DOI: http://doi.org/10.25728/cs.2021.1.6

TAX INCENTIVES FOR PROSOCIAL VOTING IN A STOCHASTIC ENVIRONMENT

V.A. Afonkin

Moscow Institute of Physics and Technology (National Research University), Dolgoprudny, Russia

✉ afonkinvadim@yandex.ru

Abstract. Three income redistribution algorithms supporting the agents with prosocial voting are considered within the Voting in Stochastic Environment (ViSE) model of social dynamics. The first algorithm is income tax; the second one ensures that the income of each agent with the prosocial strategy is not smaller than the average income; the third one ensures that the average income of prosocial agents is not smaller than that of the entire society. The social utility of prosocial voting is analyzed. The three algorithms are compared with each other. The effectiveness of income tax depends on the environment. The second and third algorithms do not suffer from this disadvantage. However, under certain conditions, the second algorithm provides too many bonuses to prosocial agents. With any of these income redistribution algorithms, the egoists get more profit than in a society without any prosocial agents. Thus, whenever such taxation schemes motivate some participants to choose the prosocial voting strategy, this will increase the expected income of all agents.

Keywords: ViSE model, altruism, voting, social dynamics, tax, pit of losses, prosocial behavior.

INTRODUCTION

Basic elements of ViSE model

In the ViSE (Voting in Stochastic Environment) model [1], a *society* consisting of *n agents* is considered. Each agent is characterized by a social attitude determining his voting strategy and current wealth (*capital*), expressed as a real number. A strategy is an algorithm for using information about a proposal and society to support (or not) the proposal put to the vote. A *stochastic environment* generates a proposal to society—a vector of realizations of independent identically distributed random variables. Each *i*th component of this vector is a *proposed capital increment* for agent *i*. The proposal is put to the vote; each agent votes for or against it, following his voting algorithm. If the number of votes “for” exceeds 50%, the proposal is approved, and the corresponding proposal components are added to the agents’ capital. (In a more general case, the number of votes “for” must exceed αn, where α is a *relative voting threshold* and n is the number of agents). Otherwise, the capital of the participants remains the same. Proposals are put to the vote consecutively; voting on one proposal is called a *move* or *step* in a sequence of decisions. In a series of votes, the parameters of the distribution generating the proposals and the voting strategies of the agents are fixed. In this paper, the Gaussian distribution is considered a generator of proposals. The research aims to analyze the effectiveness of the voting strategies of agents and collective decision procedures by the criteria for increasing the individual capitals of agents and their sum.

In the papers [1, 2] and other publications on the ViSE model, many of its variants were considered by imposing additional conditions. Specific features of the model related to the subject of this study will be discussed below.

The ViSE model refers to the theory of voting, which, in turn, is part of social choice. Unlike several game-theoretical models, agents in the ViSE model are not treated as players maximizing their utility functions. They possess capital, but their behavior is not always reduced to capital maximization: generally speaking, their behavior has an arbitrary structure given by the researcher. The main element of this behavior is the agent's personal (and not necessarily constant) voting algorithm. Society is characterized by a mechanism for making collective decisions. The researcher analyzing, within the model assumptions, the effectiveness of the individual and collective decision mechanisms is a social designer trying to understand which of the identified patterns may be useful in real life.

Pit of losses

For the ViSE model, the following scenario is known: a society consisting of agents with an egoistic social attitude acts irrationally, approving proposals that are generally disadvantageous to it since they lead to a negative total capital increment [2]. An *egoistic strategy* is a strategy in which the participant supports a proposal if and only if it increases his capital. The effect of impoverishment and ruin of society in this scenario is usually called *the pit of losses*.

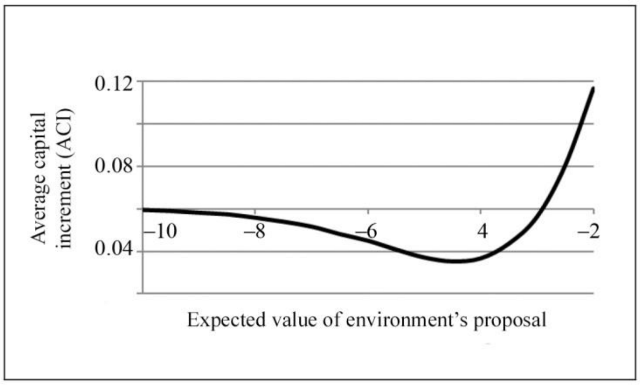

Fig. 1. Average capital increment in society of 25 agents with egoistic strategy.

