functions $J(x, t)$ for all $t \in [t_1, t_2]$ in descending order of the period number t. After obtaining the solution $J(x, t)$, we substitute the initial value x_0 in period $(t_1 - 1)$ to find, for all $t \in [t_1, t_2]$, the optimal control strategy (the sequence of optimal decisions $u^*(t)$) together with the optimal trajectory $x^*(t)$ of LCS implementation:

$$u^*(t) = \arg\max_{u|\in U}\{\underset{\omega(t)}{\mathrm{def}}\{\varphi(x^*(t), u, \omega(t), t) + J(F(x^*(t), u, \omega(t), t), t + 1)\}\}; \tag{7}$$

$$x^*(t + 1) = \underset{\omega(t)}{\mathrm{def}}\{F(x^*(t), u^*(t), \omega(t), t)\}. \tag{8}$$

The relations (5)–(8) give a rigorous algorithm for making optimal decisions on LCS management. They express formal grounds for LCS management based on the following formalism:

- The state and behavior of the LCS are modeled by the vector $x(t) \in X$.
- The managerial decisions are described by the vector $u(t) \in U$.
- The uncertainty of all kinds is represented by the vector $\omega(t) \in \Omega$.
- The dynamics of the LCS and environment are described by the function $F(\cdot)$; see the relation (1).
- The LCS effect is formalized by the function $\varphi(\cdot)$ and the relation (2).

Problem (1)–(4) and the solution algorithm (5)–(8) have several fundamental properties. Let us discuss these properties and ways to apply the approach for practical LCS management and coordination and integration of heterogeneous (engineering, financial-economic, organizational, and other) decisions when implementing LCS.

We write two special cases of the optimization problem, which are important in practice.

In many cases, the partial value function is the difference between the benefits received $h(\cdot)$ and the total costs $c_i(\cdot)$ of various kinds:

$$\varphi(x(\tau), u(\tau), \omega(\tau), \tau) = h(x(\tau), u(\tau), \omega(\tau), \tau) + \sum_i c_i(x(\tau); u(\tau); \omega(\tau); \tau). \tag{9}$$

Then the effect is defined as the sum of partial values $\varphi(\cdot)$ discounted with a constant coefficient δ:

$$\Phi(\{x(\cdot), u(\cdot), \omega(\cdot)|t_1; t_2\}) = \sum_{\tau=t_1}^{t_2}\delta^{(\tau-t_1)}\left[h(x(\tau); u(\tau); \omega(\tau); \tau) - \sum_i c_i(x(\tau); u(\tau); \omega(\tau); \tau)\right]. \tag{10}$$

The function (9) fits most (or even all) economic statements, treating benefits and costs as elements of the cash flow, profit and loss account, and accumulated value. Hence, the effect (10) can be interpreted as a *net present value* of the life cycle as an investment asset, a weighted total profit, and value added (*economic value added*, *shareholders value added*, or *market value added*).

In this case, the Bellman equations take the form

$$J(x, t_2) = \max_{u|\in U}\left\{\underset{\omega(t_2)}{\mathrm{def}}\left\{h(x(t_2); u(t_2); \omega(t_2); t_2) - \sum_i c_i(x(t_2); u(t_2); \omega(t_2); t_2)\right\}\right\}. \tag{11}$$

$$J(x, t) = \max_{u|\in U}\left\{\underset{\omega(t)}{\mathrm{def}}\left\{h(x(t); u(t); \omega(t); t) - \sum_i c_i(x(t); u(t); \omega(t); t) + \delta J\left(F\left(x, u, \omega(t), t\right), t+1\right)\right\}\right\}. \tag{12}$$

Another special case is the long life cycle with an a priori unknown completion period and stationary dynamics starting from some period:

$$x(t + 1) = F(x(t), u(t), \omega(t)). \tag{13}$$

In this case, the Bellman equations are reduced to a single equation of the form

$$J(x) = \max_{u|\in U}\left\{\underset{\omega}{\mathrm{def}}\, h(x; u; \omega) - \sum_i c_i(x; u; \omega) + \delta J\left(F\left(x, u, \omega\right)\right)\right\}\Bigg\}. \tag{14}$$

