

## A CONCEPTUAL APPLIED GEOGRAPHIC INFORMATION SYSTEM FOR MODELING SEARCH AUTONOMOUS CORRELATION-EXTREME NAVIGATION SYSTEMS

A.I. Alchinov<sup>1</sup> and I.N. Gorokhovskiy<sup>2</sup>

<sup>1</sup>Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia

<sup>2</sup>Research Center of Topographic and Navigational Support, Central Research Institute No. 27, Moscow, Russia

✉ alchinov46@mail.ru, ✉ gin\_box@mail.ru

**Abstract.** This paper presents a conceptual applied geographic information system (AGIS) for modeling search correlation-extreme navigation systems (CENSs) to control moving objects. As demonstrated below, the development and mass implementation of autonomous navigation systems of this type as the only alternative to satellite navigation systems can currently be based on subject-oriented information technology. The AGIS can be used to assemble models of a wide range of CENSs and models of technologies for adjusting their operation in specified areas with necessary computational experiments. The required software components, storage structure, and interface features are determined by constructing a general mathematical model. While preserving all specifics of the search algorithms of CENSs, this model covers the well-known image combining algorithms and, moreover, includes a synthesis scheme for search algorithms of new-type CENSs using pattern recognition and scene analysis, clustering, neural network training, and cloud data processing. Stress testing is the most important type of computational experiments with CENS models. A mathematical model of stress effects is constructed for a particular case. It describes various operating conditions for CENSs, including fatal deviations from normal operation.

**Keywords:** applied geographic information system, correlation-extreme navigation system, shooting system, pattern recognition, scene analysis, learning machines, neural network, parallel computing, cloud computing, mathematical modeling, stress testing of the system.

### INTRODUCTION

Correlation-extreme navigation systems (CENSs) serve to refine off-line information about the location, orientation, and other parameters of a moving object coming from the main navigation system. A control system uses this information to compensate the deviations in the object's motion parameters to follow a given route. Search CENSs check hypotheses about the values of motion parameters by matching the current terrain sector image received by the airborne shooting system with fragments of a reference image of the application area. The reference images are pre-

pared in advance and stored in the memory of the airborne computer. When searching for a reference image fragment close by content to the current image (in the sense of a closeness function in the onboard algorithm), a regular shift grid of the frame selecting the next fragment of the reference image is used. The hypothesis that the sought parameters have values equal to those at the grid nodes are checked. The hypothesis for which the closeness function achieves maximum is accepted. Global search schemes, gradient methods from the arsenal of numerical optimization methods, and their combinations are often used [1].

Until the late 1990s, R&D works on various aspects of CENSs were carried out intensively. At dif-

ferent stages, the solution of motion control and navigation problems was associated with general synthesis principles that would yield control parameters for moving objects in the automatic mode under specific circumstances. Directions for the further development of CENSs were defined:

- new design principles for onboard algorithms, their intellectualization and self-organization;
- application of new types of shooting systems and their combination;
- development of parallel processors, including specialized processors for implementing algorithms with a single parallel structure.

At that time, the level of information technology and the achievable characteristics of airborne computers restrained the practical implementation of the directions mentioned above. Satellite navigation systems were developed, and CENSs moved to the background.

However, the situation has changed dramatically to date. The accelerated development of CENSs in the above directions has become topical due to the following factors: the intensive development of various-purpose unmanned vehicles and the appearance of modern shooting systems, large-memory processors, and processors with parallel computing schemes in their control systems; mass distribution and development of programming tools for artificial intelligence systems; training neural networks on big data in cloud computing environments. Hence, the potential of CENSs as the only alternative to satellite navigation systems needs to be tapped more than in the existing solutions [2].

As it turned out, satellite control systems for moving objects are vulnerable in today's environment. We are increasingly aware that satellite control systems need to be protected, strengthened, and expanded. Orbital stations can be disabled or simply destroyed.

Under these conditions, the accelerated development of this problem domain can be provided by expanding R&D works using a subject-oriented computational complex. Such a complex can provide an engineer with all the necessary tools to assemble models of a wide range of CENSs and draft technologies to adjust their operation in given application areas from ready-made software components through a special interface to the component storage and conduct necessary computational experiments with them.

Note that besides the specific functionality focused on modeling search CENSs and draft technologies to

adjust their operation in given application areas, such a subject-oriented complex should provide user access to all universal means of handling geospatial information (particularly in the form of *application programming* interface (API) for external programs). In other words, the complex should provide user access to the general-purpose functionality of modern geographic information systems (GISs). Thus, it should be created as an applied geographic information system based on the extended functionality of general-purpose GISs [3].

Therefore, developing a subject-oriented complex for modeling search CENSs in the form of an applied geographic information system (AGIS CENSs) is a topical problem. First of all, it is necessary to consider new design principles of onboard algorithms (particularly, their intellectualization and self-organization), modern processors oriented at the parallel implementation of algorithms, new types of shooting systems and their integration, and other recent achievements of information technology. Such analysis is needed to justify the composition of the software components of AGIS CENSs, identify the ones with functionality implemented in related problem domains, and determine a CENS-specific assembly scheme for different onboard algorithms and draft technologies of data preparation. New variants of search CENSs will improve the dynamics of autonomous control systems using the principles of reconfigurable structures.

This paper considers two-dimensional sensing CENSs: the current and reference terrain images are compared for active and passive airborne shooting systems in the electromagnetic radiation wavelength ranges for which means of stress exposure are available or can be developed.