This situation is illustrated in Fig. 1, where the vertical axis corresponds to the average capital increment (ACI) of the participant per one step in an unfavorable environment, and the horizontal axis corresponds to the expected value of the proposals. Throughout the paper, the components of an environment proposal are realizations of independent Gaussian random variables with σ = 12. In the situation under consideration, the "unbiased" proposals of the environment lead to the same result as the agenda manipulation in the Malishevskii paradox described, for example, in [3, pp. 92–95].

Indeed, the ruin of society due to implementing the decisions made by the majority of votes of its classically rational participants is, in some sense, a paradoxical effect. It has the following explanation: in the zone of moderately negative expected values, some proposals yield a small capital increase for most agents and a total decrease by absolute value for the rest of society. Such proposals are approved by a majority of votes, but the total welfare of society decreases during their implementation.

A way to secure against the pit of losses is to select the best voting threshold α [2]. In the case of an unfavorable environment, this optimum is usually above 50%. The corresponding dependence of the total capital on the voting threshold is comparable to the results of [4], where the influence of other social mechanisms (bargaining, bribes) on the effectiveness of decisions determining social dynamics was studied.

The influence of altruistic agents on social welfare

Another factor reducing the pit of losses is the presence in society of agents who, when voting, are guided not by personal interests but by those of the entire society. An agent's strategy supporting a proposal if and only if it increases the total welfare of society is called *altruistic*. Behavior that benefits society is also called *prosocial*. In the case shown in Fig. 1, replacing three egoists with altruistic participants appreciably increases the average capital increment of society; see Fig. 2.

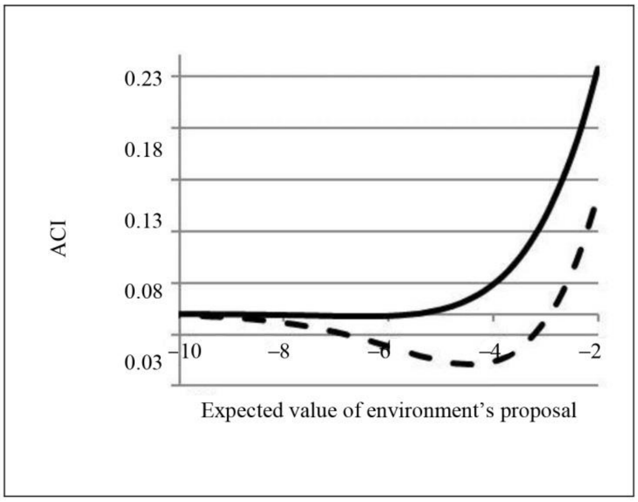

Fig. 2. Comparing effectiveness of societies composed of egoists only and egoists plus altruists:
——— 22 egoists and 3 altruists, – – – 25 egoists.

The results presented in this paper were obtained by simulations using *ViSE Experiment Module* [5]. Obtaining the same results analytically is a problem of at least high complexity: for the corresponding multiple integrals it is impossible to find a general expression in terms of standard functions.

The presence of altruistic agents always positively affects social welfare: the share of socially irrational decisions made is significantly reduced. However, the altruists themselves are outsiders in this case: according to Fig. 3, their capital is considerably smaller than the society average. This pattern is also observed for other values of the parameters.

