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DIFFERENTIAL GAMES OF PURSUIT WITH SEVERAL PURSUERS AND ONE EVADER[1]

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Аннотация. A differential game of several players is considered as follows. One player (attacker) penetrates some space, and several other players (pursuers) appear simultaneously to intercept the attacker. Upon detecting the pursuers, the attacker tries to evade them. The dynamics of each player are described by a time-invariant linear system of a general type with scalar control. A quadratic functional is introduced, and the differential game is treated as an optimal control problem. Two subproblems are solved as follows. The first subproblem is to construct a strategy for pursuing the attacker by several players having complete equal information about the game. The second subproblem is to construct such a strategy under incomplete information about the attacker actively opposing the pursuers. The simulation results are presented. The zero-sum differential game solution can be used for studying the final stage of pursuit, in which several pursuers and one evader participate.

Ключевые слова: differential games, linear dynamics, optimal feedback control, Nash equilibrium, Lyapunov functions, Riccati equation.

INTRODUCTION

The theory of differential games as a branch of mathematical control theory is closely related to the mathematical theory of optimal processes, game theory, calculus of variations, and the theory of differential equations. Problems of the theory of differential games stem from many topical applications, such as the pursuit of one controlled object by another, bringing a controlled object into a given state under unknown disturbances, and military or economic problems, to name a few. The formation of the theory of differential games is associated with R.P. Isaacs [1, 2], J.V. Breakwell [3], L.S. Pontryagin [4, 5], E.F. Mishchenko [6], B.N. Pshenichny [7], and many other foreign and Soviet scientists. Since the late 1970s, an independent area in the applied theory of differential games has appeared, dealing with the problems of pursuit, evasion, and target defense [8–19]. In the works by L.S. Pontryagin and E.F. Mishchenko [4–6], sufficient conditions for completing pursuit in linear differential games were established. In the research of N.N. Krasovskii, A.I. Subbotin [8], their students and colleagues, positional differential games were studied; for this class of games, the problems of approach and evasion were formulated, and control procedures implemented on a computer were proposed. The development of differential games theory with application to conflict-controlled systems by the 1990s was summarized by L.A. Petrosyan in his book [9]. The theory of differential games as applied to pursuit problems significantly evolved thanks to A.A. Melikyan, L.S. Vishnevetsky, N.V. Ovakimyan [10–13], and V.S. Patsko and S.S. Kumkov [14, 15]. At the 18th and 19th IFAC World Congresses, there were separate sections devoted to the theory of differential games and the practice of applying this theory to control problems in conflict states [15–19].

This paper considers a differential game with several players. One player (attacker) penetrates some space, and several other players (pursuers) appear simultaneously to intercept the attacker. Upon detecting the pursuers, the attacker tries to evade them. The dynamics of each player are described by a time-

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invariant linear system of a general type with scalar control. Note that this formulation of the game-theoretic problem is quite popular. For example, in the papers [19, 20], distributed game strategies for similar problems were developed and analyzed. The proposed solutions were based on the integration of cooperative control theory and differential game theory. As demonstrated therein, the proposed non-zero-sum game strategies are the Nash solution in terms of functionals (performance criteria) introduced to assess the players' actions. In this paper, a quadratic performance criterion is introduced, and the differential game is treated as an optimal control problem [21], i.e., a zero-sum differential game. Two subproblems are solved as follows. The first subproblem is to construct a strategy for pursuing the attacker by several players who have complete equal information about the game. The second subproblem is to construct such a strategy under incomplete information about the attacker who is actively opposing the pursuers. The simulation results are presented. The zero-sum differential game solution can be used for studying the final stage of pursuit, in which several pursuers and one evader participate.

This paper is organized as follows. Section 1 formulates the problem in which there are several pursuers and one attacker. The pursuers try to intercept the attacker, and the attacker tries to evade them. Each player can detect other players within its radius of sensitivity. Therefore, the game is a game with distributed information. Assumptions are made to exclude the cases when the attacker observes no pursuers or each pursuer observes no objects within its radius of sensitivity.

A common performance criterion is introduced in the zero-sum game to assess the actions of the pursuers and the attacker evading them. The pursuers seek to minimize this criterion, whereas the evading attacker to maximize it.

Section 2 considers the classical differential game with global information. The outcome of this game is based on optimal control theory. A theorem on the existence of solutions of the zero-sum differential game is proved. Also, Section 2 considers the differential game with distributed information.

Section 3 deals with a situation when the evading attacker artificially jams the pursuers to gain an advantage in the game. This means that the pursuers will receive information about the evader's position with some noise. Hence, the controls constructed by the pursuers will contain this noise. Thus, the trajectories along which the pursuers will intercept the evader are suboptimal. In addition, the attacker constructs its strategy for all pursuers detected, trying to escape the center of mass of all pursuers. Since their positions are subjected to noise, the attacker's trajectory will also contain a noise component.

Section 4 presents the simulation results for the differential game of pursuit in various statements considered in the previous sections.

1. PROBLEM STATEMENT

In the problem under consideration, the number of players is (n+ 1), namely, n pursuers and one attacker evading the pursuers. Each player can detect other players in its radius of sensitivity. Thus, the game is a game with distributed information. Let us make some assumptions.

Assumption 2.1. The observation between any pursuer–attacker pair is mutual, whereas the observation between two pursuers is not necessarily mutual.

Assumption 2.2. There exists at least one pursuer–attacker pair in which each member observes the other member, and each pursuer observes at least one other pursuer.

Without these assumptions, the following undesirable cases are possible in the problem: the attacker observes no pursuers, or each pursuer observes neither the attacker nor the other pursuers.

Suppose that the differential game of pursuit takes place in the m-dimensional Euclidean space. The positions of the players can be written as the vectors $y(t)=\left[y_1(t), y_2(t),..., y_m(t)\right]^{\mathrm{T}}$, $y(t)\in R^m$, for the attacker and $x_j(t)=\left[x_{j1}(t), x_{j2}(t),..., x_{jm}(t)\right]^{\mathrm{T}}$, $x_j(t)\in R^m$, for pursuer j = 1, 2,…, n, respectively. We introduce a vector $z_j(t)\in R^m$ of the form

$$z_j(t)=x_j(t)-y(t),\ \ j=1, 2, 3..., n,$$

which specifies the distance between the attacker and pursuer j. This vector determines the radius of sensitivity for each player.