In Section 1, we present a mathematical model of search algorithms and their adjustment procedures to perform their task in a given application area, which is rather general but, at the same time, preserves all specifics of search CENS algorithms. This model includes the model of image matching as a particular case and describes the scheme of assembling a significantly wider range of CENSs from the software components discussed above.

Since tough requirements are applied to the reliability of CENSs, stress testing is the most important type of computational experiments with CENS models. In Section 2, we construct a general mathematical model of stress exposures causing fatal deviations from the normal operating conditions of CENSs. Also,



we justify requirements to their modeling tools within AGIS CENSs.

## 1. MATHEMATICAL MODEL OF SEARCH CENS ALGORITHMS

Consider CENSs in which the shooting system captures a scene image  $S$  on a terrain section, and the onboard algorithm refines the planned coordinates of the carrier at the shooting instant. In other words, the parameter to be refined is  $d = (X, Y)$ . These limitations are adopted just to illustrate the main features of the mathematical model and facilitate their perception. In the final analysis, they will affect neither the set of CENS variants (and the procedures of their adjustment to perform a particular task in a given application area) covered by the model, nor the generality of the analysis results and their practical importance for the conceptual AGIS CENSs.

Let an application area be defined if the set  $D$  of all possible values of the refined parameter of the carrier motion at the shooting instant is defined, i.e.,  $d \in D$ . We denote by  $M$  the set of all possible images  $S$  coming from the shooting system to the input of the onboard algorithm of the CENS in a given application area (under the condition  $d \in D$ ).

Then the CENS is prepared to perform its task in the application area if for any  $S \in M$ , the onboard algorithm is ready to output a correct approximation  $\hat{d} = (\hat{X}, \hat{Y}) \in \hat{D}$  to the true value  $d \in D$  at the instant of receiving the image  $S$ , where  $\hat{D}$  is the set of all possible outputs of the onboard algorithm. For known search algorithms, the set  $\hat{D}$  is finite and coincides with the set of coordinates for the shift grid nodes of the frame selecting the next reference image fragment when matching the current and reference images in the application area:  $\hat{D} = \{\hat{d}_{j_1 j_2}\}$ ,  $j_1 = 1, 2, \dots, N_1$ ;  $j_2 = 1, 2, \dots, N_2$ . Since  $D \subset R^2$ , the approximation is correct if  $\rho(\hat{d}, d) \leq \varepsilon$ , where  $\rho$  is the distance function between two points on the plane  $R^2$ , and  $\varepsilon$  is an admissible error (e.g., in meters). Note that this interpretation of the CENS preparation to perform its task in a given region can be treated as the first iteration towards a general mathematical statement of the problem. In traditional terms, this problem consists in pre-

paring the actual reference image for a given application area.

Obviously, the CENS ability to perform a task in a given application area is determined by the relationship between the images obtained from the shooting system and the values of the refined parameter of the moving object at the shooting instant. We describe this relationship by a function  $f(S): M \rightarrow D$ . Note that in the general case, this function is multi-valued: the images getting into the shot under different values of the refined parameter of the moving object do not necessarily differ. Furthermore, the image content depends on other factors and parameters. For example, the moving object's height affects the image and should be considered when refining the planned coordinates. Such factors and parameters will be called disturbances. Hence, the function  $f^{-1}(d)$ , inverse to  $f(S)$ , is also multi-valued in the general case.

Other disturbances include the time of the year, weather conditions in the application area, etc. Stress exposures on the shooting systems of CENSs should be considered separately. (They can be purposeful.) Stress exposures may cause fatal deviations from the normal operating conditions of CENSs: the system will be unable to perform the task. Stress modeling in AGIS CENSs is described in Section 2.

Thus, the navigational properties of a CENS in the application area and the conditions of its orientation therein can be studied by analyzing the function  $f(S): M \rightarrow D$ .

We specify the form of initial information about the approximated function  $I_0\{f(S): M \rightarrow D\}$  when adjusting the CENS to operate in a given application area. Assume that this information is described by a computer simulation model of the shooting system:

$$I_0\{f(S): M \rightarrow D\} = \hat{f}^{-1}(d, p), d \in D, p \in P,$$

where the vector  $p \in P$  consists of the disturbing parameters included in the shooting system model. The vector  $p$  has the admissible domain  $P$ . The computer simulation model of the shooting system should approximate the function  $f^{-1}(d)$ ,  $d \in D$ , inverse to  $f(S): M \rightarrow D$ .

Now we pass to the onboard algorithms of CENSs.

If the CENS is already prepared to perform its task in a given application area, for any  $S \in M$  the

onboard algorithm will output some value  $\hat{d} \in \hat{D}$ . In other words, the algorithm is ready to calculate the value of a one-valued function  $\hat{f}: M \rightarrow \hat{D}$  for any  $S \in M$ . Hence, we may suppose the following: during the preparation process, the value of some generalized parameter  $\alpha^* \in A$  was calculated and saved in the onboard memory, adjusting the CENS to calculate the values of this particular function by separating it from a parametric family. Therefore, the CENS can be treated as a technical implementation of the parametric family of one-valued functions  $\{\hat{f}(\alpha; S)\}_{\alpha \in A}$ , where  $\hat{f}(\alpha; S): M \rightarrow \hat{D}$  is a particular function from this family uniquely defined by the generalized parameter value  $\alpha \in A$ . In the traditional interpretation, this parameter is the reference image of the application area. Under the most general assumptions, the problem of preparing the CENS for operation in a given application area turns out to be that of function approximation: as the result of preparation, a function  $\hat{f}(\alpha^*; S)$  is chosen to approximate the function  $f(S)$  in an exact sense dictated by practical requirements to the CENS. Assume that the criterion of closeness of the two functions has the form  $\rho_M(\hat{f}, f) \leq \varepsilon$ , where  $\rho_M$  is a metric in the space of such functions, and  $\varepsilon$  is a positive number.

