DOI: http://doi.org/10.25728/cs.2023.5.3

CONTROL SYSTEM DESIGN FOR MOVING OBJECTS WITH CHANNEL SWITCHING

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Abstract. This paper considers the problem of designing control systems for moving objects with control channel switching. A generalized characteristic is proposed to eliminate jumps and impulses that may occur at switching instants. This characteristic describes the entire ensemble of system trajectories under control channel switching at an arbitrary random instant. A numerical inversion method is developed for the Laplace transform based on expanding the delta function into a series of exponential polynomials. With this method, the generalized characteristic of the system can be approximated by a given time domain. The exponential series description of the generalized system characteristic allows reducing the original design problem to a system of algebraic inequalities. A particular example of designing an automatic pitch control system for an aircraft with a normal overload limit is presented; as is shown, the entire ensemble of trajectories obtained for different channel switching instants belongs to a given time domain.

Keywords: design, switched systems, selector, moving object, trajectory, angle of attack.

INTRODUCTION

Control systems of moving objects have a number of peculiarities [1, 2]. One peculiarity is their multi-mode operation. In this case, the transition to the next operating mode is accompanied by a structural change in the control part of the system. In particular, such a situation arises in the programmed control of a moving object when it is necessary to restrict the maximum allowed values of the motion parameters, e.g., the limiting angle of attack in aircraft attitude control. During operation the system structure changes according to an accepted logic of channel switching; therefore, such systems are referred to as switched systems [3–10]. This is a class of multi-mode dynamic systems consisting of a family of continuous subsystems and a device that controls mode switching.

The permanently increasing interest in switched systems is due to their wide use in applications (control of electric power systems, aircraft, industrial processes, and many other areas, up to the development of intelligent components of control systems [11–17]). Also, there are several interesting phenomena occurring in such systems. Examples show that the stability of all individual modes is not sufficient for the stability of a system with arbitrary switching [18–21]. In other words, the stability of switched systems depends on the dynamics of the system in each mode and the consistency of the modes during their switching. In this regard, research efforts are concentrated on the stability and stabilizability of switched systems [22–26], as well as methods for designing controllers with guaranteed stability and control performance [27–31]. There exist two approaches to the design of such systems. In the first case, dynamic processes are specified by a system of differential equations, which form a finite ensemble of typical trajectories. The optimal system trajectory is constructed of separate sections of these typical trajectories by a switching device (a finite automaton with memory). The memory of the switching device stores the time intervals on which the motion will follow the selected trajectory and the sequence in which the selected trajectories will be switched. In the studies of such switched systems, the main attention is paid to the development of logical operation rules for the automaton to ensure a consistent change in the system state at the switching instant without jumps and bursts of the controlled variable.

The second approach to the design of switched systems is used when the switching rules are described by constraints on time and state, or are due to external impacts applied to the controlled object. One example is the need to restrict the maximum allowed values of the motion parameters; see the discussion above. In this case, structural switching can be considered a disturbance applied to the system. Therefore, the continuous subsystems of the system's control part should compensate such disturbances by matching the system trajectories when changing its operating modes. A distinctive feature is that such systems change their properties in a jump-like manner at unknown random instants. As a result, they are treated as systems with random structure [32–34]. Consequently, there is an entire ensemble of optimal trajectories, each corresponding to a particular switching instant. In this case, the averaged or generalized characteristics of the system are used to assess its dynamics [35–37]. However, the scope of these results is often restricted to analysis problems since the system characteristics in all structural states are supposed to be known. In this regard, we propose a generalized characteristic of a system with channel switching for design purposes: it allows deriving an analytical dependence between the parameters of the control part and the ensemble of system trajectories under channel switching at an arbitrary random instant.

This paper develops a control design algorithm to match the trajectories of systems with control channel switching using a generalized characteristic that describes the entire ensemble of their trajectories under control channel switching at an arbitrary random instant.