Denoting $x^{\mathrm{T}}=\left[x_1^{\mathrm{T}}, x_2^{\mathrm{T}},\ldots, x_n^{\mathrm{T}}\right]$ and $z^{\mathrm{T}}=\left[z_1^{\mathrm{T}}, z_2^{\mathrm{T}},\ldots, z_n^{\mathrm{T}}\right]$, we compactly write the distance as

$$z(t)=x(t)-\mathbf{1}_n\otimes y(t),$$

where $\mathbf{1}_n$ is the unitary vector of dimensions $n\times 1$, and the symbol $\otimes$ indicates the Kronecker product. In the problems considered below, $t\in\left[t_0, t_f\right]$.

Assumption 2.3. Let us formulate the objectives of different players in this differential game. Consider a positive number $\varepsilon<1$:

– If at some instant t_1, $t_0 \le t_1 \le t_f$, the condition $\left\| z_j(t_1) \right\|^2 \le \varepsilon$ holds due to the actions of one or several pursuers, then the game ends because the attacker is intercepted. This outcome is the pursuers' objective in the game.

– If for any t, where $t_0 \le t \le t_f$, we have $\left\| z(t) \right\|^2 > \varepsilon$, i.e., the condition of interception is not valid, then at $t = t_f$ the game ends upon reaching the prescribed duration. This outcome is the attacker's objective in the game.

Let the game dynamics be described by an ordinary linear differential equation [9, 10] of the form

$$\frac{d}{dt} z(t) = u_p(t) - \mathbf{1}_n \otimes u_e(t), \qquad (1)$$

where $u_p(t) = \frac{d}{dt} x(t)$ and $u_e(t) = \frac{d}{dt} y(t)$ are the velocities of the pursuers and attacker, respectively.

In the non-zero-sum game for the system (1), we can introduce two performance criteria [19]:
The group of n pursuers strives to minimize the first criterion

$$J_p(z(\cdot),\ u_p(\cdot)) = \frac{1}{2} k_{pf} z^{\mathrm{T}}(t_f) z(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \begin{bmatrix} z(t) \\ u_p(t) \\ \mathbf{1}_n \otimes u_e(t) \end{bmatrix}^{\mathrm{T}} \times$$

$$\times \begin{bmatrix} q_p I & 0 & 0 \\ 0 & r_p I & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} z(t) \\ u_p(t) \\ \mathbf{1}_n \otimes u_e(t) \end{bmatrix} dt. \qquad (2)$$

The evading attacker seeks to maximize the second criterion

$$J_e(z(\cdot),\ u_e(\cdot)) = -\frac{1}{2} k_{ef} z^{\mathrm{T}}(t_f) z(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \begin{bmatrix} z(t) \\ u_p(t) \\ \mathbf{1}_n \otimes u_e(t) \end{bmatrix}^{\mathrm{T}} \times$$

$$\times \begin{bmatrix} -q_e I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & r_e I \end{bmatrix} \begin{bmatrix} z(t) \\ u_p(t) \\ \mathbf{1}_n \otimes u_e(t) \end{bmatrix} dt, \qquad (3)$$

where k_{pf}, k_{ef}, q_p, q_e, r_p, and r_e are positive parameters.

The first summand in the criterion (2) characterizes a finite value of the differential game, and the parameter ε determines the instant of successful interception, i.e., the fulfillment of the condition $\left\| z(t_1) \right\|^2 \le \varepsilon,\ t_0 \le t_1 \le t_f$. Hence, the non-execution of interception should be highly estimated by the pursuers. With this aspect in mind, in the case $\varepsilon < 1$, the parameter k_{pf} can be chosen as $k_{pf} = 1/\varepsilon$. For the evading attacker, the first summand in the criterion (3), which estimates the value of its game at the terminal instant, should be small. In other words, the parameter k_{ef} can be chosen as $k_{ef} = \varepsilon$.

According to these performance criteria, the pursuers strive to minimize the weighted distances between them and the evading attacker under the minimum energy costs. In contrast, the evading attacker seeks to maximize the weighted distances between it and the pursuers under the minimum energy costs.

Unlike [19, 20], this paper considers the zero-sum differential game. There is a common performance criterion minimized by the n pursuers and maximized by the evading attacker. Treating the differential game as an optimal control problem [21], we combine the criteria (2) and (3) as follows:

$$J_\Sigma(z(\cdot),\, u_p(\cdot),\, u_e(\cdot)) = J_p(z(\cdot),\, u_p(\cdot)) - J_e(z(\cdot),\, u_e(\cdot)) =$$

$$= \frac{1}{2} z^{\mathrm{T}}(t_f) F z(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \Big\{ z^{\mathrm{T}}(t) Q z(t) + u_p^{\mathrm{T}}(t) R u_p(t) +$$

$$- \left(\mathbf{1}_n \otimes u_e(t) \right)^{\mathrm{T}} P \left(\mathbf{1}_n \otimes u_e(t) \right) \Big\} dt, \qquad (4)$$

where $F = \left[k_{pf} + k_{ef} \right] I_n$, $Q = \left[q_p + q_e \right] I_n$, $R = r_p I_n$, and $P = r_e I_n$, the parameters k_{pf}, k_{ef}, q_p, q_e, r_p, and r_e are positive, and I_n is an identity matrix of dimensions $n \times n$.

The positive definiteness of the matrices F, Q, R, and P ensures the existence of optimal controls in this differential game [22]. As shown below, choosing the parameters r_p and r_e so that $r_p < n r_e$ corresponds to the case of "strong" pursuers. (In other words, the pursuers excel the evader by their dynamical capabilities.)

For the mathematical description of different situations (stages) in the game with distributed information, by analogy with the paper [19], we introduce the "sensitivity matrix"

$$S(t) = \begin{bmatrix} 1 & s_{01}(t) & s_{02}(t) & \dots & s_{0n}(t) \\ s_{10}(t) & 1 & s_{12}(t) & \dots & s_{1n}(t) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{n0}(t) & s_{n1}(t) & s_{n2}(t) & \dots & 1 \end{bmatrix}, \qquad (5)$$

where the subscript "0" indicates the evading attacker, and the subscripts from "1" to "n" the corresponding pursuers. For players i, j and an instant t, the parameter $s_{ij}(t)$, $t \in \left[t_0, t_f \right]$, $i,\ j = 0, 1, 2, ..., n$, $0 \le s_{ij}(t) \le 1$, is the degree of significance of the information about the latter player's state used by the former player for accomplishing its objective in the differential game.