We choose the simplest parametric approximating family of classical numerical functions, generalizing it to functions with image matrix variables  $S$  that take values in the plane  $R^2$  (the elementary case). The simplest family consists of step functions of one numerical variable. Their definitional domain on the numerical axis is divided into disjoint segments, where the function has a constant value.

We write the definitional domain of the function  $\hat{f}$  as a union of  $l$  disjoint sets (called classes):

$$M = \bigcup_{j=1}^l K_j, \text{ where } K_j \cap K_t = \emptyset \text{ for } j \neq t. \quad (1)$$

The function  $\hat{f}(S)$  will be called a generalized step function if

$$\hat{f}(S) = \sum_{j=1}^l \chi_j(S) \hat{d}_j, \quad (2)$$

where  $\chi_j(S) = 1$  if  $S \in K_j$ , and  $\chi_j(S) = 0$  otherwise, and  $\hat{D} = \{\hat{d}_1, \hat{d}_2, \dots, \hat{d}_l\}$ . (In other words,  $\chi_j(S)$  are the characteristic functions of classes  $K_j$ .) We introduce the notations

$$\mathbf{d} = (\hat{d}_1, \hat{d}_2, \dots, \hat{d}_l), \quad \mathbf{X}(S) = (\chi_1(S), \chi_2(S), \dots, \chi_l(S)).$$

Then the expression (2) takes the vector form

$$\hat{f}(S) = \langle \mathbf{X}(S), \mathbf{d} \rangle. \quad (3)$$

Due to (3), calculating the value  $\mathbf{X}(S)$  is a common problem of assigning an object to one of  $l$  disjoint classes  $K_1, K_2, \dots, K_l$ . Any algorithm for solving this problem is by definition a pattern recognition algorithm [4]. Obviously, the known search algorithms solve a particular case of the pattern recognition problem assuming that the set of images  $M$  is a union of  $l = N_1 \times N_2$  disjoint classes  $K_{ij}$ . An image  $S$  belongs to the class  $K_{j_1 j_2}$  if it is obtained in a small neighborhood  $\hat{D}_{j_1 j_2}$  of a node  $\hat{d}_{j_1 j_2}$  of the shift grid  $\hat{D} = \{\hat{d}_{j_1 j_2}\}$ , where  $j_1 = 1, 2, \dots, N_1$  and  $j_2 = 1, 2, \dots, N_2$ . Any image  $S \in K_{ij}$  coincides with the reference image fragment corresponding to the node  $\hat{d}_{j_1 j_2}$  with an accuracy to a random "term" obeying a known distribution (as a rule, Gaussian). Such assumptions apply to the original image spaces obtained from the shooting system and the image spaces preliminarily transformed by image quality improvement methods or compiled from the scene descriptions of terrain sectors got into the shot [5, 6]. Thus, we have a special case of comparison with reference objects when each class is described by a single reference object.

In the context of pattern recognition, preliminary transformations of images extract working features of recognition and must be included in the model of onboard algorithms. Under a preliminary image transformation  $\pi(S)$ , the expressions (1), (2), and (3) are written as follows:

$$\pi M = \bigcup_{j=1}^l \pi K_j, \quad (4)$$

where  $\pi K_j \cap \pi K_t = \emptyset$  for  $j \neq t$ ;



$$\hat{f}(S) = \sum_{j=1}^l \chi_j(\pi(S)) \hat{d}_j, \quad (5)$$

where  $\chi_j(\pi(S))=1$  if  $\pi(S) \in \pi K_j$ , and  $\chi_j(\pi(S))=0$  otherwise. Hence, they are characteristic functions of the classes  $\pi K_j$ .

Denoting

$$\mathbf{X}(\pi(S)) = (\chi_1(\pi(S)), \chi_2(\pi(S)), \dots, \chi_l(\pi(S))),$$

we write the expression (5) in the vector form

$$\hat{f}(S) = \langle \mathbf{X}(\pi(S)), \mathbf{d} \rangle. \quad (6)$$

In certain conditions, known search algorithms face the problem of large search zones: global schemes lead to indiscriminate enumeration, and local gradient methods may not reach the global optimum of the comparison function. At the same time, in the presence of powerful disturbances, the enumeration procedure has to be performed in the space of disturbing parameters as well. (Computational resources have to be consumed on extraneous problems.) In these conditions, a possible approach is to use hierarchical partitions of the set  $M$  into classes. Let such partitions have  $r = 2$  levels.