1. ANALYSIS OF SYSTEM DYNAMICS WITH CHANNEL SWITCHING AT RANDOM INSTANTS

Assume that in the control mode of the object's parameter $y(t)$, the system's trajectory is described by the equation

$$P_1(D)\,y(t)=Q_1(D)\,y_{\text{con}}(t), \tag{1}$$

where $y_{\text{con}}(t)$ is the control program for the given parameter; $P_1(D)=a_n^{(1)}\frac{d^n}{dt^n}+\cdots+a_1^{(1)}\frac{d}{dt}+a_0^{(1)}$ and $Q_1(D)=b_m^{(1)}\frac{d^m}{dt^m}+\cdots+b_1^{(1)}\frac{d}{dt}+b_0^{(1)}$, where $a_i^{(1)}\left(i=\overline{0,n}\right)$, $b_j^{(1)}\left(j=\overline{0,m}\right)$ are the model parameters.

When switching to the limiting channel at some instant τ, the system trajectory with respect to the parameter $y(t)$ is described by the equation

$$P_2(D)\,y(t,\ \tau)=Q_2(D)\,y_{\lim}(t) \tag{2}$$

with initial conditions determining the coincident system states at the control mode change instant:

$$y^{(r)}(t,\ \tau)\Big|_{t=\tau}=y^{(r)}(t)\Big|_{t=\tau};\ \left(r=0,1,\ldots,\ n-1\right),$$

where $y_{\lim}(t)$ is the limiting program for the given parameter; $P_2(D)=a_n^{(2)}\frac{d^n}{dt^n}+\cdots+a_1^{(2)}\frac{d}{dt}+a_0^{(2)}$ and $Q_2(D)=b_m^{(2)}\frac{d^m}{dt^m}+\cdots+b_1^{(2)}\frac{d}{dt}+b_0^{(2)}$.

To match the system trajectories when switching the control channel to the limiting channel at an arbitrary random instant $\tau\in[0,+\infty)$, we consider the following generalized characteristic of the system:

$$E\left(y(t)\right)=\int_0^{\infty}y(t,\ \tau)\,f(\tau)d\tau\,, \tag{3}$$

where $f(\tau)$ is the distribution function of the random instant τ.

On the time interval $0\le t\le\tau$, the system trajectory satisfies equation (1). In this case, $y(t,\ \tau)=y(t)$. On the time interval $0\le\tau\le t$, the limiting loop comes into operation and the system trajectory $y(t,\ \tau)$ will satisfy equation (2).

Assume that the control structure is switched with a constant intensity λ. Then the distribution function of the switching instant obeys the law $f(\tau)=\lambda e^{-\lambda\tau}$. We divide the integral (3) into two terms corresponding to the operating modes specified above:

$$E\left(y(t)\right)=\int_0^{t}y(t,\ \tau)\,f(\tau)d\tau+\int_t^{\infty}y(t)\,f(\tau)d\tau$$
$$=\int_0^{t}y(t,\ \tau)\,f(\tau)d\tau+y(t)e^{-\lambda t}.$$

To calculate the latter integral, we represent the solution of system (2) as the sum $y(t,\ \tau)=y^{(\text{I})}(t)+y^{(\text{II})}(t,\ \tau)$ of the partial solution of the inhomogeneous equation (2) with zero initial conditions and the general solution of the corresponding homogeneous equation with nonzero initial conditions. Since the initial conditions of system (2) remain valid for $y(t,\ \tau)$, it follows that

$$\left[y^{(\text{II})}(t,\ \tau)\right]^{(i)}\Big|_{t=\tau}=\left[y(t)\right]^{(i)}\Big|_{t=\tau}-y^{(I)}\left[(t)\right]^{(i)}\Big|_{t=\tau},$$
$$i=\overline{0,(n-1)}.$$

The solution of the homogeneous equation has the form [38]

$$y^{(\text{II})}(t,\ \tau)=\sum_{i=0}^{n-1}\sum_{j=0}^{n-i-1}Y^{(i)}(\tau)\,a_{i+j+1}^{(2)}\cdot\left[w_2(t-\tau)\right]^{(j)},$$

where $Y^{(i)}(\tau)=[y(\tau)-y^{(1)}(\tau)]^{(i)}$ and $w_2(t)=L^{-1}\left\{\dfrac{1}{P_2(s)}\right\}$.