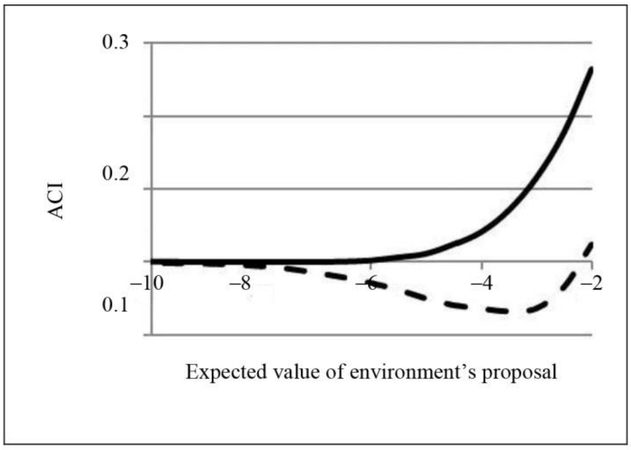

Fig. 3. Average capital increment in society of 22 egoists and 3 altrusists:
——— egoists, — — — altruists.

This is because when voting, altruists neglect the change in their capital in the case of implementing the environment's proposal. Therefore, they support, among others, the proposals enriching society as a whole but reducing their capital. Thus, the presence of altruistic agents is beneficial to society, but their role is "sacrificial."

In real life, a society not encouraging the participants who secure it against ruin seems unfair. Moreover, in a version of the model where participants can change their voting principle to an individually more profitable one, providing prosocial agents with an income of at least the average will ensure that they do not change their strategy to selfish. If the position of prosocial agents becomes better than the society average, then the share of such agents in this version will grow, leading to an increase in social welfare.

Within the ViSE model, there is no need to represent any (particularly, prosocial) behavior by maximizing the agent's utility function: this would complicate the description of complex behavioral types common in real life. For example, a social attitude aimed at supporting the entire society is conditioned by philanthropy, the need to maintain reputation, etc., but such motivation can fade into the background under serious material losses and then return without external reasons.

The goal of this study is to analyze material support mechanisms for agents with prosocial strategies. The agents are allowed to change their voting strategies to individually more profitable ones, but specific mechanisms for such a change are not considered: this is not required to achieve the goal. The paper proposes and investigates several algorithms for supporting prosocial agents based on income redistribution (in other words, prosocial voting is motivated by taxes.) Also, the paper investigates the effectiveness of the prosocial strategy under various parameters of the environment.

Voting in a society of altruistic agents was studied in the earlier paper [6]. Like in the model considered below, the agent was assumed to maximize the welfare function during voting, the value of which monotonically increases with the growth of consumption of any agent (analogy of capital increase). The progressive taxation schemes quadratically dependent on production were put to the vote, and the presence of "self-approving" equilibrium was established.

In the paper [7], as a result of laboratory experiments, it was found that monetary incentives motivate prosocial behavior in the case of its private (non-public) nature. The work [8] examined reducing internal motivation for prosocial behavior with its monetary incentives on an example of "green" (environmental) taxes. The authors concluded the following: if a tax leads to positive changes in society, its introduction is justified even under decreasing the "moral" motivation.

Choosing an appropriate taxation scheme for agents by the majority voting was studied in [9]. The main result was the conclusion that progressive tax is beneficial to the "middle class." Also, choosing a linear income tax by voting was considered in [10].

1. EFFECTIVENESS OF ALTRUISTIC STRATEGY AS FUNCTION OF ENVIRONMENT'S FAVORABLENESS

As noted above, the presence of a small share of altruistic agents in society can significantly reduce or even eliminate the pit of losses. Let us identify the environment's parameters under which the presence of altruists increases the capital of society most of all. To do this, we compare the average capital increments for the societies consisting of 25 egoists and 22 egoists plus 3 altruists under $\sigma = 12$ and different expected values of the environment's proposals. The comparison results are demonstrated in Fig. 4, where "the benefit from the altruistic strategy" is the difference between the ACIs of the two societies mentioned.

As Fig. 4, agents with the altruistic strategy bring maximum benefits to society in a *neutral* environment that generates positive and negative proposals with equal probability. In an unfavorable environment most dangerous for society (the "bottom" of the pit of losses), the help of the three altruists is less in absolute terms. At the same time, it is enough to eliminate the pit of losses almost completely.