For the generalized step function defined on a two-level hierarchical partition, the expression (5) takes the form

$$\hat{f}(S) = \sum_{j_1=1}^l \chi_{j_1}(\pi(S)) \sum_{j_2=1}^{l_1} \chi_{j_1 j_2}(\pi_{j_1}(S)) \hat{d}_{j_1 j_2}. \quad (7)$$

In the general case (an arbitrary number  $r$  of levels), we have

$$\begin{aligned} \hat{f}(S) = & \sum_{j_1=1}^l \chi_{j_1}(\pi(S)) \sum_{j_2=1}^{l_1} \chi_{j_1 j_2}(\pi_{j_1}(S)) \dots \\ & \dots \sum_{j_r=1}^{l_r} \chi_{j_1 j_2 \dots j_r}(\pi_{j_1 j_2 \dots j_{r-1}}(S)) \hat{d}_{j_1 j_2 \dots j_r}. \end{aligned} \quad (8)$$

Thus, the CENS can be adjusted to operate in a given application area by solving a function approximation problem:

– The function  $f(S): M \rightarrow D$  is given by the algorithm for calculating the values of the function  $\hat{f}^{-1}(d, p), d \in D, p \in P$ , that approximates the inverse  $f^{-1}(d): D \rightarrow M$  of the function  $f$ .

– In the parametric family of all generalized step functions  $\{\hat{f}(\alpha; S): M \rightarrow \hat{D}\}_{\alpha \in A}$  (8), it is required to

find a value  $\alpha^* \in A$  such that the function  $\hat{f}(\alpha^*; S)$  approximates the function  $f(S)$ .

– One example of the approximation criterion is  $\rho_M(\hat{f}, f) \leq \varepsilon$ , where  $\rho_M$  is the metric in the space of such functions, and  $\varepsilon$  is a positive number.

At this stage of the study, the criteria for approximating the inverse function and the approximation criterion are not required to clarify. It suffices to assume that the computer simulation model of the shooting system (the algorithm for calculating the inverse function  $\hat{f}^{-1}(d, p)$ ) yields representative sets of samples  $((d, p); S)$  describing the behavior of the approximated function on the set  $M$ .

Consider an illustrative example: implementing a family of generalized step functions (7) in the airborne computer. The term “reference terrain map” is common in the literature devoted to CENSs. The current example involves a cartographic interpretation of the concepts of the proposed mathematical model, which is justified by history: the compilation and application of maps by humans for terrain orientation underlie the development of CENSs [5, 7–9].

Figure 1 shows the operating principle of a traditional CENS supplemented with a module for the approximate preliminary estimation of the object’s coordinates. This module narrows the search area for the exact module by approximately three times. Note that the main purpose of the approximate estimation modules is not just to narrow the search area for exact algorithms in the coordinate space. Such modules are universal aggregators, bringing the uncertainty to a form for which the aggregated exact modules are effective. For example, let the terrain in the correction area be such that the function used in the exact coordinate determination module to compare the current image with fragments of the reference image has several close-value optima. Then the module will not solve the problem in the zone  $E$ . If the zoning procedure gives a unique optimum in each subzone  $E1, E2$ , and  $E3$ , and the aggregator correctly determines the one with the shot, then the problem will be solved. These changes in the onboard algorithms of CENSs generate new requirements to reference images for them. Traditional

reference images are analytical maps of geophysical fields. They are compiled for areas covering the boundaries of all landscape images possibly obtained by the shooting system during the next session of refinement and correction of aircraft motion parameters considering their deviations from the planned values. In contrast, reference images for additional hierarchical search modules of next-generation CENSs must be compiled for areas of probable locations of the aircraft (search zones) rather than the boundaries of images; they must be special synthetic maps describing the typology of landscapes in the terrain sectors that can get into the shot in the search area with the sensor

trigger. Such maps are search area zoning maps on which one area contains

the aircraft locations to which all shots correspond (survey areas) with the same-type landscape detected by the landscape characteristics automatically extracted from the image data received by the CENS sensors during a correction session. The information content is determined by the type of sensors and onboard preliminary image and pattern recognition algorithms.

Changes in the reference image content are shown in Figs. 2 and 3 on the example of a two-level CENS (see formula (7)). In this system, a simple description of the scene on the current image is formed during the