As a result,

$$E\big(y(t)\big)=y(t)e^{-\lambda t}+y^{(1)}(t)\big(1-e^{-\lambda t}\big)$$
$$+\sum_{i=0}^{n-1}\sum_{j=0}^{n-i-1}a_{i+j+1}^{(2)}\int_0^t Y^{(i)}(\tau)\lambda e^{-\lambda\tau}\cdot\left[w_2(t-\tau)\right]^{(j)}d\tau. \quad (4)$$

The generalized characteristic of the system similar to (4) was used in the papers [39, 40]. However, the desired system trajectories under channel switching were ensured therein by localizing the roots of the denominator of the image of this generalized characteristic in a given domain of the complex plane. Generally speaking, such an approach does not eliminate all jumps and bursts at the channel switching instant since the temporal characteristics depend on the image denominator and also on its numerator. In this regard, we require that the generalized characteristic $E(y(t))$ of the system lies within given limits:

$$E_1(t)\le E\big(y(t)\big)\le E_2(t). \quad (5)$$

2. MATCHING CONTROL OF THE SYSTEM WITH CHANNEL SWITCHING

To satisfy condition (5), let us find the Laplace image for $E(y(t))$:

$$E(s)=L\left\{E\big(y(t)\big)\right\}=L\left\{y(t)e^{-\lambda t}\right\}$$
$$+L\left\{y^{(I)}(t)\big(1-e^{-\lambda t}\big)\right\}$$
$$+L\left\{\sum_{i=0}^{n-1}\sum_{j=0}^{n-i-1}a_{i+j+1}^{(2)}\int_0^t Y^{(i)}(\tau)\lambda e^{-\lambda\tau}\left[w_2(t-\tau)\right]^{(j)}d\tau\right\}.$$

Consequently,

$$E(s)=y(s+\lambda)+\big(y^{(1)}(s)-y^{(1)}(s+\lambda)\big)$$
$$+\frac{\lambda[y(s+\lambda)-y^{(1)}(s+\lambda)]}{P_2(s)}\sum_{i=0}^{n-1}\sum_{j=0}^{n-i-1}a_{i+j+1}^{(2)}(s+\lambda)^i s^j.$$

The double sum in this expression can be transformed as follows:

$$\sum_{i=0}^{n-1}\sum_{j=0}^{n-i-1}a_{i+j+1}^{(2)}(s+\lambda)^i s^j=\sum_{k=1}^{n}\sum_{l=0}^{k-1}a_k^{(2)}(s+\lambda)^l s^{k-l-1}$$
$$=\sum_{k=1}^{n}a_k^{(2)}\frac{(s+\lambda)^k-s^k}{(s+\lambda)-s}=\frac{P_2(s+\lambda)-P_2(s)}{\lambda}.$$

Considering this relation, the desired image becomes

$$E(s)=y^{(1)}(s)+\big(y(s+\lambda)-y^{(1)}(s+\lambda)\big)\frac{P_2(s+\lambda)}{P_2(s)}.$$

The inverse Laplace transform should be performed to find the original function of the generalized characteristic $E(y(t))$ based on the image $E(s)$. However, it seems impossible to do in a general form because the image depends on the unknown parameters of the system's control part. Therefore, we employ a special numerical inversion method for the Laplace transform. Within this method, the delta function $\delta(t,\tau)$ is approximated by a partial sum of the series $\delta_q(t,\tau)=\sum_{i=1}^{q}d_i(t)\,\varphi_i(\tau)$, where

$$\varphi_i(t)=\sum_{j=1}^{i}c_{ij}\exp\big(-\beta(j-1)t\big),\ (\beta>0;\ i=1,2,\ldots),$$

is the set of orthonormal exponential polynomials with the weight $g(t)=\exp(-\alpha t)$, $\alpha\ge 0$.

According to the paper [41], the coefficients c_{ij} are given by

$$c_{i+1,j+1}=\frac{(-1)^{i+j}\,\Gamma(i+j+\delta+1)\sqrt{(\delta+2i+1)\beta}}{j!(i-j)!\Gamma(j+\delta+1)},$$
$$i,\ j=0,1,\ldots,$$

where $\Gamma(x)$ denotes the gamma function and $\delta=(\alpha-\beta)/\beta$.