In the case $0 < s_{ij}(t) \le 1$, player i observes player j; otherwise ($s_{ij}(t) = 0$), does not. Since each player always observes itself, the diagonal elements of the matrix (5) are constant and equal to 1. Thus, at separate stages, the players' information to accomplish their objective in the differential game may change, reflected via appropriate controls of the attacker and pursuers. This paper does not consider any methods for determining the degree of significance, i.e., the problem of finding the parameters of the matrix $S(t)$ depending on the game conditions. The criterion (4) for the zero-sum differential game with distributed information will be presented in Section 2.2.

In the game with global information, the sensitivity matrix is constant, and its elements are equal to 1.

2. CLASSICAL DIFFERENTIAL GAME AND GAME WITH DISTRIBUTED INFORMATION

2.1. Classical differential game

The classical differential game is a game with global information. It rests on the theory of optimal control: the problem is to design controls $u_p^0(t)$ and $u_e^0(t)$ for which

$$J_\Sigma(z,\ u_p^0(t),\ u_e(t)) \le J_\Sigma(z,\ u_p^0(t),\ u_e^0(t)) \le J_\Sigma(z,\ u_p(t),\ u_e^0(t)).$$

For the classical differential game with several pursuers and linear feedback controls, we have the following result.

Theorem 3.1. *Consider a differential game with n pursuers and one evading attacker with the dynamics* (1) *and the performance criterion* (4). *This game has a value under the condition $r_p < nr_e$ if the strategies of players are given by*

$$u_p^0(t) = -\frac{1}{r_p} K(t) z(t),\ \ u_e^0(t) = = -\frac{1}{n r_e} (\mathbf{1}_n^{\mathrm{T}} \otimes I_m) K(t) z(t), \tag{6}$$

where

$$\frac{d}{dt} K(t) = -K(t)\left[-\frac{1}{r_p} I_n + \frac{1}{nr_e} (\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m) \right] \times K(t) - \left[q_p + q_e \right] I_n,\ K(t_f) = \left[k_{pf} + k_{ef} \right] I_n. \tag{7}$$

This assertion is proved in the Appendix.

From equation (7) it follows that the matrix $K(t)$ is symmetric. A positive definite matrix $K(t)$ is selected from two possible solutions of equation (7). The positive definite property is established when determining the conditions for the existence of optimal controls in the classical differential game. For this purpose, we introduce the Lyapunov function with a positive definite symmetric matrix $K(t)$:

$$V(z(t)) = z^{\mathrm{T}}(t) K(t) z(t).$$

According to the Lyapunov theorem, the matrix equation (7) has a stable solution if

$$\frac{d}{dt} V(z(t)) = z^{\mathrm{T}}(t) \left\{ \frac{d}{dt} K(t) \right\} z(t) + \left\{ \frac{d}{dt} z(t) \right\}^{\mathrm{T}} K(t) z(t) + + z^{\mathrm{T}}(t) K(t) \left\{ \frac{d}{dt} z(t) \right\} \le -z^{\mathrm{T}}(t) \left[q_p + q_e \right] z(t). \tag{8}$$

Equation (1) with the controls (6) takes the form

$$\frac{d}{dt} z(t) = \left[-\frac{1}{r_p} I_n + \frac{1}{n r_e} (\mathbf{1}_n^{\mathrm{T}} \otimes I_m) \right] K(t) z(t). \tag{9}$$

Due to equation (9), we write inequality (8) as

$$z^{\mathrm{T}}(t) \left[\frac{d}{dt} K(t) + K(t) \left[-\frac{1}{r_p} I_n + \frac{1}{n r_e} (\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m) \right]^{\mathrm{T}} \times \times K(t) + \left[q_p + q_e \right] I_n \right] z(t) - - z^{\mathrm{T}}(t) K(t) \left[\frac{1}{r_p} I_n - \frac{1}{n r_e} (\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m) \right] K(t) z(t) \le 0.$$

In view of (7), we obtain the following condition for the existence of optimal controls in the differential game:

$$z^{\mathrm{T}}(t) K(t) \left[\frac{1}{r_p} I_n - \frac{1}{n r_e} (\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m) \right] K(t) z(t) \ge 0.$$

Obviously, this condition will hold if the bracketed matrix is positive definite, i.e.,

$$\frac{1}{r_p} > \frac{1}{n r_e}. \tag{10}$$

This inequality can be satisfied by tuning the parameters r_p and r_e, or the penalty matrices $R = r_p I_n$ and $P = n r_e I_n$.

Let us formulate this result as follows.

Theorem 3.2. *The differential game* (1), (4) *has a value if the penalty matrices R and P of the performance criterion* (4) *satisfy the relation* $R \prec P$.

Note that under condition (10), the performance criterion with the controls (6) achieves the saddle point, i.e.,

$$J_\Sigma(z(\cdot),\ u_p^0(\cdot),\ u_e(\cdot)) \le J_\Sigma(z^0(\cdot),\ u_p^0(\cdot),\ u_e^0(\cdot)) \le J_\Sigma(z(\cdot),\ u_p(\cdot),\ u_e^0(\cdot)).$$

Condition (10) leads to a logical conclusion: the more the pursuers are, the more successful their game outcome will be.

Theorem 3.3. *Consider a differential game with n pursuers and one evading attacker with the dynamics* (1) *and the performance criterion* (4). *Let* $J^0_{\Sigma}(t,z(\cdot))$ *denote the minimax value achieved by* $J_{\Sigma}(z(\cdot),u_p(\cdot),u_e(\cdot))$ *under the feedback optimal controls. This value is*

$$J^0_{\Sigma}(t,z(t))=\frac{1}{2}z^{\mathrm{T}}(t)K(t)z(t),\ t_0\le t\le t_f,$$

where $K(t)$ *is a symmetric positive definite matrix satisfying equation* (7) *with the right-end boundary condition.*

This assertion is proved in the Appendix.