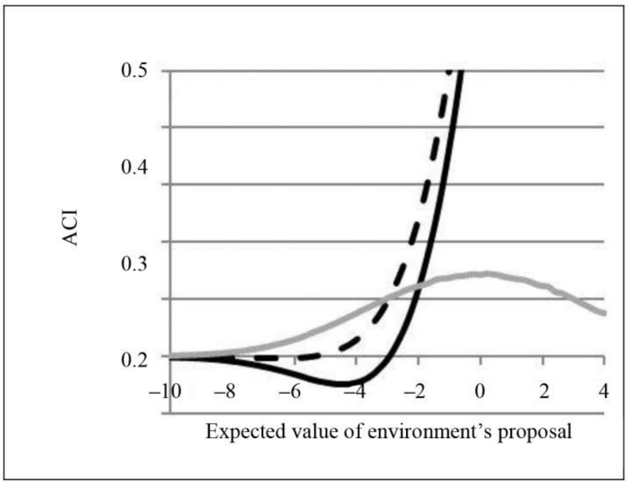

Fig. 4. Comparing average capital increments and their difference for societies of egoists only and egoists plus altruists:
25 egoists, 22 egoists and 3 altruists,
benefit from altruistic strategy.

2. TAX INCENTIVES FOR ALTRUISTIC VOTING

As noted above, altruistic agents helping society make rational decisions in a moderately unfavorable environment need support to increase the relative welfare of altruists and, presumably, prevent their shift to egoism. Consider possible schemes for redistributing society's income in their favor. This redistribution can be treated as levying a tax. The simplest scheme is "flat" income tax. After each approved proposal, the egoists who have received a positive capital increment deduct v percent of their capital increment for the current step to the fund, and the fund is equally divided among the agents with the altruistic voting strategy. The effectiveness of a support method will be assessed by the increase in the altruist's ACI after introducing the tax and by the increase in the egoist's ACI compared to his increase in a society consisting of such agents only.

Figure 5 shows the average capital increments of different participants under 13% income tax. As before, society consists of 22 egoists and 3 altruists (12% of society). An income tax rate of 13% (further called *the first income redistribution algorithm* or *the first taxation scheme*) is applied. Clearly, due to the redistribution, the income of altruists *significantly* exceeds that of egoists. At the same time, the welfare of egoists remains higher than in the society without altruists. Thus, the agents voting altruistically do good to the entire society, becoming the main beneficiaries (the wealthy stratum): being altruistic[1] is very advantageous. If agents are allowed to change their strategy, then the egoists will be willing to vote altruistically to turn from taxpayers to tax fund recipients.

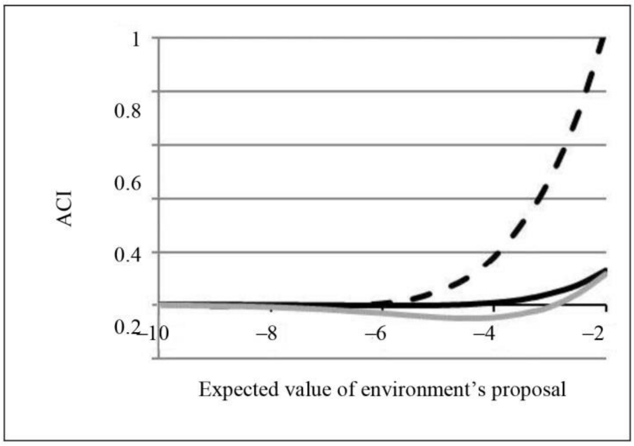

Fig. 5. Comparing average capital increments for different participants under 13% income tax:
altruists (society of 22 egoists and 3 altruists),
egoists (society of 22 egoists and 3 altruists),
egoists (society of 25 egoists).

In this regard, note that the difference between the welfare of egoists and altruists depends on the ratio of the number of agents with different strategies. The more altruists there are in the society, the less increase each of them will receive from the tax fund. As a result, the ACI curves of the two groups of participants will converge and finally match. The difference in the income of the different groups also depends on the environment's favorableness. For these reasons, it is natural to select the income tax rate depending on the parameters of society and the environment. A fixed rate can lead to insufficient or, conversely, excessive support for altruists. For example, in Fig. 5, there is an income gap that is difficult to justify.