The exponential polynomials $\varphi_i(t)$ can be obtained from any classical orthogonal polynomials $p_i(z)$ (Legendre, Laguerre, Hermite polynomials, etc. [42]) by the change of variable $z=\exp(-\beta t)$. With this change, the system trajectories are described over the entire time horizon $t\in[0,+\infty)$.

The partial-sum sequence $\delta_q(t,\tau)$ converges to the delta function $\delta(t,\tau)$. To demonstrate this fact, we introduce the auxiliary function $v(t)=g(t)E(y(t))$ for which

$$\lim_{q\to\infty}\int_0^\infty v(\tau)\delta_q(t,\tau)d\tau$$
$$=\lim_{q\to\infty}\sum_{i=1}^{q}g(t)\varphi_i(t)\int_0^\infty g(\tau)E\big(y(\tau)\big)\varphi_i(\tau)d\tau.$$

The integral on the right-hand side is the formula for calculating the coefficients of the orthogonal series when expanding the function $E(y(t))$ with respect to the system of exponential polynomials

$$e_i[E]=\int_0^\infty g(\tau)E\big(y(\tau)\big)\varphi_i(\tau)d\tau.$$

Consequently,

$$\lim_{q\to\infty}\int_0^\infty v(\tau)\delta_q(t,\tau)d\tau=\lim_{q\to\infty}g(t)\sum_{i=1}^{q}e_i(E)\varphi_i(t).$$

Since the orthogonal series is convergent for any square integrable function $E(y(t))$ with the weight $g(t)$, we obtain

$$\lim_{q\to\infty}\sum_{i=1}^{q}e_i(E)\varphi_i(t)=E(y(t))$$

and $\lim_{q\to\infty}\int_0^\infty v(\tau)\delta_q(t,\tau)d\tau=g(t)E(y(t))=v(t).$

Hence,

$$\lim_{q\to\infty}\int_0^\infty v(\tau)\delta_q(t,\tau)d\tau=\int_0^\infty v(\tau)\delta(t,\tau)d\tau=v(t),$$

and it follows that

$$\delta(t,\tau)=\lim_{q\to\infty}\sum_{i=1}^{q}g(t)\varphi_i(t)\varphi_i(\tau).$$

This expansion of the delta function can serve to invert the Laplace transform

$$E(s)=\int_0^\infty e^{-s\tau}E(y(\tau))d\tau.$$

For this purpose, we use the following transformations:

$$\begin{aligned}&\int_0^\infty e^{-s\tau}E(y(\tau))\sum_{i=1}^{q}g(t)\varphi_i(t)\varphi_i(\tau)d\tau\\&=\sum_{i=1}^{q}g(t)\varphi_i(t)\int_0^\infty e^{-s\tau}E(y(\tau))\varphi_i(\tau)d\tau\\&=\sum_{i=1}^{q}g(t)\varphi_i(t)\int_0^\infty e^{-s\tau}E(y(\tau))\sum_{j=1}^{i}c_{ij}\exp(-\beta(j-1)t)d\tau\\&=\sum_{i=1}^{q}g(t)\varphi_i(t)\sum_{j=1}^{i}c_{ij}E(s+(j-1)\beta).\end{aligned}$$

On the other hand,

$$\begin{aligned}&\lim_{q\to\infty}\int_0^\infty e^{-s\tau}E(y(\tau))\sum_{i=1}^{q}g(t)\varphi_i(t)\varphi_i(\tau)d\tau\\&=\int_0^\infty e^{-s\tau}E(y(\tau))\delta_q(t,\tau)d\tau=e^{-st}E(y(t)).\end{aligned}$$

Letting $s=\alpha$ gives

$$\begin{aligned}E(y(t))&=\lim_{q\to\infty}\sum_{i=1}^{q}\varphi_i(t)\sum_{j=1}^{i}c_{ij}E(\alpha+(j-1)\beta)\\&=\lim_{q\to\infty}\sum_{i=1}^{q}e_i(E)\varphi_i(t).\end{aligned}\tag{6}$$

Consequently, the coefficients of the series expansion of the function $E(y(t))$ with respect to the system of exponential polynomials are calculated based on the values of its image $E(s)$ at real-axis points:

$$e_i(E)=\sum_{j=1}^{i}c_{ij}E(\alpha+(j-1)\beta),\ i=1,2,\ldots$$

The generalized characteristic $E(y(t))$ written as the series expansion with respect to the system of exponential polynomials can be used to formalize the design problem of matching control in the system with channel switching.