In the case $n=1$ (only one pursuer), the controls take the form

$$u^0_p(t)=-\frac{k_p(t)}{r_p}z(t),\ u^0_e(t)=-\frac{k_e(t)}{r_e}z(t),$$

where the parameters $k_p(t)$ and $k_e(t)$ satisfy equations (7) with $K(t)=\left[k_p(t)+k_e(t)\right]I_m$ and $n=1$:

$$\frac{d}{dt}k_p(t)-\left[\frac{r_e-r_p}{r_er_p}\right]k_p^2(t)-\frac{2}{r_p}k_p(t)k_e(t)+q_p=0,$$
$$k_p(t_f)=k_{pf}, \tag{11}$$

$$\frac{d}{dt}k_e(t)-\left[\frac{r_e-r_p}{r_er_p}\right]k_e^2(t)+\frac{2}{r_e}k_p(t)k_e(t)+q_p=0,$$
$$k_e(t_f)=k_{ef}. \tag{12}$$

2.2. Differential game with distributed information

The main idea of constructing strategies in the differential game with distributed information is that each player makes a decision based on the available information at a given time instant. The dynamics of the information available to the players (the attacker and pursuers) to form their controls are described by the sensitivity matrix (5).

In general form, the distance between pursuer j, the attacker, and the other pursuers is specified by the vector

$$\tilde z_{pj}(t)=x_j(t)-\sum_{i=1}^{n}d_{ij}(t)x_i(t)-f_j(t)y(t). \tag{13}$$

If the evading attacker observes the actions of several pursuers, then the following information can be available to it:

$$\tilde z_e(t)=\sum_{i=1}^{n}e_i(t)x_i(t)-y(t). \tag{14}$$

Like in the paper [20], the coefficients $d_{ij}(t)$, $f_j(t)$, and $e_i(t)$ in (13) and (14) are composed of the elements of the matrix (5) characterizing the mutual observations of the players:

$$d_{ij}(t)=\left[1-s_{0j}(t)\right]\frac{s_{ij}(t)}{\sum_{l=1}^{n}s_{jl}(t)},$$

$$f_j(t)=s_{0j}(t),\ \ e_j(t)=\frac{s_{0j}(t)}{\sum_{i}^{n}s_{0i}(t)}.$$

The pursuers' strategies are

$$u^0_{pj}(t)=-\frac{k_p(t)}{r_p}\tilde z_{pj}(t)=$$
$$=-\frac{k_p(t)}{r_p}\left[x_j(t)-\sum_{i=1}^{n}d_{ij}(t)x_i(t)-f_j(t)y(t)\right] \tag{15}$$

for j = 1, 2,..., n.

The evading attacker forms its control using the available information (14):

$$u^0_e(t)=-\frac{k_e(t)}{r_e}\tilde z_e(t)=-\frac{k_e(t)}{r_e}\left[\sum_{i=1}^{n}e_i(t)x_i(t)-y(t)\right]. \tag{16}$$

The parameters $k_p(t)$ and $k_e(t)$ in (15) and (16) satisfy equations (11) and (12) with $K(t)=\left[k_p(t)+k_e(t)\right]I_n$ and $n=1$.

Note that these control formulas have been obtained for the system dynamics (1) with the quadratic performance criteria (2) and (3).

We write the expressions (15) and (16) compactly using the Kronecker product:

$$u^0_p(t)=-R^{-1}K(t)\{x(t)-\left[D(t)\otimes I_m\right]x(t)-$$
$$-F(t)\otimes y(t)\}, \tag{17}$$
$$u^0_e(t)=-P^{-1}(\mathbf{1}_n^{\mathrm{T}}\otimes I_m)K(t)\times$$
$$\times\left\{\left[E^{\mathrm{T}}(t)\otimes I_m\right]x(t)-y(t)\right\}. \tag{18}$$

Here $K(t)$ are solutions of equations (7), $E(t)=\left[e_1(t)\ldots e_n(t)\right]^{\mathrm{T}}$, $F(t)=\left[f_1(t)\ldots f_n(t)\right]^{\mathrm{T}}$, and $D(t)=\left[d_{ij}(t)\right]\in R^{n\times n}$.

Substituting the optimal controls (15) and (16) into the criterion (4) and performing some transformations, we obtain

$$J^0_{\Sigma}\left(z(t),\,x(t),\ y(t)\right)=\frac{1}{2}z\left(t_f\right)Fz\left(t_f\right)+$$
$$+\frac{1}{2}\int_{t_0}^{t_f}\left\{x^{\mathrm{T}}(t)\cdot H\left(D(t),E(t)\right)\cdot x(t)\right\}dt+$$
$$+\frac{1}{2}\int_{t_0}^{t_f}\left\{x^{\mathrm{T}}(t)\cdot L\left(D(t),\,E(t)\right)\cdot y(t)+\right.$$
$$\left.+y^{\mathrm{T}}(t)\cdot W\left(F(t)\right)\cdot y(t)\right\}dt, \tag{19}$$

where

$$H\left(D(t),\,E(t),\;K(t)\right)=Q+K(t)R^{-1}K(t)-K(t)\times$$
$$\times R^{-1}K(t)D(t)\otimes I_m-\left[D(t)\otimes I_m\right]^{\mathrm{T}}K(t)R^{-1}K(t)+$$
$$+\left[D(t)\otimes I_m\right]^{\mathrm{T}}K(t)\times^{-1}K(t)D(t)\otimes I_m-$$
$$-n\left[F(t)\otimes I_m\right]^{\mathrm{T}}\left[\left(1_n^{\mathrm{T}}\otimes I_m\right)K(t)\right]^{\mathrm{T}}P^{-1}\times$$
$$\times\left[\left(1_n^{\mathrm{T}}\otimes I_m\right)K(t)\right]F(t)\otimes I_m,$$
$$L\left(D(t),\,E(t),\;F(t)\right)=2\left[D(t)\otimes I_m\right]^{\mathrm{T}}\times$$
$$\times K(t)R^{-1}K(t)\left[F(t)\otimes I_m\right]-2K(t)R^{-1}K(t)\times$$
$$\times\left[F(t)\otimes I_m\right]-2\left(\mathbf{1}_n^{\mathrm{T}}\otimes I_m\right)^{\mathrm{T}}Q+2n\left[E^{\mathrm{T}}(t)\otimes I_m\right]^{\mathrm{T}}\times$$
$$\times\left[\left(\mathbf{1}_n^{\mathrm{T}}\otimes I_m\right)K(t)\right]^{\mathrm{T}}P^{-1}\left[\left(\mathbf{1}_n^{\mathrm{T}}\otimes I_m\right)K(t)\right],$$
$$W\left(F(t),K(t)\right)=nQ+\left[F(t)\otimes I_m\right]^{\mathrm{T}}\times$$
$$\times K(t)R^{-1}K(t)F(t)\otimes I_m-$$
$$-n\left[\left(\mathbf{1}_n^{T}\otimes I_m\right)K(t)\right]^{\mathrm{T}}P^{-1}\left(\mathbf{1}_n^{T}\otimes I_m\right)K(t).$$

Clearly, the integrand of the criterion (19) considers both the positions of the pursuers and evader and the mutual disposition of the pursuers and evader. Note that under the optimal controls, the value of this functional depends on the number of players and the elements $d_{ij}(t)$, $f_i(t)$, and $e_i(t)$ of the matrices $D(t)$, $F(t)$, and $E(t)$, respectively. In other words, the value depends on what information is available to the players during the game and the type of availability.