The solution of this equation, $J(\cdot)$, gives the optimal control

$$u^* = \arg\max_{u|\in U} \Big\{ \operatorname*{def}_{\omega} h(x;\, u;\, \omega) - \sum_i c_i(x;\, u;\, \omega) + \delta J\big(F(x, u, \omega)\big) \Big\}. \qquad (15)$$

As the basic elements, the extended enterprise implementing the LCS includes the employees (individuals with the ability of active choice). In other words, the EE is an active system. This peculiarity of the problem, discussed in Section 1 above (also, see [47–49]), dictates LCS optimization conditions under the hypothesis of rational behavior of employees, the assumption on bijective technological functions, and the assumption on control subject's awareness (see footnote no. 5 and [47–49]).

Moreover, the extended enterprise is a multilevel active system with a hierarchy of technologically related firms, their subdivisions, work groups, and employees. Under such conditions, the practical formation of optimal control $u^*(t_{\text{cur}})$ consists in coordinated planning in a multilevel hierarchical dynamic active system. This problem was considered in detail in sections 2.2 and 7.1 of the books [50] and [49], respectively. The algorithmic coordinated planning models developed therein can be applied to form the optimal plan $u^*(t_{\text{cur}})$ of LCS implementation.

Another important feature of the optimal control of LCS is the need to consider the nature and characteristics of the uncertain factors $\omega(\cdot)$. Following [41], we treat all possible types of uncertainty as true uncertainty (the possibility of unique or rarely recurring events, which are not explained by the existing fundamental laws and for which there is no sufficient amount of a priori observations) or measurable uncertainty (the possibility of a priori unpredictable but repeated earlier events described by fundamental laws). In the life cycle management problem, the presence of both measurable and true uncertainty is fundamental. The reasons include the long-term duration of the life cycle, the variability of the environment (technological, political, economic, etc.), the creative nature of the life cycle processes (at least, in the early stages when designing the product, i.e., creating new knowledge about the future product, its operation in the environment, and production), and the presence of individuals with their ability of active choice within the complex subject of activity. The presence of true uncertainty specifying the behavior of $\omega(\cdot)$ and, consequently, the LCS and its effect (due to the dependencies (1) and (2)) makes it difficult to eliminate the uncertainty and solve the problem.

Traditionally, a priori knowledge about the sources and generation mechanisms of uncertainty is used to eliminate it. In this problem, due to the true uncertainty, such knowledge can never be considered objective and exhaustively complete (with respect to the described objects and phenomena) and, consequently, unchangeable. Insufficient knowledge about objective regularities compels using subjective assessments and assumptions to eliminate uncertainty (the operator $\operatorname*{def}_{\omega(t)}\{\cdot\}$ in the algorithm (5)–(8)). For dynamical phenomena, such as LCS, assessments and assumptions are formed as sets of scenarios [51] describing the evolution of phenomena under flexible control calculated depending on the realized values of uncertainty factors. The scenario approach [51] is widespread in decision-making, particularly forecasting and planning, in the areas where the true uncertainty is most significant (economics and the social and political sphere). It involves expert scenarios of the behavior of the analyzed system for calculations and forecasting. This approach is a subjective (heuristic) way to form knowledge with all its inherent disadvantages. However, this approach is applied in practice when an objective instrumental study is impossible. In the problem under consideration, the scenarios $\{x_0;\ \{\Omega_n^*(t)|\ t_1 \le t \le t_2\};\ \{U_n^*(t)|\ t_1 \le t \le t_2\}\}$ consist of the initial values x_0, the sequences of the state sets $\Omega_n^*(t)$ of uncertain factors and the sets $U_n^*(t)$ of managerial decisions (depending on the vector $\omega(t)$) ordered by the period number t.

Another important aspect of this problem and the practical application of the proposed optimization approach is the variability of environment conditions and the realization of the true uncertainty of the technology, subject matter, and subject. As a result, the optimal control strategies become irrelevant over time. Therefore, it seems reasonable to solve the problem regularly, considering all currently available information, to form the optimal control strategies. Before deciding in each current period t_{cur}, it is advisable to repeat the solution of problem (1)–(15) for the time interval $t_{\text{cur}} \le t \le t_2$ with the updated a priori knowledge (scenarios and other assumptions). Among all optimal controls $\{\mathrm{u}^*(\cdot)|t_{\text{cur}};\ t_2\}$, only the nearest in time plan $u^*(t_{\text{cur}})$ is always used: generally speaking, the remaining controls $\{u^*(\cdot)|t_{\text{cur}} + 1;\ t_2\}$ can be not calculated, following the expressions (7)–(8). From the practical point of view, the optimal strategy should be updated when fixing each *baseline* [2] during the entire LCS.