Thus, an additional criterion for assessing support methods can be the dependence of the tax effect on the environment's parameters. The problem described above can be solved by more flexible taxation schemes. Here is one example, also called *the second income redistribution algorithm*, or *the second taxation scheme*.

After each approved proposal, calculate the difference ($\bar{c}$ – ACI) for the participant at the current step.

Calculate the sum of all positive excesses above the ACI over society: $S_{exc} = \sum_{i=1}^{n} I\{c_i > \bar{c}\} \cdot (c_i - \bar{c})$,

where c_i is the capital increment of agent i at the current step, and $I\{\cdot\}$ denotes the indicator function of an appropriate event. This function takes value 1 if the assertion within the curly brackets is true and 0 otherwise.

[1] In some cases, the term "altruists" is enclosed in quotation marks to emphasize that it refers to participants with the altruistic voting strategy. In view of the social support under consideration, the motivation for choosing this strategy can be mercantile, that is, selfish. The term "egoists" also refers only to the voting strategy.

Calculate the amount donated to the altruists for making their capital increments not smaller than the society average: $S_{don} = \sum_{i=1}^{n} altr(i) I\{c_i < \bar{c}\} \cdot \{\bar{c} - c_i\}$, where $Altr(i) = 1$ if agent i is altruist and $Altr(i) = 0$ otherwise.

Calculate *the income withdrawal rate* $u = \frac{S_{don}}{S_{exc}}$.

Charge the tax $(c_i - \bar{c})u$ from each agent i whose capital increment is greater than $\bar{c}$.

Redistribute the tax fund collected at this step among the altruists whose capital increments are smaller than $\bar{c}$, making them equal to $\bar{c}$.

Note that the coefficient u cannot exceed 1: the sum S_{exc} includes *all* excessive incomes, and S_{don} is the total income deficit (in relation to the average) only for the altruists who are proposed the capital increments smaller than the society average. Therefore, the income of the "lucky ones" with $c_i > \bar{c}$ cannot fall below the value $\bar{c}$.

Well, the second taxation scheme ensures that every altruist will obtain a capital increment of at least the society average from each proposal. Moreover, the tax is paid not only by egoists but also by altruists, who initially obtained a capital increment above the average. The expected capital increments of different participants under the second taxation scheme are shown in Fig. 6.

We emphasize that tax collection determines only the redistribution of capital within society: the decision-making process remains the same, and the taxes, therefore, do not affect the average capital of society. According to Fig. 6, the incomes of altruists, like in the case of the first taxation scheme, appreciably exceed those of egoists. The difference from the first taxation scheme is that in an unfavorable environment, the income of altruists turns out to be even higher. In fact, depending on the number of participants with the altruistic voting strategy, they can obtain either more or less income under the second taxation scheme compared to the first one (income tax).

Now consider *the third income redistribution algorithm* (*the third taxation scheme*), intended to reduce the gap between the incomes of altruists and egoists. This income is collected and redistributed as follows:

After each proposal approved, calculate the difference ($\bar{c}$ – ACI) of the participant at the current step and the ACIs $\bar{c}_{altr}$ of altruists. If $\bar{c}_{altr} \geq \bar{c}$, implement the current proposal without any changes; otherwise, pass to Step 2.

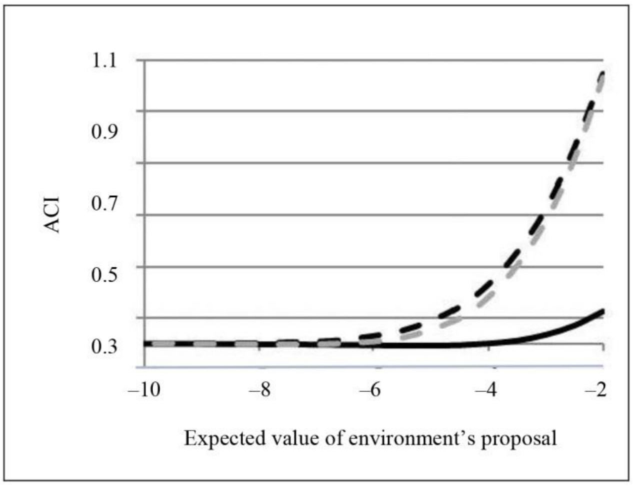

Fig. 6. Comparing average capital increments of different participants under different taxation schemes:
egoists (second taxation scheme), altruists (second taxation scheme), altruists (first taxation scheme).