In the game with one pursuer, one evading attacker, and global information (the classical differential game; see subsection 2.1), the parameters are $n=1$, $D(t)=0$, and $E(t)=1$. As a result, the controls (17) and (18) become the same as in (6). In this case, the criterion takes the form

$$J_\Sigma^0(z(t))=\frac{1}{2}z\left(t_f\right)Fz\left(t_f\right)+\frac{1}{2}\int_{t_0}^{t_f}z^{\mathrm{T}}(t)\times$$
$$\times\left\{Q+K(t)R^{-1}K(t)-K(t)P^{-1}K(t)\right\}z(t)dt.$$

According to Theorem 3.3, $J_\Sigma^0(t,z(t))$ is given by

$$J_\Sigma^0(t,z(t))=\frac{1}{2}z^{\mathrm{T}}(t)K(t)z(t),\;t_0\le t\le t_f\,.$$

Consider a particular case of the differential game with distributed information and the binary sensitivity matrix (5). The element $s_{ij}(t)$ of this binary matrix indicates whether player *i* observes player *j* at a time instant *t* or not: if $s_{ij}(t)=1$, then player *i* observes player *j*; otherwise ($s_{ij}(t)=0$), player *i* does not observe player *j*. Since each player observes itself, the diagonal elements of the matrix (5) are constant and equal to 1.

Case 1. Let pursuer *j* not observe the evading attacker, i.e., $s_{0j}=0,\ f_j=0$. From the expression (17) we therefore have

$$u_{pj}^0(t)=-\frac{k_p(t)}{r_p}\left[x_j(t)-\sum_{i=1}^n d_{ij}(t)x_i(t)\right].$$

This means that pursuer *j* will follow the nearest observable pursuers.

Case 2. Let pursuer *j* observe the evading attacker, i.e., $s_{0j}=1,\ f_j=1$, and let this player have no information about the other pursuers, i.e., $d_{ij}=0$. From the expression (16) we therefore have

$$u_{pj}^0(t)=-\frac{k_p(t)}{r_p}\left[x_j(t)-y(t)\right].$$

This means that pursuer *j* will try to intercept the evading attacker independently.

Consider the evading attacker's strategy (18), noting the following: if the evader observes several pursuers in its radius of sensitivity, then its control will be intended to "escape" the center of mass of all the pursuers detected.

3. DIFFERENTIAL GAME WITH NOISE

Consider a situation when the evading attacker artificially jams the pursuers to gain an advantage in the differential game of pursuit. This means that the pursuers will receive information about the evader's position with some noise. Hence, the controls constructed by the pursuers will contain this noise. Thus, the trajectories along which the pursuers will intercept the evader are suboptimal. In addition, the attacker constructs its strategy for all pursuers it detects, trying to escape the center of mass of all pursuers. Since their positions are subjected to noise, the attacker's trajectory will also contain a noise component. Note that the evading attacker will not be affected by the noise it creates, and its control strategy still depends only on the pursuers' positions.

The controls of the pursuer and attacker in the differential game with global information are given by

$$u_p(t)=-\frac{k_p(t)}{r_p}z(t),\;\;u_e(t)=\frac{d}{dt}y(t)\,.$$

Let $n(t)$ be the noise created by the evading attacker, representing the white noise with the mean $M\left[n(t)\right]=0$ and variance $M\left[n^{\mathrm{T}}(t)n(\tau)\right]=$ $=N(t)\delta(t-\tau)$. Under the new conditions in the differential game, the pursuers will detect the evading attacker along the trajectory $y^*(t)=y(t)+n(t)$. Note that the presence of noise may affect the condition of interception (the pursuers' objective in the game); see Assumption 2.3. We therefore introduce a new condition of interception:

$$E\left[\left\|z_j^*(t_1)\right\|^2\right] \le \varepsilon,$$

where $E\left[\left\|z_j^*(t_1)\right\|^2\right]$ is the root-mean-square distance between the attacker and pursuer j, or equivalently, $E\left[\left\|z_j(t)-n(t)\right\|^2\right] \le \varepsilon.$ Due to the white noise created by the attacker, we have $E\left[n^{\mathrm{T}}(t)z_j(t)\right]=0$, and the condition of interception takes the form

$$E\left[\left\|z(t)\right\|^2\right] \le \varepsilon - N.$$

If $N > \varepsilon$, the objective of interception cannot be accomplished.

We write the control strategies in the classical differential game:

$$u_p(t) = -\frac{k_p(t)}{r_p} z^*(t) = -\frac{k_p(t)}{r_p}\left[z(t)-n(t)\right].$$

Also, we write the control strategies in the differential game with distributed information:

$$u_e(t) = -\frac{k_e(t)}{r_e}\left[\sum_{i=1}^{n} e_i(t)x_i(t) - y(t)\right],$$

$$u_{pj}(t) = -\frac{k_p(t)}{r_p}\left[x_j(t) - \sum_{i=1}^{n} d_{ij}(t)x_i(t) - f_j(t)y^*(t)\right].$$

If a pursuer does not detect the evading attacker (therefore constructing its control strategy based on the pursuers detecting the attacker), its trajectory will still have a noisy component due to the noisy trajectories of the latter pursuers.