Within the scenario approach, the interpretation of the problem solution includes a stipulation about optimal control under the accepted assumptions (realization of one scenario). On the one hand, such stipulations reduce the value of optimization; on the other, such justification is, no doubt, the best possible, especially if the set of scenarios under consideration is large, making negligibly small the possibility of realizing the LCS along a trajectory different from all such scenarios. This remark is another condition for optimizing LCS management.

The LCS state vector $x(\cdot)$, the control action $u(\cdot)$, the uncertain factors $\omega(\cdot)$, and the corresponding sets of their admissible values (X, U, and Ω) describe the complex objects and phenomena of LCS (System, EE, technology, and their evolution and operation in a complex technological, political, and economic environment). The function $F(\cdot)$ formalizes the diverse LCS evolution (all changes in the pair <System, extended enterprise>) under the managerial decisions, design, technological, production, and other works, the formation, coordination, fulfillment, and control of plans, and implementation of other activities within the EE. In turn, the function $\varphi(\cdot)$ (as well as the benefits $h(\cdot)$ and costs $c_i(\cdot)$) reflects the dependence of the LCS effect on all significant aspects of LCS implementation.

However, generally, problem (1)–(15) cannot be solved in analytical form. Therefore, the practical implementation of the algorithm (5)–(8) and (11)–(15) requires using industry-specific models to represent the functions $F(\cdot)$, $\varphi(\cdot)$ $h(\cdot)$, and $c_i(\cdot)$ and reflect the complex relationships between the characteristics of the LCS states $x(\cdot)$, the managerial decisions $u(\cdot)$, and the uncertain factors $\omega(\cdot)$, including their impact on the LCS effect $\varphi(\cdot)$. Such models promptly yield numerical values of $F(\cdot)$, $\varphi(\cdot)$, $h(\cdot)$, and $c_i(\cdot)$ under different scenarios to apply the proposed algorithms (5)–(8) and (11)–(15). A graphic metaphor of these algorithms is shown in Fig. 4.

Finally, we again list all optimization conditions for LCS management (the conditions under which the proposed approach remains mathematically rigorous):

- The hypothesis of rational behavior of EE employees is the assumption that the subject chooses the actions yielding the most preferable for him results of activity considering all the information available to him.
- The assumption on bijective technological functions with respect to the actions of subjects and the results of their predecessors in the current period or the assumption on fully observable actions of the decision-maker (Principal).
- The assumption on Principal's awareness about the socially conditioned values of the cost function and reserved value of the subjects (holding for developed labor markets). The assumption that the set of LCS scenarios is large enough to make negligibly small the possibility of realizing an LCS trajectory is different from all such scenarios.