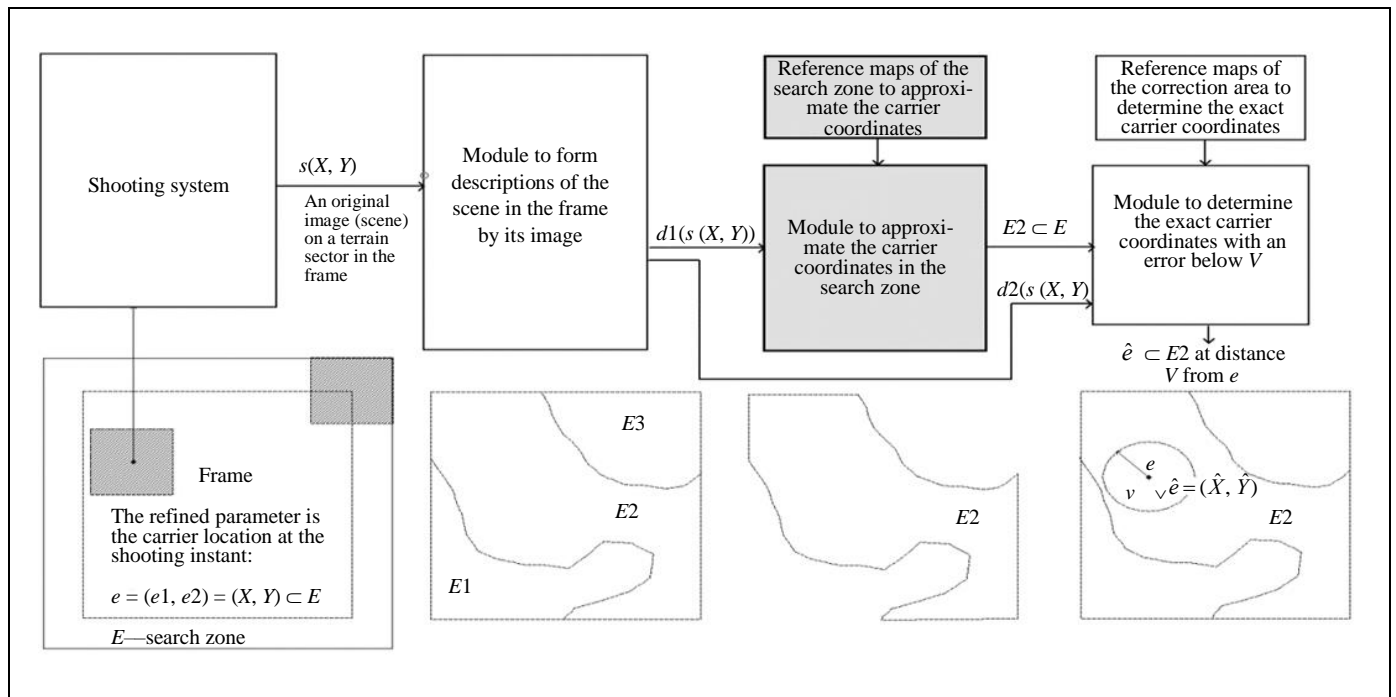


Fig. 1. Two-level onboard algorithm for CENS ( $r = 2$ ).

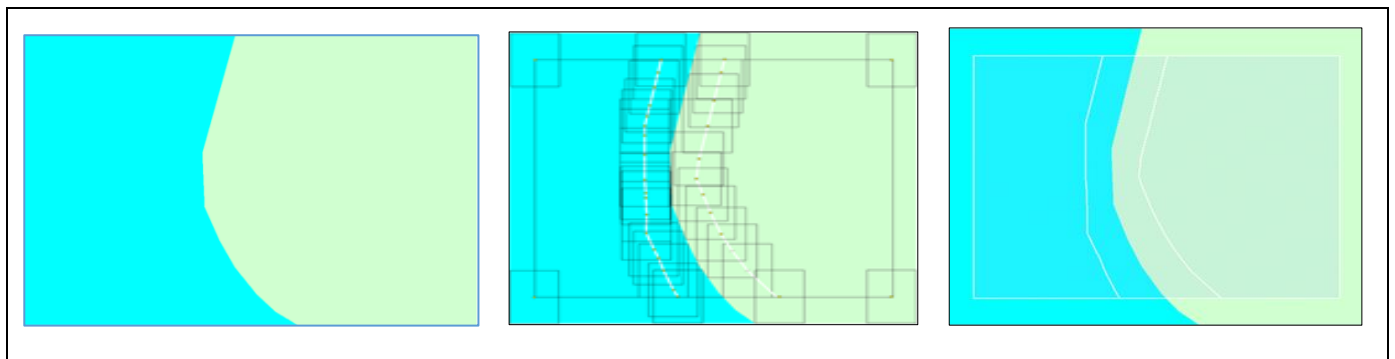


Fig. 2. Correction area, shooting sectors (shots), and search area.

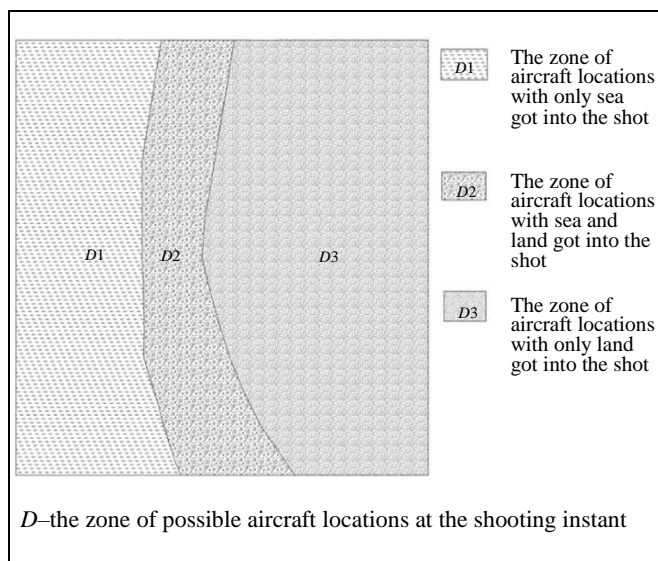


Fig. 3. A synthetic reference map of search area zoning.

preliminary transformation and extraction of two binary features,  $P1$  and  $P2$ . The feature  $P1$  takes value 1 if a hydrographic object (a morphological terrain element) is found on the image, and 0 otherwise. The feature  $P2$  takes value 1 if a land object is found on the image, and 0 otherwise.

Such scene descriptions are called morphological landmarks [4]. Synthetic reference maps with the boundaries of the zones  $D1$ – $D3$  and three references  $(0,1)$ ,  $(1,0)$ ,  $(1,1)$  have to be prepared in advance [4]. We emphasize that this elementary example illustrates the operating principle of preliminary estimation modules: an advance controlled clustering of the set of possible images, consistent with the zoning of the set of possible locations, and application of pattern recognition algorithms (attributing the input image to one of the pre-fixed classes according to their description in the language of features).