4. EXAMPLE

4.1. Classical differential game

Let us simulate a differential game in which each player has complete information (game with global information). Assume that there are one attacker and three pursuers. Then the game dynamics with the constructed controls are described by an ordinary linear differential equation of the form

$$\frac{d}{dt}x_1(t) = -\frac{k_p(t)}{r_p}(x_1(t)-y(t)),\ x_1(t_0)\ =\ [-3,\ 0]^{\mathrm{T}},$$

$$\frac{d}{dt}x_2(t) = -\frac{k_p(t)}{r_p}(x_2(t)-y(t)),\ x_2(t_0)\ =\ [3,\ 0]^{\mathrm{T}},$$

$$\frac{d}{dt}x_3(t) = -\frac{k_p(t)}{r_p}(x_3(t)-y(t)),\ x_3(t_0)\ =[4,\ 1]^{\mathrm{T}},$$

$$\frac{d}{dt}y(t) = -\frac{k_e(t)}{r_e}\left\{\frac{1}{3}\left[x_1(t)+x_2(t)+x_3(t)\right]-y(t)\right\},$$

$$y(t_0)\ =\ [0,\ 3]^{\mathrm{T}}.$$

Here the parameters $k_p(t)$ and $k_e(t)$ satisfy the equations

$$\frac{d}{dt}k_p(t) = -q_p + \frac{1}{r_p}k_p^2(t) - \frac{2}{r_e}k_p(t)k_e(t),$$

$$k_p(t_f) = k_{pf}\ = 1/\varepsilon,$$

$$\frac{d}{dt}k_e(t) = -q_e - \frac{1}{r_e}k_e^2(t) + \frac{2}{r_p}k_p(t)k_e(t),$$

$$k_e(t_f) = k_{ef}\ = \varepsilon.$$

For the performance criterion (4), we choose the parameters $r_p=1,\ r_e=2,\ q_p=1,\ q_e=2,\ k_{pf}=20,$ and $k_{ef}\ =0.05.$ For the condition of interception, we choose the parameter $\varepsilon = 0.04$. Let the game of pursuit occur for $t\in[0,4]$.

The variations of the parameters $k_p(t)$ and $k_e(t)$ over time are shown in Fig. 1.

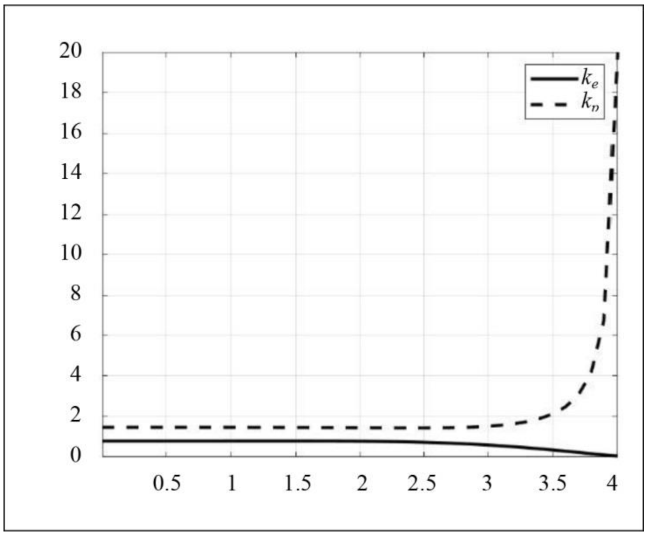

Fig. 1. Variations of parameters $k_p(t)$ and $k_e(t)$.

The graphs of the transient processes in the problems are presented below. Figure 2 shows the trajectories of the pursuers and evading attacker in the classical game with and without noise. In both of the games, the attacker has been intercepted: the condition $\left\|z(t_1)\right\| \le \varepsilon,\ t_1 < 4s$, has been satisfied at $t_1 = 3.58$ (Fig. 2*a*) and $t_1 = 3$ (Fig. 2*b*), where time is measured in conditional machine units.

The classical differential game with centered noise has been simulated using the original model with the same initial conditions.

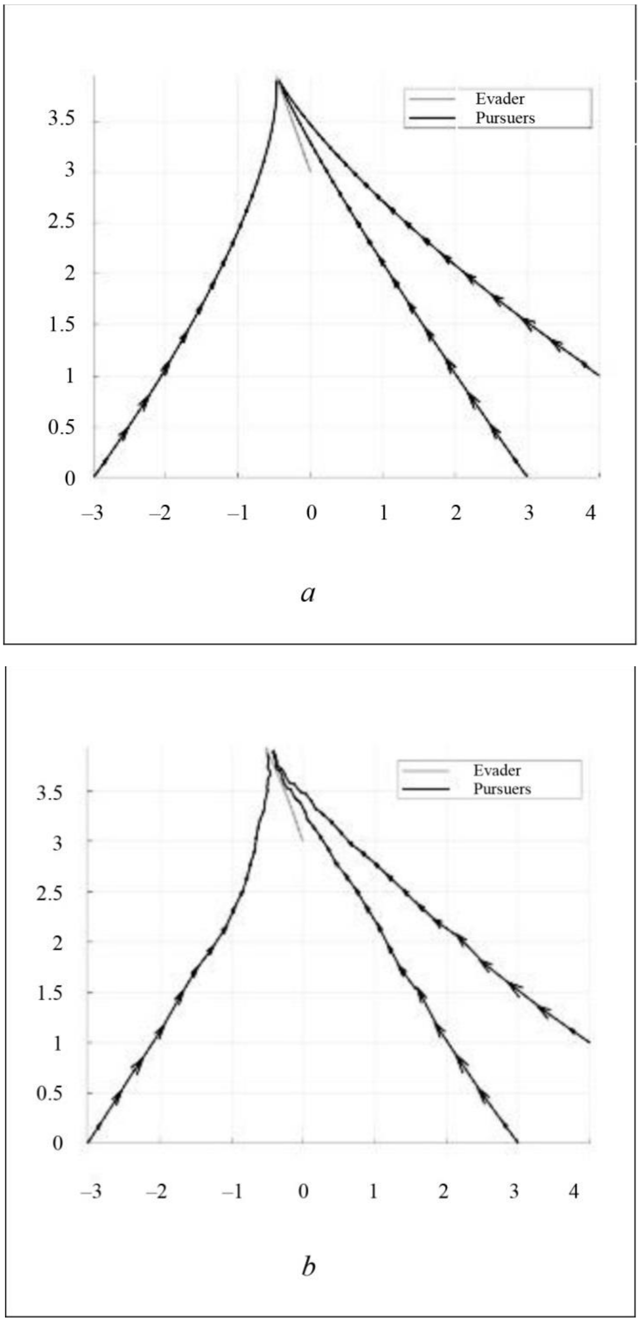

Fig. 2. Transient processes in simple game of pursuit:
(*a*) without noise (game ends at 3.58) and (*b*) with noise (game ends at 3).