Calculate the sum of all positive excesses above the average capital increment: $S_{exc} = \sum_{i=1}^{n} I\{c_i > \bar{c}\} \times$ $\times (c_i - \bar{c})$ by analogy with the second taxation scheme.

Calculate the amount donated to the altruists for making their capital increments not smaller than average over society: $S_{don} = \sum_{i=1}^{n} altr(i) I\{c_i < \bar{c}\} \cdot \{\bar{c} - c_i\}$, similar to the second taxation scheme.

Calculate *the income withdrawal rate* $\tilde{u} = \frac{(\bar{c} - \bar{c}_{altr}) n_{altr}}{S_{exc}}$, which ensures the altruists the average capital increment over society.

Calculate *the raise rate* $q = \frac{(\bar{c} - \bar{c}_{altr}) n_{altr}}{S_{don}}$.

Charge the tax $(c_i - \bar{c})\tilde{u}$ from each agent i whose capital increment is greater than $\bar{c}$.

For each altruist whose initial capital increment at this step is lower than the average one $\bar{c}$, pay the extra amount $(c_i - \bar{c})q$ from the tax fund.

This algorithm guarantees that at each step, the *average* capital increment of altruists is not smaller than $\bar{c}$ (the average capital increment in society). If this increment is initially smaller, then it is raised to the society average by payments from the tax fund; otherwise, it remains unchanged.

According to Fig. 7, the ACI of altruists under the third taxation scheme is appreciably smaller compared to the second one. The excess of the altruist's income over that of the egoist is also less.

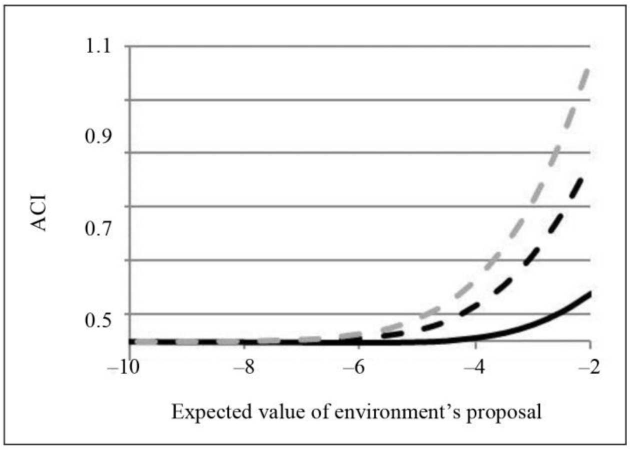

Fig. 7. Comparing average capital increments of different participants under different taxation schemes:
egoists (third taxation scheme), altruists (third taxation scheme), altruists (second taxation scheme).

We explain this pattern as follows. Suppose that the altruists obtained increments on average greater than the egoists per step. Then the third taxation scheme is not applied. At the same time, the second taxation scheme would provide a positive increase for those altruists whose initial income was below average. If a capital increment above $\bar{c}$ was only for the altruists, then the total income of the altruists under the second taxation scheme would not change; otherwise, it would increase due to egoists and become higher compared to the third taxation scheme.

Now consider the case in which the altruists obtained, on average, a smaller capital increase per step than the egoists. Under the third taxation scheme, after the redistribution of income, the ACIs of altruists and egoists will be equal to each other and the value $\bar{c}$. Under the second taxation scheme, every altruist who originally had a capital increment below $\bar{c}$ will receive an increment equal to $\bar{c}$. The increase in the altruist's capital, which initially exceeded the value $\bar{c}$, will remain above $\bar{c}$. Therefore, the average total capital increment of altruists under the second taxation scheme in each case will be not smaller compared to the third one. Due to the stochastic nature of proposals, proposals will be occasionally approved with probability 1, in which the second taxation scheme will provide altruists with a greater capital increase than the third one.