A feature space is any of those tested in a large recognition practice. A particular case is structural (morphological, syntactic) feature spaces, e.g., considered above, or mixed ones (a vector of the areas occupied by morphological terrain elements in the frame, in percentage); see [4, 5, 9].

In the general case, morphological descriptions can be built considering the set of terrain objects. The set of morphological landmark objects is limited, on the one hand, by the possibility of obtaining actual information about the boundaries of their distribution, and

on the other hand, by the possibility of automatic detection of these objects in an acceptable time by the pre-processing module of the onboard algorithm during the activation period.

Another direction of detailing morphological descriptions is to study relations and connections between objects in the scene. In this case, it is possible to consider the terrain specifics more fully when solving the CENS task. Besides the information about the boundaries of objects distribution, the information about the relations between them should be taken into account; the scene description module should be adjusted to recognize these relations and connections automatically during the activation session of the CENS. Some examples are the nesting relation (one object is located inside another object) or the order relation (the sequence of the objects when viewing the scene by given rules) [10–12].

If disturbing parameters significantly affect the scenes and their images, a series of reference maps of zoning by distribution areas of morphological landmarks may be required for certain values of these parameters. For example, let the disturbing parameter be the shooting altitude. Then the series corresponds to a specially selected set of altitudes: a multiscale zoning map by distribution areas of morphological landmarks is needed.

## 2. MATHEMATICAL MODEL OF STRESS EXPOSURES

**Problem statement.** The physical field brightness distribution  $a_{ij}$  in the test area of a CENS is known. There are  $N$  means of stress exposure. Let these means be arranged as follows: means  $k$  is placed in a point with pixel coordinates  $(x_k, y_k)$ ,  $k = 1, \dots, N$ . It is required to calculate automatically the optimal power  $A_k$  of each means of stress exposure minimizing the correlation between the current image and the stress-induced one. In the elementary case, the correlation is calculated on the window  $1 \leq i \leq Mx$ ,  $1 \leq j \leq My$ .

**Problem solution.** With this arrangement of the means of stress exposure, the field brightness at a point  $(i, j)$  is given by

$$a_{ij} + \sum_{k=1}^N A_k \delta_k(i - x_k, j - y_k). \quad (9)$$

The correlation between these images has the form

$$C = \frac{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2} \sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2}}. \quad (10)$$

To solve the problem, we find the vector of amplitudes  $(A_1, \dots, A_N)$  minimizing the correlation value  $C$ . According to the first-order optimality condition, all partial derivatives of  $C$  with respect to  $A_k$  must be equal to 0. We calculate the partial derivatives:

$$\begin{aligned} \frac{\partial C}{\partial A_s} &= \frac{1}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2}} \left\{ \frac{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s)}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2}} - \right. \\ &\quad \left. \frac{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \right)}{\left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 \right)^{\frac{3}{2}}} \right\} = \\ &\quad \frac{1}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2} \sqrt{\left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 \right)^{\frac{3}{2}}}} \times \\ &\quad \left\{ \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 - \right. \\ &\quad \left. \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \right) \right\} = \\ &\quad \frac{1}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2} \sqrt{\left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 \right)^{\frac{3}{2}}}} \times \\ &\quad \left\{ \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2 + 2 \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) + \right. \\ &\quad \left. \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 - \right. \\ &\quad \left. \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2 \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) - \right. \end{aligned}$$





$$\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) - \left. \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \right\} =$$

(Author’s note: *the underlined terms are mutually reduced; for  $A_k = 0$ , all derivatives vanish, which corresponds to the correlation maximum*)

$$\frac{1}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2} \sqrt{\left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 \right)^3}} \times \left\{ 2 \sum_{k=1}^N A_k \left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_k(i-x_k, j-y_k) \right) + \sum_{k=1}^N A_k \left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2 \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \delta_s(i-x_s, j-y_s) \delta_k(i-x_k, j-y_k) \right) - \sum_{k=1}^N A_k \left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_k(i-x_k, j-y_k) \right) + \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 - \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \right\} =$$

$$\frac{1}{\sqrt{\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2} \sqrt{\left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} + \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 \right)^3}} \times \left\{ \sum_{k=1}^N A_k \left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_k(i-x_k, j-y_k) \right) + \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right)^2 - \sum_{k=1}^N A_k \left( \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij}^2 \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \delta_s(i-x_s, j-y_s) \delta_k(i-x_k, j-y_k) \right) - \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( a_{ij} \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \left( \delta_s(i-x_s, j-y_s) \sum_{k=1}^N A_k \delta_k(i-x_k, j-y_k) \right) \right\}.$$

Thus, the optimal amplitudes satisfy a system of  $N$  quadratic equations with  $N$  unknowns. The factor at  $A_k^2$  is

$$\sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_s(i-x_s, j-y_s) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} \delta_k(i-x_k, j-y_k)^2 - \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} a_{ij} \delta_k(i-x_k, j-y_k) \sum_{i=1}^{M_x} \sum_{j=1}^{M_y} (\delta_s(i-x_s, j-y_s) \delta_k(i-x_k, j-y_k)). \tag{11}$$

For  $s = k$ , we obtain  $A_s = 0$ . Thus, the derivative  $\frac{\partial C}{\partial A_s}$  depends on  $A_s$  linearly; for fixed values  $A_k$ ,  $k \neq s$ , the optimal value  $A_s$  is unique (11).