4.2. Differential game with distributed information

Suppose that the initial position of the players is the same as in subsection 4.1. We simulate the differential game in which each player has limited information about the other players. Let the sensitivity matrix change three times during the game, which can be expressed as follows:

$$S_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix},\ S_2 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix},\ S_3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

In the first period, only one pursuer detects the attacker, and the two other pursuers, detecting the first pursuer only, follow it. In the next period, two pursuers detect the evading attacker and try to intercept it, while the remaining pursuer follows them. In the final period, each of the players detects the others, and the differential game turns into a game with global information (classical differential game).

Like in subsection 4.1, we adopt the differential game with distributed strategies as the basis model and add noise. All operations to obtain a solution are performed by analogy with the previous subsection. Figure 3 shows the trajectories of the pursuers and evader in the game with distributed strategies, without noise and with noise.

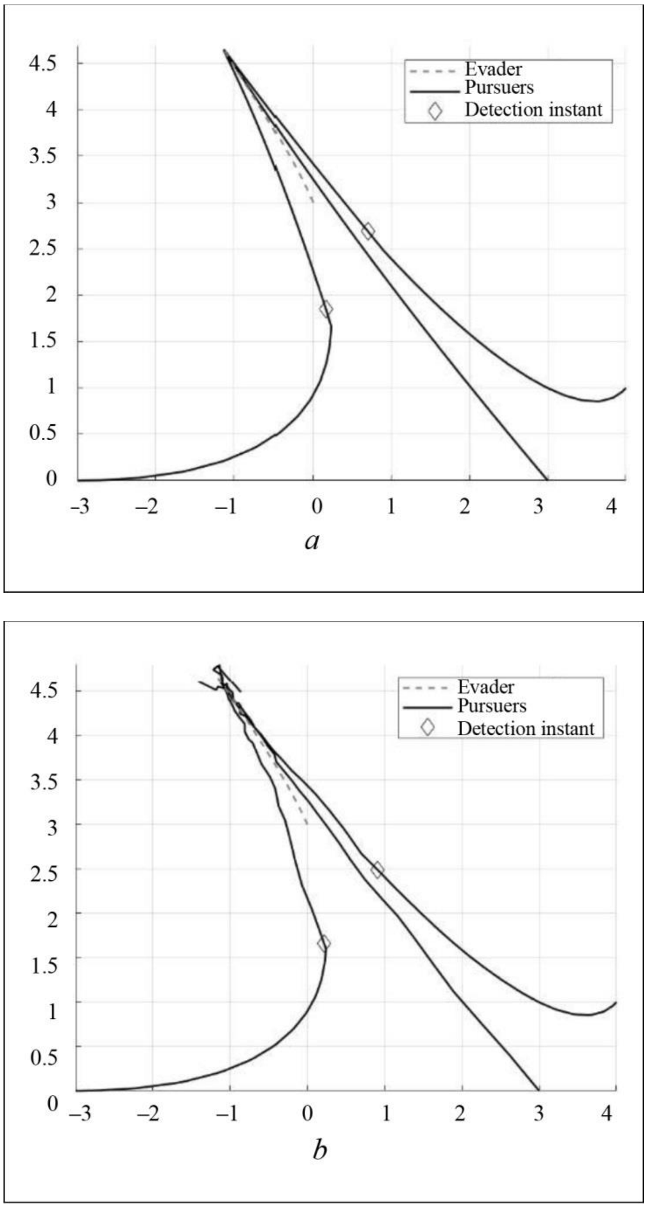

Fig. 3. Transient processes in game of pursuit with distributed information:
(*a*) without noise (game ends at 3.88) and (*b*) with noise (game ends at 4).

The graphs in Fig. 3 show the instants when different pursuers join the pursuit of the evading attacker (when the attacker enters their zones of sensitivity, i.e., at the instants of detection). Figure 3*a* corresponds to the successful interception of the attacker, i.e., the condition $\|z_j(t_1)\|^2 \le \varepsilon,\ t_1 < 4\ s$, is satisfied for $t_1 = 3.88$. Figure 3*b* corresponds to the unsuccessful interception of the attacker, i.e., the condition $\|z(t)\|^2 > \varepsilon$ holds for all $t_0 \le t \le t_f$, and the game ends upon reaching the prescribed duration $t = t_f = 4$.

CONCLUSIONS

This paper has considered a differential game of pursuit with several players. One player (attacker) penetrates some space, and several other players (pursuers) appear simultaneously to intercept the attacker. Upon detecting the pursuers, the attacker tries to evade them. The dynamics of each player are described by a time-invariant linear system of a general type. The strategies of the pursuers and evading attacker have been constructed within two subproblems: (1) all players have complete information about the state of all game participants, and (2) the pursuers have incomplete information about the evading attacker actively opposing them. The distributed strategies and some particular cases of the differential game of pursuit have also been considered. The main idea of constructing strategies for this game is that each player makes a decision based only on the available information at a given time. The simulation results have been provided to illustrate the theory.

APPENDIX

Proof of Theorem 3.1. Let us write the system's Hamiltonian

$$H(z, u_p, u_e, \lambda) = \frac{1}{2}\Big\{z^{\mathrm{T}}(t)\big[q_p + q_e\big]z(t) + r_p u_p^{\mathrm{T}}(t)u_p(t) -$$

$$-nr_e\big(\mathbf{1}_n \otimes u_e(t)\big)^{\mathrm{T}}\big(\mathbf{1}_n \otimes u_e(t)\big)\Big\} + \lambda^{\mathrm{T}}(t)\big[u_p(t) - \mathbf{1}_n \otimes u_e(t)\big].$$

Here, $\lambda(t)$ is the conjugate variable [22], which satisfies the equation

$$\frac{d}{dt}\lambda(t) = -\left\{\frac{\partial H(z, u_p, u_e, \lambda)}{\partial z}\right\}^{\mathrm{T}} = -\big[q_p + q_e\big]z(t) \quad \text{(A.1)}$$

with the boundary condition

$$\lambda(t_f) = \frac{1}{2}\frac{\partial z^{\mathrm{T}}(t_f)\big[k_{pf} + k_{ef}\big]z(t_f)}{\partial z} = \big[k_{pf} + k_{ef}\big]z(t_f).$$

The optimal controls are the stationary points of the Hamiltonian:

$$\frac{\partial H(z, u_p, u_e, \lambda)}{\partial u_p} = 0, \quad \frac{\partial^2 H(z, u_p, u_e, \lambda)}{\partial u_p^2} = r_p > 0, \quad \text{(A.2)}$$

$$\frac{\partial H(z, u_p, u_e, \lambda)}{\partial u_e} = 0, \quad \frac{\partial^2 H(z, u_p, u_e, \lambda)}{\partial u_e^2} = -nr_{ep} < 0. \quad \text{(A.3)}$$