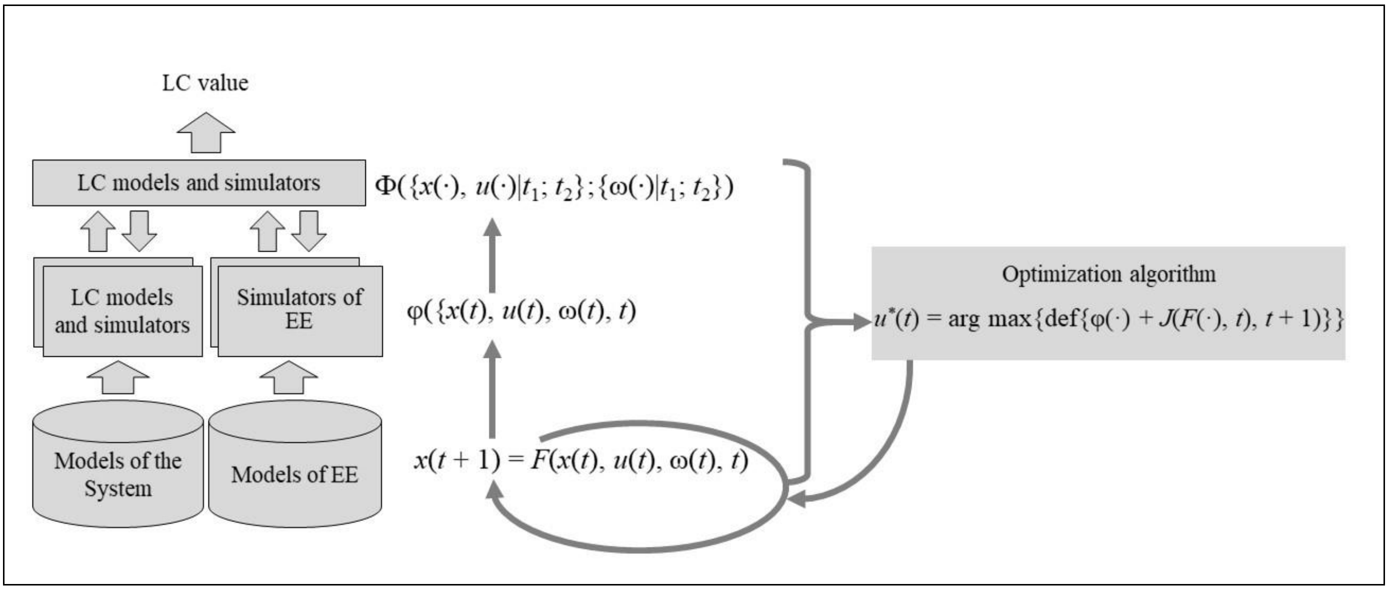

Fig. 4. Logic of the optimization algorithm.

CONCLUSIONS: AN OPTIMAL CONTROL TOOL FOR LIFE CYCLE

We summarize the results of this paper.

- A mathematically rigorous optimal control problem for the life cycle of complex products of aerospace, power, nuclear, transport, and other complex entities, capital objects and systems of the power, telecommunications, transport, agriculture, raw material, and other industries, as well as information and technological systems has been stated.
- A formal algorithm for solving the corresponding optimal control problem has been presented.
- A scenario approach to apply this algorithm in practice has been proposed; optimization conditions for LCS management (the conditions under which optimization is possible) have been listed.

These results form *an optimal control tool for the LCS*.