The above reasoning proves that the expected capital increment of altruists under the third taxation scheme (and non-zero variance σ^2) is always lower compared to the second one.

The results for all societies considered are summarized in Fig. 8.

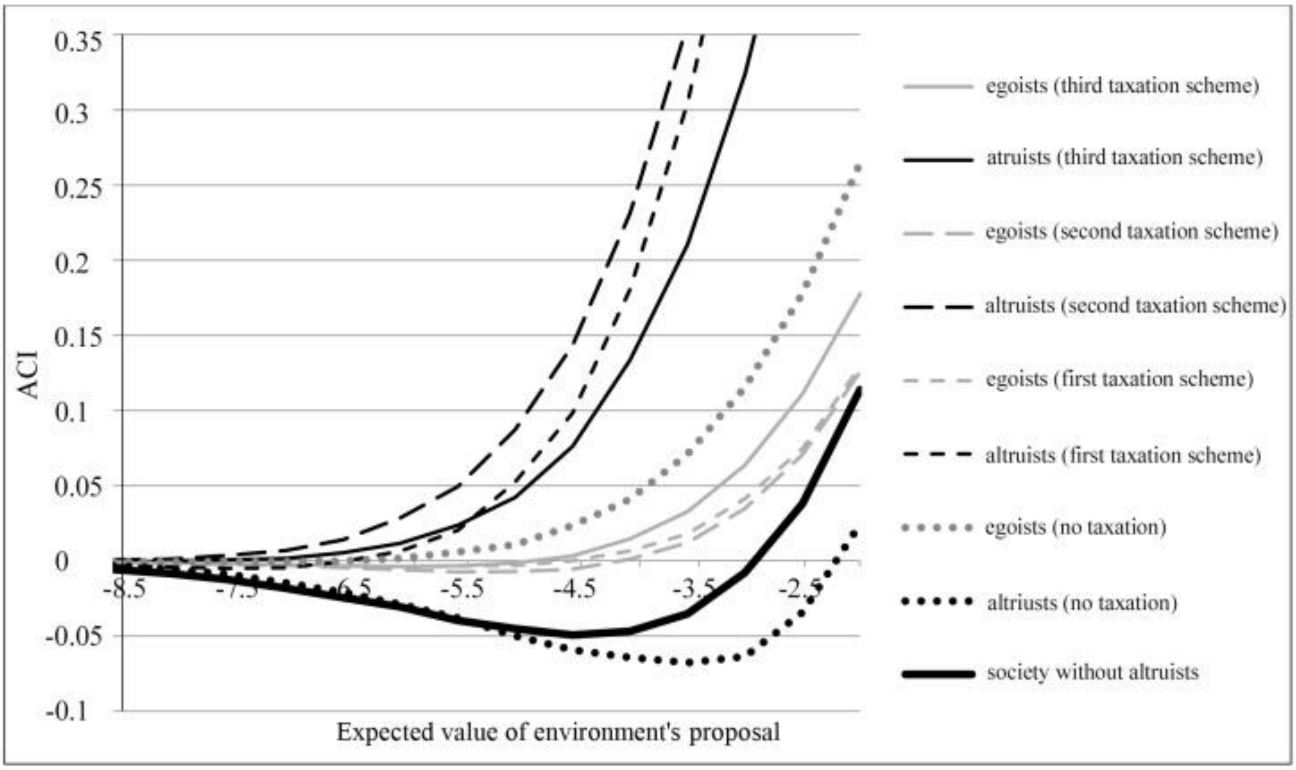

Fig. 8. Summarized data on average capital increments of different participants under different taxation schemes.

CONCLUSIONS

This paper has proposed and investigated some ways to support agents with prosocial behavior within the ViSE model. It has been established that altruists increase the capital of society, which helps to eliminate the pit-of-losses paradox. Without income redistribution, the welfare of prosocial agents is significantly smaller than that of egoistic ones. Hence, by a logical assumption, such agents would think about changing their strategy, thereby worsening society's state. Three income redistribution algorithms have been considered: income tax (the first taxation scheme), a tax with "pulling" at each step *each altruist's income* to the society average (the second taxation scheme), and a tax with "pulling" *the average income of all altruists* to the society average (the third taxation scheme). The application of each taxation scheme mentioned provides altruists with a greater average capital than egoists, which creates a material incentive for them to choose the altruistic voting beneficial for society. In this case, the benefit of society is that all participants, both egoists and altruists, obtain a greater average capital than in a society without altruists.