3. CONTROL SYSTEM DESIGN WITH CHANNEL SWITCHING IN THE DESIRED DOMAIN OF TEMPORAL CHARACTERISTICS

We represent the partial sum of the exponential series (6) as

$$E(y(t))=\sum_{i=1}^{q}e_i(E)\varphi_i(t)=\sum_{k=1}^{q}r_k(E)\exp(-\beta(k-1)t),$$

where $r_k(E)=\sum_{i=1}^{q}\lambda_{ik}e_i(E)$.

Recommendations on choosing q (the number of series terms) to achieve the required approximation accuracy can be found in several sources (e.g., see the papers [42, 43]). Let us expand the boundaries of the desired domain of temporal characteristics into similar exponential series:

$$E_1(t)=\sum_{k=1}^{q}r_k(E_1)\exp(-\beta(k-1)t),$$
$$E_2(t)=\sum_{k=1}^{q}r_k(E_2)\exp(-\beta(k-1)t).$$

Then the system of inequalities (5) can be written as

$$E(y(t))-E_1(t)=\sum_{k=1}^{q}R_k^{(\mathrm{I})}(E)\exp(-\beta(k-1)t)\ge 0,$$
$$E_2(t)-E(y(t))=\sum_{k=1}^{q}R_k^{(\mathrm{II})}(E)\exp(-\beta(k-1)t)\ge 0,$$

where $R_k^{(\mathrm{I})}(E)=[r_k(E)-r_k(E_1)]$ and $R_k^{(\mathrm{II})}(E)=[r_k(E_2)-r_k(E)]$.

With the change of variable $z=\exp(-\beta t)$, the system of constraints takes the form

$$P_1(z)=\sum_{k=1}^{q}R_k^{(\mathrm{I})}(E)z^{(k-1)}\ge 0,\ P_2(z)=\sum_{k=1}^{q}R_k^{(\mathrm{II})}(E)z^{(k-1)}\ge 0.$$

For the polynomials $P_1(z)$ and $P_2(z)$ to be non-negative on the interval $[0, 1]$, it suffices to require the following: the polynomials must take a positive value at least at one point of this interval, and, in addition, all their real roots must lie to the right of the point $z = 1$.

According to Newton's theorem on the bounds of polynomial roots, the number $z = 1$ is a lower bound for the positive roots of the polynomials $P_1(z)$ and $P_2(z)$ if

$$\left[z^{q-1} P_1\left(\frac{1}{z}\right)\right]_{z=1}^{(p)} \geq 0, \left[z^{q-1} P_2\left(\frac{1}{z}\right)\right]_{z=1}^{(p)} \geq 0, \qquad (7)$$
$$p = 0, 1, ..., q-1.$$

We require the polynomials $P_1(z)$ and $P_2(z)$ to be positive at $z = 0$, i.e., $P_1(0) = r_1(E) - r_1(E_1) > 0$, $P_2(0) = r_1(E_2) - r_1(E) > 0$.

Together with inequalities (7), these conditions lead to the set of constraints determining the belonging of the generalized characteristic $E(y(t))$ to the given domain:

$$\begin{gathered} r_1(E) - r_1(E_1) > 0, r_1(E_2) - r_1(E) > 0; \\ \sum_{i=1}^{q-p}\left[\frac{(q-i)!}{(q-p-i)!}\right]\left(r_i(E) - r_i(E_1)\right) \geq 0, \\ \sum_{i=1}^{q-p}\left[\frac{(q-i)!}{(q-p-i)!}\right]\left(r_i(E_2) - r_i(E)\right) \geq 0, \\ p = 0, 1, \ldots, q-2. \end{gathered} \qquad (8)$$

Heuristic zero-order search algorithms (e.g., the Hooke–Jeeves pattern search method) are recommended to solve this system of algebraic inequalities.