REFERENCES

1. *The Guide to the Systems Engineering Body of Knowledge* (*SEBoK*), ver. 1.2, Pyster, A. and Olwell, A., Eds., Hoboken: The Trustees of the Stevens Institute of Technology, 2013. http://www.sebokwiki.org. Accessed August 29, 2021.
2. *ISO/IEC/IEEE 15288:2015: Systems and Software Engineering - System Life Cycle Processes*, 2015.
3. Novikov, D.A., *Cybernetics: From Past to Future*, Springer, 2016.
4. Peregudov, F.I. and Tarasenko, F.P., *Vvedenie v sistemnyi analiz* (Introduction to Systems Analysis), Moscow: Vysshaya Shkola, 1989. (In Russian.)
5. *GOST* (State Standard) *R 54871-2011*: *Project Management. Requirements for Program Management*, 2011.
6. *GOST* (State Standard) *R 56136-2014*: *Life Cycle Management of Military Products. Terms and Definitions*, 2014.
7. Mesarović, M.D., Macko, D., and Takahara, Y., *Theory of Hierarchical, Multilevel Systems*, New York Academic, 1970.
8. Moiseev, N.N., *Matematicheskie zadachi sistemnogo analiza* (Mathematical Problems of Systems Analysis), Moscow: Nauka, 1981. (In Russian.)
9. *Enterprise Systems Engineering: Advances in the Theory and Practice*, Rebovich, G. and White, B., Eds., Boca Raton: CRC Press. 2011.
10. Farr, J.V., *Systems Life Cycle Costing: Economic Analysis, Estimation, and Management* (*Engineering Management*), 1st ed., CRC Press, 2011.
11. Gupta, Y. and Chow, W.S., Twenty-Five Years of Life Cycle Costing Theory and Application: A Survey, *The Intern. Journal of Quality and Reliability Management*, 1985, vol. 2, pp. 51–76.
12. Leszczynski, Z. and Jasinski, T., Comparison of Product Life Cycle Cost Estimating Models Based on Neural Networks and Parametric Techniques - A Case Study for Induction Motors, *Sustainability*, 2020, vol. 12, art. no. 8353, pp. 2–14. DOI: 10.3390/su12208353.
13. Liu, H., Gopalkrishnan, V., Ng, W.K., et al., An Intelligent System for Estimating Full Product Life Cycle Cost at the Early Design Stage, *Intern. Journal of Product Lifecycle Management*, 2008, no. 2(2-3), pp. 96–113.
14. Loyer, J.L. and Henriques, E., A MBSE Probabilistic Framework for Preliminary Lifecycle Costing of Mechanical Products, *INCOSE International Symposium*, Las Vegas, NV, 2014, vol. 24, iss. 1, pp. 182–195.
15. Oduyemi, O., Okoroh, M., and Dean, A., Developing an Artificial Neural Network Model for Life Cycle Costing in Buildings, *Proc. 31st Annual ARCOM Conference*, Raidén, A.B. and Aboagye-Nimo, E., Eds., Lincoln, UK, 2015, pp. 843–852.
16. Wang, G., Roedler, G.J., Pena, M., and Valerdi, R.A., A Generalized Systems Engineering Reuse Framework and Its Cost Estimating Relationship, *INCOSE International Symposium*, Las Vegas, NV, 2014, vol. 24, iss. 1, pp. 274–297.
17. Captain, T., *Can We Afford Our Own Future? Why R&D Programs Are Late and Over-budget - And What Can Be Done to Fix the Problem*, Deloitte Development LLC, 2009.
18. *NASA Cost Estimating Handbook*, ver. 4.0, NASA Cost Analysis Division, Washington, DC: National Aeronautics and Space Administration, 2015.
19. *GAO Cost Estimating and Assessment Guide. Best Practices for Developing and Managing Capital Program Costs*, GAO-09-3SP, US Government Accountability Office, Washington, DC, 2009.
20. Holland, J., Studying Complex Adaptive Systems, *Journal of Systems Science and Complexity*, 2006, vol. 19(1), pp. 1–8.
21. North, M., A Theoretical Formalism for Analyzing Agent-based Models, *Complex Adaptive Systems Modeling*, 2014, vol. 2, art. no. 3. DOI: 10.1186/2194-3206-2-3.
22. Rzevski, G. and Skobelev, P., *Managing Complexity*, London: WIT Press, 2014.
23. Sanchez-Anguix, V., Tunalı, O., Aydoğan, R., and Julian, V., Can Social Agents Efficiently Perform in Automated Negotiation?, *Applied Sciences*, 2021, vol. 11, art. no. 6022. https://doi.org/10.3390/app11136022.
24. Alagar, V.S. and Periyasamy, K., Calculus of Communicating Systems, in *Specification of Software Systems*, London: Springer, 2011. https://doi.org/10.1007/978-0-85729-277-3_15.
25. Bergstra, J., Process Algebra for Synchronous Communication, *Information and Control*, 1984, vol. 60, pp. 109–137.
26. Hoare, C., *Communicating Sequential Processes*, New York: Prentice-Hall, 1985.
27. Milner, R., *A Calculus of Communicating Systems*, Lecture Notes in Computer Science, vol. 92, Heidelberg: Springer, 1980.
28. Estefan, J., Survey of MBSE Methodologies, Seattle: INCOSE, 2008. http://www.omgsysml.org/mbse_methodology_survey_revb.pdf.
29. *The Oxford Handbook of the Economics of Networks*, Oxford: Oxford University Press, 2016.
30. Jackson, M., *Social and Economic Networks*, Princeton: Princeton University Press, 2010.
31. Burkov, V.N., Gorgidze I.A., and Lovetskii, S.E., *Prikladnye zadachi teorii grafov* (Applied Problems of Graph Theory), Tbilisi: All-Union Center of the Academy of Sciences of the Georgian SSR, 1974. (In Russian.)
32. Golenko-Ginzburg, D.I., *Stokhasticheskie setevye modeli planirovaniya i upravleniya razrabotkami* (Stochastic Network Models to Plan and Manage R&D Activities), Voronezh: Nauchnaya Kniga, 2010. (In Russian.)
33. Matveev, A.A., Novikov, D.A., and Tsvetkov, A.V., *Modeli i metody upravleniya portfelyami proektov* (Models and Methods of Project Portfolio Management), Moscow: Trapeznikov Institute of Control Sciences RAS, 2005. (In Russian.)