Due to the linear dependence, we can control the correlation  $C$ ; its minimum admissible value can be set in advance according to the frequency-metric properties of the testing field. Arranging the means of stress exposure with the amplitudes  $A_k$  at specific field points, we construct the vector of their optimal amplitudes minimizing the correlation  $C$ . The sufficient condition of the minimum correlation between the current image and the stress-induced one is determined by the vector  $A_k$  and its dimension corresponding to the number  $N$  of the means of stress exposure, depending on the field properties of the testing area.

The algorithm for finding the optimal parameters of the means of stress exposure is as follows:

1. Calculating the quadratic functions in the numerators of the partial derivatives.
2. Organizing a loop on convergence. In each loop:
  - 2.1. Organizing a loop on  $s$  from 1 to  $N$ . In each cycle:
    - 2.1.1 Calculating the unique values  $A_s$  for fixed values  $A_k$ ,  $k \neq s$ .

Each iteration minimizes the correlation by selecting one amplitude (the correlation decreases with each iteration). Thus, the monotonicity of the objective function ensures the convergence of this algorithm.

The points  $(i, j)$  are selected sequentially where the field brightness in a given wavelength range  $\lambda$  significantly exceeds the average field brightness of the test area. In each iteration cycle, the value  $A_k$  changes until reaching the minimum correlation between the current image and the stress-induced image on the testing area in a window with dimensions  $1 \leq i \leq Mx$ ,  $1 \leq j \leq My$ . A specific range of  $A_k$  is set for the selected image type to control loop calculations.

This problem is solved using an iterative process of generalized coordinate-wise descent. Each iteration involves:

- Minimizing the correlation by selecting the optimal amplitudes under the current arrangement of the means of stress exposure. This problem is now solved.
- Minimizing the correlation by choosing the optimal locations of the means of stress exposure

under their current amplitudes. This problem is also solved. However, its solution is omitted here.

The values  $A_k$  for particular means of stress exposure under given wavelength ranges of electromagnetic radiation are presented in special sources. As a rule, the means of stress exposure are calibrated by the shooting system of CENSs before the testing procedure.

When implementing this algorithm, each means of stress exposure should be placed according to the calculated coordinates. A necessary means of stress exposure for the environment of CENSs is selected depending on the amplitude at the point with the given coordinates. Due to the considerable volume of such studies, their results are not included in this paper. However, control of the means of stress exposure for CENSs is a significant problem that needs to be solved [13, 14].

The means of stress exposure for the shooting systems of CENSs should change the parameters of the Earth's physical fields in different wavelength ranges of electromagnetic radiation. As a rule, these are optical, thermal, radio-thermal, radar, geomagnetic, and other geospatial fields of the Earth. The mathematical model of stress exposures on CENSs should be considered when designing new navigation systems to create modern and high-efficient CENSs.

---

## CONCLUSIONS

---

A mathematical model of search algorithms—a parametric family of algorithms—for calculating generalized step functions has been obtained. This model includes an image matching model as a special case and describes a scheme to assemble a wider range of onboard algorithms of CENSs from the algorithms of feature extraction on images, scene description, and classification.

The problem of adjusting the onboard algorithm of a CENS to perform its task in a given area has been reduced to that of approximating a given function of images by generalized step functions in the space of the object's refined parameters. This problem is solved by training and self-training of the hierarchical classification of images.

An applied GIS for the developers of CENSs and technologies of their adjustment to perform their tasks in given areas (the AGIS CENS) accelerates the development of control systems for moving objects



equipped with CENSs. As shown by the analysis of the obtained mathematical model, the existing image matching modules (which estimate the motion parameters with the required accuracy by correlation methods) should be supplemented with preliminary estimation modules for motion parameters using hierarchical pattern recognition methods. At each iteration, such modules attribute images to one of several classes by solving nondegenerate pattern recognition problems. Image matching modules are applied at the last step. The additional modules are intended to reduce the level of uncertainty so that the image matching modules can effectively solve their problem with the required accuracy (which is impossible under the initial level of uncertainty). Hence, the software components of the AGIS CENS should include components implementing known algorithms for pattern recognition, feature extraction on images, and scene description.

A CENS cannot be adjusted to perform its task in a given area without computer simulation models of the shooting systems. Such models should synthesize images similar to shooting systems for all possible areas and conditions of their application. The software components implementing these models, geospatial data, and terrain models must be included in the AGIS CENS.

The adjustment problem of CENSs is reduced to nondegenerate pattern recognition problems. It involves hierarchical partitioning into classes and tuning of recognition algorithms consistent with the zoning of the application area using the entire arsenal of modern training and self-training tools of recognition systems, including neural networks. Therefore, the software components of the AGIS CENS should include libraries of components from the related R & D areas.

The set of views on synthesizing the conditions of application of CENSs in a variable environment with stress exposures forms an interconnected system. Without due consideration of this system, it is impossible to achieve the necessary effect of CENS application.

Further research can be focused on the following issues:

- Refining the approximation problem statement in terms of approximation criteria, requirements to the simulation models of shooting systems and their complexes, and in-depth mathematical study of this problem.

- Constructing a general scheme to assemble onboard algorithms of CENSs and procedures for their adjustment to perform their tasks in given areas based on the above expressions and schemes for training and self-training in pattern recognition and classification.