To illustrate the proposed approach, we construct a pitch control system for an aircraft with a normal overload limit.

4. AUTOMATIC PITCH CONTROL SYSTEM DESIGN FOR AN AIRCRAFT WITH A NORMAL OVERLOAD LIMIT

Consider the pitch control system for an aircraft with the normal overload-limiting channel originally proposed in the paper [44]. The structural diagram of this system is shown in Fig. 1.

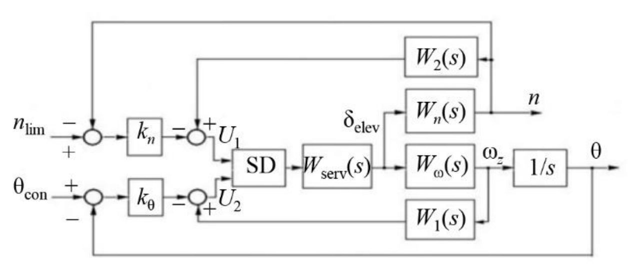

Fig. 1. The pitch control system with the normal overload-limiting channel.

The switching device (SD) supplies the maximum absolute value of the control signal to the servo drive:

$$U = \begin{cases} U_1 \text{ if } |U_1| > |U_2| \\ U_2 \text{ if } |U_2| > |U_1|. \end{cases}$$

The transfer functions of the aircraft to control the elevator δ_{elev} have the following form:

– for the pitch velocity ω_z, $W_\omega(s) = \dfrac{-(s+2.012)}{s^2 + 4.107s + 25.256}$;

– for the normal overload n, $W_n(s) = \dfrac{-1}{s^2 + 4.107s + 25.256}$.

The transfer function of the elevator servo drive is $W_{servo}(s) = \dfrac{10}{s}$.

Selecting a physically implementable astatic pitch autopilot with velocity feedback, we obtain the transfer function

$$W_1(s) = \frac{k_2 s^2 + k_1 s + k_0}{T_2 s^2 + T_1 s + 1},$$

where k_0, k_1, k_2, and k_θ are the autopilot gains; see Fig. 1.

In turn, we choose the transfer function $W_2(s)$ for the normal overload-limiting automaton in the form

$$W_2(s) = \frac{k_4 s^2 + k_3 s}{T_4 s^2 + T_3 s + 1},$$

where k_3, k_4, and k_n are the automaton's gains; see Fig. 1.

Using the introduced characteristics, we find the transfer function of the pitch control loop:

$$\Phi_1(s) = \frac{B_1(s)}{A_1(s)}$$
$$= \frac{b_3^{(1)} s^3 + b_2^{(1)} s^2 + b_1^{(1)} s + b_0^{(1)}}{a_6^{(1)} s^6 + a_5^{(1)} s^5 + a_4^{(1)} s^4 + a_3^{(1)} s^3 + a_2^{(1)} s^2 + a_1^{(1)} s + a_0^{(1)}},$$

where

$$b_3^{(1)} = 10k_\theta T_2,\ b_2^{(1)} = 10k_\theta\left(2.012T_2 + T_1\right),$$

$$b_1^{(1)} = 10k_\theta\left(2.012T_1 + 1\right),\ b_0^{(1)} = 20.12\,k_\theta,\ a_6^{(1)} = T_2,$$

$$a_5^{(1)} = 4.107T_2 + T_1,$$

$$a_4^{(1)} = 25.256T_2 + 4.107T_1 + 10k_2 + 1,$$

$$a_3^{(1)} = 25.256T_1 + 10k_1 + 20.12k_2 + 10k_\theta T_2 + 4.107,$$

$$a_2^{(1)} = 10k_0 + 20.12k_1 + 20.12k_\theta T_2 + 10k_\theta T_1 + 25.256,$$

$$a_1^{(1)} = 20.12k_0 + 20.12k_\theta T_1 + 10k_\theta,\ \text{and}\ a_0^{(1)} = 20.12k_\theta.$$