34. Kelly, F. and Yudovina, E., *Stochastic Networks*, Cambridge: Cambridge University Press, 2014.
35. Kleiner, G.B., Evolution and Modernization of Enterprise Theory, *Trudy 5-go Mezhdunarodnogo simpoziuma po evolyutsionnoi ekonomike* (Proc. 5th International Symposium on Evolutionary Economics), Moscow, Institute of Economics RAS, 2004. (In Russian.)
36. Vozhakov, A.V., Gitman, M.B., and Stolbov, V.Yu., Models of Collective Decision-Making in Production, *Large-Scale System Control*, 2015, no. 58, pp. 161–178. (In Russian.)
37. Mintzberg, H., *Structure in Fives: Designing Effective Organizations*, Englewood Cliffs, NJ: Prentice-Hall, 1983.
38. Adizes, I., *Managing Corporate Lifecycles: An Updated and Expanded Look at the Corporate Lifecycles*, The Adizes Institute Publishing, 2004.
39. *A Guide to the Project Management Body of Knowledge* (*PMBOK® Guide*), 6th. ed., Project Management Institute, 2017.
40. Belov, M.V. and Novikov, D.A., *Modeli deyatel'nosti* (Models of Activity), Moscow: Lenand, 2021. (In Russian.)
41. Belov, M.V. and Novikov, D.A., *Methodology of Complex Activity: Foundations of Understanding and Modelling*, Cham: Springer, 2020.
42. *ISO/IEC/IEEE 24748-1:2018: Systems and software engineering - Life cycle management - Part 1: Guidelines for life cycle management*, 2018.
43. Germeier, Yu.B., *Non-Antagonistic Games*, Springer, 1986.
44. Burkov, V.N., *Osnovy matematicheskoi teorii aktivnykh sistem* (Fundamentals of the Mathematical Theory of Active Systems), Moscow: Nauka, 1977. (In Russian.)
45. Novikov, D.A., *Theory of Control in Organizations*, New York: Nova Science Publishers, 2013.
46. Bolton, P. and Dewatripont, M., *Contract Theory*, Cambridge: MIT Press, 2005.
47. Belov, M.V., Incentive-Compatible Control in Dynamic Multi-Agent Systems. Part 1. Contracts in Dynamic System with One Principal and Multiple Agents, *Control Sciences*, 2020, no. 1, pp. 39–47. (In Russian.)
48. Belov, M.V., Incentive-Compatible Control in Dynamic Multi-Agent Systems. Part 2. Contracts in Dynamic Hierarchical Multi-Agent System, *Control Sciences*, 2020, no. 2, pp. 36–46. (In Russian.)
49. Belov, M. and Novikov, D., *Optimal Enterprise. Structures, Processes and Mathematics of Knowledge, Technology and Human Capital*, CRC Press, 2021.
50. Belov, M.V. and Novikov, D.A., *Upravlenie zhiznennymi tsiklami organizatsionno-tekhnicheskikh system* (Management of Life Cycles of Organizational and Technical Systems), Moscow: Lenand, 2020. (In Russian.)
51. Dewar, J., *Assumption Based Planning: A Tool for Reducing Avoidable Surprises*, Cambridge: Cambridge University Press, 2002.

This paper was recommended for publication by D.A. Novikov, a member of the Editorial Board.

Received September 25, 2021.
Accepted November 22, 2021.

Author information

Belov, Mikhail Valentinovich. Dr. Sci. (Eng.), Skolkovo Institute of Science and Technology, Moscow, Russia
✉ mbelov59@mail.ru

Cite this paper
Belov, M.V. Optimal Control of the Life Cycle of Complex Systems. *Control Sciences* **1,** 15–26 (2022).
http://doi.org/10.25728/cs.2022.1.2

Translated into English by *Alexander Yu. Mazurov*,
Cand. Sci. (Phys.–Math.),
Trapeznikov Institute of Control Sciences,
Russian Academy of Sciences, Moscow, Russia
✉ alexander.mazurov08@gmail.com