- Developing methods to construct hierarchical partitions of images into classes consistent with the zoning of the application area of CENSs.

- Developing the mathematical model of generalized step functions to clarify the parallel structure specifics of the onboard algorithms of CENSs and design special processors for airborne computers.

- Solving the problem of stress exposures for different types of CENSs using the physical fields of the Earth in different wavelength spectra of electromagnetic radiation.

## REFERENCES

1. Beloglazov, I.N., Dzhanzhgava, G.I., and Chigin, G.P., *Osnovy navigatsii po geofizicheskim polyam* (Fundamentals of Navigation by Geophysical Fields), Moscow: Nauka, 1985. (In Russian.)
2. Avgustov, L.I., Orientation by Geophysical Fields Provides Autonomous Navigation of a Combat Aircraft, *Kommersant-Science*, 2015, no. 2, pp. 34–35. (In Russian.)
3. Zhangcai, Y., A Multi-Scale GIS Database Model Based in Petri Net, *Proceedings of ISPRS Workshop on Service and Application of Spatial Data Infrastructure, XXXVI(4/W6)*, Hangzhou, China, 2005, pp. 271–275.
4. Goodfellow, I., Bengio, Y., and Courville, A., *Deep Learning*, Cambridge: MIT Press, 2016. <http://www.deeplearningbook.org/>
5. Ali, R.A. and Hardie, R.C., Recursive Non-local Means Filter for Video Denoising, *EURASIP Journal on Image and Video Processing*, 2017, article no. 29.
6. Almahdi, R.A. and Hardie, R.C., Recursive Non-local Means Filter for Video Denoising with Poisson-Gaussian Noise, *Proceedings of 2016 IEEE National Aerospace and Electronics Conference (NAECON) and Ohio Innovation Summit (OIS)*, Dayton, OH, USA, 2016, pp. 318–322.
7. Duda, R.O. and Hart, P.E., *Pattern Classification and Scene Analysis*, New York: John Wiley & Sons, 1973.
8. Alchinov, A.I., Beklemishev, N.D., and Kekelidze, V.B., *Metody tsifrovoi fotogrammetrii* (Methods of Digital Photogrammetry), Moscow: Tekhnologiya "Talka," 2007. (In Russian.)
9. Maiorov, A.A., Materukhin, A.V., and Kondaurov, I.N., The Structure of a Streaming Data Processing System in Geosensor Networks, *Izv. vuzov. Geodesy and Aerophotosurveying*, 2018, vol. 62, no. 6, pp. 712–719. (In Russian.)
10. Gorokhovskiy, I.N. and Balin, B.M., On the Computer-Aided Design and Investigation of a Family of Correlation-Extreme Navigation Systems, in *Avtomatizatsiya proektirovaniya i issledovaniya korrelyatsionno-ehkstremal'nykh system* (Computer-Aided Design and Investigation of Correlation-Extreme Sys-

- tems), Tarasenko, V.P., Ed., Tomsk: Tomsk State University, 1987. (In Russian.)
11. *Spravochnik po teorii avtomaticheskogo upravleniya* (Handbook on the Theory of Automatic Control), Krasovskii, A.A., Ed., Moscow: Nauka, 1987. (In Russian.)
  12. Syryamkin, V.I. and Shidlovsky, V.S., *Korrelatsionno-ekstremal'nye radionavigatsionnye sistemy* (Correlation-Extremal Direction-Finding Systems), Tomsk: Tomsk State University, 2010.
  13. Gaspar, J., Ferreira, R., Sebastião, P., and Souto, N., Capture of UAVs through GPS Spoofing, *Proceedings of 2018 Global Wireless Summit (GWS)*, Chiang Rai, Thailand, 2018, pp. 21–26. DOI: 10.1109/GWS.2018.8686727.
  14. Zhang, D., Zhou, X., Chang, E., Wa, H., and Chen, Y., Investigation on Effects of HP Pulse on UAV's Datalink, *IEEE Transactions on Electromagnetic Compatibility*, 2020, vol. 62, pp. 829–839.

*This paper was recommended for publication by B.V. Pavlov, a member of the Editorial Board.*

*Received October 5, 2021, and revised November 11, 2021.  
Accepted December 23, 2021.*

#### Author information

**Alchinov, Alexander Ivanovich.** Dr. Sci. (Eng.), Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia  
✉ alchinov46@mail.ru

**Gorokhovskiy, Igor Nikolaevich.** Dr. Sci. (Eng.), Research Center of Topographic and Navigational Support, Central Research Institute No. 27, Moscow, Russia  
✉ gin\_box@mail.ru

#### Cite this paper

Alchinov, A.I., Gorokhovskiy, I.N., A Conceptual Applied Geographic Information System for Modeling Search Autonomous Correlation-Extreme Navigation Systems. *Control Sciences* **1**, 43–54 (2022). <http://doi.org/10.25728/cs.2022.1.4>

Original Russian Text © Alchinov, A.I., Gorokhovskiy, I.N., 2022, published in *Problemy Upravleniya*, 2022, no. 1, pp. 54–66.

Translated into English by *Alexander Yu. Mazurov*,  
Cand. Sci. (Phys.–Math.),  
Trapeznikov Institute of Control Sciences,  
Russian Academy of Sciences, Moscow, Russia  
✉ alexander.mazurov08@gmail.com