Similarly, for the normal overload-limiting loop, we find the transfer function

$$\Phi_2(s) = \frac{B_2(s)}{A_2(s)} =$$

$$= \frac{b_3^{(2)}s^3 + b_2^{(2)}s^2 + b_1^{(2)}s + b_0^{(2)}}{a_6^{(2)}s^6 + a_5^{(2)}s^5 + a_4^{(2)}s^4 + a_3^{(2)}s^3 + a_2^{(2)}s^2 + a_1^{(2)}s},$$

where

$$b_3^{(2)} = 10k_nT_4,\ b_2^{(2)} = 10k_n\left(T_3 + 2.012T_4\right),$$

$$b_1^{(2)} = 10k_n\left(2.102T_3 + 1\right),\ b_0^{(2)} = 10k_n,\ a_6^{(2)} = T_4,$$

$$a_5^{(2)} = 4.107T_4 + T_3,\ a_4^{(2)} = 25.256T_4 + 4.107T_3 + 1,$$

$$a_3^{(2)} = 25.256T_3 + 10k_nT_4 + 10k_4 + 4.107,$$

$$a_2^{(2)} = 10k_3 + 10\,k_nT_3 + 25.256,\ \text{and}\ a_1^{(2)} = 10k_n.$$

Let the control channel be switched with the constant intensity $\lambda = 1\ \text{s}^{-1}$. We require that the generalized characteristic $E(y(t))$ of the system lies in the domain bounded by the functions $E_1(t) = 0.9\left(1 - 2\exp(-0.5t) + \exp(-t)\right)$ and $E_2(t) = 1.1\left(1 - \exp(-4t)\right)$. For the boundaries of this domain to belong to the exponential series basis, we select its parameters as follows: $\alpha = \beta = 0.5$ and $q = 9$.

The exponential series for the boundaries of the domains have the following coefficients:

$$r_1(E_1) = 0.9,\ r_2(E_1) = -1.8,\ r_3(E_1) = 0.9,\ r_4(E_1) = 0,$$

$$r_5(E_1) = 0,\ r_6(E_1) = 0,\ r_7(E_1) = 0,\ r_8(E_1) = 0,\ r_9(E_1) = 0,$$

$$r_1(E_2) = 1.1,\ r_2(E_2) = 0,\ r_3(E_2) = 0,\ r_4(E_2) = 0,$$

$$r_5(E_2) = 0,\ r_6(E_2) = 0,\ r_7(E_2) = 0,\ r_8(E_2) = 0,$$

and $r_9(E_2) = -1.1$.

In turn, for the generalized characteristic $E(y(t))$, these coefficients are calculated as

$$r_k(E) = \sum_{i=k}^{q}\sum_{j=1}^{i} c_{ij}c_{ik}E(0.5j),\ k = 1, 2, \ldots, 9,$$

where

$$E(0.5j) = \frac{0.5jQ_1(0.5j+1)P_2(0.5j+1)}{0.5j(0.5j+1)P_1(0.5j+1)P_2(0.5j)}$$

$$+\frac{\left[(0.5j+1)Q_2(0.5j) - 0.5jQ_2(0.5j)\right]P_1(0.5j+1)}{0.5j(0.5j+1)P_1(0.5j+1)P_2(0.5j)}.$$

Substituting these expressions into the system of inequalities (8) yields the set of constraints on the parameters of the control part. Solving the system of inequalities, we find the following parameter values:

$$k_\theta = 60.49,\ k_n = 50.42,\ k_0 = 17.76,\ k_1 = 11.11,$$

$$k_2 = 1.11,\ k_3 = 16.49,\ k_4 = 1.99,\ T_1 = 0.50,$$

$$T_2 = 0.0005,\ T_3 = 0.002,\ \text{and}\ T_4 = 0.000001.$$

Figure 2 shows the model of the control system designed in Matlab\Simulink.