The problem of excessive bonuses to altruists may arise. Of the approaches considered, the third taxation scheme best secures against it, rewarding altruists on average in a smaller volume than the second taxation scheme. The consequences of introducing a flat-rate income tax (the first taxation scheme) strongly depend on the environment's favorableness and the share of altruistic agents, which indicates its inflexibility. At the same time, the administration of the second and third taxation schemes requires complete information on the income of participants and more complex calculations, which makes these taxes less transparent and somewhat complicates their practical application. In all income redistribution algorithms considered, egoists obtain a higher income than in a society without prosocial agents, which makes the appearance of altruists supported by tax attractive, particularly for egoists.

In the paper [11], an optimal taxation scheme was intended to ensure social welfare by maximizing the total utility function of society. In this paper, another criterion of tax optimality has been proposed and investigated: the degree of support for the agents whose strategy contributes to increasing social welfare. The patterns identified during this study can be used to develop real taxation algorithms.

REFERENCES

1. Chebotarev, P.Yu., Loginov, A.K., Tsodikova Ya.Yu., et al., Analysis of Collectivism and Egoism Phenomena within the Context of Social Welfare, *Automation and Remote Control*, 2010, vol. 71, no. 6, pp. 1196–1207.
2. Chebotarev, P.Yu., Malyshev, V.A., Tsodikova, Ya.Yu., Loginov, A.K., Lezina, Z.M., and Afonkin, V.A., The Optimal Majority Threshold as a Function of the Variation Coefficient of the Environment, *Automation and Remote Control*, 2018, vol. 79, no. 4, pp. 725–736.
3. Mirkin, B.G., *Group Choice*, Fishburn, P.C., Ed., New York: Wiley, 1979.
4. Gomes, A. and Jehiel, P., Dynamic Processes of Social and Economic Interactions: On the Persistence of Inefficiencies, *J. Polit. Econom.*, 2005, vol. 113, pp. 626–667.
5. Afonkin, V.A., VISE Experiment Module, *Certificate of State Registration of the Computer Program no. 2018615940 RF*. Registered May 18, 2018.
6. Kranich, L., Altruism and the Political Economy of Income Taxation, *Journal of Public Economic Theory*, 2001, vol. 3 (4), pp. 455–469.
7. Ariely, D., Bracha, A., and Meier, S., Doing Good or Doing Well? Image Motivation and Monetary Incentives in Behaving Prosocially, *American Economic Review*, 2009, vol. 99, pp. 545–555.
8. Nyborg, K., Will Green Taxes Undermine Moral Motivation, *Public Financ. Manag.*, 2010, vol. 110 (2), pp. 331–351.
9. Snyder, J.M. and Kramer, G.H., Fairness, Self-Interest, and the Politics of the Progressive Income Tax, *Journal of Public Economics*, 1988, vol. 36 (2), pp. 197–230.
10. Roberts, K., Voting over Income Tax Schedules, *J. Public Econom.*, 1977, vol. 8, pp. 329–340.
11. Berliant, M. and Gouveia, M., On the Political Economy of Income Taxation, *MPRA Paper No. 92528*, 2019. https://mpra.ub.uni-muenchen.de/92528/1/MPRA_paper_92528.pdf.

This paper was recommended for publication by F.T. Aleskerov, a member of the Editorial Board.

Received April 10, 2019, and revised November 16, 2020.
Accepted November 16, 2020.

Author information

Afonkin, Vadim Aleksandrovich. Postgraduate, Moscow Institute of Physics and Technology, Dolgoprudny,
✉ afonkinvadim@yandex.ru

Cite this article

Afonkin, V.A. Tax Incentives for Prosocial Voting in a Stochastic Environment. *Control Sciences* **1**, 53–59 (2021). http://doi.org/10.25728/cs.2021.1.6