Conditions (A.2) and (A.3) determine the optimal controls

$$u_p^0(t) = -\frac{1}{r_p}\lambda(t), \quad u_e^0(t) = -\frac{1}{n r_e}(\mathbf{1}_n^{\mathrm{T}} \otimes \lambda(t)). \quad \text{(A.4)}$$

Therefore, the variable $\lambda(t)$ is the solution of the two-point boundary value problem (the Euler–Lagrange equations)

$$\frac{d}{dt}z(t) = \left[-\frac{1}{r_p}I_m + \frac{1}{n r_e}(\mathbf{1}_n^{\mathrm{T}} \otimes I_m)\right]\lambda(t),$$

$$z_0(t) = z_0,$$

$$\frac{d}{dt}\lambda(t) = -\big[q_p + q_e\big]z(t),$$

$$\lambda(t_f) = \big[k_{pf} + k_{ef}\big]z(t_f).$$

The auxiliary variable $\lambda(t)$ will be calculated using the sweep method [22]. Let us find $\lambda(t)$ in the form

$$\lambda(t) = K(t)z(t), \quad \text{(A.5)}$$

where $K(t)$ is an unknown matrix. The total derivative of the expression (A.5) is given by

$$\frac{d}{dt}\lambda(t) = \left\{\frac{d}{dt}K(t)\right\}z(t) + K(t)\left\{\frac{d}{dt}z(t)\right\} =$$

$$= \left[\left\{\frac{d}{dt}K(t)\right\} + K(t)\left[-\frac{1}{r_p}I + \frac{1}{nr_e}\times\right.\right.$$

$$\left.\left.\times(\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m)\right]K(t)\right]z(t)\Big]. \quad \text{(A.6)}$$

Equalizing the expressions (A.1) and (A.6), we obtain:

$$\frac{d}{dt}K(t) = -K(t)\left[-\frac{1}{r_p}I_m + \frac{1}{nr_e}(\mathbf{1}_n \otimes \mathbf{1}_n^{\mathrm{T}} \otimes I_m)\right]\times K(t) -$$

$$-\big[q_p + q_e\big]I_m, \quad K(t_f) = \big[k_{pf} + k_{ef}\big]I_m.$$

Due to (A.4) and (A.5), the equations take the form:

$$u_p^0(t) = -\frac{1}{r_p}K(t)z(t),$$

$$u_e^0(t) = -\frac{1}{nr_e}(\mathbf{1}_n^{\mathrm{T}} \otimes I_m)K(t)z(t). \blacklozenge$$

Proof of Theorem 3.3. Consider the integrand of the performance criterion

$$J_\Sigma(z(\cdot), u_p(\cdot), u_e(\cdot)) = \frac{1}{2}z^{\mathrm{T}}(t_f)Fz(t_f) +$$

$$+\frac{1}{2}\int_t^{t_f}\Big\{z^{\mathrm{T}}(t)Qz(t) + u_p^{\mathrm{T}}(t)Ru_p(t) - \big(\mathbf{1}_n \otimes u_e(t)\big)^{\mathrm{T}} \times$$

$$\times P\big(\mathbf{1}_n \otimes u_e(t)\big)^{\mathrm{T}} P\big(\mathbf{1}_n \otimes u_e(t)\big)\Big\}dt.$$

Substituting $d\big[z^{\mathrm{T}}(t)K(t)z(t)\big]/dt$ into this integrand and compensating the result outside the integral by $0.5\big[z^{\mathrm{T}}(t)K(t)z(t) - z^{\mathrm{T}}(t_f)K(t_f)z(t_f)\big]$, we have:

$$J_{\Sigma}(z(\cdot),u_p(\cdot),u_e(\cdot))=\frac{1}{2}z^{\mathrm{T}}(t_f)Fz(t_f)+$$
$$+\frac{1}{2}\Big[z^{\mathrm{T}}(t)K(t)z(t)-z^{\mathrm{T}}(t_f)K(t_f)z(t_f)\Big]+$$
$$+\frac{1}{2}\int_t^{t_f}\Big\{z^{\mathrm{T}}(t)Qz(t)+u_p^{\mathrm{T}}(t)Ru_p(t)-$$
$$-\left(\mathbf{1}_n\otimes u_e(t)\right)^{\mathrm{T}}P\left(\mathbf{1}_n\otimes u_e(t)\right)\Big\}dt+$$
$$+\frac{1}{2}\int_t^{t_f}\left\{z^{\mathrm{T}}(t)\left\{\frac{dK(t)}{dt}\right\}z(t)+\right.$$
$$\left.+\left\{\frac{dz^{\mathrm{T}}(t)}{dt}\right\}K(t)z(t)+z^{\mathrm{T}}(t)K(t)\left\{\frac{dz(t)}{dt}\right\}\right\}dt. \quad (\mathrm{A}.7)$$

Note that under the optimal controls

$$u_p^0(t)=-R^{-1}K(t)z(t),\ \ u_e^0(t)=-P^{-1}\left(\mathbf{1}_n^{\mathrm{T}}\otimes K(t)z(t)\right), \quad (\mathrm{A}.8)$$

where

$$\frac{d}{dt}K(t)+K(t)\Big[-R^{-1}+P^{-1}(\mathbf{1}_n\otimes\mathbf{1}_n^{\mathrm{T}})\Big]\times$$
$$\times K(t)+Q=0,\ K(t_f)=F, \quad (\mathrm{A}.9)$$

the system's dynamics are described by

$$\frac{d}{dt}z(t)=\Big[-R^{-1}+P^{-1}(\mathbf{1}_n^{\mathrm{T}}\otimes I_m)\Big]K(t)z(t),$$
$$z(t_0)=z_0. \quad (A.10)$$

Recall that $J_{\Sigma}^0(t,z(t))$ denotes the minimax value of the criterion $J_{\Sigma}(z(t),\, u_p(t),\, u_e(t))$. Substituting (A.8) and (A.10) into (A.7) and taking (A.9) into account, we finally arrive in

$$J_{\Sigma}^0(t,z(t))=\frac{1}{2}z^{\mathrm{T}}(t)K(t)z(t),\ \ t_0\le t\le t_f. \ ♦$$

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