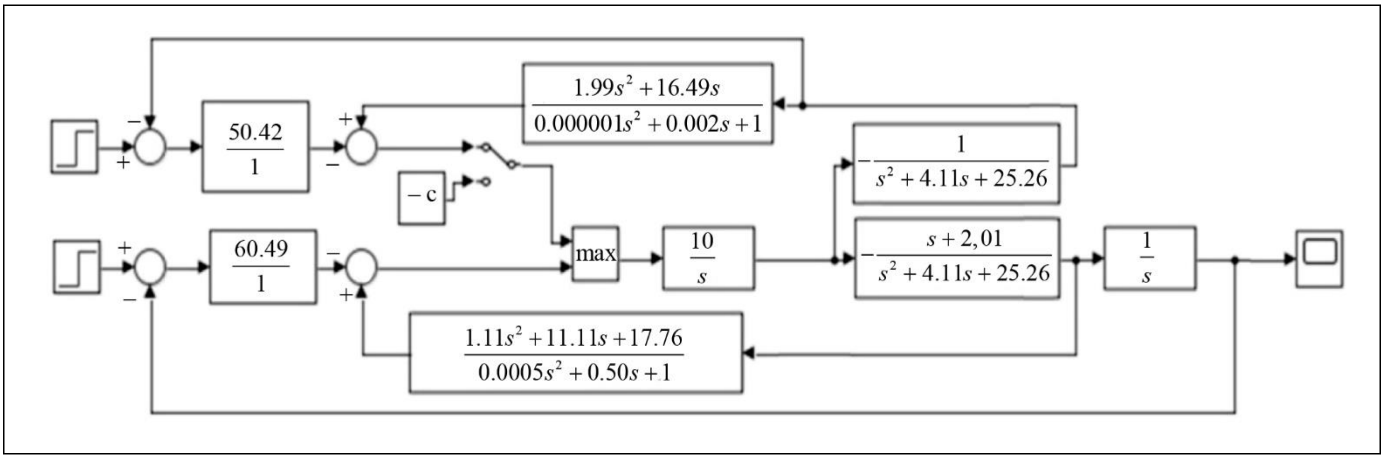

Fig. 2. The model of the control system.

The simulation results for this control system are demonstrated in Fig. 3.

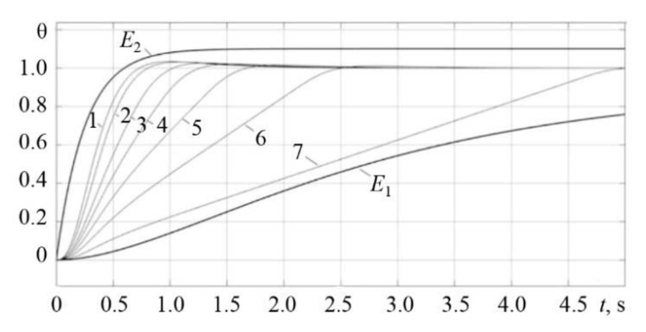

Fig. 3. Transients in the pitch control system with the normal overload-limiting channel.

The graphs in Fig. 3 correspond to the following switching instants: (1 – τ) = 0.1 s, (2 – τ) = 0.3 s, (3 – τ) = 0.6 s, (4 – τ) = 0.8 s, (5 – τ) = 1.3 s, (6 – τ) = 2.1 s, and (7 – τ) = 4.6 s.

According to the simulation results, the system transients for different channel switching instants belong to the specified domain and keep the aperiodic

CONCLUSIONS

With the approach presented in this paper, the processes occurring in the control systems of moving objects with channel switching can be studied from a common point of view. For this purpose, we have proposed a generalized characteristic describing the entire ensemble of all output responses of the control system under all possible structural change instants (switching instants) of its control part. This characteristic can be used to design control systems for moving objects owing to a special numerical inversion method of the Laplace transform. As one example, an automatic pitch control system designed for an aircraft with a normal overload limit has illustrated the effectiveness of this approach.

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This paper was recommended for publication by V.B. Pavlov, a member of the Editorial Board.

Received April 13, 2023,
and revised August 17, 2023.
Accepted August 30, 2023.

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Cite this paper
Abdullina, E.Yu., Efanov, V.N., Control System Design for Moving Objects with Channel Switching. *Control Sciences* **5,** 32–40 (2023). http://doi.org/10.25728/cs.2023.5.3

Original Russian Text © Abdullina, E.Yu., Efanov, V.N., 2023, published in *Problemy Upravleniya*, 2023, no. 5, pp. 40-49.

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